Scale Dependency of Hydraulic Conductivity in Heterogeneous Media

by Dirk Schulze-Makuch, Douglas A. Carlson, Douglas S. Cherkauer, and Peter Malik

Abstract

Various types of sediments and rocks were analyzed for the relationship between hydraulic conductivity (K) and scale of measurement. No variations of K with scale were observed for homogeneous media such as quartz-arenites (quartz sandstones). However, hydraulic conductivity increased with scale of measurement in heterogeneous media. The scaling behavior can be described with the equation \( K = c (V)^m \), where \( c \) is a parameter characteristic of the geological medium that relates to geological variables such as average pore size and pore interconnectivity in porous media, and probably fracture opening and fracture interconnectivity in fractured media. \( V \) is the volume of tested material (used as scale measure), and \( m \) is the exponent of the relationship (slope of the line on a log-log plot). The value of the exponent depends on the type or types of flow present. Porous flow media have an exponent of 0.5, multiple flow media an exponent between 0.5 and 1.0, and fracture and conduit flow controlled media an exponent of about 1.0. The more dominant fracture/conduit flow is relative to porous flow, the closer the exponent is to 1.0. \( K \) increases with scale up to a rock volume after which the aquifer approaches the properties of an equivalent homogeneous medium and \( K \) remains constant with scale. This volume (upper bound of the relationship) is related to the degree of heterogeneity in a medium. It is at a much larger scale in karstic media (if encountered at all) than in nonkarstic and more homogeneous media. Both confined and unconfined aquifers exhibit a similar scale dependence.

Introduction

Hydraulic conductivity (K) is known to vary with scale of measurement for a variety of geologic media. Clauser (1992) showed that K values for various granitic rocks increase with scale over several orders of magnitude until a tested volume of rock is reached (upper bound of relationship) beyond which K remains constant. An increase of K with scale of measurement from the laboratory scale to the regional scale without an upper bound was found by Sauter (1992) in a karstic limestone aquifer in southwestern Germany. Hydraulic conductivity variations with scale of measurement are also present in unconsolidated sediments (Herzog and Morse 1984; Bradbury and Muldoon 1990). Vertical hydraulic conductivity (Kv) differences of up to four orders of magnitude were observed by Hanor (1993) when comparing laboratory values with Kv's derived from water balance calculations for clay beds at a hazardous waste site in Louisiana. These differences were attributed to intercalated sands and zones of pedogenic secondary porosity and fracturing developed during periods of subsaerial weathering and not to the matrix properties of the clay.

A single exponential relationship of horizontal hydraulic conductivity to scale of measurement was proposed by Neuman (1994) for all types of geologic media. However, Rovey (1994) and Schulze-Makuch (1996) analyzed the scale behavior of hydraulic conductivity for various geological units and found media-dependent relationships. In the carbonate aquifer of southeastern Wisconsin, the log-log exponent of the increase of \( K \) with scale was about 0.5 for porous flow media and higher if the analyzed unit contained fractures (Schulze-Makuch 1996). Also, most of the carbonate facies had an upper bound beyond which \( K \) remained constant with scale.

This type of scale behavior, exponential with upper bounds, was theoretically proposed by Wheatcraft and Tyler (1988) for the longitudinal dispersivity parameter in heterogeneous media. Longitudinal dispersivity and hydraulic conductivity (horizontal K) appear to be intrinsically related (Carrera 1993; Schulze-Makuch and Cherkauer 1997). Carrera (1993) concluded that the scale dependency of \( K \) is directly or indirectly caused by heterogeneities. He provides an excellent overview of how hydraulic conductivity variations with scale affect solute transport modeling. As larger blocks of the subsurface are tested for subsurface flow, preferred pathways are encountered that increase the average measured value of hydraulic conductivity. These preferred pathways can be provided by facies heterogeneities, fractures, or flow conduits. An intuitive argument against the scaling of hydraulic conductivity being dependent on heterogeneity is that the observed behavior is actually caused by different methods of measurement (Butler and Healey 1998). However, Schulze-Makuch and Cherkauer (1998) observed the same value of exponential scale increase whether using one or

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several methods of measurements within a given geologic unit. Tidwell and Wilson (1997) observed distinct and consistent trends diagnostic of permeability upscaling using a gas permeameter.

The objective of this paper is to establish the relationship of hydraulic conductivity to scale of measurement and investigate whether it is a function of the type of subsurface medium tested. The main focus is on whether the scaling relationship is affected by the heterogeneity and the flow type within a medium. The approach used was to obtain aquifer test data from the laboratory to the regional scale for a large number of different geological media and analyze them for their scaling relationship.

**Methods**

**Categorization of Hydraulic Conductivity Tests**

Information that forms the basis of the scale relationships presented was obtained from carbonate rocks of southeastern Wisconsin, northeastern Wisconsin, northeastern Illinois, Nevada, Puerto Rico, southwestern Germany, and Slovakia; from unconsolidated sediments of southern Wisconsin and Slovakia; from rock units of the Cambrian Sandstone Aquifer of western Wisconsin; from the Floridian Aquifer of southwestern Florida and Georgia; and from Devonian Shale of the Appalachian Basin. A total of 39 different geological media were analyzed. A brief description of each geological unit analyzed in this study and a list of data sources is provided in the appendix.

**Separation by Dominant Flow Type**

Data collected from different formations and aquifers were subdivided by the type of flow present in the geological medium. The separation was based on (1) time-drawdown responses in pumping tests; (2) modality of hydraulic conductivity histograms; and (3) core analysis.

For example, a porous medium typically has a Priest type time-drawdown response, a fractured medium shows a log-log linear response, and a double-porosity medium responds with a com-
posite response of the other two (Figure 1a). Histograms of hydraulic conductivity are suited to distinguish between double-porosity media and porous or fractured rock. Double-porosity media typically exhibit bimodality, one mode representative of porous flow, the other of fracture flow (Figure 1b). A unimodal K distribution indicates that there is only one primary type of flow (unless both types of flow have a similar hydraulic conductivity value, which is uncommon). Core analysis can then be used to identify which type of flow(s) are the primary ones in a given medium based on the presence of facies heterogeneities, fractures, or solution conduits (Figure 1c). However, results on a core scale have to be evaluated with care, because fractures common in a core might not contribute much to the overall flow on a field scale due to a lack of interconnectivity or due to the dominance of high-K porous facies zones. Breaks in cores can also be the result of the core drilling and handling processes.

Separation by Type of Medium

After the first separation was made by flow type, the hydraulic conductivity data were separated by the type of medium. Unconsolidated sediments were subdivided into alluvial deposits, outwash deposits, lacustrine sediments, beach deposits, volcanic effusives, and till-related deposits based on mode of deposition and grain-size distribution. Consolidated deposits were subdivided into sandstones, carbonate rocks, and shales.

Separation by Scale of Measurement

Once the data were categorized by flow type and geologic origin or lithology, the relation of K to scale of measurement within each group was examined. This required the selection of a suitable measure of scale. The chosen measure has to be applicable for many types of tests. Historically, measurement scales for hydraulic conductivity were either the distance water travels during a test, radius of influence (Rovey and Cherkauer 1995), or volume of tested material (Bradbury and Muldoon 1990). In this study, the latter measure was used because it is thought to provide a more accurate and consistent measure (Table 1). For example, consider comparing piezometer tests (slug or baildown tests) with permeameter tests. The flow distance of a piezometer test (the dimension parallel to flow) is generally only several centimeters into the geologic medium, but the procedure is testing several meters of material transverse to flow (along the screen). In contrast, the flow distance through a core plug in a permeameter test is also several centimeters, but the transverse radius is only in the centimeter range. Using a distance-parallel-to-flow approach would project both tests on the same scale, although they are not. With a volume of tested material as the scale measure, these two types of tests are separated by orders of magnitude.

The formulas used to calculate the volume of material tested by each method considered are listed in Table 1. For lab or field tests, the tested volume of the medium can generally be calculated by dividing the volume of water used in the test by the unit’s effective porosity. Porosity may also vary with scale, but any such variations can be assumed minor compared to that of K, because the latter’s values commonly range over orders of magnitude while porosity’s do not.

