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SHORT NOTES

ON SCALING RELATIONS FOR LARGE EARTHQUAKES

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A change in "b-value", or slope in the frequency-magnitude relation, around $M \sim 7.2$, was recently documented (Pacheco *et al.*, 1992), based on the analysis of the recently published catalog of large earthquakes since the beginning of this century (Pacheco and Sykes, 1992). Pacheco *et al.* (1992) have interpreted this change from $b \sim 1$ to $b \sim 1.5$ as resulting from the change of scaling expected when the downdip width (W) of the fault reaches saturation, as a result of the finite thickness of the brittle zone. In this interpretation, they invoked a self-similarity argument proposed by Rundle (1989).

Pacheco *et al.* (1992) also imply that their observation is compatible with Scholz's "L-model" (Scholz, 1982), in which slip (u) scales with the length (L) of the fault, and, as a consequence, for large earthquakes, seismic moment (M_0) scales with L^2. On the other hand, Rundle's (1989) derivation was based on the "W-model", a model in which slip scales with the small dimension (W) of the fault, which is compatible with results of dynamic elastic modeling (Madariaga, 1976). How can we reconcile these contradictions?

In fact, we demonstrate here that L-models and W-models imply very different scaling relations. We follow the tiling argument of Rundle (1989) to show that the observed change in "b-value" in the Pacheco and Sykes (1992) catalog is consistent with W-models and is inconsistent with L-models.

Rundle (1989) derived magnitude-frequency relations based upon an idealized model of a fault, some simple assumptions, and an important empirical result. The latter is that, for any magnitude band centered on magnitude $m(m - \Delta m/2, m + \Delta m/2), \Delta m \to 0$, the average rate of rupture of fault area is constant. It is therefore postulated that the total amount of area available for rupture in events of magnitude m is the constant Σ_T. It is not explicitly required that Σ_T be a continuous or simply connected surface, it may in fact be the union of a great number of disjointed surfaces. In fact, there will be $N = \Sigma_T / S_m$ areas of size S_m. It is further assumed that the average stress drop for events of any magnitude m is $\Delta\sigma_T$. However, it should be noted that the stress decrease due to slip on a patch of area S_m averaged over all of Σ_T will be somewhat less than $\Delta\sigma_T$ because slip on the patch can increase the stress on its neighbors, and vice versa. Therefore, each patch needs to slip some number of times (possibly a large number) before the stress decrease averaged over all of Σ_T is $\Delta\sigma_T$. If the average frictional properties remain roughly the same through time, then one can speak of an average recurrence interval T_m for the patch. One can also define the time τ_T, the cycle interval, for events of size S_m to reduce the stress over Σ_T by the amount $\Delta\sigma_T$. The patch must therefore slip on average τ_T / T_m times during the time interval τ_T.

Let us start from the fundamental relation (e.g., Aki, 1972):

$$M_0 = \mu u L W = \mu u S \tag{1}$$

where μ is rigidity and S is the surface of the fault affected by the earthquake. Consider a fault (or a union of faults) of total area Σ_T and let N be the number of earthquakes of size $S = LW$ required to completely cover Σ_T, according to the argument presented above. Then:

$$\Sigma_T = NS \tag{2}$$

For small earthquakes, let $L \sim W \sim r$, hence $S \alpha r^2$, $N \alpha r^{-2}$, and, in both W and L cases, $u \alpha r$, hence, from (1):

$$\text{Log}\, N = a - 2/3\, \text{Log}\, M_0 = a' - M_w$$

where a and a' are appropriate constants, and M_w is moment magnitude (Hanks and Kanamori, 1979). Under the hypothesis that the rate of occurrence $\langle n \rangle$ for events of size S obeys a power law $\langle n \rangle = N\gamma/\tau_T$, where τ_T is the cycle interval as defined above, and integrating $\langle n \rangle$ over the magnitude range $[M_w, \infty]$, we obtain a "b-value" of γ. We note that if $\gamma \neq 1$, the amount of time, τ, it takes to reduce the stress over area Σ_T by $\Delta \sigma_T$ in events of magnitude m will depend on m. For observations averaged over long time intervals, $\gamma = 1$ (strict self-similarity).

