Scale Dependency of Hydraulic Conductivity Measurements

by Charles W. Rovey II[a] and Douglas S. Cherkauer[b]

Abstract

The hydraulic conductivity of five stratigraphic units in a carbonate aquifer has been measured with slug, pressure, and pumping tests, and with two calibrated digital models. The effective test radii range from less than one to greater than 10,000 meters. On log-log plots hydraulic conductivity increases approximately linearly with test radius to a range between 20 and 220 meters, but thereafter, it is constant with scale.

The increase in magnitude of hydraulic conductivity is similar to scaling effects reported at seven additional sites in a variety of geologic media. Moreover, the increase in magnitude correlates with an increase in variance of log-hydraulic conductivity measured at successively greater separation distances.

The rate of increase in both parameters, and particularly the range, have characteristic values for different pore systems. The larger ranges are consistently present in units with greater secondary porosity. Therefore, scaling effects provide a qualitative measure of the relative importance of secondary and primary permeability, and they can potentially be used to distinguish the dominant type of pore system.

Introduction

Considering the effort devoted to studying scale effects on dispersivity, it seems strange that hydraulic conductivity, a more fundamental parameter, has not been similarly investigated. The lack of attention is even more puzzling, given the numerous anecdotal reports, amounting to common knowledge, that lab tests consistently give hydraulic conductivities less than field tests. A compilation by Herzog and Morse (1984) remains one of the few sources where the scale of measurement was specifically recognized as a factor for these differences.

The relationship between hydraulic conductivity and scale, however, is more complex than a simple correction factor between lab and field measurements. Bredehoeft et al. (1983) compared hydraulic conductivities of a shale as measured by lab, slug, and pumping tests with that from a calibrated digital model. The long-term pumping test in an underlying sandstone had a radius of influence of approximately 10,000 m, and gave a similar value for the overlying shale as the calibrated model value. The lab tests had values approximately one thousandth that of the regional value, while small-scale field measurements, slug tests, had values approximately one tenth of the regional value. Therefore, hydraulic conductivity appears to increase with scale regardless of the method of measurement. Based on field measurements at different scales, we first quantify how hydraulic conductivity varies with test radius for five hydrostratigraphic units within a dolomite aquifer in southeast Wisconsin (Figure 1). These results are then compared with published data of hydraulic conductivity from additional sites in a variety of geologic media and with variograms of log-hydraulic conductivity distribution. The increase in hydraulic conductivity with measurement scale appears to be a general phenomenon which is correlated to an increase in the variance of its distribution.

Previous Work

The most complete report on the scale dependency of hydraulic conductivity measurements is by Bradbury and Muldoon (1990). They measured hydraulic conductivity at scales from approximately 10^{-2} to 10^{4} meters in both glacial outwash and mixed outwash-diamicton (fine-grained glacigenic) sediment. In both media regional estimation methods (pumping tests, digital models) gave hydraulic conductivities approximately three to five times greater than small-scale field measurements (slug tests) and nearly 10 times greater than lab tests (Figure 2a, b). They also noticed that scale effects vary with the nature of heterogeneity. Hydraulic conductivity of outwash sands increases with test radius at a log-log slope of 0.38, whereas the mixed outwash-diamicton increases at a greater slope, 0.92.

Bruner and Lutenegger (in press) and Keller et al. (1986) measured hydraulic conductivity in jointed, clay-rich glacial tills with lab, slug, and pumping tests (Figure 2c, d). The measurement scale is not as accurate in these cases, but using the best estimates, hydraulic conductivity increases with a slope of approximately 1.0 to a range between two and five meters on log-log graphs.

Sauter (1991) investigated a mature karstic limestone (Figure 2e). The rate of increase in hydraulic conductivity (0.66) is intermediate between the jointed tills and the porous outwash, but the most notable difference is the range in scale effect. Hydraulic conductivity increases with mea-

[a]Department of Geography, Geology and Planning, Southwest Missouri State University, 901 S. National Ave., Springfield, Missouri 65804.

[b]Department of Geosciences, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201.

Received December 1993, revised September 1994, accepted October 1994.

surement scale to at least 3200 meters, a much greater distance than the jointed or granular media. Thus, an increase in hydraulic conductivity over many orders of magnitude in test radius appears typical of karst aquifers in contrast with shorter ranges in other media.

In summary, data available to date suggest that different geologic media have characteristic measures of scale effects (slope and range). Further, these measures may be useful in distinguishing different types of flow (i.e. granular, fracture, or conduit) in cases where it is unknown. In the remainder of this paper we test this hypothesis, first with data from a carbonate aquifer comprised of numerous stratigraphic units (Figure 3a), and then with values gathered from previously published reports.

Data Base

Hydrostratigraphy

Rovey (1990) and Rovey and Cherkauer (1994a, b) divided the carbonate aquifer of southeastern Wisconsin into nine major hydrostratigraphic units by correlating hydraulic conductivity from pressure-injection tests with stratigraphic intervals (Figure 3b). Each hydrostratigraphic unit has a regionally consistent hydraulic conductivity related to depositional environment, lithology, and mode of secondary porosity. Fine-grained (mudstone) lithologies have little macroporosity, and ground-water flow in these units is predominantly through joints. Coarse-grained units contain both intergranular porosity in grainstone facies and moldic porosity in packstone facies, produced by selective dissolution of fossil grains.

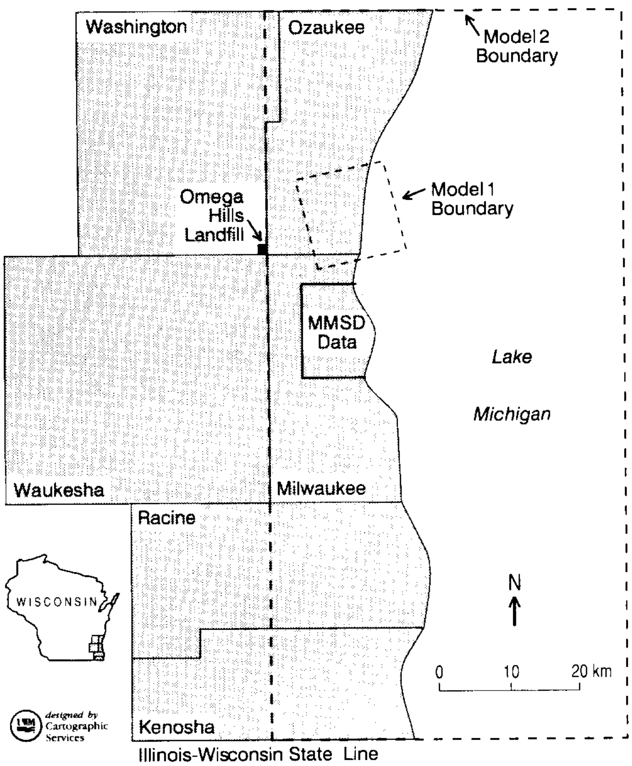

Fig. 1. Study area, southeast Wisconsin. Insets show data collection sites and digital model boundaries.

The upper 6 meters of rock (weathered zone on Figure 3b) are characterized as incipient epikarst and constitute a separate hydrostratigraphic layer. The weathered zone cuts across formation boundaries, but is developed almost exclusively in the fine-grained, joint-dominated strata which comprise the majority of the aquifer. It is the only unit with abundant nonselective dissolution features, including abundant vugs and nominal ($< .5$ cm) solutional widening along joints and hairline fractures. However, it has no geomorphic expression of karst such as dolines, karren, or conduits, and it also lacks hydrologic characteristics of karst such as rapid recharge, spring discharge, and erratic fluctuations in potentiometric surface and carbonate saturation.

MMSD Data

The Milwaukee Metropolitan Sewerage District (MMSD) performed numerous slug, pressure-injection, single-well, and multiwell pumping tests to develop a geotechnical database for an extensive tunneling project (MMSD, 1981; 1984a, b; 1988; Table 1). The geometric mean of hydraulic conductivity increases steadily with the scale of testing. For example, the Thiensville value increases from 2.5×10^{-4} to 10^{-3} to about 10^{-2} cm/sec as measured by slug, pressure, and pumping tests. The same general pattern holds for the other units, although pressure-injection values are not consistently greater than slug test values. Two explanations account for this minor inconsistency. The small number of slug and pressure tests within some units introduces some inaccuracy. Also, as shown later, the two test methods have overlapping ranges in test radii.