In this study, the scale measure for computer models of regional flow was assigned to be the effective volume bounded by the average distance flow travels, a representative transverse distance, and the thickness of the geological unit analyzed. Flow distance within a model was selected as the average distance between recharge to discharge from the model at steady state. A transverse distance of 1/10 of the distance in flow direction was assumed because of the commonly assumed ratio of 1:10 for transverse dispersivity to longitudinal dispersivity, which has been reported by various authors (Domenico and Schwarz 1990; Gelhar et al. 1992). As mentioned, the hydraulic parameters of hydraulic conductivity and longitudinal dispersivity are interrelated (Schulze-Makuch and Cherkauer 1997). Only calibrated or verified regional flow models were included in the data analysis. It is common practice to use K values derived from pumping tests as initial hydraulic conductivities, thus noncalibrated K values do not represent the model domain. After the regional flow model is calibrated to the real hydraulic data set (usually hydraulic head data), K values are thought to be representative for the modeled domain.

| Table 1 |

Parameters Used in This Study as Measure of Scale—All Produce a Measure of the Total Volume of Geological Medium Affected by the Test

<table>
<thead>
<tr>
<th>Type of Test</th>
<th>Measure of Scale (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeameter tests</td>
<td>V₆</td>
</tr>
<tr>
<td>Piezometer tests</td>
<td>V₆/nₑ</td>
</tr>
<tr>
<td>Packer tests</td>
<td>V₆/nₑ</td>
</tr>
<tr>
<td>Single and multiple well pumping tests</td>
<td>Qₚnₑ</td>
</tr>
<tr>
<td>Specific capacity data</td>
<td>Qₚnₑ</td>
</tr>
<tr>
<td>Tracer tests</td>
<td>Vₚ/nₑ</td>
</tr>
<tr>
<td>Passive infiltration tests</td>
<td>L₆/L₄/B</td>
</tr>
<tr>
<td>Regional flow computer models</td>
<td>L₆/L₄/B</td>
</tr>
</tbody>
</table>

Symbols used:

- V₆ = volume of the rock or sediment sample (m³)
- V₆ = volume of water introduced or removed during recorded time interval (m³)
- nₑ = effective porosity of geological unit (dimensionless)
- Q = average pumping rate (m³/min)
- t = total time of recorded pumping test (min)
- Vₚ = volume of water removed from the pumped well until the appearance of the advective front (m³)
- L₆ = flow distance (m)
- L₄ = transverse spreading distance, assumed to be L₆/10 (m)
- B = thickness of geological unit (m)

Analysis of Tests for Scale Effect

After available measurements were sorted by flow type and geologic origin or lithology, the K values were plotted relative to scale of measurement. For each set plotted, all data below any observed upper bound on scale effect were used to calculate a regression line and a 95% confidence envelope. Measurements at the lab scale are usually provided by permeameter tests; by slug or baildown tests at small field scales; by tracer, specific capacity, or pumping tests (both single and multiple well tests) at large field scales; and by infiltration data and regional flow models at the regional scale. Specific locations were distinguished only if significant differences appeared between sample sets from different source areas.

Large aquifers comprising of various types of rocks were separated into formations and the formations were analyzed for the scale effects. Unconsolidated sediments were subdivided by hydrofacies based on depositional origin; consolidated rocks were subdivided based on stratigraphy (with the exception of the reef carbonate). Ideally, consolidated rocks should also be subdivided into hydrofacies and be analyzed on a facies level. However, available data
were generally insufficient to allow separation by hydrofacies except on the laboratory scale.

A distinction was sought between homogeneous media and heterogeneous media. A quantitative measure of heterogeneity relevant to ground water flow is difficult to choose because the standard deviation and variance of hydraulic conductivity data are known to be a function of scale of measurement, with standard deviation being largest on the laboratory scale and decreasing toward the field and regional scale (Margolin et al. 1998). In addition, all hydraulic conductivity tests in a data set have to be conducted at the same scale of measurement so as not to induce an increased variance in the test population, which is derived from the scaling behavior of K. This study used the variance of K values obtained from permeameter tests as the most appropriate measure for heterogeneity because (1) all K determinations from one data set are conducted on the same scale of measurement (standardized sample that fits into a permeameter), and (2) the variance obtained is a conservative estimate because the largest K variations are to be expected on a laboratory scale. If several permeameter data sets were available, the variance of K was determined from each of the data sets and an average was calculated. If the variance of K measurements was below 0.5 log cycles, then a geological unit was considered homogeneous; otherwise, a geological unit was considered heterogeneous.

Error and Uncertainties

Several errors are introduced by analyzing data as described. The limited number of measurements introduces uncertainty about the slope of the regression line (the scaling effect), the position of the upper bound (the point beyond which the scaling effect ceases), and the y-intercept of the regression line (e, hydraulic conductivity at a rock volume of 1 m³). The first uncertainty was quantified by calculating the 95% confidence envelope about the regression slope. The uncertainty about c can only be estimated, because data generally don’t allow the calculation of a standard deviation (or 95% confidence interval) of hydraulic conductivity at a rock volume of 1 m³ (the standard deviation is generally an unknown function of scale of measurement as previously discussed). Thus, it is possible to estimate c only from the 95% confidence envelope about the regression line. Determining the relationship of the standard deviation of any type of hydraulic test to scale of measurement is beyond the scope of this study. The uncertainty about the position of the upper bound was expressed by giving the position as a range value. This uncertainty might also influence the slope of the regression if measurements beyond the limit of the scale effect are included. To minimize this possibility, only points that were clearly below a possible upper bound were used in the regression analysis.

Another type of error is introduced by the uncertainty in selecting the controlling type of fluid flow. The amount of uncertainty depends on the quality of geological and hydrological data available. The more descriptive core analyses or boring logs are, the more certain is the separation of an aquifer into distinct geological units. The more aquifer tests conducted in a unit, the more certain is the determination of the controlling fluid flow type.

A further uncertainty occurs if hydraulic conductivity measurements are not all taken from the same sampling area. Piezometer scale measurements are often made in a different locality than aquifer pumping tests, in which case it is not possible to separate spatial variation of K from any difference induced by scale of measurement. This problem has been overcome by working only with large data sets that include K measurements at all scales from within the same locality. Another uncertainty is that hydraulic conductivity values may vary as a function of depth as previously reported in some fractured rocks (LeGrand 1954; Summers 1972). Another type of error can occur if vertical hydraulic conductivity values (e.g., laboratory measurements) were adjusted to horizontal K values by using known anisotropy ratios of the geological material and assuming that anisotropy ratios are scale invariant. This latter assumption may or may not be true.

Observations

Scale Behavior of Hydraulic Conductivity in Heterogeneous Media Exhibiting Porous Flow

Media that produced a Theis-type response to a pumping test and had a unimodal hydraulic conductivity distribution and/or had a lack of fractures in the core analysis were categorized as having pore-dominated flow. From the data sets available porous-flow conditions were identified in both consolidated and unconsolidated media. One example of an unconsolidated medium is glacial outwash sediments of southeastern Wisconsin. These sediments are rich in coarse-grained sediment consisting of more than 70% sand and gravel (Milwaukee Metropolitan Sewage District [MMSD] 1981, 1984a, 1984b, 1988; Rodenback 1988; Simpkins 1989). Nevertheless, outwash should be considered heterogeneous because its properties change rapidly both parallel and transverse to the depositional channels. A total of 49 measurements of K have been obtained by a variety of researchers using different methods. For the sake of clarity, in Figure 2 all permeameter and piezometer test results conducted by a single group using a consistent method have been used to calculate a geometric mean K. That, in turn, is plotted against the arithmetic mean of the scale of measurement for the same set. The 95% confidence bars are added on each mean of a single group to quantify its uncertainty.

