24 Toward Global Waveform Tomography with the SEM: Improving Upper-Mantle Images at Shallow Depths

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24.1 Introduction

The SEMum upper-mantle \( V_S \) model (Lekic and Romanowicz, 2011) was developed using the spectral element method (SEM; e.g. Komatitsch and Vilotte, 1998) to invert long-period \( (T \geq 60 \text{ s}) \) waveforms of fundamental and overtone mode surface waves. SEM global waveform inversion was made feasible partly through use of a mode-coupled SEM (Capdeville et al., 2003), combined with an innovative smooth crustal model. The 60 \( \text{km} \) crustal layer allows the SEM to take long time steps, thus speeding computation, at the expense of complicating interpretation of the shallowest upper-mantle structure.

Our goal has been to produce an updated model (SEMum2) using a more geologically-plausible fictitious Moho for the crustal layer. This update would have the immediate benefit of easing interpretation of some uppermost upper-mantle structure. In the future, we intend to include SEMum2 in the initial model for a forthcoming whole-mantle inversion, using shorter period waveform data \( (T \geq 40 \text{ s}) \). Thus, it is advantageous to perform the update now, while still using the 60 \( \text{s} \) data set so that SEM simulation is comparatively inexpensive.

24.2 Crustal model development

SEM solution accuracy is strongly affected by mesh fidelity to the underlying earth model, requiring interior boundaries (Moho, 410 \( \text{km} \), etc...) to be matched with element faces. Time-stability of the SEM is determined by the minimum ratio between spatial discretization and wave speed (the Courant-Friedrichs-Lewy, or CFL condition), with the maximum stable time-step determining the overall cost of time-integration.

In a pure global SEM (e.g. Tromp, et al., 2008), the CFL condition is dominated by high \( V_P \) in the core. If the core is replaced with a modal solution (Capdeville, et al., 2003), then small spatial discretization in the thin oceanic crust dominates - leading to a less restrictive, but still prohibitively small, time-step. As a solution, Lekic and Romanowicz (2011) developed a new crustal model: a radially-anisotropic smooth crustal layer of uniform 60 \( \text{km} \) thickness, designed to fit observed surface-wave dispersion maps (Shapiro and Ritzwoller, 2002). This scheme provides a simpler alternative to a true “homogenization” of an \textit{a priori} crustal model (e.g. Capdeville and Marigo, 2007), while also supplying independent constraints on the effect of crustal structure on the wavefield.

For SEMum2, we adopt a more geologically plausible laterally-varying layer thickness, at the expense of a more restrictive CFL condition. Starting from Crust2.0 Moho depth (Bassin and Masters, 2000), we restrict crustal thickness \( H \) to the interval \( H \in [30, 60] \text{ km} \) and filter at 2x lateral resolution of the SEM mesh. We seek “crust-equivalent” anisotropic \( V_S \) structure (e.g. Backus, 1962) that fits observed surface-wave dispersion and is parameterized with depth in Gauss-Lobatto-Legendre interpolants (GLL), as used in the SEM. The crustal layer is generated following a two-step procedure:

(1) We define a space of admissible radially-anisotropic GLL models \( V_{S,i,\text{iso}} \in [3, 4.5] \text{ km s}^{-1}, V_{S,H}/V_{S,V}^2 \in [0.8, 1.2] \) from which we draw \( \sim 20k \) realizations \( \{m_i\} \) and calculate dispersion curves for layer thickness \( H \in [30, 60] \text{ km} \) and bathymetry \( h \in [0, 6] \text{ km} \). Group-velocity dispersion maps \( (25 - 60 \text{ s}) \) are resampled on a uniform grid of knots, and dispersion curves are estimated for all \( \{m_i\} \) through interpolation to local seafloor and Moho topography \((H, h)\). Best-fitting GLL models are selected for each knot, where misfit is measured in the \( L_1 \) norm to reduce sensitivity to outlier measurements common at short periods, with additional weighting by measurement uncertainty and tunable anisotropy regularization.

(2) We next perform a linearized inversion in a neighborhood surrounding the selected model using the generalized least-squares formalism (Tarantola and Valette, 1982). We use individual group-velocity kernels for each model knot, reflecting local bathymetry and fictitious Moho, which are recalculated following each iteration. Upper-mantle structure from SEMum is assumed. After 3 iterations, mean absolute misfit over all periods and model knots fell into the target range of 50 - 60 \( \text{m s}^{-1} \). The resulting model allows a 5x time-step prolongation over direct meshing of Crust2.0.

24.3 Preliminary update and discussion

We refer the reader to Lekic and Romanowicz (2011) for a detailed discussion of the waveform inversion scheme. The SEM is used to forward model fundamental and overtone mode surface waves \( (T \geq 60s) \), which are inverted for upper-mantle structure using NACT waveform sensitivity kernels following the generalized least-squares formalism. Group-velocity data \( (T \leq 150s) \) is included in the inversion for consistency. We follow an identical procedure, substituting the new crustal model as well as new crustal corrections for use with NACT, developed in a manner similar to that of Lekic, et al. (2010).

To date, we have performed one update iteration, and SEM simulation for the second is ongoing. We antici-
Table 2.3: $||u_{obs} - u_{pred}||^2 / ||u_{obs}||^2$ for SEMum2 and [SEMum] by data-type and component. Values reflect 80% of the total data set, but are considered representative.

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<th>L</th>
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<tbody>
<tr>
<td>fund</td>
<td>0.360 (0.388)</td>
<td>0.283 (0.413)</td>
<td>0.326 (0.354)</td>
</tr>
<tr>
<td>high</td>
<td>0.248 (0.327)</td>
<td>0.293 (0.385)</td>
<td>0.275 (0.310)</td>
</tr>
<tr>
<td>mixed</td>
<td>0.299 (0.320)</td>
<td>0.212 (0.316)</td>
<td>0.283 (0.296)</td>
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In the near future, we will complete the second update iteration, at which point we anticipate SEMum2 will have stabilized. Thereafter, we will begin data collection for the whole-mantle inversion, potentially using new $(T \geq 40$ s) SEMum2 synthetics for waveform selection.

### 24.4 Acknowledgements

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### 24.5 References


