solution, cataclasis, etc.), and the degree of interaction among the objects.

In the examples that follow we will assume that the objects have a uniform composition and are not significantly different in strength from their matrix (e.g., quartz pebbles in a quartz-sand matrix or ooids in a limestone). We will not attempt to analyze objects that have a significant strength contrast with their matrix, such as granite pebbles in a shaly matrix, because in such rocks the strain becomes distributed differently in the two lithologies.

First, consider what we would see on a two-dimensional plane cut through a group of three-dimensional ellipsoids. There will be elliptical shapes with two variables to consider: R_f, the final shape ratio, and ϕ, the final orientation of the long axis of each ellipse. Note that the observed area variations do not necessarily indicate original variations in the volume of the objects. A two-dimensional cut goes through the centers of some objects but may just skim the edges of others. We must also consider the initial distribution of the objects in space - Was ϕ random in the undeformed state? Or was there some preferred orientation of the ellipsoid axes, such as is often found in pebble layers in river beds or on beaches?

We can also think of the objects as a collection of central points, ignoring the shape of the objects for the moment. The strain of the whole rock may be represented by the spatial distribution of the centers. In a two-dimensional section the majority of pebbles or ooids are a certain minimum distance apart. Rarely, very small pebbles or ooids may occur side by side to give a very small separation of their centers. This kind of distribution of points that tend to be a minimum distance apart is called an *anticlustered* distribution. If the ellipsoids have a *random orientation* in three dimensions, then the minimum distance tends to be *uniform*, i.e., the same in all directions. If the sediment is imbricated, then the minimum distance may be longer in one direction than another.

There are various methods for the analysis of deformed ellipsoids. We will use the three simplest and yet most practical of them: the Fry (1979) method, which uses particle center distributions; the R_f/ϕ method (e.g., Lisle, 1985) and De Paor's (1987) adaptation of it, which use axial ratios and their orientations; and the Robin (1977) method, which uses irregularly shaped objects such as pillow forms in lava flows or enclaves in granitic rocks.

The Fry Method

The Fry method is based on the assumption that an initially uniform anticlustered distribution of points will change after deformation into a nonuniform distribution. Distances between points become increased in the extensional field and decreased in the contractional field of strain (Fig. 15-34). Maximum distances between points occur parallel to the principal stretch direction, S_1; minimum distances occur parallel to S_2. So if we can measure the distances between the centers of objects in the deformed state, we can use these data to calculate the finite strain.

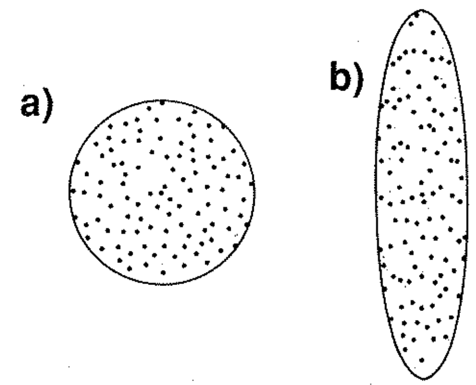

Figure 15-34. The principle behind Fry's method. (a) Initial uniform anticlustered distribution of points inside a unit circle; (b) after strain the distribution becomes non- uniform.

Exercise 15-13

Figure 15-35 is a thin-section photomicrograph of deformed ooids from South Mountain, Maryland. Note that few of the particles have truly elliptical shapes, so measuring axial ratios directly would give an incorrect result.

Step 1: Take a tracing paper sheet and mark the center point of each ooid on it. Number the points as you go, and draw a reference line somewhere to the side of the points. In some of the examples, calcite crystals obscure the ooid center, so you will have to estimate the central position.

Step 2: Mark a central reference cross on a separate sheet of tracing paper. Overlay this sheet onto the first so that the cross coincides with point 1. Trace the reference line through onto the top sheet. Trace the position of all other points (2, 3, 4, . . .) onto the overlay.

Step 3: Move the top sheet so that the cross is on point 2 and the reference lines remain parallel. Trace the position of all other points (1, 3, 4, 5, . . .).