K increases with scale of measurement for glacial outwash sediments up to the position of an upper bound, after which the medium behaves as a homogeneous medium and K remains con-
Table 2
Relationship of Hydraulic Conductivity to Scale of Measurement in Heterogeneous, Porous-Flow Controlled Geological Media

<table>
<thead>
<tr>
<th>Geological Unit</th>
<th>N°</th>
<th>Exponent m</th>
<th>r°</th>
<th>95% CF</th>
<th>Upper Bound</th>
<th>c°</th>
<th>Geographic Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconsolidated Media</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alluvium</td>
<td>48</td>
<td>0.50</td>
<td>0.47</td>
<td>0.27</td>
<td>&gt;1000</td>
<td>-4.8</td>
<td>Southern Wisconsin</td>
</tr>
<tr>
<td>Glaciolacustrine sediments</td>
<td>49</td>
<td>0.48</td>
<td>0.40</td>
<td>0.31</td>
<td>&gt;200</td>
<td>-5.7</td>
<td>Southern Wisconsin</td>
</tr>
<tr>
<td>Glacial outwash sediments</td>
<td>49</td>
<td>0.53</td>
<td>0.87</td>
<td>0.09</td>
<td>2000-20,000</td>
<td>-4.5</td>
<td>Southern Wisconsin</td>
</tr>
<tr>
<td>Consolidated Media</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edgewood Formation</td>
<td>84</td>
<td>0.49</td>
<td>0.62</td>
<td>0.08</td>
<td>&gt;1</td>
<td>-7.8</td>
<td>Northern Illinois</td>
</tr>
<tr>
<td>Franconia/Lone Rock Formation</td>
<td>22</td>
<td>0.52</td>
<td>0.95</td>
<td>0.07</td>
<td>100-2000</td>
<td>-5.1</td>
<td>Northeastern Illinois and southeastern Wisconsin</td>
</tr>
<tr>
<td>Mayville Formation</td>
<td>104</td>
<td>0.55</td>
<td>0.83</td>
<td>0.08</td>
<td>10,000-100,000</td>
<td>-6.6</td>
<td>Southeastern Wisconsin</td>
</tr>
<tr>
<td>Thiensville Formation</td>
<td>190</td>
<td>0.52</td>
<td>0.79</td>
<td>0.06</td>
<td>1000-20,000</td>
<td>-5.7</td>
<td>Southeastern Wisconsin</td>
</tr>
<tr>
<td>Reef carbonate</td>
<td>65</td>
<td>0.45</td>
<td>0.65</td>
<td>0.13</td>
<td>&gt;100</td>
<td>-5.6</td>
<td>Northeastern Illinois and southeastern Wisconsin</td>
</tr>
</tbody>
</table>

a Total number of hydraulic conductivity measurements.
b Correlation coefficient of the relationship based on all single data points below the identified upper bound.
c 95% confidence interval about the exponent.
d Upper bound of the relationships in m³ of tested material given as range.
e Medium characteristic parameter c given as log value (m/s).

Note: A brief description of each geological unit analyzed and a list of references used as data sources are provided in the Appendix.

Figure 3. Relationship of hydraulic conductivity to scale of measurement in the Thiensville Formation of the carbonate aquifer of southeastern Wisconsin. Permeameter, piezometer and packer tests were plotted as geometric means with 95% confidence intervals; pumping tests, specific capacity tests, and modeling values as single values. Number of observations are given adjacent to means. The regression line is derived from all individual values (n = 190) below the model scale. The 95% confidence interval about the slope is 0.52 ± 0.06, and r is the correlation coefficient.

Figure 4. Relationship of hydraulic conductivity to scale of measurement in the Sinnipee Group of northeastern Illinois and southern Wisconsin. Permeameter and packer tests were plotted as geometric means with 95% confidence intervals; specific capacity tests and modeling values as single values. Notice the large scale packer test within the cloud of the specific capacity tests. Specific capacity tests were derived from two different sources and geographical areas. Number of observations are given adjacent to means. The regression line is derived from all individual values (n = 357) below the model scale. The 95% confidence interval about the slope is 0.77 ± 0.05, and r is the correlation coefficient.

K = c(V)^m

where c is the y-intercept of the regression line, V the volume of geological material tested, and m the scaling exponent (Figure 2). The parameter c equals the hydraulic conductivity value at a sediment volume of 1 m³. Low-conductivity materials such as silts or clays have low c values, while high-conductivity material such as coarse-grained sediments have high c values. Based on a regression analysis the scaling exponent is 0.53 ± 0.09 at a 95% confidence level. The correlation coefficient of the relationship is 0.87, using all of the individual data points below the upper bound. The uncertainty of the upper bound is approximately one order of magnitude, between 2000 and 20,000 m³. The parameter c has a value of 10^{-4.5} as determined by the regression analysis.

A similar analysis was conducted on other porous-flow controlled unconsolidated and consolidated media (Table 2). The expo-
Table 3
Relationship of Hydraulic Conductivity to Scale of Measurement in Heterogeneous Double-Porosity Media

<table>
<thead>
<tr>
<th>Geological Unit</th>
<th>N(^a)</th>
<th>Exponent m</th>
<th>r(^b)</th>
<th>95% CI(^c)</th>
<th>Upper Bound(^d)</th>
<th>c(^e)</th>
<th>Geographic Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconsolidated Media</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quaternary till-rich deposits</td>
<td>264</td>
<td>0.71</td>
<td>0.88</td>
<td>0.05</td>
<td>20-200</td>
<td>-5.8</td>
<td>Southern Wisconsin</td>
</tr>
<tr>
<td>Oak Creek till*</td>
<td>200</td>
<td>0.64</td>
<td>0.62</td>
<td>0.11</td>
<td>20-200</td>
<td>-6.1</td>
<td>Southern Wisconsin</td>
</tr>
<tr>
<td>New Berlin till*</td>
<td>24</td>
<td>0.74</td>
<td>0.85</td>
<td>0.19</td>
<td>&gt;50</td>
<td>-5.6</td>
<td>Southern Wisconsin'</td>
</tr>
<tr>
<td>Clayey beach deposits</td>
<td>133</td>
<td>0.75</td>
<td>0.73</td>
<td>0.12</td>
<td>&gt;20,000</td>
<td>-6.9</td>
<td>Slovakia</td>
</tr>
<tr>
<td>Hron River alluvium</td>
<td>260</td>
<td>0.81</td>
<td>0.69</td>
<td>0.10</td>
<td>&gt;100,000</td>
<td>-6.5</td>
<td>Slovakia</td>
</tr>
<tr>
<td>Ipel River alluvium</td>
<td>61</td>
<td>0.69</td>
<td>0.77</td>
<td>0.14</td>
<td>&gt;20,000</td>
<td>-5.9</td>
<td>Slovakia</td>
</tr>
<tr>
<td>Volcanic deposits</td>
<td>119</td>
<td>0.82</td>
<td>0.71</td>
<td>0.16</td>
<td>100,000-10(^9)</td>
<td>-7.6</td>
<td>Slovakia</td>
</tr>
<tr>
<td>Consolidated Media</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alexandrian series</td>
<td>23</td>
<td>0.59</td>
<td>0.80</td>
<td>0.07</td>
<td>2000-10,000</td>
<td>-5.2</td>
<td>Northeastern Wisconsin</td>
</tr>
<tr>
<td>Burnt Bluff Sequence</td>
<td>145</td>
<td>0.37</td>
<td>0.47</td>
<td>0.40</td>
<td>&gt;2000</td>
<td>-4.9</td>
<td>Northeastern Wisconsin</td>
</tr>
<tr>
<td>Engadine Formation(^f)</td>
<td>37</td>
<td>0.76</td>
<td>0.82</td>
<td>0.17</td>
<td>200-2000</td>
<td>-6.3</td>
<td>Northeastern Wisconsin</td>
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<tr>
<td>Manistique Formation</td>
<td>97</td>
<td>0.66</td>
<td>0.69</td>
<td>0.11</td>
<td>&gt;500</td>
<td>-5.6</td>
<td>Northeastern Wisconsin</td>
</tr>
<tr>
<td>Prairie du Chien Group</td>
<td>15</td>
<td>0.83</td>
<td>0.96</td>
<td>0.12</td>
<td>&gt;2000</td>
<td>-6.9</td>
<td>Northeastern Illinois and southern Wisconsin</td>
</tr>
<tr>
<td>Romeo Beds</td>
<td>65</td>
<td>0.68</td>
<td>0.82</td>
<td>0.12</td>
<td>&gt;1000-10,000</td>
<td>-7.0</td>
<td>Southeastern Wisconsin</td>
</tr>
<tr>
<td>Sinnipee Group</td>
<td>357</td>
<td>0.77</td>
<td>0.87</td>
<td>0.05</td>
<td>2000-20,000</td>
<td>-6.6</td>
<td>Northeastern Illinois and southern Wisconsin</td>
</tr>
<tr>
<td>Bedded limestone, ox2</td>
<td>106</td>
<td>0.55</td>
<td>0.69</td>
<td>0.11</td>
<td>&gt;1000</td>
<td>-6.2</td>
<td>Swabian Alb, southwestern Germany</td>
</tr>
<tr>
<td>Mesozoic carbonates</td>
<td>11</td>
<td>0.81</td>
<td>0.94</td>
<td>0.21</td>
<td>&gt;200,000</td>
<td>-8.4</td>
<td>Slovakia</td>
</tr>
</tbody>
</table>

\(^a\)Total number of hydraulic conductivity measurements.
\(^b\)Correlation coefficient of the relationship based on all single data points below the identified upper bound.
\(^c\)95% confidence interval about the exponent.
\(^d\)Upper bound of the relationships in m\(^2\) of tested material given as range.
\(^e\)Medium-characteristic parameter c given as log value (m/s).
\(^f\)Geological unit with some conduit flow.

