ELSEVIER

Earth and Planetary Science Letters 5971 (2001) 1–5

EPSL

www.elsevier.com/locate/epsl

Fractal power spectra plotted upside-down Comment on "Scaling of power spectrum of extinction events in the fossil record" by V.P. Dimri and M.R. Prakash

James W. Kirchner *

Department of Earth and Planetary Science, University of California, Berkeley, CA 94720-4767, USA

Received 2 May 2001; accepted 30 July 2001

Fossil extinction power spectra, recently published by Dimri and Prakash [1], appear to exhibit power-law scaling in which spectral power is inversely proportional to frequency. Dimri and Prakash interpret their results as demonstrating a fractal pattern in the fossil record, with long-range correlations that suggest self-organized critical dynamics. Here I point out that their methods are vulnerable to biases and artifacts, and I show that their conclusions are based on power spectra that have been plotted upside-down.

Dimri and Prakash analyze marine family extinction data [2] using rescaled-range (R/S) analysis and three different spectral techniques: the fast Fourier transform (FFT), the maximum entropy method (MEM), and the Lomb–Scargle Fourier transform (LSFT). Their use of R/S analysis overlooks the fact that particularly with small data sets like theirs, R/S analysis exhibits 'very large bias' [3], often yielding results similar to their figures 3d and 4d whether or not the underlying data are fractal. Similarly, their use of interpolated time series (in their figures 1b,d, 2a,b, 3a,b, and 4) overlooks previous work showing that interpolation can introduce significant artifactual correlation, yielding apparent fractal behavior even when the underlying data are random white noise [4].

Dimri and Prakash correctly point out that the LSFT [5,6] offers significant advantages over conventional spectral methods, because it can be used on unevenly spaced data. Previous studies have used the LSFT to calculate fossil extinction power spectra [7] and autocorrelation functions [8], as well as cross-correlations between extinction and origination rates [9]. However, this previous body of work contradicts Dimri and Prakash's conclusion that fossil extinction rates are fractal. Analyses of four different extinction metrics and three different fossil data sets (including a longer and more complete version of the data used by Dimri and Prakash) using the LSFT show that fossil extinction rates do not exhibit fractal scaling [7], and are not significantly correlated over timescales longer than 5 Myr [8]. In contrast to this previous work, Dimri and Prakash present no statistical tests of their finding of fractal structure in the fossil record. That is, they do not show that the fossil record yields significantly different results than would be obtained from appropriate null hypotheses (such as random re-shuffles of the original data), when subjected to the same pre-processing and analysis.

What, then, is Dimri and Prakash's evidence, and how do they arrive at it? Their R/S analysis

* Tel.: +1-510-643-8559; Fax: +1-510-643-9980.
E-mail address: kirchner@seismo.berkeley.edu (J.W. Kirchner).

0012-821X/01/$ – see front matter © 2001 Published by Elsevier Science B.V.
PII: S0012-821X(01)00467-8

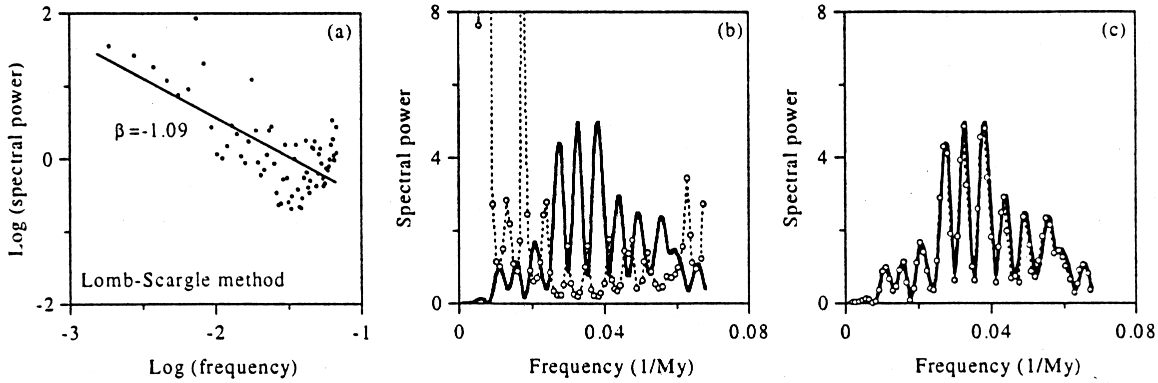

Fig. 1. (a) LSFT power spectrum on log–log axes, digitized from Dimri and Prakash's figure 3c. The apparent log–log slope is −1.09, consistent with fractal $1/f$ scaling. (b) The same power spectrum on linear axes, digitized from Dimri and Prakash's figure 1d (solid line), with points from panel a, transformed to linear axes, superimposed (open circles). Note that the solid line and the open circles appear to be reciprocals of each other, and that the high spectral power at low frequencies on the log–log plot (panel a) exceeds the limits of the linear axes (panel b). (c) The same power spectrum, with reciprocals of points from panel a superimposed. The points and the line agree within the digitizing error, implying that the log–log plot shown in Dimri and Prakash's figure 3c has been plotted upside down.