Step 4: Repeat the procedure for all the points on the lower sheet. You will end up with an empty space around the reference cross and a concentration of points just outside this space. It is the shape of the space that is of interest to us; a circular space means a uniform distribution of points (i.e., no strain), and an elliptical shape is a direct representation of the strain ellipse. After moving the overlay four or five times you will see at each station that those points at a considerable distance from the cross do

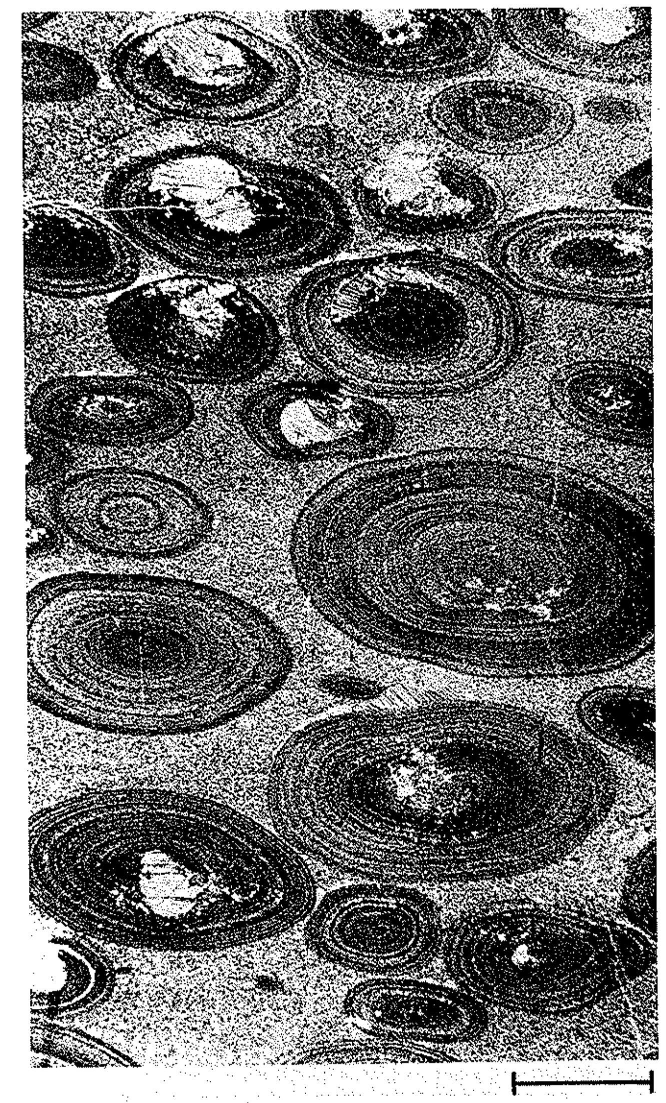

Figure 15-35. Thin section photograph of deformed ooids, South Mountain, Maryland. Scale bar 0.5 mm. From the archives of E. Cloos, Department of Earth and Planetary Sciences, Johns Hopkins University, with permission.

not play a role in defining the space and so need not be plotted.

Step 5: After all the points have been covered, remove the upper tracing sheet and find the best fit ellipse to the space around the reference cross.

Step 6: Measure the strain ratio, R_s, and note its orientation with respect to Figure 15-35.

The Fry method is extremely simple and relatively rapid. It can be carried out on rocks that have considerable pressure solution along particle-particle contacts. However, it requires at least 25 points, and preferably many more, to produce a reasonably ellipse-shaped space. Ellipticity estimates can be quite inaccurate, especially at low point concentrations or where there is a significant size distribution of particles. It is quite common to find "stray" points inside the elliptical space, and one must decide whether to draw the ellipse inside all points, to include the first points at the edge of the space, or to draw the ellipse just outside the space. Finally, the method cannot be used where it is suspected that the ellipsoidal particles had an initial preferred orientation of their axes before deformation.