*Oak Creek till and New Berlin till are subdivisions of the Quaternary till-rich deposits.

Note: A brief description of each geological unit analyzed and a list of references used as data sources are provided in the Appendix.

Figure 5. Relationship of hydraulic conductivity to scale of measurement in volcanic deposits of Slovakia. The regression line is derived from all values (n = 119) below an upper bound of 120,000 m\(^3\). The 95% confidence interval about the slope is 0.82 ± 0.16, and r is the correlation coefficient.

The relationship for one consolidated medium, the Thiensville Formation, is shown in Figure 3. The exponent m of other consolidated porous-flow controlled media is close to the exponent determined for the Thiensville Formation and the glacial outwash sediments. Values of c for porous-flow controlled media range from 10\(^{-7.8}\) to 10\(^{-4.5}\). The location of the upper bound ranges from a small to a large field scale, although large-scale data are insufficient to establish the position of the upper bound for some geological units.

Scale Dependency of Hydraulic Conductivity in Heterogeneous Media Exhibiting Both Porous and Fracture Flow

Commonly both porous and fracture flow are significant in heterogeneous media. Fracture flow is a common occurrence in subsurface media and is reported in nearly every type of geologic medium (Snow 1969; Summers 1972; Gale 1982; Prudic 1982; D'Astous et al. 1989). Media in which fracture flow is usually negligible compared to porous flow are coarse unconsolidated sediments and sandstones. Media in which porous flow is usually negligible are clay, shale, and intrusive rocks. In all other types of rocks, significant flow can be transmitted by both the pores and the fractures (or conduits) within a medium. These media are called double-porosity media.

An example of a double-porosity medium is the Sinnipee Group of northeastern Illinois and southern Wisconsin. The Sinnipee Group consists of high-magnesium limestones that commonly

909
contain variable amounts of shale and have facies changes. The lowermost portion of the group is a fine-to-medium-grained, medium-bedded to massive dolomite that grades upward into a fine-grained limestone with thin shale beds in the upper central portion of the rock unit (Young 1992). High-magnesium limestone and interbedded limestone and dolomite compose the uppermost portion of the Sinnipee Group.

The relationship of hydraulic conductivity with scale of measurement for the Sinnipee Group is shown in Figure 4. For the sake of clarity, the means of single groups of permeameter or packer tests using a consistent method are plotted with 95% confidence bars. The position of the upper bound is approximately located at a rock volume of about 10,000 m³. The relationship can be described with Equation 1. The scaling exponent is 0.77 ± 0.05 at a 95% confidence level (Figure 4; Table 3).

Volcanic deposits from central Slovakia are an example of a double-porosity medium that primarily consists of unconsolidated sediments but also includes consolidated subunits. The relationship of hydraulic conductivity to scale of measurement for these volcanic deposits is not based on different types of aquifer tests (as for the other geological media shown) but only on pumping tests. Figure 5 shows 119 pumping tests plotted as single data points. The pumping tests were conducted under a wide range of conditions from small-field scale tests (small pumping rate, short duration of test) to large-field scale tests (high-capacity pumping tests for long time periods), thus allowing analysis of the scaling behavior of K for a large range of scales. The many outliers and the large spread in data are probably due to the inclusion of both pyroclastic and extrusive flows in the same unit. An upper bound of the relationship appears to be likely at an affected rock volume of approximately 120,000 m³. Below it, the scaling exponent of the relationship is 0.82 ± 0.16 at a 95% confidence level. The correlation coefficient of the relationship is 0.71 reflecting the spread in the data set (Figure 5).

The relationship of hydraulic conductivity to scale of measurement for other double-porosity media is given in Table 3. The scaling exponent of these units ranges between 0.55 and 0.83 with the exception of the Burnt Bluff Sequence. The regression analysis produces an exponent of 0.37 for the data from the Burnt Bluff Sequence, which is likely due to the limited data set on the aquifer test scale (the correlation coefficient r of this relation is 0.46; the 95% confidence interval is ±0.40). The parameter c for the double-porosity media ranges from 10⁻⁴⁻¹ to 10⁻⁴⁻⁹. The upper bound of the double-porosity media is generally located on the field scale; however, for some media only a lower limit of the upper bound could be established due to a lack of large-scale data.

![Image](image-url)

**Figure 6.** Relationship of hydraulic conductivity to scale of measurement in the Racine Formation of the carbonate aquifer of southeastern Wisconsin. Permeameter, piezometer, packer, and passive infiltration tests were plotted as geometric means with 95% confidence intervals; pumping tests and specific capacity data as single values. Number of observations are given adjacent to means. Passive infiltration tests are derived from the infiltration of Lake Michigan water into the Racine Formation due to the construction of a sewage tunnel. The regression line is derived from all individual values (n = 160) below the infiltration scale. The 95% confidence interval about the slope is 0.91 ± 0.06, and r is the correlation coefficient.

<table>
<thead>
<tr>
<th>Geological Unit</th>
<th>N</th>
<th>Exponent m</th>
<th>r²</th>
<th>95% CI</th>
<th>Upper Bound</th>
<th>c</th>
<th>Geographic Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Devonian Shale</td>
<td>89</td>
<td>1.08</td>
<td>0.53</td>
<td>0.80</td>
<td>0.01 - 10⁻⁷</td>
<td>-14.3</td>
<td>West Virginia and Ohio</td>
</tr>
<tr>
<td>Joliet Formation</td>
<td>33</td>
<td>1.00</td>
<td>0.89</td>
<td>0.18</td>
<td>&gt;10</td>
<td>-6.4</td>
<td>Northeastern Illinois and southeastern Wisconsin</td>
</tr>
<tr>
<td>Kankakee Formation</td>
<td>49</td>
<td>1.13</td>
<td>0.94</td>
<td>0.11</td>
<td>&gt;1</td>
<td>-6.2</td>
<td>Northeastern Illinois</td>
</tr>
<tr>
<td>Lower Carbonate Unit</td>
<td>10</td>
<td>0.82</td>
<td>0.94</td>
<td>0.30</td>
<td>&gt;10⁵</td>
<td>-8.5</td>
<td>Nevada</td>
</tr>
<tr>
<td>Maquoketa Formation</td>
<td>93</td>
<td>0.80</td>
<td>0.88</td>
<td>0.09</td>
<td>&gt;2000</td>
<td>-7.3</td>
<td>Northeastern Illinois and southern Wisconsin</td>
</tr>
<tr>
<td>Racine Formation</td>
<td>160</td>
<td>0.91</td>
<td>0.93</td>
<td>0.06</td>
<td>1000-10,000</td>
<td>-7.3</td>
<td>Southeastern Wisconsin</td>
</tr>
<tr>
<td>Weathered zone</td>
<td>88</td>
<td>0.97</td>
<td>0.90</td>
<td>0.11</td>
<td>1000-2000</td>
<td>-6.2</td>
<td>Southeastern Wisconsin</td>
</tr>
</tbody>
</table>

*Total number of hydraulic conductivity measurements.

*Correlation coefficient of the relationship based on all single data points below the identified upper bound.

*95% confidence interval about the exponent.

*Upper bound of the relationships in m³ of tested material given as range.

*Medium - characteristic parameter c given as logarithm (m/s).

*Geological unit with some conduit flow.

Note: A brief description of each geological unit analyzed and a list of references used as data sources are provided in the Appendix.
Table 5
Relationship of Hydraulic Conductivity to Scale of Measurement in Heterogeneous Conduit Flow Media

<table>
<thead>
<tr>
<th>Geological Unit</th>
<th>N</th>
<th>Exponent m</th>
<th>r</th>
<th>95% CI</th>
<th>Upper Bound</th>
<th>c</th>
<th>Geographic Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aguada Limestone</td>
<td>19</td>
<td>1.01</td>
<td>0.91</td>
<td>0.34</td>
<td>$10^5$-$10^6$</td>
<td>-8.7</td>
<td>Puerto Rico</td>
</tr>
<tr>
<td>Floridan Aquifer</td>
<td>70</td>
<td>1.11</td>
<td>0.67</td>
<td>0.49</td>
<td>$1000$-$10,000$</td>
<td>-5.9</td>
<td>Southwestern Florida</td>
</tr>
<tr>
<td>Massive Limestone k12/3</td>
<td>94</td>
<td>0.80</td>
<td>0.71</td>
<td>0.16</td>
<td>$&gt;10^3$</td>
<td>-8.1</td>
<td>Swabian Alb, southwestern Germany</td>
</tr>
<tr>
<td>Tertiary Limestone Aquifer</td>
<td>36</td>
<td>0.67</td>
<td>0.89</td>
<td>0.12</td>
<td>$20,000$-$120,000$</td>
<td>-5.7</td>
<td>Georgia, U.S.A.</td>
</tr>
</tbody>
</table>

*Total number of hydraulic conductivity measurements.
Correlation coefficient of the relationship based on all single data points below the identified upper bound.
95% confidence interval about the exponent.
Upper bound of the relationships in m$^3$ of tested material given as range.
Median characteristic parameter c given as log value (m/s).

