Problem 3:

Reduced time axis: $t_r = t - X/6$

Scale: 13.65 cm ≈ 18 sec → 1 sec ≈ 0.76 cm
1cm ≈ 1.32 sec

Direct Arrival: $t_1 = X / V_1$

Head Wave: $t_2 = \tau_1 + X / V_2$ → $\tau_1 = (1 / V_1) \cdot 2 \cdot h \cdot Cos(i_c)$
$i_c = Sin^{-1}(V_1 / V_2)$

PROFILE 1:

Direct Arrival: @ 100km: t_r = 4.35cm · 1.32sec/cm = 5.74 sec

@ 0km: t_r = 3.90cm · 1.32sec/cm = 5.15 sec

$t_{100km} = (t_{r100} - t_{r=0 \text{ in plot}})$ + 100km / 6km/sec = (5.74 – 4.0) + 100km / 6km/sec = 18.41 sec

$t_{0km} = (t_{r0} - t_{r=0 \text{ in plot}})$ + 0km / 6km/sec = (5.15 – 4.0) + 0km / 6km/sec = 1.15 sec

$$Slope = \frac{1}{V_1} = \frac{18.41 - 1.15}{100 - 0} = 0.1726 \, sec/km$$ → $V_1 = 5.80 km/sec$

Head Wave: @ 300km: t_r = 0.90cm · 1.32sec/cm = 1.19 sec

@ 200km: t_r = 4.20cm · 1.32sec/cm = 5.54 sec

t_{300km} = 47.19sec ; t_{200km} = 34.88sec

$$Slope = \frac{1}{V_2} = \frac{47.19 - 34.88}{300 - 200} = 0.1231 \, sec/km$$ → $V_2 = 8.12 km/sec$

Zero intercept: τ_{r1} = 10.55cm · 1.32 sec/cm = 13.93 sec

$\tau_1 = \tau_{r1}$ - 4.0 + 0/6 = 9.93 sec

$$i_c = Sin^{-1}\left(5.80/8.12\right)$$ → $i_c = 45.58^o$

$$h = \frac{\tau_1 \cdot V_1}{2 \cdot Cos(i_c)} = \frac{9.93 \cdot 5.80}{2 \cdot Cos(45.58)}$$ → h = 41.15 km

PROFILE 2:

Direct Arrival: @ 100km: $t_r = 3.70\text{cm} \cdot 1.32\text{sec/cm} = 4.89$ sec

@ 0km: $t_r = 3.30\text{cm} \cdot 1.32\text{sec/cm} = 4.36$ sec

$t_{100km} = (t_{r100} - t_{r=0\ \text{in plot}}) + 100\text{km} / 6\text{km/sec} = (4.89 - 4.0) + 100\text{km} / 6\text{km/sec} = 17.55$ sec

$t_{0km} = (t_{r0} - t_{r=0\ \text{in plot}}) + 0\text{km} / 6\text{km/sec} = (4.36 - 4.0) + 0\text{km} / 6\text{km/sec} = 0.36$ sec

$$Slope = \frac{1}{V_1} = \frac{17.55 - 0.36}{100 - 0} = 0.1717\ \text{sec}/km \quad \rightarrow \quad \boxed{V_1 = 5.82\text{km/sec}}$$

Head Wave: @ 300km: $t_r = 1.10\text{cm} \cdot 1.32\text{sec/cm} = 1.45$ sec

@ 200km: $t_r = 4.15\text{cm} \cdot 1.32\text{sec/cm} = 5.48$ sec

$t_{300km} = 47.45\text{sec}$; $t_{200km} = 34.81\text{sec}$

$$Slope = \frac{1}{V_2} = \frac{47.45 - 34.81}{300 - 200} = 0.1264\ \text{sec}/km \quad \rightarrow \quad \boxed{V_2 = 7.91\text{km/sec}}$$

Zero intercept: $\tau_{r1} = 9.92\text{cm} \cdot 1.32\ \text{sec/cm} = 13.09$ sec

$\tau_1 = \tau_{r1} - 4.0 + 0/6 = 9.09$ sec

$$i_c = Sin^{-1}\left(5.82/7.91\right) \quad \rightarrow \quad i_c = 47.37^o$$

$$h = \frac{\tau_1 \cdot V_1}{2 \cdot Cos(i_c)} = \frac{9.62 \cdot 5.82}{2 \cdot Cos(47.37)} \quad \rightarrow \quad \boxed{h = 39.06\ \text{km}}$$

Problem 4:

Profile 1	Profile 2
$V_1 = 5.80$ km/sec	$V_1 = 5.82$ km/sec
h = 41.15 km	h = 39.06 km
$V_2 = 8.12$ km/sec	$V_2 = 7.91$ km/sec

The difference in the two results suggests that there is a dipping layer. In fact, the top velocity (which was determined with the direct arrival) agrees very well between the two profiles, but this is not the case for depth and lower velocity of the half space.

Based on the deeper depth to V2 in the first profile, it is expected that the “apparent” velocity would be higher than in the second profile, and this is what is observed.