The R_f/ϕ Method

A principal aim of strain analysis is to find the initial shape of deformed objects such as fossils and pebbles. In a deformed conglomerate, initial pebble shapes, if known, could give important clues to the sedimentary and tectonic environments at the time of their deposition. The R_f/ϕ method of strain analysis assumes that the ellipsoidal object is deformed together with its matrix. On a two-dimensional cut through the rock, the initial *shape factor*, R_i, is changed to the finally observed axial ratio, R_f. The orientation of the ellipse long axis is changed from θ in the undeformed state to ϕ after deformation (Fig. 15-36). The equations that govern this transformation are presented in Lisle (1985) and derived by Ramsay (1967, pp. 205-9). The R_f/ϕ method allows calculation of the ellipticity of the strain ellipse (R_s) by measuring the

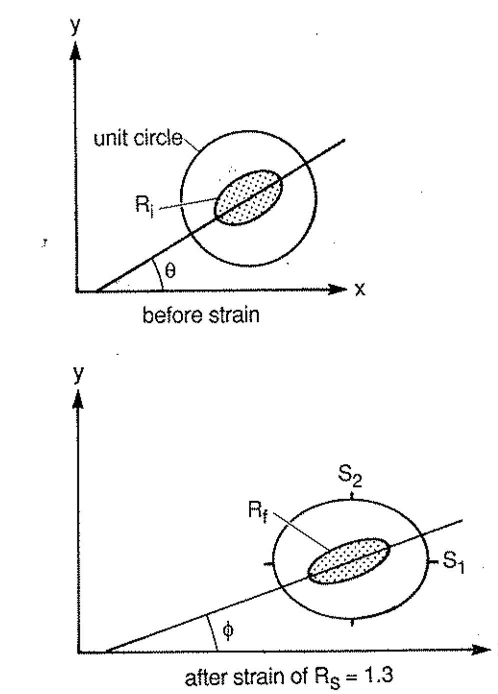

Figure 15-36. Deformation of elliptical shape, R_i, by strain of $R_s = 1.3$ to produce a new ellipse, R_f. (Modified from Lisle, 1985.)

observed ellipticities (R_f) and orientations (ϕ) of a number of objects. Note that in the undeformed state, $R_s = 1.0$.

The final axial ratio, R_f, and orientation, ϕ, of any object will depend on the relative orientations of the initial ellipse, R_i, and the strain ellipse, R_s. Refer back to Figure 15-33 to verify this for yourself. If we start with a group of ellipses such as in Figure 15-37a, having identical R_i values but variable orientations, θ, and deform them by an amount R_s, the final R_f and ϕ values will all vary as in

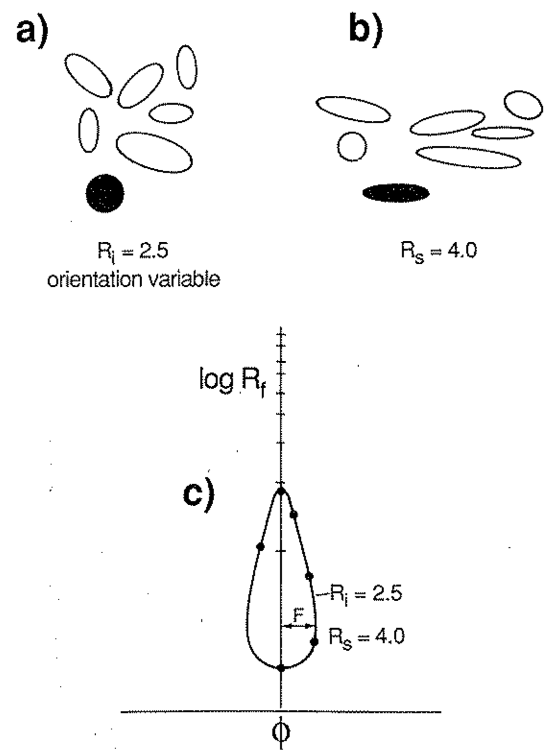

Figure 15-37. Elliptical markers with a constant R_i but variable orientation and their representation after strain. (Modified from Lisle, 1985.) (a) Undeformed state, $R_i = 2.5$; (b) after a strain of $R_s = 4.0$; (c) R_f and ϕ values for ellipses in (b) plotted on log R_f/linear ϕ graph paper. F = fluctuation.

