EPS130 – Strong Motion Seismology Laboratory 3
Wave Motions

Exercise 1. The seismic wavefield due to an isotropic point-source may be represented as $\varphi(t,r,z)=f(t-R/v)/R$, where v is the seismic wave velocity, t is time, R is the radial distance from the source defined as $R=\text{sqrt}(r^2 + z^2)$. The quantity R/v represents the arrival time of a wave at distance=R, and therefore $f(t<R/v, R)=0$.

The P-wave scalar wave equation, $\ddot{\phi} - v^2 \cdot \nabla^2 \phi = 0$, has the following form in cylindrical coordinates.

$$\frac{\partial^2 \phi}{\partial t^2} - v^2\left[\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}\right] = 0$$

Verify that $\varphi(t,r,z)=f(t-R/v)/R$ is a solution to the scalar wave equation in cylindrical coordinates. (Hint: the use of a Laplace Transform is recommended).

Exercise 2. Given the elastic displacement potentials,

$\varphi=7\exp(2i(0.5x_1+\text{sqrt}(3)x_2/2-6t))$ and $\psi_j=\langle\text{sqrt}(3),-1,6\rangle\exp(3i(0.5x_1+\text{sqrt}(3)x_2/2-4t))$

Determine the amplitudes of the components of the P and S waves, and the angles describing the wave propagation direction and particle motion. (Note that the coordinate system is (x1-north, x2-east, x3-vertical(positive up))

Exercise 3. Given the amplitudes of the displacements for P and S waves as follows,

$U_i^P=\langle 4/(3\text{sqrt}(2)),\ 4/(3\text{sqrt}(2)),\ 4/\text{sqrt}(3)\rangle$

$U_i^S=\langle -\text{sqrt}(3),\ -\text{sqrt}(3),\ \text{sqrt}(2)\rangle$

The velocities of the two waves are $V_p=6$ km/s and $V_S=4$ km/s, and the angular frequency is $\omega=4$. Find the expressions for the potentials φ, ψ_i.