Miscellaneous Data

Rovey (1990) analyzed four additional fully penetrating multiwell pumping tests tapping the entire Silurian portion of the aquifer. Files of the Wisconsin Geological and Natural History Survey (WGNHS) also contain unpublished drawdown data from 29 single-well, fully penetrating tests throughout the study area (within the digital model-2 boundaries, Figure 1). We have collected these data and analyzed the single-well tests with the Bradbury and Rothschild (1985) specific capacity conversion program using an average storage coefficient of 5×10^{-4} from the multiwell pumping tests (Rovey, 1990). Use of a single average value is justified, because it is largely determined by the degree of confinement by the overlying glacial sediments which are fairly uniform throughout the area. Moreover, the final value is relatively insensitive to the storage coefficient value, so even large errors have little effect on calculated hydraulic conductivity.

When hydraulic conductivities calculated with this procedure can be compared to values calculated using type-curve matching or semilog analysis on observation wells monitored during the same test, the results are encouraging. The average difference in value is approximately a factor of two with no apparent bias toward an over or underestimation (Rovey, 1990). Moreover, the average bulk Silurian value from the single-well tests is virtually identical with that from multiwell tests (Table 1). Therefore, hydraulic conductivities converted from specific capacities are accepted as

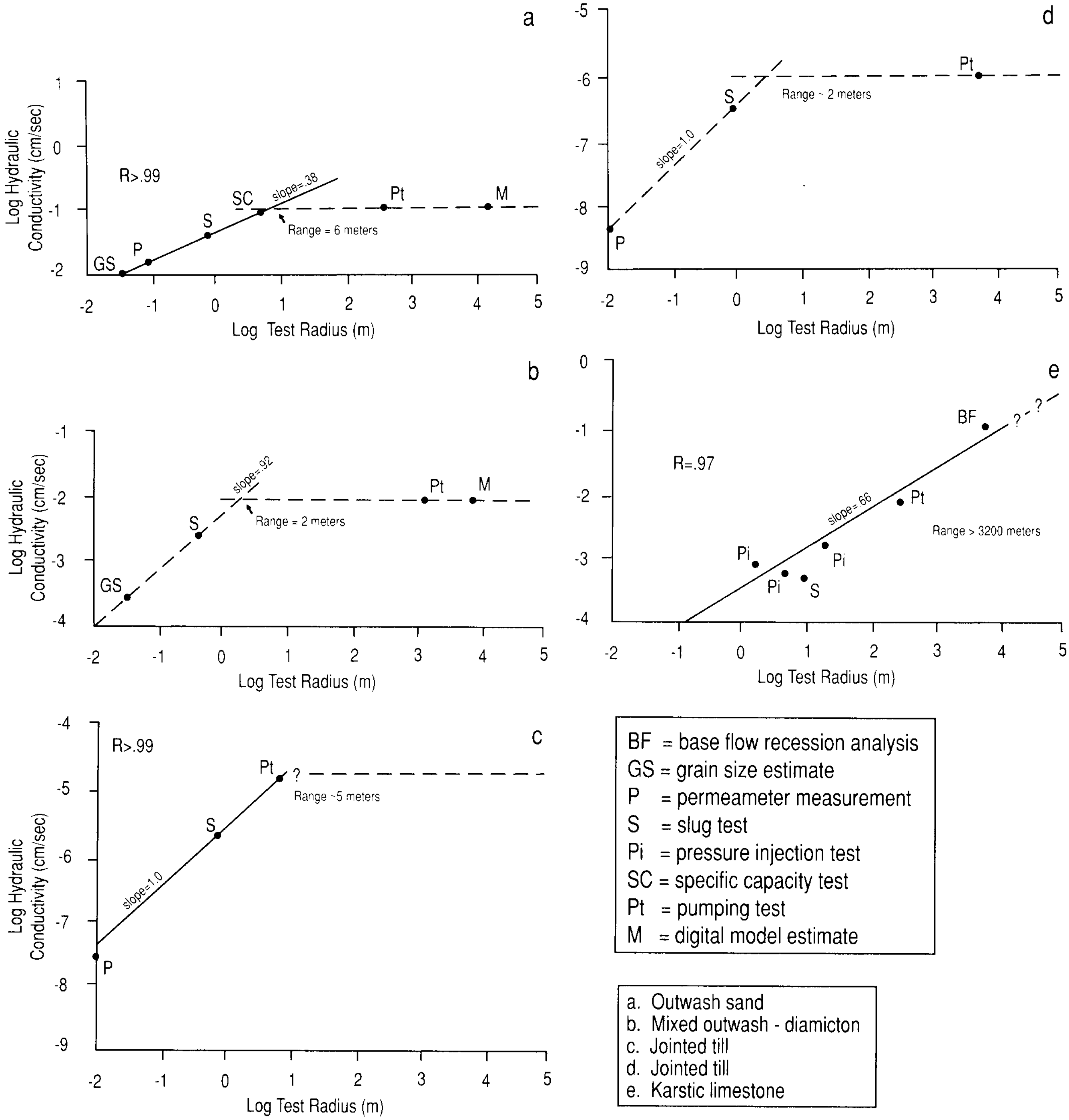

Fig. 2. Increase in hydraulic conductivity with test radius. Solid lines are linear regressions between 3 or more points; dashed lines are fit by hand. The measured range listed is the test radius beyond which hydraulic conductivity is approximately constant. a: Outwash sand. Modified from Bradbury and Muldoon (1990). Test radius is calculated using thicknesses and test intervals from Bradbury (1993). b: Mixed outwash-diamicton. Modified from Bradbury and Muldoon (1990). c: Jointed till. Modified from Bruner and Luteneggar (in press). Test radius estimate is from Bruner (1993). d: Jointed till. Calculated from data in Keller et al. (1986). The pumping test value is calculated from diffusivity and storage coefficient values measured during pumping test of an underlying sand. Test radius for the pumping test is calculated assuming hydraulic conductivity in the sand is .1 cm/sec. See text for discussion of radius of slug tests. e: Karstic limestone. Modified from Sauter (1991) using field test values only. The hydraulic conductivities shown are the midpoints of the "common range" in the original.

valid, and the term "pumping test" will hereinafter refer to both multiwell and single-well tests unless a distinction is specifically made.

Pearson (1993) also collected specific capacity values from the Wisconsin Department of Natural Resources for wells near the Omega Hills Landfill (Figure 1). Many of these wells are open to multiple strata. However, he was able to calculate hydraulic conductivities for individual stratigraphic units, using the Bradbury and Rothschild (1985) conversion in tandem with the equation for effective hydraulic conductivity in a layered medium.

Finally, two digital flow models have been independently calibrated within the study area. Rovey (1983) simulated flow over a 110 square kilometer area using separate bulk parameters within the Silurian and Devonian portions of the aquifer. Mueller (1992) modeled the entire study area