Figure 15-37b. If we now measure R_f and ϕ for each ellipse and plot the values on a log R_f / linear ϕ graph such as in Figure 15-37c, the points should form a characteristically onion-shaped plot (in practice, the angle ϕ is measured with respect to an arbitrary reference line; in Figure 15-37c the reference line was drawn parallel to the principal stretch). The long axes of the deformed ellipses *fluctuate* by an amount F on either side of the principal stretch direction. F is least at the greatest R_s values (i.e., where the "onion" is thinnest). Maximum R_f values occur where R_i and R_s ellipse long axes coincide ($R_{f\,max} = R_s R_i$), and minimum R_f values occur where R_s and R_i ellipses are mutually perpendicular ($R_{f\,min} = R_s/R_i$ *or* R_i/R_s, whichever is the greater). In these two extreme cases the ϕ value will be zero; hence, the onion-shaped curve is ideally biaxially symmetrical about the $\phi = 0°$ axis (Fig. 15-37c).

If we take a second series of ellipses with R_i values identical to one another, but different from those in Figure 15-37a, and subject them to the same R_s, we will find that their R_f and ϕ values also plot as an onion-shaped distribution. We can build up a series of "onion-ring" R_i curves for any given value of R_s. Figure 15-38 shows one typical series of curves for $R_s = 4.0$ and $R_i = 1.25$, 1.5, 1.75, 2.0, 7.5, 3.0, 4.0, and 6.0.

Suppose that the series of ellipses are all in the same orientation initially but that they have different initial shape factors (Fig. 15-39a). Superposition of a strain, R_s (Fig. 15-39b), will produce R_f/ϕ values that fall on a differently shaped curve (Fig. 15-39c) known as a *theta-curve* (Lisle, 1977). A series of theta-curves for values $\theta = 0°$ to $\theta = 90°$ (Fig. 15-40) can be drawn for every value of R_s. The curve $\theta = 45°$ is also known as the 50%-of-data curve (Fig. 15-40). Together with the $\phi = 0°$,

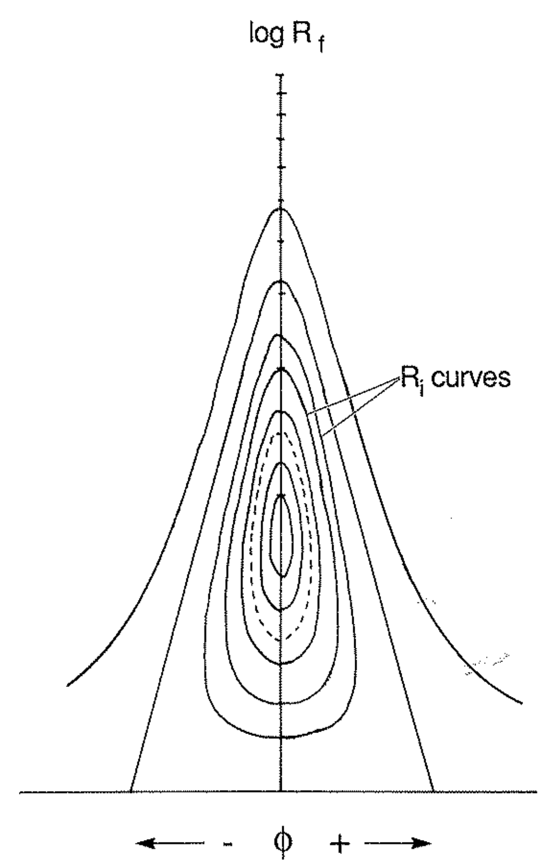

Figure 15-38. R_f/ϕ curves for a strain ratio $R_s = 4.0$. Innermost curve is for $R_i = 1.25$, dashed curve is $R_i = 1.75$, outermost is $R_i = 6.0$. (Adapted from Lisle,1985.)