Note: A brief description of each geological unit analyzed and a list of references used as data sources are provided in the Appendix.

Figure 7. Scale invariance of hydraulic conductivity with scale of measurement in a very homogeneous medium: the St. Peter Sandstone of the Upper Midwest, U.S.A. In Illinois and especially Chicago, the St. Peter Sandstone is more rich in silt leading to a subdivision of the formation into the Chicago subfacies. Means are plotted that were obtained from different types of tests and sampling areas. Letters indicate different data sources; n = number of observations. For the sake of clarity, error bars are not included. Full citations of the references are given in the reference section of this paper.

Scale Dependency of Hydraulic Conductivity in Heterogeneous Media Exhibiting Fracture Flow
Geological units in which the rock matrix is so tight that only an insignificant amount of flow is transmitted by the matrix are categorized here as fracture-flow media. These media fractures serve as both primary fluid pathways and storage locations. A geological unit that fits this description is the Racine Formation of the carbonate aquifer of southeastern Wisconsin. The Racine Formation consists primarily of a white to light gray, finely crystalline dolomite. Vugs are poorly connected with a volumetric percentage of less than
Table 6
Relationship of Hydraulic Conductivity to Scale of Measurement in Homogeneous Media

<table>
<thead>
<tr>
<th>Geological Unit</th>
<th>N^a</th>
<th>Exponent m</th>
<th>Upper Bound(^b)</th>
<th>Geographic Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brent Group</td>
<td>308</td>
<td>&gt;0.19</td>
<td>&lt;10(^{-5})</td>
<td>North Atlantic, near Norway</td>
</tr>
<tr>
<td>Dresbach Formation</td>
<td>40</td>
<td>0.65*</td>
<td>10(^{-3})</td>
<td>Upper Midwest of U.S.A.</td>
</tr>
<tr>
<td>Mt. Simon Formation</td>
<td>151</td>
<td>NA</td>
<td>&lt;10(^{-4})</td>
<td>Upper Midwest of U.S.A.</td>
</tr>
<tr>
<td>St. Peter Formation</td>
<td>305</td>
<td>NA</td>
<td>&lt;10(^{-3})</td>
<td>Upper Midwest of U.S.A.</td>
</tr>
</tbody>
</table>

\(^a\) Total number of hydraulic conductivity measurements.
\(^b\) Upper bound of the relationships in m/2 of tested material given as approximate value.
NA = not available, scaling exponent could not be determined from available data.
*Based on a narrow range of scales, 18 permeability tests located below the upper bound (r = 0.23).
Note: A brief description of each geological unit analyzed and a list of references used as data sources are provided in the Appendix.

Table 7
Relationship of Hydraulic Conductivity to Scale of Measurement in Geological Media Dominated by Various Flow Types

<table>
<thead>
<tr>
<th>Type of Medium</th>
<th>N^a</th>
<th>Exponent m</th>
<th>95% CI(^b)</th>
<th>Upper Bound(^c)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous media</td>
<td>4</td>
<td>ND</td>
<td>&gt;0.19 - 0.65(^#)</td>
<td>N/A</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Heterogeneous porous flow media</td>
<td>8</td>
<td>0.51</td>
<td>0.45 - 0.55</td>
<td>0.02</td>
<td>&gt;1 - &gt;100,000</td>
</tr>
<tr>
<td>Double-porosity media</td>
<td>16</td>
<td>0.72</td>
<td>0.55 - 0.83(^*)</td>
<td>0.04</td>
<td>&gt;50 - 10(^b)</td>
</tr>
<tr>
<td>Fracture-flow media</td>
<td>7</td>
<td>0.96</td>
<td>0.80 - 1.13</td>
<td>0.09</td>
<td>&gt;1 - 20,000</td>
</tr>
<tr>
<td>Conduit flow media</td>
<td>4</td>
<td>0.90</td>
<td>0.67 - 1.11</td>
<td>0.20</td>
<td>&gt;1000 - &gt;10(^b)</td>
</tr>
</tbody>
</table>

\(^a\) Number of different geological media evaluated.
\(^b\) 95\% confidence interval about the exponent.
\(^c\) Upper bound of the relationships in m/2 of tested material.
\(^#\) Medium-characteristic parameter \(c\) given as log value (m/s).
ND = Not determined, data set was insufficient to calculate parameter.
N/A = Not applicable, an exponent could be determined for only two of the four media.
\(^*\) Based on a very limited scale range.
\(^+\) After removal of outlier at a 99\% significance level.
\(^\) Not including a statistical outlier (Devonian Shale, -14.3) discussed in text.

1\% and a diameter smaller than 0.25 mm (Schulze-Makuch 1996) with the exception of a more porous zone at the base of the formation referred to as the Romeo Beds. The Romeo Beds are a zone of predominantly packstones with vuggy and fossil moldic pores, some enlarged by dissolution. While pumping tests in the rest of the Racine Formation typically exhibit fracture-flow responses (linear response on a log-log plot), some test responses in the Romeo Beds indicate porous flow (Thesis-type response). However, based on an analysis of 2951 m of core, the Romeo Beds comprise only about 10\% of the total Racine Formation (Schulze-Makuch 1996).

Fractures in the Racine Formation are locally highly transmissive. Based on more than 500 aperture measurements, the mean aperture width of vertically oriented fractures (joints) was measured to be 10.5 \times 10^{-4} m for open joints and 15.7 \times 10^{-4} m for partially filled joints. Horizontally oriented fractures (bedding plane breaks) in the Racine Formation had a mean aperture width of 16.0 \times 10^{-4} m for open fractures and 45.6 \times 10^{-4} m for partially filled fractures (Carlson 1997). The most common filling materials are clay, shale, or calcite.

The relationship of hydraulic conductivity to scale of measurement can be described by Equation 1 and is shown in Figure 6. The scaling exponent of the relationship is 0.91 \pm 0.06 at a 95\% confidence level. The relationship of hydraulic conductivity to scale of measurement for some other fracture-flow controlled geological media is shown in Table 4. Scaling exponents for these media range from 0.80 to 1.13, and values for c range from 10^{-14.3} to 10^{-6.2}. The c value of 10^{-14.3} for Devonian Shale is a statistical outlier. However, this value may not be a geological outlier because of the makeup of the test population. Shales constitute a major portion of fracture-flow controlled media and are known to have extremely low-hydraulic conductivity values (Freeze and Cherry 1979). In this test population, the Devonian Shale is the only "true" shale, while all other units are carbonate units with low to moderate transmissivities. A hydraulic conductivity value of 10^{-14.3} m/s for a shale at a rock volume of 1 m^3 is not entirely unreasonable. The upper bound of the fracture-flow media seems to be on the large field scale, although data for many of these media are not available on a large enough scale to determine the boundary's location.

Scale Dependency of Hydraulic Conductivity in Heterogeneous Media Exhibiting Conduit Flow

Conduit flow is a common occurrence in carbonate rocks that are exposed to the surface or near surface. Dissolution processes enhance fractures to solution conduits and result in turbulent flow
Table 8
Values of c for Various Types of Geological Media

<table>
<thead>
<tr>
<th>Type of Medium</th>
<th>Unconsolidated Sediments</th>
<th>Consolidated Rocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meana 95% CIb</td>
<td>Meana 95% CIb</td>
</tr>
<tr>
<td>Heterogeneous porous</td>
<td>5.0 0.06</td>
<td>6.2 0.79</td>
</tr>
<tr>
<td>flow media</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double-porosity media</td>
<td>6.5 0.53</td>
<td>6.5 0.68</td>
</tr>
<tr>
<td>Fracture-flow media</td>
<td>NA NA</td>
<td>7.0 0.90</td>
</tr>
<tr>
<td>Conduit flow media</td>
<td>NA NA</td>
<td>7.1 1.49</td>
</tr>
</tbody>
</table>

*aArithmetic mean of c values presented in Tables 2 through 6, given as log-value (m/s).  
b95% confidence interval about c in log-cycles.
NA = not available.
*Not including a statistical outlier (Devonian Shale, −14.3) discussed in text.

within the solution conduits. Table 5 provides the relationship of hydraulic conductivity to scale of measurement for four conduit flow media. Scaling exponents vary between 0.67 and 1.11, and c values range from $10^{-8.7}$ to $10^{-5.7}$. An upper bound at the regional scale appears to be typical for conduit-flow rocks. Mature karst media such as the massive limestones of the Swabian Alb may have no upper bound.

