

# Hillslope evolution by nonlinear creep and landsliding: An experimental study: Comment and Reply

## COMMENT

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Roering et al. (2001) describe very careful and interesting experiments that beautifully illustrate the transition from steady downhill creep at low gradients to highly dynamic transport on steep slopes. They interpret this behavior in terms of a single nonlinear diffusion coefficient, suggesting that transport on steep and gentle slopes is governed by common underlying physics. However, Figure 2A of Roering et al. (2001) shows that there is a transition from a state of mostly continuous flux on gentle slopes, to strongly pulsed behavior with avalanches on steep slopes. It does not seem adequate to describe transport on steep slopes by a larger effective diffusion constant, because the nature of this transport is fundamentally nondiffusive. Such a treatment does not lead to a proper understanding of the complexity of the processes involved.

Basically, in a diffusive system any particle (or grain) describes a random walk (Brownian motion), and its total distance grows as the square root of the traveling time. However, avalanches travel at constant velocity (Hwa and Kardar, 1992; Carreras et al., 1996), as can be verified experimentally using tracer grains. This constitutes a new element in the transport of particles that cannot be described correctly using a diffusive equation, unless the diffusion “constant” is chosen to be a nonlinear function in order to cancel out the square-root behavior. However, there is no justification for this when the propagation can simply be described using a constant velocity. Moreover, avalanches seen in sandpile models show finite-size scaling behavior (Christensen, 1996), i.e., behavior dependent on the system size, implying that the effective diffusion derived for one system is not valid in another larger or smaller system. This annuls the predictive power of the effective diffusion coefficient for any system other than the one for which it was derived.

So, while at gentle slopes transport is diffusive, at steep slopes “ballistic” transport with avalanches takes over. Here, the flux self-organizes (adjusts its value) in order to maintain the slope below the critical value. It is natural that the flux should increase sharply when approaching the critical, highly unstable slope. At high values of the slope (near the critical gradient), the flux is mainly determined by the sediment supply rates, rather than by the slope itself. Naturally, there is a smooth transition between the two regimes.

The transport flux might be described by a formula of the following type (cf. Sánchez et al., 2001):

$$q = K S + \alpha v \Theta(S - S_c), \quad (1)$$

where  $q$  is the mass flux,  $S$  the slope or gradient,  $K$  the diffusion coefficient,  $\alpha$  a fit parameter,  $v$  the ballistic velocity,  $S_c$  the critical slope, and  $\Theta(x)$  a sharply (i.e., nonlinearly) growing function of  $x$  when  $x > 0$ , and small when  $x \leq 0$  (as a first approximation, the Heaviside function may be used). Equation (1) describes the local transport flux, which increases strongly when the critical gradient is exceeded locally. Then, the increased flux steepens the slope further down the hill, lead-

ing to an increase of flux there, thus generating a propagating avalanche.

The spectra shown by Roering et al. (2001) are also indicative of the suggested transport regime transition. The logarithmic spectral slope increases from near 0 in the diffusive case (white noise, random walk) to near 1 in the critical case ( $S \approx 0.42$ ). For higher supply rates the spectral slope is seen to increase further, related to the quasi-periodic behavior also seen in Figure 2A. At the highest supply rates ( $S = 0.48\text{--}0.52$ ), the large flux that is imposed by the external supply rate forces all avalanches to be large, so that the small avalanches cease to exist. One can test this idea experimentally by determining the distribution of avalanche sizes or flux amplitudes, which should be power-law in the critical case ( $S \approx 0.42$ ) and be peaked at large sizes when  $S > 0.42$ . Thus, it would seem that at moderate values of the supply rate ( $S \approx 0.42$ ) the system is in a self-organized critical state, free of any characteristic lengths and time scales (Bak et al., 1987, 1988), reminiscent of the behavior of numerical sandpiles described in the ample literature on self-organized critical state (Bak et al., 1988; Hwa and Kardar, 1992).

It is interesting to note that the external noise source, applied in the form of a speaker, may be a crucial ingredient to resolve the intermediate critical region adequately (cf. Rosendahl, 1994, where noise was absent). To further investigate the possible correspondence between experiment and models, we would suggest that the experiment data be subjected to the relevant analyses: detection of self-similarity (Hurst, 1951; Mandelbrot and Wallis, 1968); estimation of the probability distribution functions of the transport events and the flux (Bak et al., 1988; Frette et al., 1996); and the transport of tracer grains to distinguish between diffusive and ballistic behavior (Christensen et al., 1996).

In conclusion, while the nonlinear diffusion model of Roering et al. (2001) may adequately predict an average hillslope profile in the given experimental situation, we believe, based on the arguments given, that it does not allow extrapolation to other situations. Correct recognition of self-organized critical transport is crucial to understanding, modeling, and predicting the flux in all its highly dynamic aspects, and to be able to make adequate risk assessments and perform hazard mitigation in real-life situations.