(their figures 3d and 4d) and their analyses of interpolated time series (their figures 2 and 4) are not conclusive, given the biases and artifacts known to be inherent in these methods [3,4]. In any case, Dimri and Prakash primarily base their conclusions on two power spectra (their figure 3b,c) that were calculated by the MEM and LSFT after polynomial detrending of the data. They emphasize these particular methods, holding that they "are more appropriate", "are more reliable", and "give better resolution". Dimri and Prakash's log–log plots appear to show power-law scaling, with spectral power declining proportionally to frequency. This is the pattern one would expect for a fractal time series exhibiting long-range correlations. I will examine both of these figures in detail.

Their log–log LSFT power spectrum (their figure 3c, reproduced here as Fig. 1a) appears to show that the spectral power is highest at the lowest frequencies, but paradoxically, their own plot of the same power spectrum on linear axes (their figure 1d, reproduced here as the solid line in Fig. 1b,c) shows spectral power converging toward zero at the lowest frequencies. To explore this apparent discrepancy, I digitized the points in the log–log plot, transformed them back to linear axes, and re-plotted them superimposed on the linear plot (Fig. 1b). Two features are immediately apparent. First, the high spectral power at low frequencies on the log–log plot would be off-scale on the linear axis used by Dimri and Prakash. Second, every low point on the log–log plot corresponds to a spectral peak on the linear plot, suggesting that the two plots are reciprocals of one another. I tested this conjecture by calculating the reciprocals of each point on the log–log plot; these agree exactly with the linear plot, within the precision with which I can digitize their graphs (Fig. 1c).

This comparison shows that Dimri and Prakash have plotted the reciprocal of spectral power (rather than spectral power itself) in either their log–log plot or their linear plot. Intuition suggests that it is their linear plot that is correct, since they have pre-processed their data using third-order polynomial detrending; this should suppress the lowest-frequency variations in the time series, leading to very low spectral power (particularly for the lowest frequencies they show, which correspond to wavelengths two to four times as long as their entire data set). To test this conjecture I tried to reproduce Dimri and Prakash's linear plot. I detrended their source data [2] using a third-order polynomial (Fig. 2a,b), and then calculated the power spectrum using the LSFT (Fig.

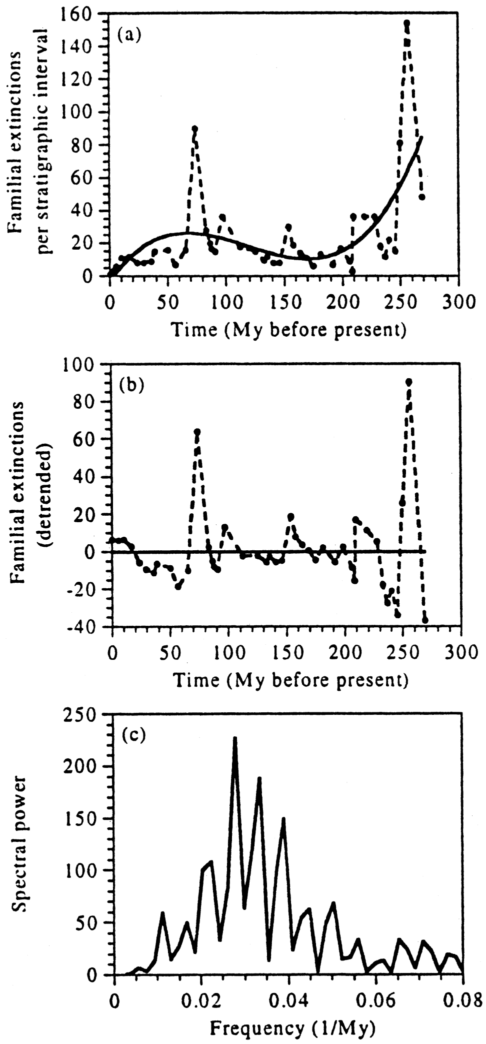

Fig. 2. (a) Marine family extinction time series used by Dimri and Prakash, with cubic polynomial fit. (b) The same extinction data after polynomial detrending. (c) The power spectrum of the detrended data by the LSFT. The good agreement with the solid lines in Fig. 1b,c indicates that Dimri and Prakash's power spectra have been plotted correctly on linear axes, and their log–log plots have been inverted.