Problem 5:
In general, the continental crust varies from 20 to 70 km deep. Its general p-wave velocity is less than about 6.5km/sec. Below the crust, an increase of velocity is observed to about 8.0 km/s for the upper mantle. This general earth structure agrees well with the above results.

Problem 6:

Direct Arrival:

Assume: $V_1 = \frac{1}{2}(V_{1\text{-profile 1}} + V_{1\text{-profile 2}}) = 5.81$ km/s

$t = X / V_1 = 350\text{km} / 5.81\text{km/sec} = 60.24$ sec

Head Wave: $\tau_{1\text{-profile 1}} = (1/V_1) \cdot 2 \cdot h_{\text{profile 1}} \cdot \text{Cos}(i_c)$; $t_{2\text{-profile 1}} = \tau_1 + X / V_1$; $\text{Sin}(i_c + \delta) = V_1 / V_{2\text{-profile 1}}$

$\tau_{1\text{-profile 2}} = (1/V_1) \cdot 2 \cdot h_{\text{profile 2}} \cdot \text{Cos}(i_c)$; $t_{2\text{-profile 2}} = \tau_1 + X / V_1$; $\text{Sin}(i_c - \delta) = V_1 / V_{2\text{-profile 2}}$

where δ = dip of boundary

Using the two equations with δ we can solve for this dip angle and obtain:

$$\left.\begin{array}{l} i_c + \delta = 45.69 \\ i_c + \delta = 45.27 \end{array}\right\} \quad 47.27 + 2\,\delta = 45.69 \;\rightarrow\; \boxed{\delta = -0.8^{\circ}} \;\rightarrow\; \boxed{i_c = 46.48^{\circ}}$$

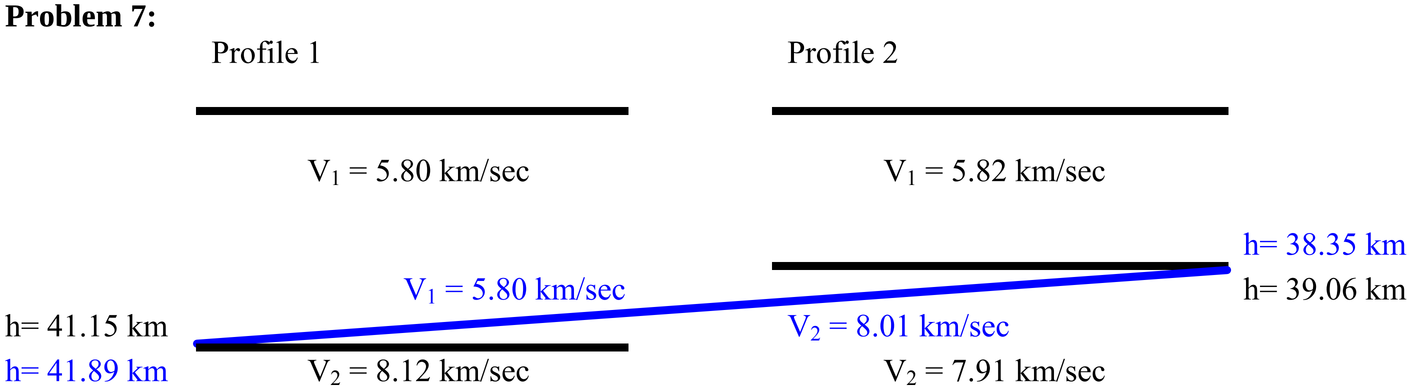
$$Sin(i_c) = V_1 / V_2 \;\rightarrow\; V_2 = 5.81 / Sin(46.48) \;\rightarrow\; \boxed{V_2 = 8.01\text{km/sec}}$$

To determine the depths, use the intercept times:

$$h_{profile\ 1} = \frac{\tau_{1-profile\ 1} \cdot V_1}{2 \cdot Cos(i_c)} = \frac{9.93 \cdot 5.81}{2 \cdot Cos(46.48)} \;\rightarrow\; \boxed{h_{\text{profile 1}} = 41.89 \text{ km}}$$

$$h_{profile\ 2} = \frac{\tau_{1-profile\ 2} \cdot V_1}{2 \cdot Cos(i_c)} = \frac{9.09 \cdot 5.81}{2 \cdot Cos(46.48)} \;\rightarrow\; \boxed{h_{\text{profile 2}} = 38.35 \text{ km}}$$

Problem 7:



The dipping reversed profile compares well with 1-D cases at each end. The depth to the moho is a little less at the profile 2 location and a little greater in the profile 1 location. The moho velocity is between the two estimates.

Problem 8:
The direct arrivals do not have a zero intercept. In fact for:

Profile 1: t_{0km} = 1.15 sec

Profile 2: t_{0km} = 0.36 sec

In the plots, an initial short travel time segment can be drawn, which has approximately a zero intercept.