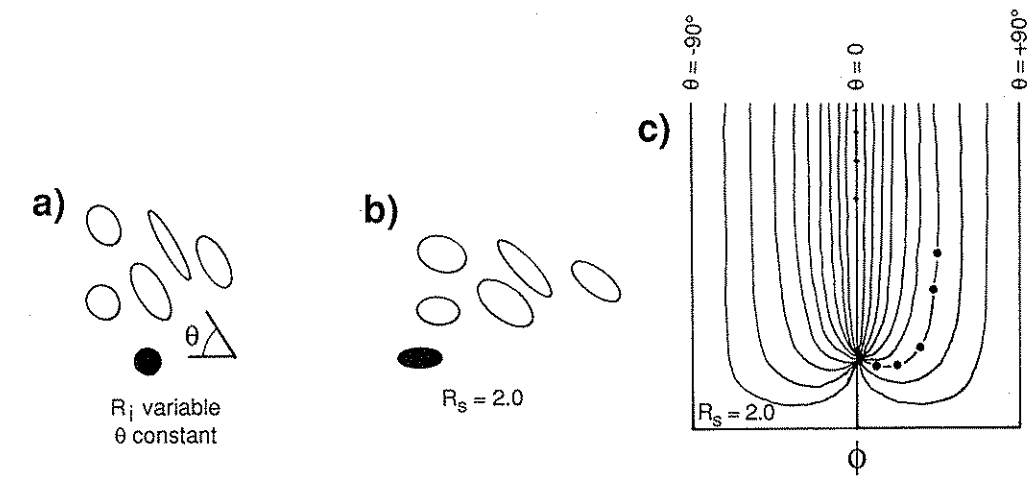

Figure 15-39. Elliptical markers with a constant initial orientation but variable R_i and their representation after strain. (Modified from Lisle,1985.) (a) Undeformed state; (b) after a strain of $R_s = 2.0$; (c) R_f and ϕ values for ellipses in (b). When plotted on log R_f / linear ϕ graph paper, the points fall on a θ curve.

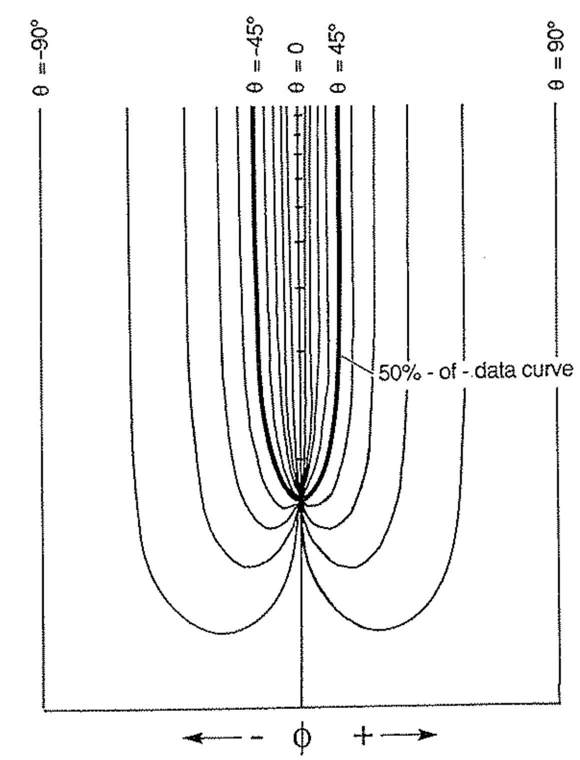

Figure 15-40. A typical set of θ curves for R_f/ϕ analysis. Strain ratio $R_s = 4.0$. (Modified from Lisle,1985.)

$\theta = 0°$ line it divides the plot into four quadrants (Fig. 15-41), each of which should contain 25% of the data points *provided* that the original ellipsoids were randomly oriented. Thus, a *symmetrical* distribution of points about the $\phi = 0°$ line (Fig. 15-41a) is usually taken to indicate the absence of an original sedimentary fabric, whereas an *asymmetrical* distribution (Fig. 15-41b) is thought to result from an initial fabric such as imbricated pebbles in a stream bed. For a further discussion of interpretation of R_f/ϕ patterns, see Lisle (1985).