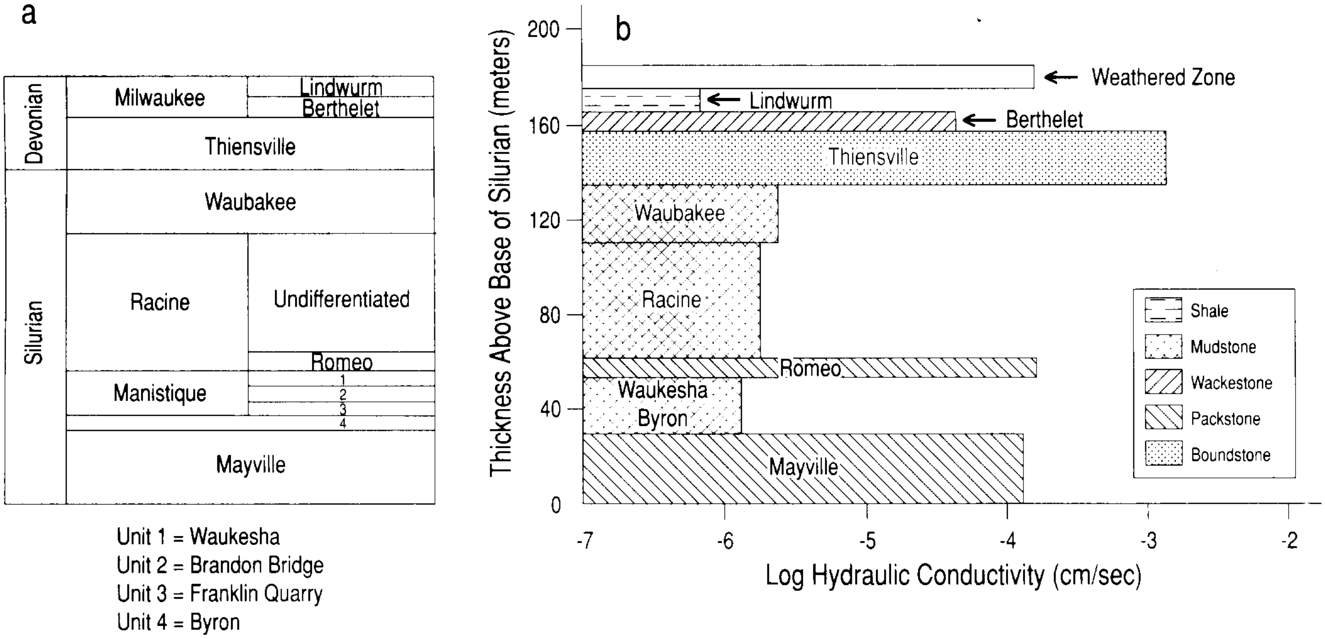

Fig. 3. Relation of hydraulic conductivity to stratigraphy, dolomite aquifer, southeastern Wisconsin (Rovey and Cherkauer, 1994a). Geometric means and number of tests are also listed in Table 1. a: Stratigraphy. Primary subdivision is at the formation level. Member subdivisions are informal, except within the Milwaukee Formation. b: Hydraulic conductivity measured by pressure-injection tests. Each bar is the average thickness of a particular unit and extends to the geometric mean hydraulic conductivity. Patterns depict the dominant texture of each unit.

Table 1. Geometric Mean Hydraulic Conductivity Grouped by Test Method and Stratigraphic Unit for the Dolomite Aquifer of Southeast Wisconsin

Stratigraphic Unit	*Slug Tests*[a] *(Hvorlsev Method)*	*Pressure-Injection Tests*[b] *(5 Min.)*	*Short-Term Single-Well Tests*[c] *(53 Min Ave)*	*Long-Term Single-Well Tests*[d] *(5 Hrs)*	*Multiwell Pumping Tests*[e] *(12-24 Hrs)*	*Digital Models*[f]
Weathered Zone	-4.7 (16)	-4.3 (37)	-3.4 (6)	-1.9 (78)		-2.0 (1)
Lindwurm	-5.8 (2)	-6.2 (5)				
Berthelet	-5.1 (2)	-4.4 (8)				
Thiensville	-3.6 (8)	-3.0 (49)	-2.1 (3)		-1.7 (2)[a]	-2.0 (1)
Waubakee	-6.1 (1)	-5.6 (50)	-4.8 (5)			-4.4 (1)
Racine	-5.7 (7)	-5.8 (129)	-4.8 (5)	-4.3 (171)		-4.4 (1)
Romeo		-3.8 (17)	-4.1 (1)			-2.7 (1)
Waukesha-Byron	-5.8 (4)	-5.9 (23)				-4.7 (1)
Mayville	-5.1 (7)	-3.9 (11)	-3.8 (3)	-3.3 (18)		-3.3 (1)
Bulk Silurian				-3.6 (21)	-3.7 (6)	-3.4 (1)
Approximate R_i (meters)[g]	1	1-10	5-20	100	100-500	30,000

(Values are the $\log_{10}$ in cm/sec. Numbers in parentheses are the number of tests in a respective unit for a given test method.)

[a]Unpublished field data from MMSD.

[b]MMSD (1981, 1984a, b). Additional values for the weathered zone are taken from MMSD (1988), changing the mean value presented in Rovey and Cherkauer (1994a) who also discuss the pressure test methods and limitations.

[c]MMSD (1984a). Specific capacity values are converted to hydraulic conductivities using the Bradbury and Rothschild (1985) conversion. Tests from the Racine and Waubakee are averaged together, because test intervals intercepted both formations.

[d]Individual unit values are from Pearson (1993). Bulk Silurian value is calculated from WGNHS well records. Both are converted to hydraulic conductivity using Bradbury and Rothschild (1985).

[e]MMSD (1984a) and Rovey (1990).

[f]Individual unit values are from Mueller (1992). Bulk Silurian value is from Rovey (1983).

[g]Actual values are given in Table 2.

Table 2. Calculated Radius of Influence Grouped by Test Method and Stratigraphic Unit, Dolomite Aquifer, Southeast Wisconsin

Stratigraphic Unit	*Slug Tests*	*Pressure-Injection Tests*	*Short-Term Single-Well Tests*	*Long-Term Single-Well Tests*	*Multiwell Pumping Tests*	*Digital Models*
Weathered Zone	-0.03	-0.08	1.2	2.4		4.8
Lindwurm	-0.05	-1.2				
Berthelet	0.02	0.58				
Thiensville	0.15	1.0	1.9		3.0	4.6
Waubakee	-0.07	-0.32	0.73			4.3
Racine	-0.04	-0.40	0.73	1.6		4.8
Romeo		0.56	0.90			4.8
Waukesha-Byron	-0.03	-0.48				4.8
Mayville	-0.02	0.52	1.3	2.0		4.8
Bulk Silurian						3.3

(Values are base 10 logs in meters. For example, the value −0.03 is the exponent in $10^{-0.03}$ which is a radius of influence of 0.93 meters.)

with a fully three-dimensional model using six layers corresponding closely to the major hydrostratigraphic units delineated by Rovey and Cherkauer (1994a, b).

Radius of Influence Calculations

Based on Table 1 and the preceding discussion, measured hydraulic conductivity depends on the scale of measurement. To quantify that dependency some length parameter must be associated with each test. Bradbury and Muldoon (1990) and Bruner and Lutenegger (in press) used test volume as the measure of scale. However, in the more numerous reports on dispersivity, travel distance, a one-dimensional measure, is routinely used. Also, established geostatistical methods of quantifying spatial variability use a one-dimensional separation distance (lag) term. Therefore, to facilitate comparison with results from mass transport studies and measures of aquifer heterogeneity, a radius of influence (R_i) is used here.

Calculating a meaningful R_i is problematic. For digital model values R_i is taken as the square root of the modeled area, or the area of the layer, if it is smaller. For consistency, the R_i for all other field tests is estimated using a form of the Cooper-Jacob distance-drawdown equation:

$$R_i^2 = \frac{2.25\ Tt}{S} \quad (1)$$

where T = transmissivity [measured hydraulic conductivity (m^2/day) multiplied by thickness of tested interval]; t = time duration of test (days); and S = storage coefficient [mean value = 5×10^{-4} (dimensionless) measured from multiwell pumping tests; Rovey, 1990]. Thus, the assumption is made that any variation of storage coefficient with location or scale is negligible compared to hydraulic conductivity.

The use of the Cooper-Jacob equation is well-established for pumping tests, but the assumptions of a constant injection/withdrawal rate and local homogeneity may not be reasonable for every small-scale test. For example, the rate of inflow or outflow during a slug test decreases with time. Therefore, the effective time duration of the slug tests is taken as the basic time lag (measured from the field plots), the length of time at which recovery would be complete if the initial rate of inflow/outflow remained constant.

The use of the Cooper-Jacob equation is also not entirely consistent with the assumption of a steady state employed in calculating hydraulic conductivities during pressure testing (Cedergren, 1977). In practice, however, the necessary conditions are less restrictive and only a quasi-steady state is reached during the test, where there is no significant change in gradient or injection rate during the short (5 minute) test duration.

Where comparison is possible, calculated values of R_i are similar to those based on other methods. Bliss and Rushton (1984) simulated pressure tests for an aquifer with hydraulic conductivity averaging approximately 10^{-3} cm/sec, similar to the Thiensville pressure-injection mean (Table 1). At the midpoint of the test interval the modeled R_i was approximately 12 meters which is very close to the 10 meter R_i estimated for the Thiensville (Table 2).

The calculated values of R_i for the slug tests are approximately one meter (Table 2). Bruner (1993) estimated a slug test R_i between 0.5 and 1.0 meters by directly monitoring responses in adjacent wells during tests. Guyonnet et al. (1993) generated a series of theoretical type curves and related regression equations showing effective R_i of slug tests for combinations of dimensionless wellbore storage and dimensionless head. In a 5 cm diameter well, the dimensionless head decreases to 0.1, the typical value at the end of a slug test, by the time a measurable disturbance has propagated a distance approximately 25 times the wellbore radius. The wellbore radius is approximately 10 cm in this case, giving an approximate R_i of 2.5 m.

