Statistical inevitability of Horton's laws and the apparent randomness of stream channel networks: Comment and Reply

COMMENT

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In Kirchner's (1993) statistical study of stream networks, he argued on the basis of Monte Carlo simulations, that Horton's ratios are not diagnostic of any particular mode of formation but are nearly universal attributes of all networks. Although we agree with the primary conclusion of the paper, we dispute the assertion that the inevitability of Horton's ratios is "an artifact of stream-ordering methods" (Kirchner, 1993, p. 594). Alternatively, we suggest that fractal analysis provides a more useful way to interpret the results of Kirchner (1993).

It has long been noted that river networks are classic examples of scale-invariant (fractal) trees (Mandelbrot, 1982). In fact, satisfying Horton's bifurcation and length-order ratios are necessary conditions for any drainage to be fractal. Thus, Kirchner (1993) has really shown that the majority of drainages created with the Monte Carlo simulation are, in fact, fractal. In terms of Horton's bifurcation ratio (R_B) and length-order ratio (R_L), the fractal dimension (D) of a drainage network may be written as

$$D = \frac{\log(R_B)}{\log(R_L)}.$$

Using the modal values for R_B and R_L from Kirchner (1993), we find that $D \approx 2.0$. The fractal dimension can be related to the Euclidian dimension in a direct way, such that a dimension of $D = 2$ corresponds to the space-filling geometry of a plane. Given the conclusion that most networks are inherently fractal, it should not be surprising that, if extended to infinite order, the networks tend toward the space-filling geometry of $D = 2$. Indeed, in nature, a fractal dimension close to 2 is a necessary condition for river networks to drain any point on the land surface. This was also first noted by Mandelbrot (1982).

We agree with Kirchner (1993) that satisfying Horton's ratios is not an adequate check on any particular model for network formation. We emphasize, however, that the basic mechanics behind the formation of fluvial networks is still poorly understood, and the problem merits further investigation. In the past ten years, several physical models have been proposed to generate fractal networks. Kondoh and Matsushita (1986), Meakin et al. (1991), and Masek and Turcotte (1993) have introduced diffusion-limited aggregation models for drainage networks. Stark (1991) developed a model based on self-avoiding percolation clusters. Chase (1992), Willgoose et al. (1991), and Kramer and Marder (1992) developed advection-diffusion models coupled with topography. To a greater or lesser extent, these models make unique predictions about the way in which networks self-organize in response to precipitation and topography. Although these models presumably generate fractal networks, they differ in terms of network shape, density, and evolution. Thus, although structural differences *between* natural networks are certainly of interest, the universal pattern of network evolution common to all drainages deserves further study as well.

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REPLY

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The numbers, lengths, and areas of streams in natural channel networks vary systematically with stream order. These relations, known as Horton's "laws of drainage network composition," have been widely interpreted as evidence that natural channel networks are fractal. Masek and Turcotte apply this interpretation to my recent theoretical study of network structure, which shows that Horton's laws, and the typical values of the Horton ratios, are not specific to natural channel networks but instead are shared by virtually all possible networks. My work also shows that Horton's bifurcation, length, and area ratios (R_B, R_L, and R_A) are profoundly insensitive to pronounced changes in network structure. Because R_B and R_L contain little geomorphological information, I am skeptical that the ratio $\log(R_B)/\log(R_L)$ will be geomorphologically informative when interpreted as a fractal dimension, as Masek and Turcotte suggest and as others have suggested previously (e.g., Tarboton et al., 1988; Hinrichsen et al., 1989; LaBarbera and Rosso, 1989; Liu, 1992).

Masek and Turcotte argue that because nearly all of my hypothetical networks obey Horton's laws, these networks are fractal. I cannot agree, because logically they have the cart before the horse. *If* a network is statistically self-similar (or "fractal"), *then* it must obey Horton's laws. To my knowledge, the converse has not been shown—i.e., that networks conforming to Horton's laws (however closely or loosely they may do so in the real world) are *necessarily* self-similar. How nonfractal can a network be while still conforming to Horton's laws as closely as do real stream networks? If both fractal and nonfractal networks obey Horton's laws, then one cannot use Horton's laws as an indicator of whether or not stream channel networks are fractal.