Scale Behavior of Hydraulic Conductivity in a Homogeneous Medium

If it is true that intrinsic heterogeneity in a medium causes the scale effects described, then no scale variations should be observed in a perfectly homogeneous medium. Unfortunately such a perfectly homogeneous medium does not exist in nature. One unit, however, that is close to the ideal homogeneous medium is the St. Peter Sandstone of the Upper Midwest of the United States. The St. Peter Sandstone is a pure quartzose, fine- to medium-grained sandstone. The sand grains are usually well rounded and consist of more than 99% quartz in its dominant sedimentological facies, the Tonti Member (Young 1992). The average variance of K values obtained from permeameter tests is extremely small (0.07 log cycles), which qualifies it as a homogeneous medium as defined previously.

Hydraulic conductivity measurements from various areas in the Upper Midwest were obtained from various authors and a scale of measurement calculated. For the sake of clarity geometric means of a single group using a consistent method are plotted in Figure 7. No variation of K with scale of measurement can be observed, except perhaps at the laboratory scale. Hydraulic conductivity values are lower in the Chicago, Illinois area, but this is likely caused by a facies change and these values likewise appear to be scale invariant (Figure 7). Young (1992) observed that the St. Peter Sandstone is homogeneous over large areas but becomes more silty in Illinois, especially in the Chicago area. The relationship of K to scale of measurement for some other homogeneous media, all of them sandstones, is shown in Table 6. The position of the upper bound appears to be on the permeameter scale for each of the sandstones.

Discussion

The relation of hydraulic conductivity to scale of measurement for 39 geological media is summarized in Table 7. These media were subdivided into five categories: homogeneous, heterogeneous porous-flow controlled, double-porosity, fracture-flow controlled, and conduit-flow controlled. Geological media classified as homogeneous do not exhibit variation of K on a field or regional scale. On the other hand, all heterogeneous media analyzed exhibit a hydraulic conductivity increase with scale of measurement. The empirical relation is in the form of Equation 1:

$$K = c (V)^m$$

where K [LT⁻¹] is hydraulic conductivity, c [L¹T⁻₃m⁻¹] is a coefficient characteristic of the medium, (V) [L³] is the volume of material tested, and m the exponent of the relationship.

This relationship is valid up to the position of an upper bound value, after which K remains constant with scale. The scaling exponent m was 0.51 ± 0.02 for the heterogeneous porous-flow medium (n = 8), 0.96 ± 0.09 for fracture-flow controlled media (n = 7), 0.90 ± 0.20 for conduit-flow media (n = 4), and ranged from 0.55 to 0.83 for double-porosity media (n = 15 plus 1 outlier). The scaling exponent was independent of whether a geological unit was under confined or confined conditions.

Media with fractures and solution conduits have larger scaling exponents than media that are solely controlled by porous flow. The more predominant fracture or conduit flow is relative to porous flow, the larger is the scaling exponent. Fractures and solution conduits in a medium can present more preferred pathways than the porous matrix, allowing a fluid to travel faster through the subsurface. This, in turn, results in larger K values for these rocks when a larger volume of subsurface is tested on a field or regional scale translating into a larger scaling exponent. A good example of this effect is the Racine Formation, which was discussed previously. The Racine Formation is a fracture-flow controlled geological unit, but it includes the Romeo Beds, a subunit with significant porous-medium flow. The Racine Formation as a whole unit has a scaling exponent of 0.91 ± 0.06, while the Romeo Beds have a scaling exponent of 0.68 ± 0.12. If data derived from the Romeo Beds are excluded from the analyses, the exponent of the Racine Formation increases to a value of 0.97 ± 0.13. Thus, exclusion of the porous-flow subunit of the Racine Formation increases the scaling slope to nearly one.

The parameter c, the y-intercept of Equation 1, is a measure of the hydraulic conductivity at the scale of 1 m³ for each medium studied. Because it is medium-dependent, no general pattern appears in its distribution (Tables 2 through 6). Instead, it exhibits a wide range of values in the results categorized by flow type (Table 8). However, c values appear to decrease for media with larger fracture and conduit influence although this relation is statistically significant only for unconsolidated sediments. Also, c values differ between consolidated media and unconsolidated media, but only for porous media. The c value appears to be related to pore size and pore interconnectivity. For example, c is larger for the Thiensville Formation than for the Romeo Beds and the Mayville Formation (Table 2 and 3; Appendix). These geological units are all part of the carbonate aquifer of southeastern Wisconsin. The Thiensville Formation is predominantly a medium crystalline rock (Folk 1965) with vugs of a diameter of up to 1 cm or more. Based on core observations, pores in the Thiensville Formation have a high degree of interconnectivity. Vugs in the Romeo Beds are only 0.5 mm in diameter on average and are generally moderately connected in an otherwise tight matrix. The Mayville Formation has medium- to coarse-grained crystalline areas similar to parts of the Thiensville Formation, but these zones are embedded within a finer matrix with a low degree of interconnectivity (Schulze-Makuch 1996). The relationship of c
values to pore diameter and degree of interconnectivity is also supported by the larger c values observed in sand- and gravel-rich sediments (glacial outwash) with larger pore sizes and a higher degree of interconnectivity than in the more clay- and silt-rich alluvial and lacustrine deposits of southern Wisconsin (Table 2; MMSSD, 1981, 1984a, 1984b, 1984c, 1988). In a fractured medium the parameter c is likely related to the fracture aperture of the medium and fracture interconnectivity.

The upper bound represents the scale above which a medium can be described as quasi-homogeneous. The upper bound depends on the degree of heterogeneity present in a medium. In a perfectly homogeneous medium, all observations will be at the upper bound and K is invariant with scale. In a very homogeneous medium, such as the St. Peter Sandstone (Figure 7), the upper bound is at the scale of the permeability tests or somewhat below. In heterogeneous media the upper bound is located at a field scale. In conduit-flow media, the upper bound is sometimes observed at the regional scale. In other cases, no bound has been found within the investigated range of scales (massive limestones of the Swabian Alb, southwestern Germany). Mature karst media are highly heterogeneous at multiple scales, (1) because of the variable types of heterogeneities present (vugs, intercrystalline pores, fractures, channel conduits) that might allow all types of fluid flow (porous, fracture, and conduit flow) to play a significant role; and (2) because of the presence of channel conduits and other cavities that may inhibit the notion of a quasi-homogeneous medium at any scale.

Conclusions

Evidence collected from 39 geological media demonstrates that the relationship of hydraulic conductivity (K) to scale of measurement is a function of (1) the type of fluid flow present in a medium, and (2) the degree of heterogeneity in the medium. In heterogeneous porous media, K increases by half an order of magnitude with each order of magnitude increase in scale of measurement (using volume of tested material as the scale measure). In fracture and conduit flow media, K increases by about one order of magnitude with each order of magnitude increase in scale of measurement. In double-porosity media with an equal proportion of porous flow and fracture/conduit flow K increases by about 0.75 order of magnitude with each order of magnitude increase in scale of measurement. The more dominant fracture or conduit flow is relative to porous flow in a double-porosity medium, the closer the scaling behavior of K resembles that of a fracture/conduit flow media. The scaling effect terminates at the laboratory scale for homogeneous sandstones and at the field or regional scale for heterogeneous media. It may not cease at all for mature karst media. We believe that the relationships presented can be used to better predict hydraulic conductivity values on the field and regional scale for units from which only lab scale measurements have been made.

Appendix

A brief description of the geological media analyzed is given in the appendix. This background information is subdivided into porous flow media, double-porosity media, fracture-flow media, conduit-flow media, and near-homogeneous geological media. Geological media within each subdivision are listed as in Tables 2 through 6.

I. Porous-Flow Media

A. Unconsolidated Sediments


2. Glaciolacustrine sediments of southern Wisconsin. Stratified clay, silt and sand in a glaciolacustrine environment. Sand content varies from 31% to 67% (MMSSD 1981, 1984b, 1988). Based on the studies by Simpkins (1989) and Rodenback (1988), silt content ranges between 50% and 55% and clay content between 10% and 15%.