For large earthquakes, for which the width has ceased to grow ($W = W_0$), we have $\Sigma_t = NW_0L$, hence $N \alpha L^{-1}$. Here we need to distinguish two cases:

1—"*L-model*"

Here, $M_0 \alpha L^2 W_0$, which implies

$$\text{Log}\, N = a - 0.5\, \text{Log}\, M_0 = a' - 0.75 M_w.$$

Hence a "b-value" of 0.75γ and we note that M_0 scales with L^2 (Fig. 1).

2—"*W-model*"

Here $M_0 \alpha L W_0$, which implies:

$$\text{Log}\, N = a - \text{Log}\, M_0 = a' - 1.5 M_w$$

Hence a "b-value" of 1.5γ, and we note that M_0 scales with L^1 (Fig. 1).

We conclude that we should observe a different change of slope both in the frequency-moment (or frequency-magnitude) relation and in the moment-length relation, depending on whether slip scales with W or with L.

Figure 1 shows that the data of Pacheco *et al.*, for which the "b-value" changes from 1 to 1.5 with increasing size of earthquakes, are in good agreement with models in which slip scales with the small dimension of the fault. Relaxing the constraint $\gamma = 1$, Rundle's (1989) derivation, which was made under the assumption of a W-model, implies a ratio $b2/b1 = 1.5$, where $b1$ is the "b-value" for smaller earthquakes, $b2$ that for larger earthquakes. Retabulation of the data of Pacheco *et al.* indicates that $b2/b1 = 1.5 \pm 5\%$, not 0.75 as expected for "L models".

We consider that this is worth stressing, because different investigators seeking to understand the physics underlying tectonic strain release rely heavily on certain scaling relations to support their derivations, in particular the scaling of a slip. For example, in a recent paper by Sornette and Virieux (1992), the authors illustrate how a nonlinear diffusion equation can explain the

