Fall 2007
EPS 236: Geological fluid mechanics

Homework number 4

Due Wednesday September 19

7. a) Non-dimensionalize (normalize) the Navier-Stokes equations

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = -\nabla p + \mu \nabla^2 \mathbf{u}$$

and

$$\nabla \cdot \mathbf{u} = 0$$

in order to obtain the Reynolds number $Re = \rho U l / \mu$ where U is a characteristic velocity and l is the characteristic length scale.

b) What equations do we get in the two limits $Re \gg 1$ and $Re \ll 1$ (these will be Euler's equations and the Stokes equations, respectively)?

For c) and d) you will need to get some estimate of U: *explain* how you estimate U. One possibility is to try a simple experiment at home (use syrup or honey to simulate the lava).

c) What equations of motion will describe flow for a small crystal (radius 1 mm) settling in a lava flow (viscosity 1000 Pa s)? Why?

d) What equations of motion will describe flow for a sand grain (radius 1 mm) settling in water? Why?

8. When we build models of river systems and ships, care is usually taken to scale the Froude number

$$Fr = U/\sqrt{gl},$$

where l is the length scale and g is gravity (this is the ratio of velocity to the speed of shallow water waves; we might talk about the Froude number in more detail later).

Suppose we would like to study the motion of a boat (length 100 m, speed 10 m/s) in the lab. Due to budget and space constraints, we can only make a model boat that is 1 m long.

a) How fast does the model boat have to move for Froude number scaling to hold?

b) What can we do if we want the Reynolds number to **also** be the same for the real boat and the model?