B. Consolidated Rocks

1. Edgewood Formation of northern Illinois. The Edgewood Formation is a fine- to medium-grained dolomite with thin to medium thick beds. Within this unit are both chert beds and chert nodules up to 10 cm thick and some thin dolomitic shale beds. In general, the dolomite is composed of grains 0.02 to 0.05 mm in diameter. The few scattered quartz grains present are 0.01 to 0.05 mm in diameter (Harza Engineering Co. 1976).

2. Franconia/Lone Rock Formation of northeastern Illinois and southern Wisconsin. The Franconia Formation is a complex of poorly sorted, glauconitic fine-grained clastics and dolomites in the Upper Midwest (Young 1992). The Franconia Formation is referred to as Lone Rock Formation in Wisconsin and Iowa (Cherkauer 1997; Harza Engineering Co. 1976; Popadopolus et al. 1969).

3. Mayville Formation of southeastern Wisconsin. The Mayville Formation is a vuggy and moldic crystallized dolomite that is typically fully dolomitized and commonly consists of two phases of crystallization. Fine to medium or medium crystallized yellowish areas (probably burrows) are embedded in the finer crystallized whitish matrix (Schulze-Makuch 1996). Interconnectivity of the coarser crystalline areas varies greatly but is generally modest. K was found to correlate (with a correlation coefficient of 0.58) to the estimated percentage of coarser crystalline areas (>0.25 mm crystal size) based on permeameter tests and core analysis conducted on 11 samples (Schulze-Makuch 1996). Pumping tests responses in the Mayville Formation were predominantly Theis-type responses; however, fracture responses in the drawdown record were sometimes observed early (Cherkauer and Mulkic 1992; Limmenansons 1995; Mequon Engineering Department 1987–1993; MMSSD 1984a, 1984b, 1984c, 1988; Mueller 1992; Mulvey 1995; Nader 1990; Rovey 1990; Wisconsin Geological and Natural History Survey [WGNHS] 1936-1988 [Ozaukee County]).

4. Thiensville Formation of southeastern Wisconsin. The Thiensville Formation is the most transmissive unit within the carbonate aquifer of southeastern Wisconsin and was intensively tested in conjunction with the construction of the Milwaukee Deep Tunnels through Silurian and Devonian dolomites. The Thiensville Formation is a Devonian dolomite with a facies distribution of 40% packstone/grainstone, 30% mudstone, 15% grainstone with large vugs enhanced by dissolution, and 15% poorly consolidated calcium-rich breccia. Therefore, the Thiensville Formation is considered a hetero-
geneous, grainstone-dominated material that includes portions of finer-grained material (variance of K measurements = 2.37 log-cycles). Despite the heterogeneity, the Thiensville Formation displays standard Theis-like responses to single and multiple well pumping tests in all test responses analyzed, indicating that the formation behaves as a porous medium. In addition, packer tests conducted through the Thiensville Formation produced a unimodal K distribution (Rovey 1990), another indication of a porous-flow controlled medium (Clite 1992; Cherkerau and Mikulic 1992; Linnemanstons 1995; Meeuwen Engineering Department 1987–1993; MMSD 1984a, 1984b, 1984c, 1988; Mueller 1992;Mulvey 1995; Nader 1990; Schulze-Makuch 1996; WGNHS 1936–1988 [southern Ozaukee County]).

5. Reef carbonate of northeastern Illinois and southeastern Wisconsin. Typically a fossiliferous boundstone. However, core samples obtained from the Milwaukee Metropolitan Sewage District also included wackestones that were fine to medium crystalline (Schulze-Makuch 1996). Secondary porosity such as framework porosity and vuggy porosity is common in the reef carbonate, but fracture porosity also occurs. Available pumping test indicated Theis-type (porous-flow) responses (Harza Engineering Co. 1976; MMSD 1984a, 1984b, 1988).

II. Double-Porosity Media

A. Unconsolidated Sediments

1. Quaternary till-rich deposits of southern Wisconsin. Unconsolidated sediments with a silt and clay content ranging from 40% to 80% (MMSD 1981, 1984a, 1984b, 1988; Rodenback 1988; Simpkins 1989). Fracture transmission of flow is prevalent in the silty clayey till, while porous flow is prevalent in the coarser grained intertill deposits. The influence of fractures on these till-rich deposits is well documented. Both Connell (1984) and Fleming (1986) observed systematic pattern of joints and fractures within till-deposits of southeastern Wisconsin at more than 15 sites. Connell (1984) pointed out that the joint spacing in these deposits ranged from 2 to 14 cm. Simpkins (1989) observed that fractures common in the upper portion of the till increased the average hydraulic conductivity by a factor of about 20, compared to till located stratigraphically at a lower interval that is less fractured. Taylor and Fleming (1988) observed that the glacial till deposits have a significant anisotropy in electrical resistivity measurements as a possible result of joint-fracture influence. Carlson (1997) showed a consistent anisotrophy of electrical resistivity for 20 sites in southeastern Wisconsin, which is roughly parallel with existing joint sets (Gonthier 1975).

2. Oak Creek till of southern Wisconsin. Predominantly silty clayey till with a silt and clay content of above 50% (subunit of Quaternary till-rich deposits) (Carlson 1997; Gonthier 1975; Rodenback 1988; Simpkins 1989).

3. New Berlin till of southern Wisconsin. A till with an unusually high sand and gravel content of up to 75% (subunit of Quaternary till-rich deposits) (Carlson 1997; Gonthier 1975; Simpkins 1989).

4. Clayey beach deposits of Slovakia. Heterogeneous Miocene and Pliocene sandy deposits. These sediments range in grain size from clay to sand. Sands are present as extensive lenses interbedded within clay deposits (Benkova-Dulovicova 1997).

5. Hron River alluvium of Slovakia. The gravels, sandy gravels, and clayey gravels can reach thicknesses of up to 30 m and are usually covered by a loamy layer of 0.1 to 3 m thickness. Hron River gravels are of crystalline rock material (granites and metamorphites) from the upper part of the river basin as well as of andesite, basalt, and rhyolite material from the middle part of the basin. Hydraulic properties in the alluvial sediments were reported to change within short distances (Kertes 1987, Benkova-Dulovicova 1997).

6. Ipel River alluvium of Slovakia. The gravels, sandy gravels, and clayey gravels can reach thicknesses of up to 30 m and are usually covered by a loamy layer of 0.1 to 3 m thickness. Ipel river gravels are more andesitic than Hron gravel (described previously), because crystalline rocks are present only in the uppermost portion of the watershed. Hydraulic properties in the alluvial sediments were reported to change within short distances (Kertes 1987; Benkova-Dulovicova 1997).

7. Volcanic deposits of Slovakia. These deposits are derived from central Slovakian stratovolcanoes in the Upper Miocene – Badenian and Sarmatian period (Hemphillian stages). They produced pyroclastic material that was deposited and redeposited in the shallow sea and lakes in the vicinity of the volcanoes. Grain sizes change horizontally and vertically from bomb and pebble size (lahar flows) to fine-grained and often hydrothermally altered sediment within a few tens of meters. The pyroclastic sediments are interlayered with andesitic lava flows that are primarily located in the upper part of the deposits. Due to the abundance of andesitic lava flows, it was not possible to distinguish the two types of volcanic materials for pumping test analysis, and consequently they were treated as one hydrogeological unit (Benkova-Dulovicova 1997).

B. Consolidated Rocks


5. Prairie du Chien Group of northeastern Illinois and southern Wisconsin. The lower Ordovician Prairie du Chien Group consists primarily of a sandy fine-grained dolomite. Locally, it includes sandstone beds and shale or sandy shale. The base
of the Prairie du Chien Group is commonly a sandy and cherty coarse-grained dolomite (Harza Engineering Co. 1976; Popadopoulos et al. 1969; WGNHS 1936–1988).

6. **Romeo Beds of southeastern Wisconsin.** The Romeo Beds are a more permeable zone located on the base of the tight mudstones of the Racine Formation. The Romeo Beds are predominantly packstones with vuggy and maldic porosity, some enlarged by dissolution. Interconnectivity of vugs and molds depends largely on the area of investigation but is usually modest (Clite 1992; Cherkauer and Mikulic 1992; Linnemanstons 1995; Mequon Engineering Department 1987–1993; MMSD 1984a, 1984b, 1984c, 1988, 1995a, 1995b; Mueller 1992; Mulvey 1995; Nader 1990; Rovey 1990; Schulze-Makuch 1996).