2c). As Fig. 2 shows, my power spectrum is generally consistent with Dimri and Prakash's linear plot, declining toward zero at low frequencies. Our scales are different, owing to different normalization pre-factors in our respective algorithms, and the details of the spectra differ slightly, possibly because I used a simple polynomial rather than Dimri and Prakash's Chebyshev polynomial (which cannot be estimated straightforwardly from unevenly spaced data).

Despite these minor differences, comparison of Fig. 2c with Fig. 1b,c confirms that Dimri and Prakash have correctly plotted the spectrum in their linear plot, and therefore shows that they have plotted the reciprocal of spectral power on their log–log plot. That is, the data points on their log–log plot are upside-down, relative to the numbering on the axes. What their log–log plot shows as a spectral power of 10 (log = 1) is actually 0.1 (log = −1), and vice versa. Therefore, if their log–log plot were plotted correctly, it would show spectral power increasing proportionally to frequency (rather than decreasing), in direct contradiction to conclusions that they draw from it. Their log–log plot, in the inverted form in which it is shown, suggests fractal dynamics with long-range correlations. Plotted right-side-up, it would instead be consistent with so-called blue noise, which is characterized by large fluctuations at high frequencies and small fluctuations at low frequencies. Blue noise is almost never observed in nature, and probably arises here as an artifact of their polynomial detrending procedure, which systematically suppresses the low-frequency fluctuations in the source data (see Fig. 2b).

Dimri and Prakash's log–log plot of the MEM power spectrum (their figure 3b, reproduced here as Fig. 3a) also appears to show fractal log–log scaling, similar to their LSFT spectrum. Here again, the log–log plot of the power spectrum appears to show that the spectral power is highest at the lowest frequencies (Fig. 3a), whereas the same spectrum on linear axes shows the lowest power at those same frequencies (solid lines in Fig. 3b,c). Digitizing the points from the log–log plot and superimposing them on the linear plot reveals that the peak on the linear plot corresponds to the lowest point on the log–log plot (Fig. 3b). This suggests that, as with their LSFT spectrum, their log–log plot of the MEM spectrum does not show spectral power, but instead shows the reciprocal of spectral power. As before, I tested this conjecture by inverting the digitized data values from the log–log plot (by reversing

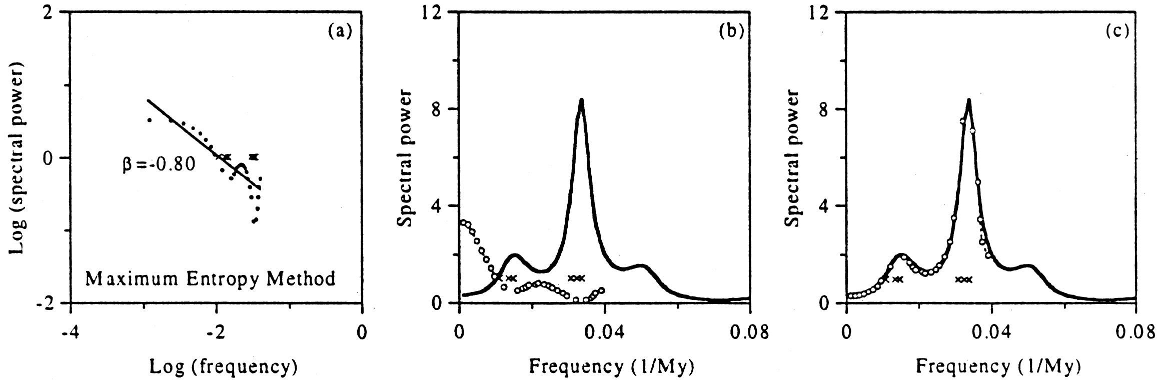

Fig. 3. (a) MEM power spectrum on log–log axes, digitized from Dimri and Prakash's figure 3b. The apparent log–log slope is −0.80, consistent with fractal 1/*f* scaling. (b) The same power spectrum on linear axes, digitized from Dimri and Prakash's figure 1c (solid line), with points from panel a, transformed to linear axes, superimposed (open circles). Note that the solid line and the open circles are reciprocals of each other. (c) The same power spectrum with reciprocals of points from panel a superimposed. The open circles and the line agree within the digitizing error, implying that the log–log plot shown Dimri and Prakash's figure 3b has been plotted upside down. Note also that the right half of the spectrum (frequency range 0.04–0.08) has been omitted entirely from the log–log plot. The 'x' symbols show several anomalous points that do not follow a consistent pattern with the rest of the values. These correspond to a logarithmic value of zero (within the digitizing precision), and perhaps represent a further plotting error in Dimri and Prakash's figure 3b.

