EPS 236: Geological fluid mechanics
Fall 2007

Homework number 6

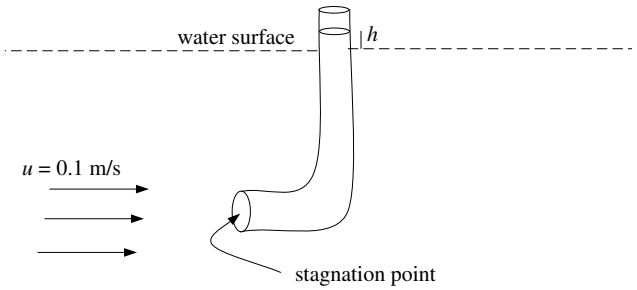
Due Monday, October 1

12. Show that the Navier-Stokes equations can be written as

$$\frac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega} \cdot \nabla \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega}; \quad \nabla \cdot \mathbf{u} = 0$$

where $\boldsymbol{\omega} = \nabla \wedge \mathbf{u}$ is the vorticity. The term $\boldsymbol{\omega} \cdot \nabla \mathbf{u}$ describes the stretching and twisting of vortex lines (see Tritton §6.6 for a detailed discussion).

13. In inviscid fluids, flow speed is sometimes measured using a *Pitot tube*, see illustration below. The idea is that a stagnation points occurs at the entrance to the tube, and the pressure at this point will be equal to the hydrostatic pressure at the base of the tube; as a result the water level will rise in the tube, and the height of the water can be used to determine the flow velocity using Bernoulli's equation. Assuming the flow speed is 0.1 m/s, how high will the water level rise in the tube?



14. Consider two equal-size spherical particles in a very viscous fluid (e.g. a magma) sinking because they are more dense than the surrounding fluid (see picture below). Assuming the Reynolds number is $\ll 1$, what can you say about the change in their relative orientation and separation distance? Why?