Therefore, based on comparisons with field measurements, models, and theory, the calculated values of R_i based on the Cooper-Jacob equation, are accurate to within a factor of two to three. This magnitude of possible error is much smaller than the range of values that were investigated (approximately five orders of magnitude, Table 2). Therefore, any errors in calculated R_i should have little effect on the overall comparison of results.

Results from Southeastern Wisconsin

Hydraulic Conductivity Magnitude

The relationship between hydraulic conductivity and scale of measurement (R_i) is plotted on log-log coordinates

(Figure 4) for units which have at least four independent methods of measurement. All plots have an initial linear increase in hydraulic conductivity, with slopes varying between 0.86 and 1.0 before hydraulic conductivity reaches a constant value.

The range over which hydraulic conductivity increases varies considerably among units. Hydraulic conductivity in the Waubakee and Racine Formations is joint-controlled and increases with R_i to approximately 20 meters. The Thiensville and Mayville Formations both contain complex pore systems. The Mayville contains both intergranular and secondary porosity as solution-enlarged molds of fossil grains. The Thiensville also contains intergranular porosity, but has horizons of nonselective dissolution beneath several minor paleo-weathering surfaces. Hydraulic conductivity increases with scale to 50 and 125 meters in the Mayville and Thiensville, respectively, coinciding with the greater degree of dissolution. The weathered zone, with the greatest degree of secondary effects, has the greatest range of hydraulic conductivity increase, 220 meters. Summarizing, the range of scale increase correlates with the degree, and possibly the type, of secondary porosity.

Explanations for Measured Scale Effects

Inspection of Table 1 and Figure 4 reveals that the measured hydraulic conductivity of a given unit increases with the scale of measurement. The increase is too uniform and too large to be coincidence. The increase also cannot be attributed to systematic differences or inaccuracy among the different measurement techniques. The ratio of values as measured by different methods varies considerably from one formation to another.

Measured hydraulic conductivity does increase with the scale of measurement. However, the factor(s) causing the correlation is not yet clear. In principle, several factors besides scale dependency could cause or contribute to the observed relationships, such as limitation of test methods, skin effects, and borehole storage effects.

The first possibility is that the small-scale tests may be incapable of measuring extremely high values. Thus the calculated means would be biased toward low values. For example, two slug tests in the Weathered Zone and one in the Thiensville recovered fully within the time required to make the first measurement. Therefore, they are averaged with the remaining slug tests as "greater than" values, and the true means would be somewhat greater than those in Table 1. However, all slug tests within the remaining units had finite values. Thus, a test bias could not cause slug test means in these units to be lower than those of pumping tests.

Similarly, the upper measurable limit of the pressure-injection tests was somewhere in the range 10^{-3} to 10^{-2} cm/sec (MMSD, 1984a). Therefore, the means of some of the high conductivity units may have been underestimated, particularly the Thiensville which has numerous values within that range (MMSD, 1984a). However, this explanation is again inconsistent with results in other units. The lower measurable limit of the pressure-injection tests was 10^{-7} cm/sec, and the low conductivity units (Waubakee, Racine, Waukesha-Byron) occasionally tested at this boundary, causing their calculated means to be too high. Consequently, accuracy limitations in the pressure-injection tests cannot account for the low conductivity units lower values relative to the pumping tests. To summarize, a bias against large values among the small-scale tests cannot be a major factor contributing to the observed scale increase.

A second possibility also relates to test methodology. If the wellbore is damaged during drilling, skin effects dampen the borehole response, particularly during shorter times and at small radial distance, lowering calculated hydraulic conductivities (Streltsova, 1988). Such an effect could systematically bias the measured hydraulic conductivities of the small-scale, shorter duration tests toward lower values.

Based on available information, however, the skin effect in the wells considered here is negligible. First, none of the wells considered here were drilled with mud; hence, a significant invaded zone around the well bore would be unlikely. Second, the Silurian bulk value measured by the shorter duration single-well tests is actually slightly larger than the value from the longer duration, multiwell tests (Table 1). Third, early drawdown measurements were taken within the pumped well during two of the multiwell tests. These measurements allow calculation of the skin factor using type curve analysis (Earlougher, 1977), and in these two wells at least, the skin factor is zero. Finally, similar increases in hydraulic conductivity with measurement scale have been simulated with digital models of heterogeneous media where skin effects are absent (Rayne, 1993). Therefore, it is reasonable to conclude that skin effects are not an important factor contributing to the increase of hydraulic conductivity.

For pumping tests, borehole storage effects can lower the calculated hydraulic conductivity at early times and small radial distance, much like a positive skin factor (Tongpenyai and Raghavan, 1981). However, neither the Hvorslev method of slug test analysis nor the pressure-injection tests assume a line source/sink when calculating hydraulic conductivity. Instead these methods account for the initial volume of water in the borehole. Therefore, borehole storage effects would lower only the pumping test values, but this is inconsistent with the pumping test values, exceeding the slug and pressure test values in all cases.

As listed in Table 1, the values of the short-term single-well tests are less than values from the longer pumping tests. To determine if the difference in values could be caused by storage effects, the short-term tests were analyzed using the following equation (Driscoll, 1986):

$$t = \frac{.017\,(d_c^2 - d_p^2)}{Q/s} \tag{2}$$

where t = time in minutes, after which borehole storage is negligible; d_c = diameter of borehole (cm); d_p = diameter of discharge pipe (cm); and Q/s = specific capacity of the well at time t in liters/minute/cm of drawdown. The short-term single-well tests were conducted in 10 cm diameter boreholes with an average duration of 53 minutes. Conservatively assuming a discharge pipe of 2.5 cm, the minimum specific capacity for negligible storage effects at 53 minutes is calculated to be .02 liters/minute/cm. This value is below the