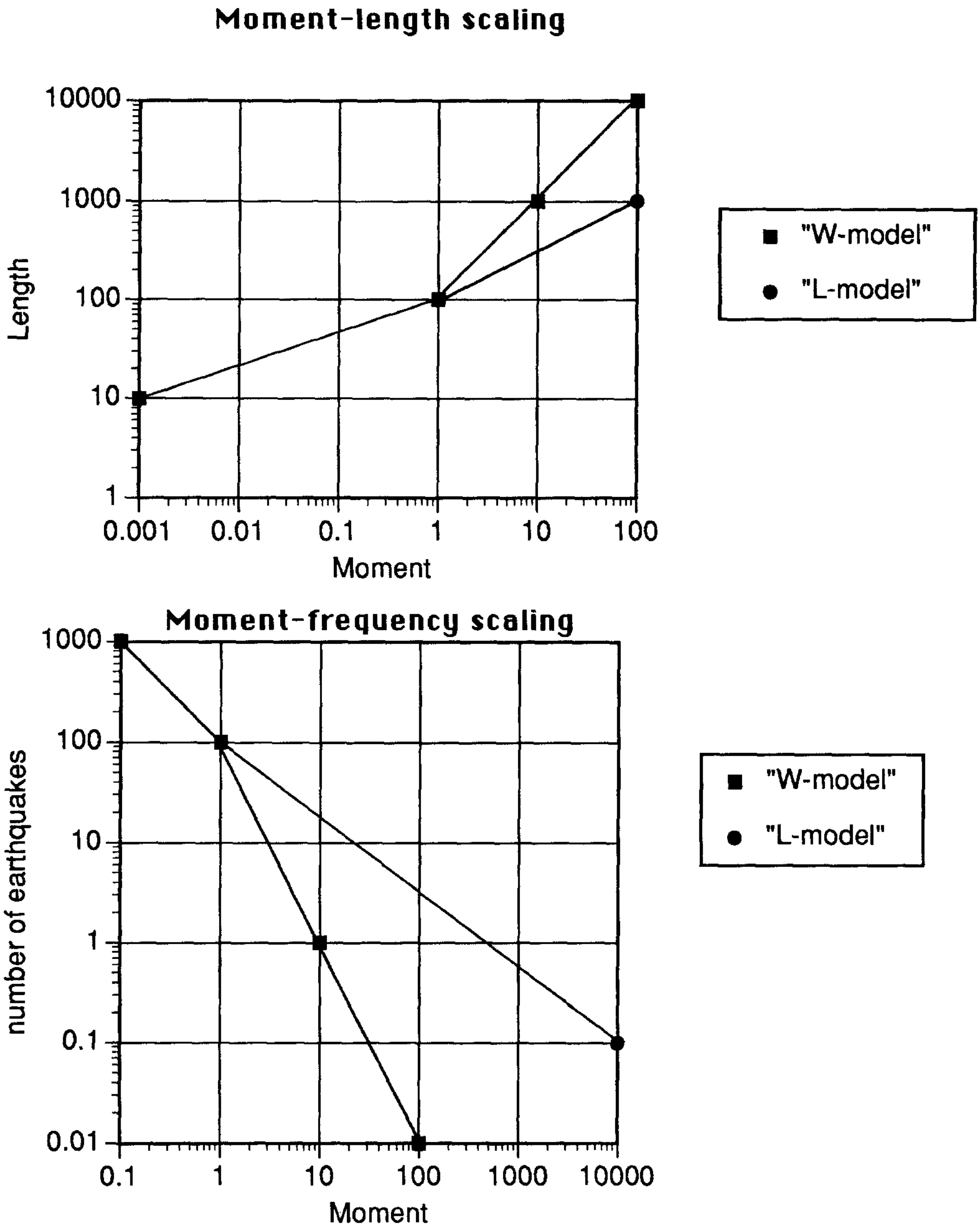

FIG. 1. Sketch illustrating the different behavior of scaling associated with the saturation of the width W of the fault zone, for the "L-model" and for the "W-model". *Top*: Moment-Length relations. *Bottom*: Moment-Frequency relations.

change of "b value" from 1 to 1.5 observed for large earthquakes by Pacheco *et al.* (1992). Sornette and Virieux start from the hypothesis that slip scales with length and consequently seismic moment with L^2 for large earthquakes, and they conclude, after some algebra, that this is compatible with the observed "b-value" change, thereby validating their modeling approach. We note however that their derivation is only valid under the hypothesis that W grows with L, in a manner where $W \sim L\zeta$ and $\zeta \neq 0$, and therefore is irrelevant to the question of what happens when the width of the fault zone reaches the saturation value $W = W_0$.

We believe that it is time to reconsider the widely assumed scaling of slip with length, for large earthquakes (those for which W has saturated), that is based on a small set of observations relying heavily on pre-1960's data, with poor control over parameters (Scholz, 1982). Observed slip is a poor parameter to use to obtain insight into scaling relations for large earthquakes. The few existing measurements are generally based on surface observations that give no insight on what the slip distribution is at depth. Geodetic measurements are rare and possibly contaminated by afterslip effects. Although all major earthquake parameters are difficult to measure, M_0, L and W, which can be directly estimated from seismological data, are the least uncertain, especially for the last 25 years, with the increase in number and quantity of globally distributed seismological stations. Recent compilations of these parameters (Pacheco *et al.*, 1992; Romanowicz, 1992) suggest that their scaling is consistent with what is expected from dynamic elastic models.

More high quality data are clearly needed to completely resolve this question, and the reality may, of course, not be as simple as a "W-model" or an "L-model".

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