EPS 236, Fall 2007: Geological fluid mechanics

Homework number 2

Due Monday September 10

The purpose of these questions is to learn, or recall, some basic applied math ideas and notation. Fluids mechanics literature uses all this math and notation without explanation! One of the goals of this class is to provide sufficient preparation to read and understand papers in fluid mechanics.

1. Consider two vectors, $\mathbf{A} = 2\mathbf{e}_x + 4\mathbf{e}_y + 6\mathbf{e}_z$ and $\mathbf{B} = 3\mathbf{e}_x - 2\mathbf{e}_y - 2\mathbf{e}_z$ (here, and throughout the course, $\mathbf{e}_i$ indicates a unit vector in the i direction). What is the angle between these two vectors?

2. Show that $\mathbf{A} \cdot \mathbf{B} \wedge \mathbf{C} = \mathbf{A} \wedge \mathbf{B} \cdot \mathbf{C} = -\mathbf{C} \cdot \mathbf{B} \wedge \mathbf{A}$.

3. Using the very useful identity (you are also encouraged to try deriving this identity) called the $\epsilon\delta$ identity

$$\epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$$

to show that

$$(\mathbf{a} \wedge \mathbf{b}) \wedge \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}.$$

Here we use ϵ to denote the permutation symbol and δ is the Kronecker delta.

4. Solve the differential equation

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}; \quad y(0) = -1.$$

Provide an explicit solution for y, i.e. $y = \ldots$

5. Find a solution to the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$$

that satisfies $y = 0$ and $dy/dx = 1$ at $x = 0$.