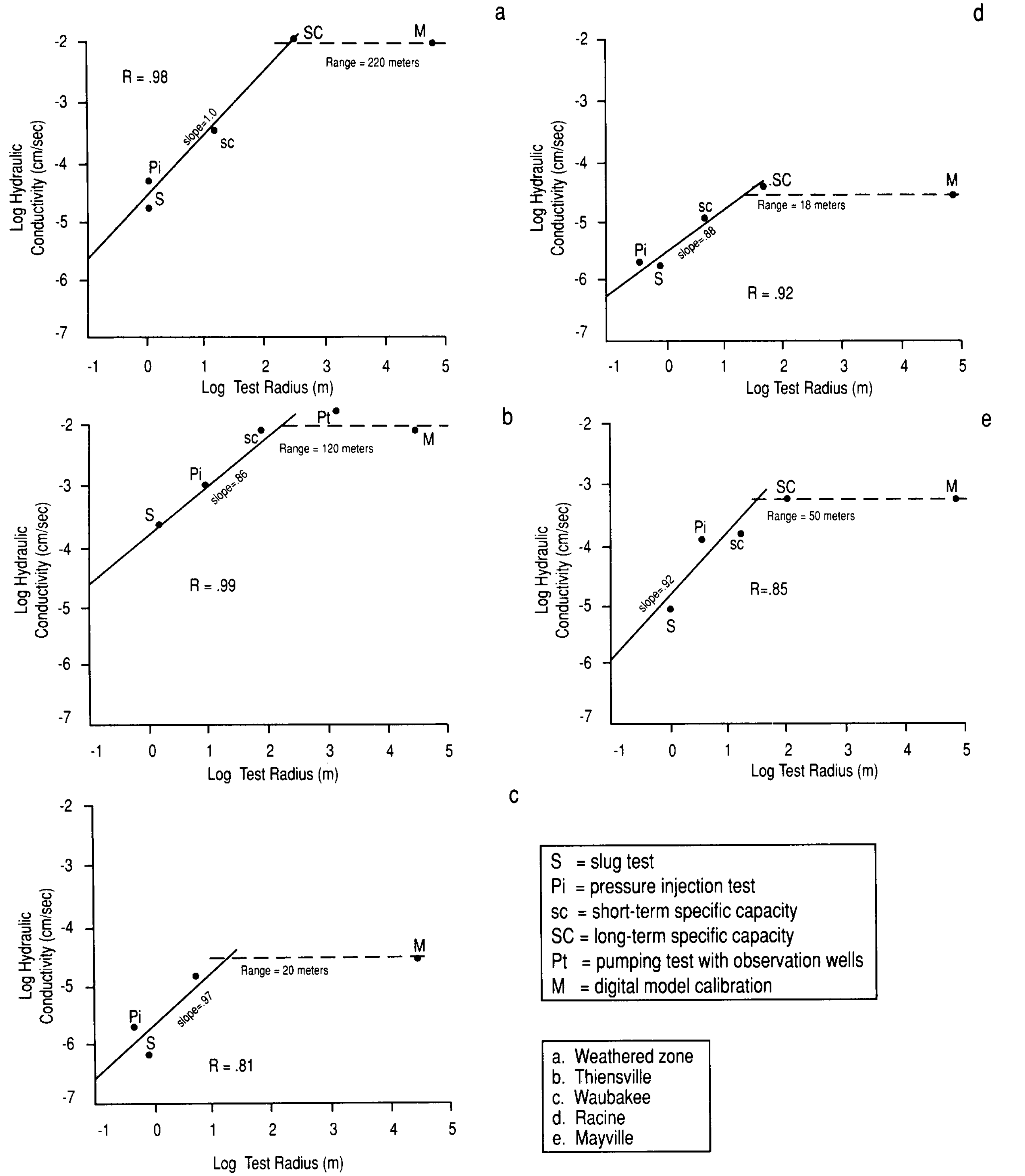

Fig. 4. Relation between measured hydraulic conductivity and effective test radius, dolomite aquifer. Hydraulic conductivities are from Table 1; test radii from Table 2. Solid lines are linear regressions between 3 or more points; dashed lines are fit by hand. The measured range listed is the test radius beyond which hydraulic conductivity is approximately constant.

measured specific capacity for the majority of tests, except those of the Waubakee/Racine. Therefore, borehole storage effects may have lowered the Racine/Waubakee value of the short-term single-well tests. However, this value of 1.6×10^{-5} cm/sec, which is admittedly low, is still much larger than the slug and pressure test values of approximately 2×10^{-6} cm/sec.

In summary, the various alternatives can be used to explain the observed variability in measured hydraulic conductivity in some instances. However, for every case in which an alternative could be valid, there is at least one contradictory relationship. The one explanation which is universally consistent with the measured increase is that hydraulic conductivity increases with the scale of measure-

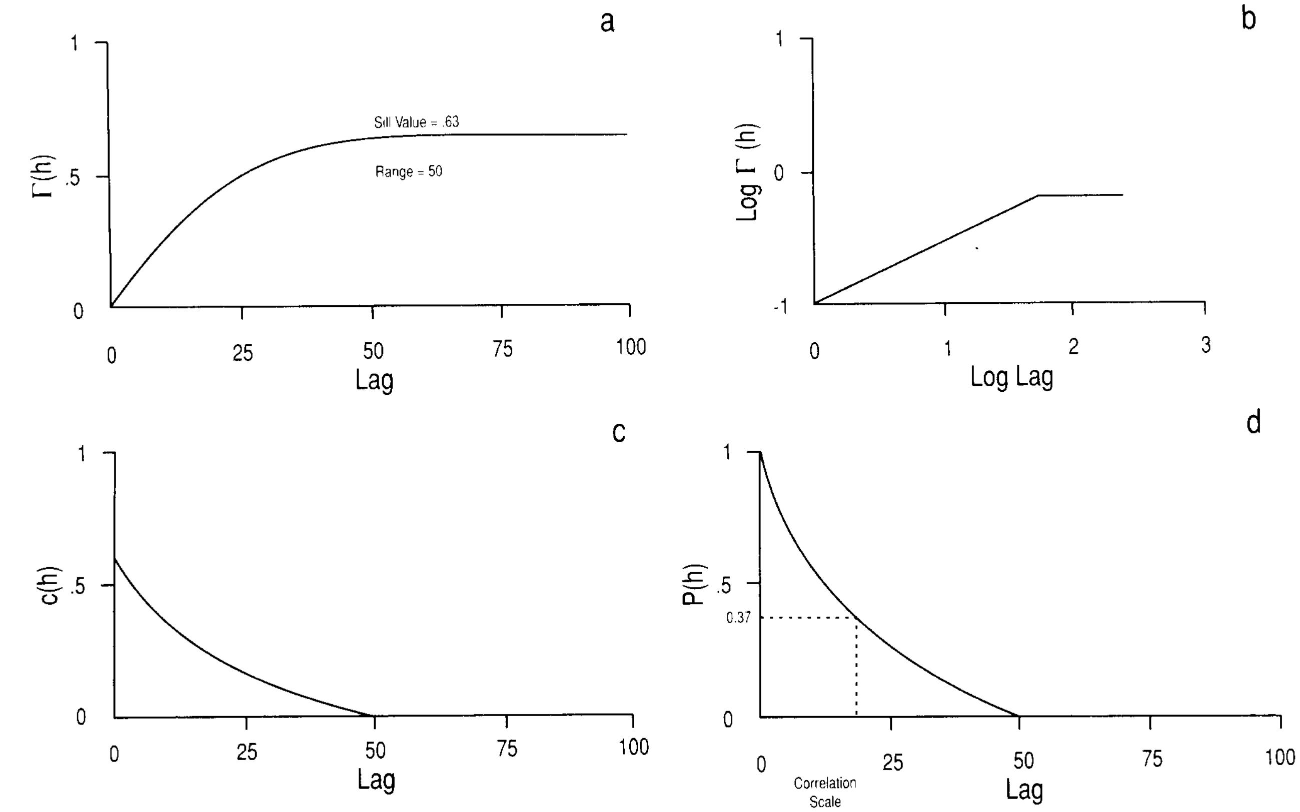

Fig. 5. Graphical measures of spatial correlation. All parameter values are arbitrary, for illustration only. a: Variogram, arithmetic scale. b: Variogram, log-log scale. c: Covariance function, arithmetic scale. d: Correlation function, arithmetic scale.

ment, much like dispersivity. At small scales hydraulic conductivity and ground-water flow tend to be uninfluenced by rare heterogeneities which raise conductivity and flow rates over a regional scale. Stated otherwise, the chances of a small-scale test encountering an extremely rare high-conductivity heterogeneity are disproportionately small relative to the degree with which that heterogeneity raises regional hydraulic conductivity.

Comparison to Scale Increase in Dispersivity

In the previous section the hypothesis was introduced that the increase in hydraulic conductivity with scale is somehow related to heterogeneity. In this section we expand this hypothesis by showing similarities with scale increases in dispersivity and variance of hydraulic conductivity.

Like hydraulic conductivity, dispersivity increases linearly on log-log plots, generally to distances between 10 and 100 meters. Thereafter, the spread in values is nearly constant and a best-fit line is approximately horizontal. The same general pattern is apparent from data or plots of dispersivity from single sites (Freyberg, 1986; Garabedian et al., 1991) and on plots with values combined from multiple sites (Gelhar et al., 1992; Neuman, 1990).

Stochastic theories dealing with dispersivity generally relate scaling effects to increases in spatial variability of hydraulic conductivity with distance (Dagan, 1982, 1984; Gelhar and Axness, 1983). They also predict that dispersivity should approach a constant value as the hydraulic conductivity becomes statistically uncorrelated at increased distances, that is, as the scale of an equivalent homogeneous medium is reached. These conclusions are reasonably consistent with results of intensive field investigations employing geostatistical methods to describe spatial variability in hydraulic conductivity (Sudicky, 1986; Freyberg, 1986; Hess et al., 1992; LeBlanc et al., 1991; Garabedian et al., 1991). Therefore, we further hypothesize that a (or the) common factor between the scale effects in dispersivity and hydraulic conductivity is variability in the hydraulic conductivity field.