Based on those two lines, the velocities for the shot points are:

Profile 1: $Slope = \frac{1}{V_{low\ 1}} = \frac{\left(1.15 + {}^{11.33}\!/_{6}\right) - 0}{11.33 - 0} = 0.2682\,{}^{\sec}\!/_{km}$ → $\boxed{V_{low\ 1} = 3.73\text{km/sec}}$

Profile 2: $Slope = \frac{1}{V_{low\ 2}} = \frac{\left(0.36 + {}^{12.53}\!/_{6}\right) - 0}{12.53 - 0} = 0.1954\,{}^{\sec}\!/_{km}$ → $\boxed{V_{low\ 2} = 5.12\text{km/sec}}$

As expected the near surface velocities are smaller.

Profile 1: τ_1 =1.15 sec

$$i_c = Sin^{-1}\left({}^{3.73}\!/_{5.80}\right) \quad \rightarrow \quad i_c = 40.0^{o}$$

$$h = \frac{\tau_1 \cdot V_1}{2 \cdot Cos(i_c)} = \frac{1.15 \cdot 3.73}{2 \cdot Cos(40.0)} \quad \rightarrow \quad \boxed{h = 2.80 \text{ km}}$$

Profile 2: τ_1 =0.36 sec

$$i_c = Sin^{-1}\left({}^{5.12}\!/_{5.82}\right) \quad \rightarrow \quad i_c = 61.6^{o}$$

$$h = \frac{\tau_1 \cdot V_1}{2 \cdot Cos(i_c)} = \frac{0.36 \cdot 5.12}{2 \cdot Cos(61.6)} \quad \rightarrow \quad \boxed{h = 1.94 \text{ km}}$$

These superficial slow layers increase the travel time for a model in which they are taken into account.

Also, the presence on them implies that what was considered before (in problem 3) as direct arrival is in fact a head wave traveling at the edge of this interface.

Furthermore, this slow shallow velocities will have an effect on our estimation of the depth to the moho, thought this effect will be minor.

Problem 9:
There are some parts of the profile in which curvature of the first arrival can be clearly seen. This implies a non-homogeneous medium. Also, other train of arrivals can be observed in the profiles, besides the "direct" arrival and the head wave. A line was drawn on profile 2, showing one of these trains of arrivals. For this case, the velocity can be estimated as:

$$Slope = \frac{1}{V_{\text{int}}} = \frac{\left(3.3 \cdot 1.32 + {}^{300}\!/_{6}\right) - \left(4.56 \cdot 1.32 + {}^{200}\!/_{6}\right)}{300 - 200} = 0.1467\,{}^{\sec}\!/_{km} \quad \rightarrow \quad \boxed{V_{low\ 2} = 6.82\text{km/sec}}$$

Having this intermediate faster layer means that the time it actually takes for the ray to arrive to the mantle is smaller, but ignoring this larger velocity layer, we interpret the record as if the mantle was shallower than in reality → *we underestimate the real depth of the mantle.*

Problem 10:

$XC = 2 \cdot h \cdot t \cdot \tan(i_c)$; $h = h_{profile\ 1\ -\ or\ -\ profile\ 2} - XC/2 \cdot \tan(\delta)$

$$XC_{profile\ 1} = \frac{2 \cdot h \cdot \tan(i_c)}{1 + \tan(\delta) \cdot \tan(i_c)} = \frac{2 \cdot 41.89 \cdot \tan(46.48)}{1 + \tan(0.8) \cdot \tan(46.48)} \rightarrow \boxed{XC_{profile\ 1} = 86.95\ km}$$

$$XC_{profile\ 2} = \frac{2 \cdot h \cdot \tan(i_c)}{1 + \tan(\delta) \cdot \tan(i_c)} = \frac{2 \cdot 38.35 \cdot \tan(46.48)}{1 + \tan(0.8) \cdot \tan(46.48)} \rightarrow \boxed{XC_{profile\ 2} = 79.60\ km}$$

It is not so easy to see this on the records. On profile 2, there seems to be a train of large secondary arrivals at around 90km, which is relatively close to the 80km calculated above.

Problems 11 & 12:

- We are assuming a 2D profile.

- The profile measured could be oblique to the strike of the discontinuity, in which case we would be measuring an apparent dip.

- We are assuming that between discontinuities the layers are homogeneous. This is an approximation.
- In the interpretation we are accepting the assumptions of Ray Theory.

- Although some assumptions may not seem completely valid, for the level of detail in which we are interpreting the earth structure (just a few major layers), the assumptions seem reasonable in the sense that there is probably greater uncertainty in the interpretation (e.g. line fitting) than the ones caused by the simplifying assumptions made in the analyses.

Problem 13:

Yes, the analysis could be repeated for an S-wave model. The problem in this case would be that the s-waves are never a first arrival ($\beta < \alpha$) and therefore it would be harder to select the arrivals of the wave trains. In the profiles, for the first 100km, the arrival of the S-waves can be observed.

If the records of the tangential component of motion had been measured, the waves would have been decoupled from the p-waves, and would be easier to interpret them.

Finally, since these waves were generated by chemical explosions, the source mainly generated compressional waves (more details on source mechanisms will be discussed later in the class).