the sign of the logs of the spectral power). Within the digitizing precision, the corrected data values agree exactly with the MEM spectrum on linear axes (Fig. 3c). This shows that Dimri and Prakash's figure 3b (my Fig. 3a) is the reciprocal of the power spectrum, rather than the power spectrum itself. That is, the data have been plotted upside-down relative to the numbering on the *y*-axis. If the data values were plotted correctly, they would show spectral power increasing with frequency, inconsistent with the long-range correlations that the authors infer.

These plotting errors have occurred in some cases and not in others. Whereas Dimri and Prakash's figures 3b and 3c have been plotted upside-down, their figure 3a has been plotted correctly (see Fig. 4 of this comment), although it would not support their conclusions in either orientation. Likewise, the log–log plots in Dimri and Prakash's figure 4 are all right-side-up; in this

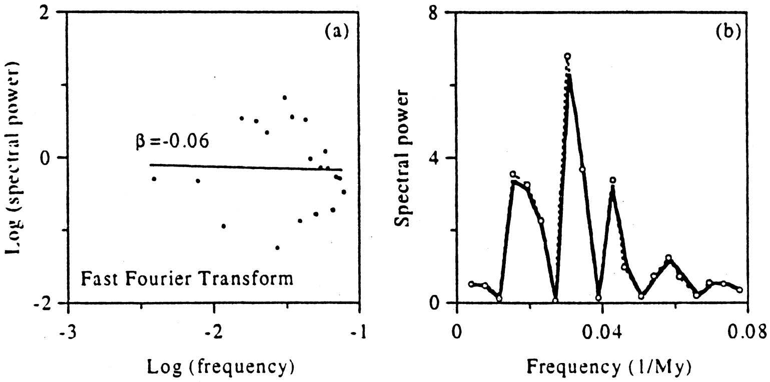

Fig. 4. (a) Power spectrum by FFT of interpolated data, plotted on log–log axes, digitized from Dimri and Prakash's figure 3a. (b) The same power spectrum plotted on linear axes, digitized from Dimri and Prakash's figure 1b (solid line), with points from panel a, transformed to linear axes, superimposed (open circles). Note that in this case the solid line and the open circles agree, indicating that the log–log plot has been plotted right-side-up.

orientation they are consistent with the authors' conclusions (although, as noted above, they are compromised by interpolation artifacts).

Dimri and Prakash's finding of fractal scaling in extinctions is largely based on power spectra that, if plotted correctly, would have contradicted their conclusions. These spectra have somehow been plotted upside-down, while other plots that appear to support the authors' conclusions have not been similarly inverted.

Acknowledgements

I thank Anne Weil for her suggestions on the text.***[SK]***

References

[1] V.P. Dimri, M.R. Prakash, Scaling of power spectrum of extinction events in the fossil record, Earth Planet. Sci. Lett. 186 (2001) 363–370.

[2] J.J. Sepkoski, Jr., D.M. Raup, Periodicity in marine extinction events, in: D.K. Eliot (Ed.), Dynamics of Extinction, Wiley, New York, 1986, pp. 3–35.

[3] C.P. North, D.I. Halliwell, Bias in estimating fractal dimension with the rescaled-range (R/S) technique, Math. Geol. 26 (1994) 531–555.

[4] J.W. Kirchner, A. Weil, No fractals in fossil extinction statistics, Nature 395 (1998) 337–338.

[5] N.R. Lomb, Least-squares frequency analysis of unequally spaced data, Astrophys. Space Sci. 29 (1976) 447–462.

[6] J.D. Scargle, Studies in astronomical time series analysis. II. Statistical aspects of spectral analysis of unevenly spaced data, Astrophys. J. 263 (1982) 835–853.

[7] J.W. Kirchner, A. Weil, The fossil record does not support fractal extinctions or self-organized criticality of the biosphere, GSA Abstr. Programs 30 (1998) 328.

[8] J.W. Kirchner, A. Weil, Correlations in fossil extinctions and originations through geological time, Proc. R. Soc. London B Biol. Sci. 267 (2000) 1301–1309.

[9] J.W. Kirchner, A. Weil, Delayed biological recovery from extinctions throughout the fossil record, Nature 404 (2000) 177–180.