Three common graphical measures of spatial variability are the semivariogram (or simply variogram), the covariance (autocovariance) function, and the correlation function (Figure 5; Isaaks and Srivastava, 1989). The functions for each respective measure are given by:

$$\Gamma(h) = 1/2E[K(z+h) - K(z)]^2 \qquad (3)$$

(equals semivariance between points at various lags)

$$C(h) = E[K(z+h) * K(z)] - E^2[K(z)] \qquad (4)$$

(equals covariance between points at various lags)

$$\rho(h) = \frac{C(h)}{C(0)} \qquad (5)$$

(equals covariance function divided by variance)

where: E[ ] denotes an average value over all paired samples at a given lag; z is a spatial coordinate location; h is a distance or lag from z; and K(z) = hydraulic conductivity measured at z. The three measures are interrelated, and the covariance function is converted to the variogram by:

$$\Gamma(h) = C(0) - C(h). \qquad (6)$$

Of these, the covariance and correlation functions are the most widely used in mass transport studies, and the usual correlation scale is defined for convenience as the distance or lag at which $\rho(h)$ declines to e^{-1} or 0.37 (Figure 5). Note, however, that this distance is shorter than that at which the covariance declines to zero (complete uncorrelation) or alternatively, the distance (range) at which the variogram reaches a constant variance (sill). This latter measure is used here because it facilitates direct comparison with the plots of hydraulic conductivity presented earlier.

Additional Site Data

Geostatistical parameters and hydraulic conductivity measurements at different scales are available, or can be calculated, for several additional systems (Figure 6). Pertinent results from an outwash sand at the Borden Site in Ontario, Canada were presented by Sudicky (1986) and Mackay et al. (1986). Variance in log hydraulic conductivity increases with scale at a low (0.24) log-log slope with a range of 10 meters (Figure 6a). Any scale increase in the magnitude of hydraulic conductivity cannot be accurately determined,

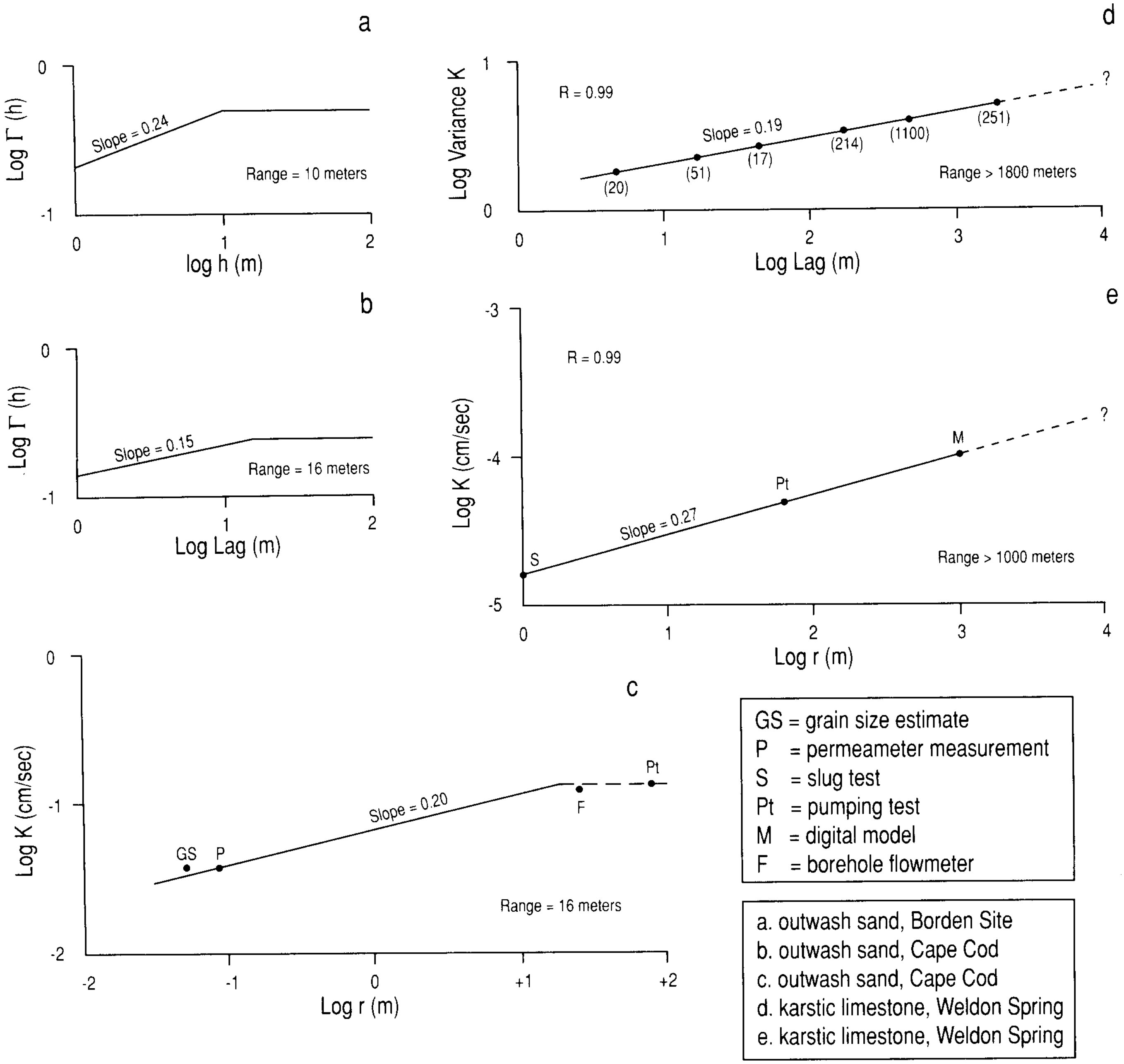

Fig. 6. Scaling effects in mean and variance of log hydraulic conductivity at other study sites. a: Glacial outwash, Borden Site. Generated from modeled correlation function, line A-A′ in Sudicky (1986). b, c: Glacial outwash, Cape Cod, Massachusetts. The variogram (b) is from the modeled isotropic variogram with nugget effect using flowmeter data in Hess et al. (1992). Hydraulic conductivity plot (c) is generated from values in LeBlanc et al. (1991) and Wolf et al. (1991), assuming test durations of 1 and 12 hours for the borehole flowmeter and pumping tests. d, e: Karstic limestone, Weldon Springs, Missouri. Variogram (d) is nondirectional, generated from slug test values calculated from raw field data and surveyed locations supplied by MK-Ferguson Company, St. Charles, Missouri. Lag tolerance is .5 log units; annotated values are number of data pairs. Hydraulic conductivity plot (e) is generated using data from MK-Ferguson and Jacobs (1990) and Durham (1991). The model conductivity is the area-weighted average of values from three separate modeled zones.

but is minimal, close to zero. Mean hydraulic conductivities from grain-size analyses, permeameters, and slug tests are essentially equal.

Another glacial outwash sand was investigated at Cape Cod, Massachusetts (LeBlanc et al., 1991; Garabedian et al., 1991; Hess et al., 1992). As a qualification, there is less control on the R_i for some of these tests, but both hydraulic conductivity and its variance again have small slopes and ranges (Figures 6b and c; approximately 0.2 and 16 meters, respectively).

Data from an additional karstic limestone aquifer are also available from Weldon Spring, Missouri (Price, 1991; Carman, 1991; Durham, 1991; MK-Ferguson and Jacobs, 1990). Hydraulic conductivity was measured with slug tests, pumping tests, and a calibrated digital model. Both hydraulic conductivity and variance increase with measurement scale (Figures 6d, e); however, the slopes (0.27 and 0.19, respectively) are significantly less than the 1.0 slope typical of fractured media (Figures 2c, d). However, the most striking contrast is not the slope, but the range of the scale increase. In a second karstic carbonate aquifer the range in both hydraulic conductivity and variance exceeds the maximum scale of investigation, in this case, approximately 2000 meters.

The indefinite range in karstic aquifers contrasts sharply with finite ranges in nonkarstic carbonates and unconsolidated media (Table 3). In nonkarstic carbonates the maximum range of hydraulic conductivity is approximately 200 meters, with distances less than 50 meters typical for units with the smallest degree of dissolution. The range in variance also is finite for the nonkarstic carbonates (Figure 7). Although the variogram shape is questionable at small lags, the nonkarstic carbonates all have distinct sills, and their maximum range is approximately 200 m, similar to the maximum range in hydraulic conductivity.

In summary, the variograms are strikingly similar to the hydraulic conductivity plots of the same geologic unit. As heterogeneity increases, so does mean hydraulic conductivity, and as statistical homogeneity is reached, hydraulic conductivity becomes constant.

Summary

The hydraulic conductivities of five carbonate hydrostratigraphic units were measured over radial distances ranging from less than one to greater than 10,000 m. The observed increase in hydraulic conductivity with scale is consistent with results from a variety of geologic media, including glacial outwash, jointed clay-rich tills, and karstic limestones. The results reinforce Bradbury and Muldoon's (1990) conclusion that hydraulic conductivities based on small-scale field measurements will generally be less than regional values, even if they are based on 100 or more individual tests.

