

Homework number 5

Due Monday, September 24

9. One final practice question on vector calculus involving gradients: show that

$$\nabla \wedge (\nabla \wedge \mathbf{u}) = \nabla(\nabla \cdot \mathbf{u}) - \nabla^2 \mathbf{u}.$$

10. Show that the flow rate Q of a fluid with viscosity μ and density ρ through a tube with radius R is given by

$$Q = \frac{\pi R^4}{8\mu} G$$

where G is the pressure gradient driving the flow. You will first need to calculate the velocity u as a function of radial position r (solve the momentum equation in *cylindrical coordinates*). This type of flow is often called Poiseuille flow or pipe flow (after a French Physicist (1799-1869); the cgs unit of viscosity, Poise, is also named after Poiseuille).

11. If the eruption rate of the Columbia River flood basalts was $1 \text{ km}^3/\text{day}$, what is the radius of the conduit (here assumed to be cylindrical and smooth) that transported the magma? Assume a viscosity of 100 Pa s , a density difference between the magma and surrounding rocks of 300 kg/m^3 , and a magma density of 2600 kg/m^3 ; also assume that only the density difference between the magma and surrounding rocks drives the flow.

Given the assumptions made above, would the flow of the magma through the conduit be laminar or turbulent?