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## REPLY

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We appreciate the interest that van Milligen and Bons have shown in our experimental study of hillslope evolution (Roering et al., 2001a). They argue that particle diffusion is a poor model for disturbance-driven creep and landsliding processes on hillslopes. We agree with this assessment and nowhere in our paper did we suggest otherwise. Their comments on our work appear to arise from several fundamental misconceptions, which we welcome the chance to clarify.

van Milligen and Bons misinterpret our slope-dependent transport law as implying that sand grains are transported by diffusion (even though that term does not appear in our paper). Any slope-dependent transport law, when combined with the continuity equation, yields a differential equation that describes the evolution of a hillslope profile through time (e.g., Culling, 1960). Because this differential equation resembles the diffusion equation, slope-dependent transport models are often termed “diffusive” by geomorphologists. However, such differential equations describe diffusion of the hillslope surface (Roering et al., 2001, Figure 3A) and do not refer to the transport of individual particles as van Milligen and Bons suggest. Although the rate constant in such equations has the dimensions of diffusivity, it is not intrinsically related to diffusion at the grain scale, nor is grain-scale diffusion required to generate slope-dependent transport.

Individual particle trajectories are distinct from the time-averaged particle motions that determine the net flux of sediment. van Milligen and Bons correctly point out that the displacements of individual particles undergoing random walks grow as the square root of time, but the average displacement of a group of such particles, and thus the net sediment flux, would be exactly zero. Instead, the nonrandom component of particle motions, which reflects the forces acting on an ensemble of grains, generates net downslope transport. van Milligen and Bons incorrectly assume that particle motions in our experimental hillslope would be diffusive (and thus random) at low gradients. Figure 1B of our paper clearly shows (through the use of tracer particles) that grain transport is steady, downslope-directed, and distinctly nonrandom (and thus categorically not diffusive).

It appears that the primary aim of van Milligen and Bons’s Comment is to point out the inadequacy of equation 1 (Roering et al., 2001a) for describing the nature of transport on steep slopes. The complexity of the transition between granular creep and landsliding is compelling, and we appreciate van Milligen and Bons’s interest in developing a conceptual model for the propagation of avalanches. However, the use of equation 1 to describe the empirical flux-gradient curve (Fig. 1C, Roering et al., 2001a) does not implicitly suggest “that transport on steep and gentle slopes is governed by common underlying physics,” as stated by van Milligen and Bons. In fact, a careful reading of our paper reveals that we intended equation 1 only to be used to represent the flux-gradient curve (which it does well) because it is a continuum model and “cannot be used to predict how hillslope gradient affects the temporal variability of flux or triggers the transition from granular creep to landsliding” (Roering et al., 2001a). Our experiments

do confirm that downslope sediment fluxes—due to continuum creep processes—increase sharply and nonlinearly with gradient, well below the gradients that trigger landsliding.

van Milligen and Bons propose an alternative model, in which sediment fluxes increase linearly with gradient until a critical slope is reached, whereupon landsliding dominates. Models of this type have been proposed before (e.g., Kirkby, 1984), but are inconsistent with our experimental data, which show nonlinear slope-dependence at gradients that are too shallow for landsliding to occur. Furthermore, van Milligen and Bons’s proposed flux law has at least four parameters (plus those implicit in their unspecified function  $\theta$ ), such that it would likely be difficult to calibrate and use for simulating the evolution of natural landscapes.

van Milligen and Bons interpret the  $1/f$  scaling we observed in the power spectrum at an intermediate slope ( $S = 0.42$ ) as indicating self-organized critical behavior. Experimental support for  $1/f$  scaling is elusive in the granular-flow literature (Manna, 1999), and it should be emphasized that such fractal scaling can result for reasons other than self-organized critical behavior. Our results are not consistent with self-organized critical dynamics for several reasons. In our experiments, the power-law slope of the power spectra varies continuously with hillslope gradient, so there is no indication that the system has any tendency to maintain itself in a critical state characterized by  $1/f$  scaling. Whereas self-organized critical theory implies that  $1/f$  scaling should arise from avalanche dynamics, we observed  $1/f$  scaling only under conditions in which discrete landsliding did not occur. For the case  $S = 0.42$ , sediment transport was characterized by a continuous layer of creeping grains and not the initiation and propagation of sediment waves.

van Milligen and Bons conclude by asserting that our nonlinear model “does not allow extrapolation to other situations.” They are apparently unaware of our previous work showing that the same nonlinear flux law explains the topographic form of steep, soil-mantled hillslopes in the western Oregon Coast Range (Roering et al., 1999). In our study area, hillslopes (which are orders of magnitude larger than our experimental sandpile) tend to be convex near the drainage divide and become increasingly planar in the downslope direction, consistent with our proposed nonlinear model and inconsistent with the commonly used linear transport model. The systematic decrease in convexity with increasing gradient appears to be a common feature of soil-mantled hillslopes, suggesting that our nonlinear flux model may be broadly useful for modeling sediment transport and hillslope evolution (e.g., Roering et al., 2001b).

Our experimental results suggest that there may be a complex process involved in transition between nonlinear continuum creep and episodic landsliding on steep slopes. We did not suggest, as van Milligen and Bons imply, that our simple nonlinear transport model could account for process dynamics and the onset of landsliding in our experimental hillslope. Our experiments (1) documented how sediment flux increases with gradient under controlled conditions, (2) documented that continuum creep processes generate a rapid nonlinear increase in sediment flux with increasing gradient well before the onset of episodic landsliding, and (3) demonstrated that disturbance-driven sediment transport generates convex hillslopes, the evolution of which is well described by nonlinear slope-dependent transport models.

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