7. **Simmeppe Group of northeastern Illinois and southern Wisconsin.** This geological unit was introduced in the text (Harza Engineering Co. 1976; McLeod 1975; Popadopoulos et al. 1969; WGNHS 1936–1988).


9. **Mesozoic carbonates of Slovakia.** Interlayered limestone and dolomite beds of Middle and Upper Triassic Age that belong to the Central Western Carpathian Group of nappes. Some hydraulic descriptions point out the presence of fracture and karstic flow (Kullman and Petras 1979; Kullman 1990; Benkova-Dulovicova 1997).

### III. Fracture Flow Media

1. **Devonian Shale of the Appalachian Basin.** The Devonian shales of the Appalachian Basin contain coaly organic material and appear either gray or black. Devonian Shale is composed mainly of tiny quartz grains < 0.005 mm diameter with sheets of thin clay flakes (Soeder 1982). Another particles size study (Zielinski and McVey 1982) notes median particle size is 0.0069 ± 0.00141 mm and the grain size distribution is < 2% sand, 73% silt, and 25% clay. Primary pores are typically 5 x 10^{-5} mm in diameter (Soeder 1982). Matrix porosity is typically 1% to 4.5% (Kuuskraa and Wicks 1984; Salamy et al. 1987; Ning et al. 1993), fracture porosity is typically 0.078% to 0.09% (Kuuskraa and Wicks 1984; Salamy et al. 1987; Alam et al. 1982; Kuuskraa and Wicks 1983; Yost et al. 1987).

2. **Joliet Formation of northeastern Illinois and southeastern Wisconsin.** This Silurian dolomite is a cherty, finely crystalline mudstone and is referred to as Joliet Formation in northeastern Illinois. Locally, zones with high maldic porosity occur in the upper part of the Joliet Formation. The Joliet Formation equivalent in eastern Wisconsin are the Waukesha and Brandon Bridge Members of the Manistique Formation (Harza Engineering Co. 1976; Schulze-Makuch 1996).

3. **Kankakee Formation of northeastern Illinois.** This Silurian dolomite is a finely crystalline tight mudstone. It is referred to as the Kankakee Formation in northeastern Illinois and as Byron Formation and Franklin Member of the Manistique Formation in eastern Wisconsin (Harza Engineering Co. 1976).

4. **Lower Carbonate Unit of Nevada.** Limestones and dolomites with some quartzite, siltstone and claystone. Complexly fractured aquifer of Cambro-Ordovician Age that supplies major springs throughout eastern Nevada. Intracrystalline porosity and permeability is negligible. Solution caverns are present locally, but regional ground water movement is controlled by fracture transmissivity (Winograd and Thordarson 1975).

5. **Maquoketa Formation of northeastern Illinois and southern Wisconsin.** A primarily calcereous, silty shale with variable amounts of interbedded dolomite and limestone (Young 1992). The Maquoketa Shale is a regional aquitard (Harza Engineering Co. 1976; WGNHS 1936–1988 [southern Wisconsin]).


7. **Weathered zone of southeastern Wisconsin.** In this study and in previous studies (e.g., Rovey 1990) the upper 3.1 m (10 ft) of the dolomite aquifer of southeastern Wisconsin have been separated from the rest of the aquifer because of its higher permeability. The weathered zone has a significantly higher degree of secondary porosity, fracturing, and solution-enhanced fractures, and also displays the beginning stages of epikarst. Most of the pumping tests in this unit revealed a log-log linear time-drawdown response typical of a fracture-flow controlled medium (Clite 1992; Cherkauer and Mikulic 1992; Mequon Engineering Department 1987–1993; MMSD 1984a, 1984b, 1984c, 1988; Mulvey 1995; Schulze-Makuch 1996; WGNHS 1936–1988 [southern Ozaukee County]).

### IV. Conduit-Flow Media

1. **Aguada Limestone of Puerto Rico.** The Aguada Limestone consists of rubble to finely crystalline limestone alternating with beds of clayey limestone, chalk, and rubbly calcareous claystone (Monroe 1963, in Sepulveda 1996). Hydraulic conductivity data were obtained for this aquifer from the Quebrada Honda Karst Valley and the southern karst uplands (Sepulveda 1996).

2. **Florida Aquifer of southwestern Florida.** The Florida Aquifer consists of Middle Eocene–Lower Miocene granular limestone. The limestone is interbedded with dolomite and clay. The Suwannee Limestone is part of the aquifer and consists of loosely cemented foraminifers (Wolansky and Corral 1985; Barr 1996; Bush and Johnston 1988; Hickey 1982; Klein et al. 1964; Menke et al. 1961; Robinson 1995; Wolansky et al. 1985).

3. **Massive limestone, ki23, of southwestern Germany.** Upper Jurassic micritic limestones in an algal-sponge bioherm facies (Gwinner 1976). High rates of dolomitization and recrystallization resulted in a high percentage of secondary aquifer porosity. The ki23 includes the “Lochfels” facies, which is a cavernous limestone with large vugs and sucrosic dolomite (Sauter 1992; Sauter 1995).

4. **Tertiary Limestone Aquifer of Georgia.** The aquifer comprises consolidated marine and marginal limestone and dolomite and lesser amounts of evaporites, clay, sand, and marl (Krause 1982). The degree of secondary porosity and permeability is extremely variable. Faults, joints, fractures, solution-enhanced fractures, and other conduits are present within the carbonate aquifer. Hayes et al. (1983) pointed out large interconnected solution channels that may account for only a small part of the cross-sectional flow area, but carry a major part of the flow.
Hayes et al. (1983) observed the “waterhammer phenomenon” in aquifer test responses, which suggests that flow occurs primarily in large, solution-derived conduits. The Tertiary Limestone Aquifer is termed Floridan Aquifer in Florida (Matthews and Krause 1984; Randolph et al. 1985).

V. Homogeneous Media

1. **Brent Group of the North Atlantic (near Norway).** The Brent Group is a middle Jurassic sandstone with a thickness of about 250 m and is composed of five formations: Broom, Etive, Ness, Tarbert, and Rannoch (Livera and Gdula 1990). Brent Group sandstones are fine- to coarse-grained sandstones, siltstones, and mudstones including significant thicknesses of coal, up to 10 m in the Ness Formation. The facies are generally marine, deltaic, shoreface, or lagoonal. Coarse facies tend to be sandstones where porosity is about 27% ± 4%; fine-grained facies have a porosity of about 21% ± 7%. The Brent Group sandstones typically have a quartz content of 93%, a feldspar content of 4.6%, and a mica content of 2.4% (Livera and Gdula 1990; Corbett and Jensen 1991).

2. **Dresbach Formation of the Upper Midwest, U.S.A.** The Dresbach Sandstone is generally a medium- to coarse-grained quartzite sandstone (Young 1992). It is locally silty, glauconitic, or dolomitic. In some later classifications this Cambrian sandstone is divided into the Ironont and Galesville Sandstone (Cherkauer 1997; Emmons 1987; Gross 1980; Harza Engineering Co. 1976a; Mandle and Kontis 1992; McLeod 1975; Nicholas et al. 1987; Southeastern Wisconsin Planning Commission 1976; Walton and Csallany 1962).

3. **Mt. Simon Formation of the Upper Midwest, U.S.A.** The Mount Simon Sandstone is generally a medium- to coarse-grained, poorly to moderately sorted, sometimes pebbly quartzose sandstone. Sand grains range in size from fine to very coarse and in degree of roundness from angular to rounded. This Cambrian formation tends to be arkosic, coarse-grained, and locally conglomeratic with igneous pebbles near its base. Lenticular shale commonly occurs in the upper part of the formation (Young 1992; Becker et al. 1978; Emmons 1987; Mandle and Kontis 1992; Southeastern Wisconsin Regional Planning Commission 1976; Nicholas et al. 1987; Vatican 1981).

4. **St. Peter Formation of the Upper Midwest, U.S.A.** This geological unit was introduced in the text. See Figure 7 for data sources.

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References


Cherkauer, D.S. 1997. Personal communication on specific capacity tests and permeameter tests on rocks from the Cottage Grove area, Dane County, Wisconsin. June 10. Professor of Geosciences, University of Wisconsin-Milwaukee.


Sauter, M. 1995. Personal communication on the scale behavior of hydraulic conductivity in the karst aquifer of the Swabian Alb, April 11. Lehrstuhl für Angewandte Geologie, Institut für Geologie und Paläontologie, Universität Tübingen, Germany.