Scaling effects vary consistently with the type of geologic medium and degree of secondary porosity (Table 3). Glacial outwash, with primary porosity only, generally has the smallest rate and range of scale increase. Thus, small-scale field measurements such as slug tests will be closest to regional values, generally within a factor of three (Figures 2a, 6a, 6c). The rate of scale increase is much greater in consolidated/joint-dominated media. Slug tests in these media may underestimate regional values by factors ranging from 2 to 500, depending on the range in effects (Figures 2c, d; 6) which correlate to the degree of secondary dissolution. In mature karst aquifers hydraulic conductivity increases without apparent bound, so it may not even be possible to speak of a unique regional hydraulic conductivity.

Finally, the increase of hydraulic conductivity with

Table 3. Characteristic Values of Slope and Range for Various Geologic Media

	Glacial Outwash[a]	*Jointed Till*[b]	*Carbonates Joint-Dominated*[c]	*Carbonates Secondary Moldic Porosity*[d]	*Carbonates Incipient Dissolution Along Joints*[e]	*Carbonates Mature Karst*[f]
Range, K	7.3 (0-16) [3]	3.5 (2-5) [2]	19 (18-20) [2]	50 — [1]	170 (120-220) [2]	— (>1000, >3200) [2]
Range, Variance	13.0 (10-16) [2]	— —	<100 m [2]	— —	<200 m [2]	>1800 — [1]
Slope, K	0.19 (0-0.38) [3]	1.0 (1.0) [2]	0.92 (0.88-0.97) [2]	0.92 — [1]	0.93 (0.86-1.0) [2]	0.46 (0.27-0.66) [2]
Slope, Variance	0.20 (0.15-0.24) [2]	— —	— —	— —	— —	0.19 — [1]

(Range and slope are taken from Figures 2, 4, and 6. The first number is the arithmetic mean, that in parentheses is range of all values, that in brackets is the number of different geologic units summarized.)
[a]From Figures 2a, 6a, 6b, 6c.
[b]From Figures 2c, d.
[c]Racine and Waubakee Formations, Figures 4c, d; 7c, d.
[d]Mayville Formation, Figure 4e.
[e]Thiensville Formation and Weathered Zone, Figures 4a, b; 7a, b.
[f]Figures 2e, 6d, 6e.

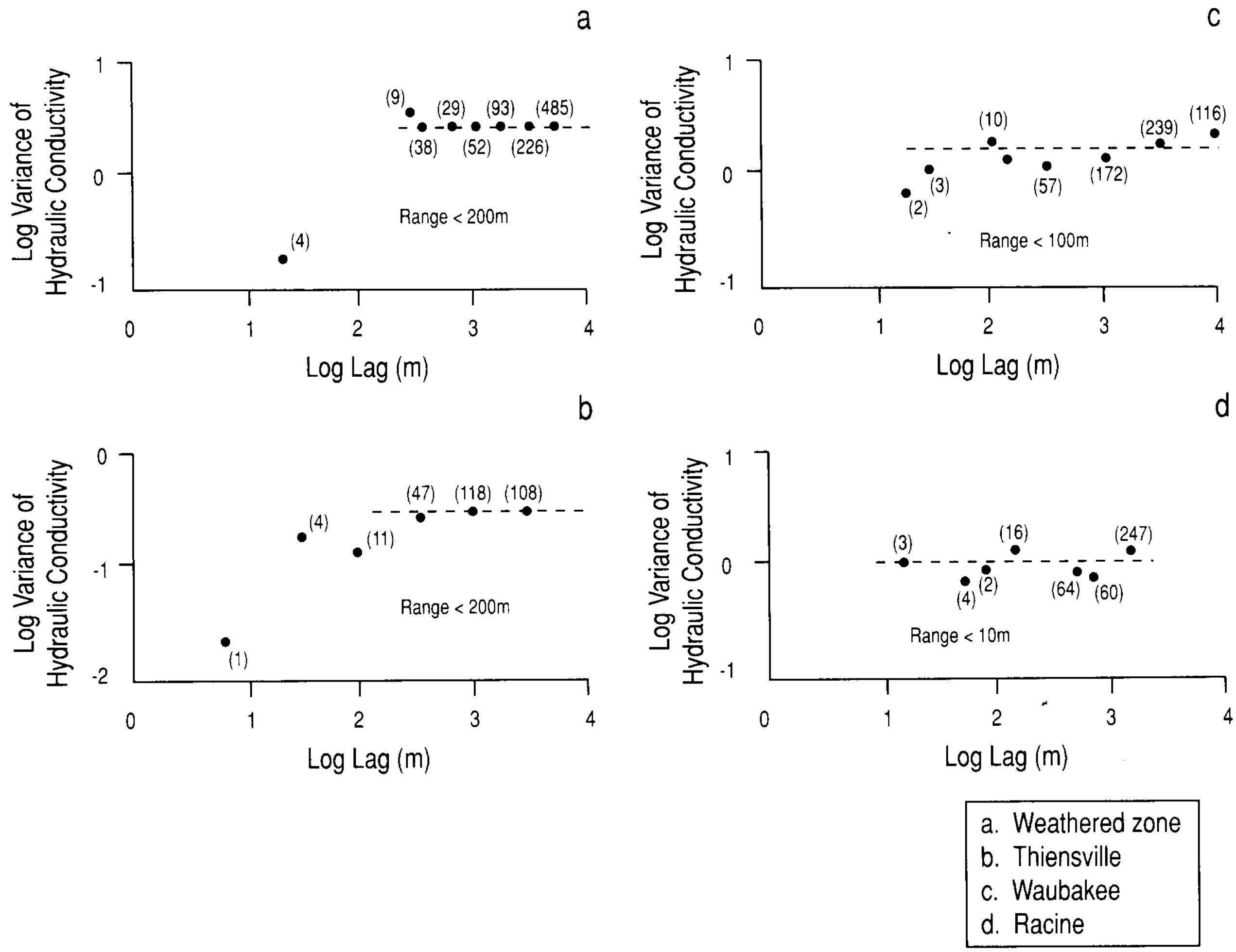

Fig. 7. Log-log variograms of log hydraulic conductivity distribution in dolomite aquifer. Variograms are nondirectional, generated from pressure test values (Table 1) using surveyed borehole locations. Annotated values are the number of data pairs at a given lag. Lag tolerance is 0.5 log units.

radius of influence in the test method is related to an increase in variance of log-hydraulic conductivity at greater lag distances between measurement points. This correlation suggests that increases in hydraulic conductivity and dispersivity are related through a common dependency on variance.

Acknowledgments

We thank Steve Fradkin of STS Inc., formerly at the Project Management Office, Milwaukee Metropolitan Sewerage District, who often went to great lengths to make various project data available. We also acknowledge the help and data supplied by Ken Bradbury of the Wisconsin Geologic and Natural History Survey, Roger Bruner of Foth and Van Dyke, Minneapolis, Minnesota, and the MK-Ferguson Company, St. Charles, Missouri. Carry McConnell of the University of Missouri-Rolla reviewed an early version of the manuscript, resulting in significant improvements. This research was funded by the University of Wisconsin Sea Grant Institute under grants from the National Sea Grant College Program, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, and from the State of Wisconsin Federal Grant NA90AA-D-SG469, project R/MW-35.

References

Bliss, J. C. and K. R. Rushton. 1984. The reliability of packer tests for estimating the hydraulic conductivity of aquifers. Quarterly Journal of Engineering Geology. v. 17, pp. 81-91.

Bradbury, K. R. 1993. Personal communication. Wisconsin Geological and Natural History Survey.

Bradbury, K. R. and E. R. Rothschild. 1985. A computerized technique for estimating the hydraulic conductivity of aquifers from specific capacity data. Ground Water. v. 23, pp. 240-246.

Bradbury, K. R. and M. A. Muldoon. 1990. Hydraulic conductivity determinations in unlithified glacial and fluvial materials. In: D. M. Nielsen and A. I. Johnson (eds.). Ground Water and Vadose Zone Monitoring. ASTM STP 1053, pp. 138-151.