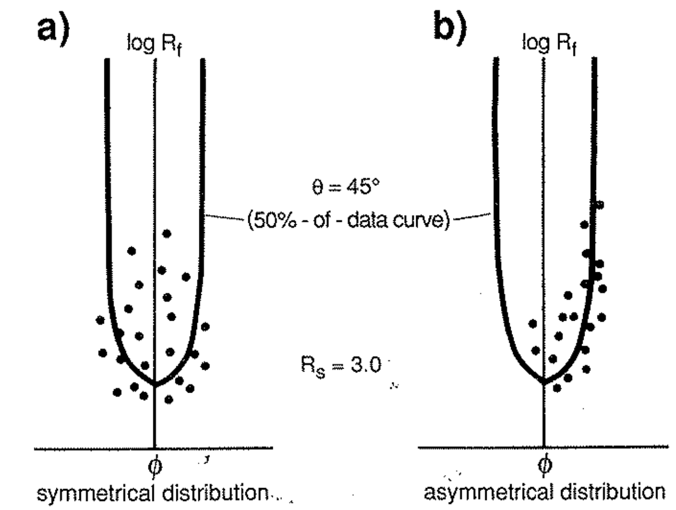

Figure 15-41. Division of the R_f/ϕ plot into four quadrants by use of the 50%-of-data curve ($\theta = 45°$). (a) A symmetrical distribution - 25% of points fall into each quadrant; (b) an asymmetrical distribution. (Modified from Lisle, 1985.)

Exercise 15-14

A single set of R_f/ϕ curves for the value $R_s = 4.0$ is shown in Figure 15-42. We need similar curves for all