It is important to recognize that networks obey Horton's laws largely because of the particular way that the conventional stream-ordering rules divide the network into separate streams and assign them their various orders. The streams for which numbers, lengths, drainage areas, and orders are compared in Horton's laws do not begin and end at points determined by nature. Instead, they begin and end at points determined by an externally imposed classification scheme. This classification scheme filters the geomorphological information in the network such that Horton's laws are, as I have put it, "statistically inevitable": one can contrive examples that violate Horton's laws, but such examples are rare. The commonly used stream-ordering rules require that two streams of a given order must meet for a stream of the next higher order to begin. This hierarchical definition of stream order, combined with a small degree of structural randomness or random mapping error, can create the power-law relations that are commonly observed, whether or not the networks themselves possess a systematic underlying configuration. These relations explain why nearly all possible networks obey Horton's laws and why wide variation in network structure has little effect on the calculated Horton ratios and, thus, the estimated fractal dimensions.

Because Horton's laws and the "typical" values of $R_B = 4$ and $R_L = 2$ are largely an artifact of stream ordering, I am reluctant to interpret Horton's laws as the "signature" of self-similarity in stream networks. I agree with Masek and Turcotte that "the universal pattern of network evolution common to all drainages deserves further study," *if such a pattern actually exists*. Whether such a pattern exists, and what its character is, remain open questions. To determine whether a universal pattern exists, we need sensitive measures of network configuration, ones that will yield similar measurements for diverse networks only if the networks themselves share a common structure.

It is tempting to assume that invariance in some descriptive statistic, such as fractal dimension, implies uniformity in the things to which that statistic is applied. Sometimes this assumption turns out to be correct, but sometimes the statistic is simply insensitive, or is controlled artifactually by the way we have classified or measured the objects of our analysis. We have been down that road before. For decades many geomorphologists interpreted the uniformity in Horton's ratios as reflecting a general underlying property of stream networks themselves, and they have used Horton's laws to justify diverse theories of network evolution. Now we better understand the limitations of Hortonian analysis, but I am concerned that we may unwittingly repeat this experience, this time with fractal geometry. One can now find phrases in the fractals literature echoing Horton's (1945) argument that his laws "evolve from physical processes which Nature follows rather closely in the development of stream systems. . . ." One also sees statements implying that particular theories of network evolution must be correct because they predict the same fractal dimensions that are observed for real networks. For such inferences, appropriate null hypotheses are crucial; we should know what values we expect for the descriptive statistic of interest (bifurcation ratio, fractal dimension, and so forth) both if the theory holds *and* if it does not.

Fractal geometry merits serious attention as a technique for describing many geomorphological phenomena, including stream networks. However, I hope we will not uncritically assume that fractal dimensions are meaningful in every case where a fractal dimension can be calculated. We should be able to find out quickly whether or not fractal dimension is a useful index of stream channel network structure, particularly if we remember that descriptive statistics can mislead as well as inform.

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Post–125 Ma carbon storage associated with continent-continent collision: Comment and Reply

COMMENT

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Selverstone and Gutzler (1993) concluded that the transfer of carbon from the atmosphere to deep-seated metamorphic reservoirs during the Tethyan continent-continent collision may have contributed to post–125 Ma global cooling. However, because the long-term (>1 m.y.) atmospheric CO_2 content is controlled by the interplay between the flux of CO_2 from Earth degassing vs. CO_2 consumed by silicate-rock weathering (Berner et al., 1983), carbon sequestered in orogenic environments affects atmospheric CO_2 *only* if it influences the rates of degassing (Kerrick and Caldeira, 1993) and/or weathering (Raymo and Ruddiman, 1992). We agree that there is net carbon storage in orogenic belts; nevertheless, collisional orogenesis can enhance CO_2 degassing to the atmosphere because metamorphism in orogenic belts can liberate to the atmosphere CO_2 that would have otherwise remained locked up in kerogen and carbonate rocks.

Selverstone and Gutzler (1993) concluded that carbon was retained in carbonate minerals and graphite and that there was little or no CO_2 loss from metamorphic devolatilization reactions involving these minerals. However, residual carbonate may remain in metamorphic rocks even after significant CO_2 loss by decarbonation. Quantitative estimates of reaction progress suggest that extensive decarbonation occurred in many impure metacarbonate rocks. Marls lose between 5 and 30 wt% CO_2 during prograde metamorphism (Frank, 1983; Kerrick and Caldeira, 1993). Abundant calc-silicates in metamorphosed siliceous dolomites of the Alps (Trommsdorff, 1966; Franz and Spear, 1983), Mediterranean Tethys, and Himalaya orogens (Kerrick and Caldeira, 1993) attest to significant decarbonation. Using a model orogenic carbonate