Bredehoeft, J. D., C. E. Neuzil, and P.C.D. Milly. 1983. Regional flow in the Dakota Aquifer: A study of the role of confining layers. U.S. Geological Survey Water-Supply Paper 2237. 45 pp.

Bruner, D. R. and A. J. Lutenegger. (In press). Measurement of saturated hydraulic conductivity in fine-grained glacial tills in Iowa: Comparison of in situ and laboratory methods. In: D. E. Daniel and S. J. Trautwein (eds.). Hydraulic Conductivity and Waste Contaminant Transport in Soils. ASTM STP 1142.

Bruner, D. R. 1993. Personal communication. Foth and Van Dyke, Minneapolis, MN.

Carman, J. 1991. Aquifer characteristics of the shallow

Burlington-Keokuk Limestone at the Weldon Spring Site. Weldon Spring Site Remedial Action Project, DOE Contract DE-AC05-86OR21548. pp. 155-201.

Cedergren, H. R. 1977. Seepage, Drainage and Flow Nets. Wiley Interscience. 534 pp.

Dagan, G. 1982. Stochastic modeling of groundwater flow by unconditional and conditional probabilities, 2. The solute transport. Water Resources Research. v. 18, no. 4, pp. 835-848.

Dagan, G. 1984. Solute transport in heterogeneous porous formations. Journal of Fluid Mechanics. v. 145, pp. 151-177.

Driscoll, F. G. 1986. Groundwater and Wells. Johnson Division, St. Paul, MN. 1089 pp.

Durham, L. 1991. Groundwater modeling at the Weldon Spring chemical plant/raffinate pits and vicinity properties. Weldon Spring Site Remedial Action Project, DOE Contract DE-AC05-86OR21548. pp. 203-228.

Earlougher, R. C. Jr. 1977. Advances in well test analysis. Society of Petroleum Engineers Monograph. v. 5

Freyberg, D. L. 1986. A natural gradient experiment on solute transport in a sand aquifer, 2. Spatial moments and the advection and dispersion of nonreactive tracers. Water Resources Research. v. 22, no. 13, pp. 2031-2046.

Garabedian, S. P., D. R. LeBlanc, L. W. Gelhar, and M. A. Celia. 1991. Large-scale natural gradient tracer test in sand and gravel, Cape Cod, Massachusetts, 2. Analysis of spatial moments for a nonreactive tracer. Water Resources Research. v. 27, no. 5, pp. 911-924.

Gelhar, L. W., C. Welty, and K. R. Rehfeldt. 1992. A critical review of data on field-scale dispersion in aquifers. Water Resources Research. v. 28, no. 7, pp. 1955-1974.

Gelhar, L. W. and C. L. Axness. 1983. Three dimensional stochastic analysis of macrodispersion in aquifers. Water Resources Research. v. 19, no. 1, pp. 161-180.

Guyonnet, D., S. Mishra, and J. McCord. 1993. Evaluating the volume of porous medium investigated during slug tests. Ground Water. v. 31, no. 4, pp. 627-633.

Herzog, B. L. and W. J. Morse. 1984. A comparison of laboratory and field determined values of hydraulic conductivity at a waste disposal site. Proceedings, Seventh Annual Madison Waste Conference, Dept. of Engineering Professional Development, Univ. of Wisconsin-Madison, Madison, WI. pp. 30-52.

Hess, K. M., S. H. Wolf, and M. A. Celia. 1992. Large-scale natural gradient test in sand and gravel, Cape Cod, Massachusetts, 3. Hydraulic conductivity variability and calculated macrodispersivities. Water Resources Research. v. 28, no. 8, pp. 2011-2027.

Isaaks, E. H. and R. M. Srivastava. 1989. An Introduction to Applied Geostatistics. Oxford University Press. 561 pp.

Keller, C. K., G. Van Der Kamp, and J. A. Cherry. 1986. Fracture permeability and groundwater flow in clayey till near Saskatoon, Saskatchewan. Canadian Geotechnical Journal. v. 23, pp. 229-240.

LeBlanc, D. R., S. P. Garabedian, K. M. Hess, L. W. Gelhar, R. D. Quadri, D. G. Stollenwerk, and W. W. Wood. 1991. Large-scale natural gradient tracer test in sand and gravel, Cape Cod, Massachusetts, 1. Experimental design and observed tracer movement. Water Resources Research. v. 27, no. 5, pp. 895-910.

Mackay, D. M., D. L. Freyberg, and P. V. Roberts. 1986. A natural gradient experiment on solute transport in a sand aquifer, 1. Approach and overview of plume movement. Water Resources Research. v. 22, no. 13, pp. 2017-2029.

MK-Ferguson Company and Jacobs Engineering Group. 1990. Aquifer characteristics data report for the Weldon Spring site chemical plant/raffinate pits and vicinity properties. DOE Contract No. DE-AC05-86OR21548. 111 pp.

Milwaukee Metropolitan Sewerage District (MMSD). 1981. Inline Storage Facilities Plan. v. 4, Borehole Logs.

Milwaukee Metropolitan Sewerage District (MMSD). 1984a. North Shore Interceptor Geotechnical Report. v. 3, Contract Documents.

Milwaukee Metropolitan Sewerage District (MMSD). 1984b. Cross Town Interceptor Geotechnical Report. v. 3, Contract Documents.

Milwaukee Metropolitan Sewerage District (MMSD). 1988. Kinnickinnic/Lake Michigan Tunnel System Geotechnical Report. v. 3, Contract Documents.

Mueller, S. 1992. Three-dimensional digital simulation of the ground-water contribution to Lake Michigan from the Silurian aquifer of southeastern Wisconsin. Unpublished M.S. thesis, Univ. of Wisconsin-Milwaukee. 231 pp.

Neuman, S. P. 1990. Universal scaling of hydraulic conductivities and dispersivities in geologic media. Water Resources Research. v. 26, no. 8, pp. 1749-1758.

Pearson, R. 1993. The controls on hydraulic conductivity distributions in the Silurian aquifer at Omega Hills Landfill, southeastern Wisconsin. Unpublished M.S. thesis, Univ. of Wisconsin-Milwaukee. 110 pp.

Price, P. 1991. Shallow groundwater investigations at Weldon Spring, Missouri. Weldon Spring Site Remedial Action Project, DOE Contract DE-AC05-86OR21548. pp. 141-153.

Rayne, T. 1993. Variability of hydraulic conductivity in sandy till: The effects of scale and methodology. Unpublished Ph.D. dissertation, Univ. of Wisconsin-Madison. 135 pp.

Rovey, C. W. II. 1990. Stratigraphy and sedimentology of Silurian and Devonian carbonates, eastern Wisconsin, with implications for ground-water flow into Lake Michigan. Unpublished Ph.D. dissertation, Univ. of Wisconsin-Milwaukee. 427 pp.

Rovey, C. W. II. 1983. Computer modeling of the interaction between Lake Michigan and the dolomite aquifer at Mequon, Wisconsin. Unpublished M.S. thesis, Univ. of Wisconsin-Milwaukee. 271 pp.

Rovey, C. W. II and D. S. Cherkauer. 1994a. Relation between hydraulic conductivity and texture in a carbonate aquifer: Observations. Ground Water. v. 32, no. 1, pp. 53-62.

Rovey, C. W. II and D. S. Cherkauer. 1994b. Relation between hydraulic conductivity and texture in a carbonate aquifer: Regional continuity. Ground Water. v. 32, no. 2, pp. 227-238.

Sauter, M. 1991. Assessment of hydraulic conductivity in a karst aquifer at local and regional scale. Proceedings of Third Conference on Hydrogeology, Ecology, Monitoring and Management of Ground Water in Karst Terranes. National Ground Water Assoc. pp. 39-56.

Streltsova, T. D. 1988. Well Testing in Heterogeneous Formations. Exxon Monograph, John Wiley & Sons.

Sudicky, E. A. 1986. A natural gradient experiment on solute transport in a sand aquifer: Spatial variability of hydraulic conductivity and its role in the dispersion process. Water Resources Research. v. 22, no. 13, pp. 2069-2082.

Tongpenyai, Y. and R. Raghavan. 1981. The effect of wellbore storage and skin on interference test data. Journal of Petroleum Technology. January, 1981, pp. 151-160.

Wolf, S. H., M. A. Celia, and K. M. Hess. 1991. Evaluation of hydraulic conductivities calculated from multiport-permeameter measurements. Ground Water. v. 29, no. 4, pp. 516-525.