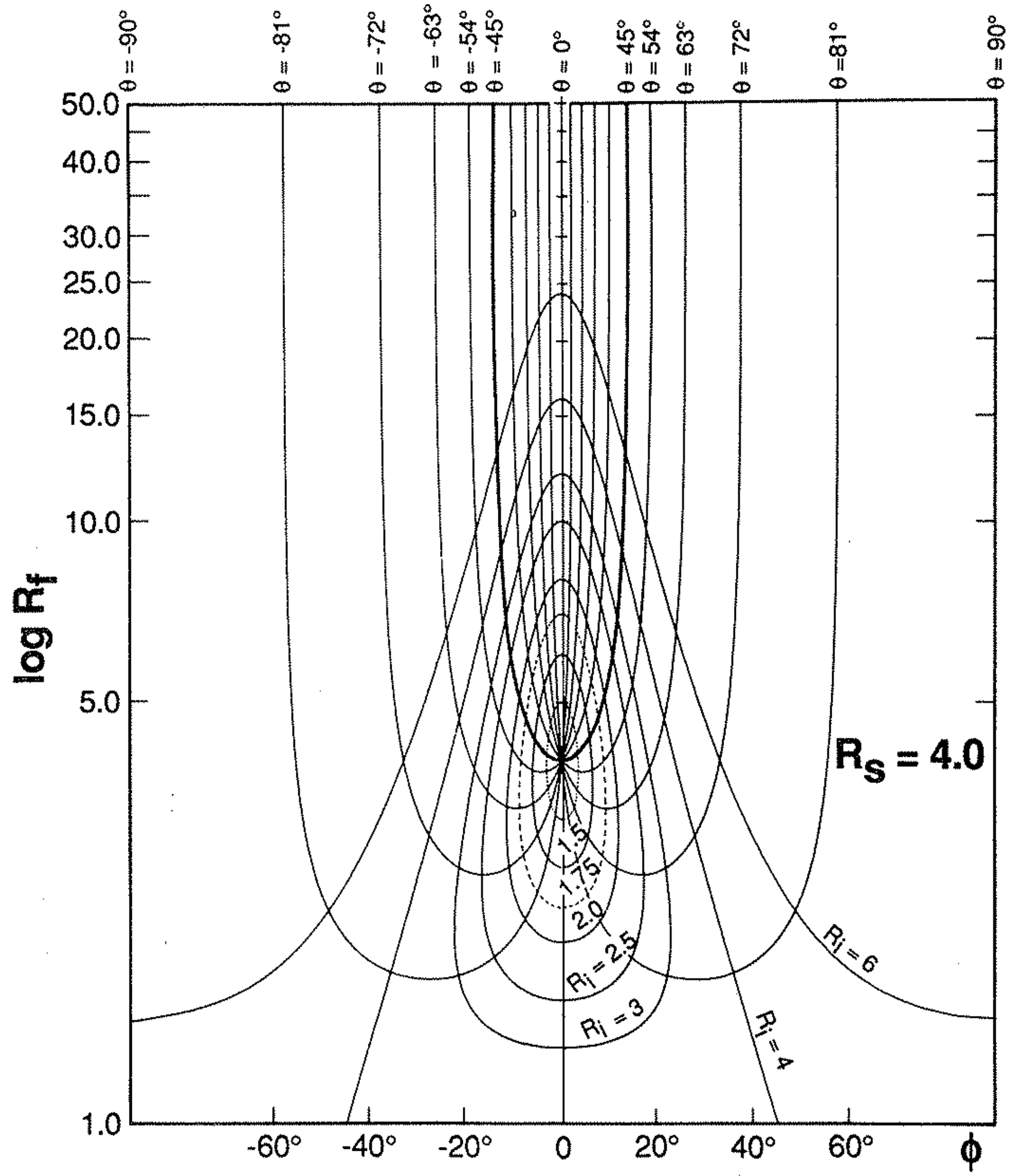

Figure 15-42. R_f/ϕ and θ curves for R_s = 4.0. (Modified from Lisle, 1985.)

values of R_s, and a complete set is provided by Lisle (1985). The method for their use is as follows:

Step 1: Use a sheet of tracing paper over Figure 15-35 and draw a reference line in any orientation. You will find it helpful to draw the line close to the apparent maximum principal stretch axis, but the orientation is not critical.

Step 2: Measure the longest and shortest axis of each ooid and the orientation, ϕ, of each long axis with respect to the reference line (ϕ measured clockwise is positive, counterclockwise is negative). Tabulate your data and calculate R_f for each ooid.

Step 3: Plot the R_f and ϕ values for each ooid onto transparent log/linear graph paper (or use an overlay) with axes as shown in Figure 15-41. For use with Lisle's (1985) curves, you will need a ϕ scale of 10° = 1 cm and an R_f logarithmic axis with a 12.5-cm cycle.

Step 4: Fitting the R_f/ϕ curves to the data. First, we need to find the orientation of the long axis of the strain ellipse. If your reference line was exactly parallel to S_1, then the data should be symmetrical about the ϕ = 0° line. If your reference line lay at some angle to S_1, then find the axis of symmetry of the plot and this will give ϕ_s, the orientation of S_1 with respect to your reference line. Now center your plot so that its symmetry axis ϕ_s lies along the ϕ = 0° line.

You must now go through the different R_f/ϕ curves of Lisle (1985) for the various R_s values and find the set that divides the data equally. In other words, 25% of the data should fall in each of the four quadrants formed by the θ = 0° and θ = 45° curves. Ideally, an equal number of points should fall between each pair of adjacent θ curves. This is a time-consuming exercise and one that cannot be carried out if the data are not symmetrically distributed about the ϕ_s line. In this case more sophisticated techniques are required (see Dunnet and Siddans, 1971, and Lisle, 1985).

Step 5: When the appropriate R_f/ϕ curves have been found, simply record their R_s value.

Step 6: The initial shape factor, R_i, for each pebble can now be read directly from the graph by noting the position of each point plotted relative to the R_i contours.

This now traditional approach to strain analysis can be extremely accurate but it is not rapid, nor is it easy to use if the sediment had an initial fabric, as many sediments do! However, the detailed information obtained is well worth the extra effort involved. One drawback is that a suite of different graphs must be examined before the R_s value can be found, and even then it is seldom easy to select among two or three that could equally well fit the data.

De Paor's Adaptation of the R_f/ϕ Method

A simpler, more rapid technique for dealing with the R_f/ϕ data has been developed by De Paor (1988). The complete set of θ curves for all values of R_s have been combined onto one diagram known as a *hyperbolic net* (Fig. 15-43). There are two halves to the net. One hemisphere is labeled R and the other ε, the natural strain, where ε = 0.5 ln(R). We need only concern ourselves here with the R side of the net. The R-axis of the net is divided on a logarithmic scale and represents any axial ratio (R_s, R_i, or R_f). ϕ is measured evenly around the periphery of the net (Fig. 15-44). Points are plotted in this "R_f/ϕ space" just as they would be on a more familiar stereonet: R_f = 1.0 occurs at the net center; high values of R_f are nearest the periphery. The acute hyperbolas represent Lisle's θ = 45° (50%-of-data) curves for every value of R_s from 1.0 to 10.0.

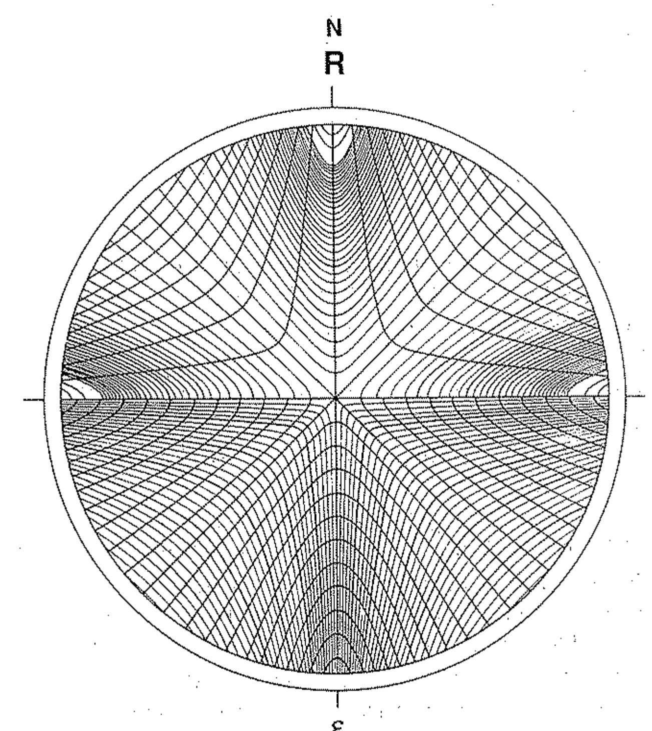

Figure 15-43. The hyperbolic net. (After De Paor, 1988.)

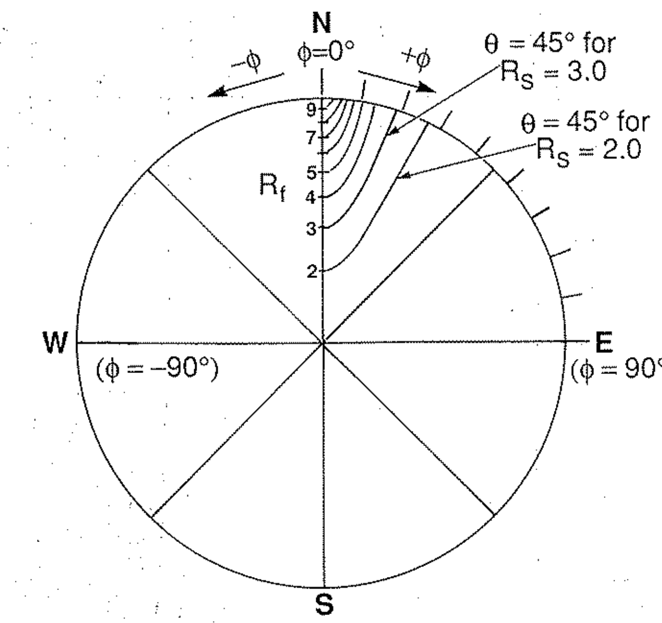

Figure 15-44. Axes and scales of the R side of the hyperbolic net shown in Figure 15-43. (After De Paor, 1988.)

Exercise 15-15

We will use the R_f/ϕ data that you collected in Exercise 15-14.

Step 1: Place a tracing sheet over the hyperbolic net and mark the north axis, N. Plot the R_f and ϕ values for each ooid as follows: (a) Mark the ϕ value on the periphery of the net; (b) rotate the tracing paper until the desired ϕ value is at the north axis of the underlying net; (c) count out the R_f value along the north-south axis starting from the center pin which is at R_f = 1.0 (note that any R_f values greater than 10.0 would plot between the two perimeter circles). Repeat this procedure for each point.

Step 2: After all points are plotted, rotate the tracing paper until the N-S axis becomes the axis of symmetry for the data set. Draw this line through the data set and label it ϕ_s (Fig. 15-45a).

Step 3: Examine the θ = 45° curves until you find one that divides the population of points in half (Fig. 15-45b). The intersection of this 50%-of-data curve with the north-south axis gives the value R_s for the outcrop. Mark this point and read R_s (*Note:* the scale for R_s is the same as that for R_f).

Step 4: Rotate the tracing paper back to its starting position and read the angle ϕ_s around the periphery. You now have the strain ratio, R_s, and its orientation with respect to your reference line.