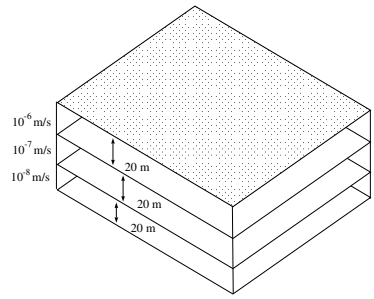
EPS 236: Flow in porous materials, optional questions

1. Adapted from question 11 on page 56 in Domenico and Schwartz.

a) Three horizontal, homogeneous, and isotropic formations overlie one another. Each is 20 m thick, with conductivities of 10^{-6}, 10^{-7}, and 10^{-8} m/s. Determine the horizontal and vertical components of hydraulic conductivity for an equivalent homogeneous, anisotropic formation.



b) Why are horizontal permeabilities usually greater than vertical permeabilities?

2. Consider an aquifer with porosity $n = 10$ % and thickness $b = 100$ m. When the hydraulic head decreases 10 m it is found that the aquifer thickness decreases 10 cm. What is the specific storage? What is the relative importance of matrix and water compressibility?

3. Hydraulic conductivity is, in general, not a scalar quantity, but a second rank tensor. Represented in matrix form it looks like

$$\mathbf{K} = \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix}$$

where x, y and z are our three coordinate directions. I will use bold symbols to denote vectors and tensors. Darcy's equation now looks like

$$\mathbf{q} = -\mathbf{K} \cdot \nabla h$$

where ∇ is the gradient and $\cdot$ denotes the dot product.

$\mathbf{K}$ is a symmetric tensor (i.e., the matrix above is symmetric so that, for example, $K_{xy} = K_{yx}$).

The question here is number 12 on page 56 in Domenico and Schwartz. A sample is subjected to a hydraulic gradient of $\partial h/\partial x = 0.1$ cm/cm in the x direction and a flux q_y of 0.001 cm/s in the y direction in response to this gradient (there is no gradient in the y direction). The same sample is then subjected to a hydraulic gradient of $\partial h/\partial y = 0.01$ cm/cm in the y direction and a flux q_x is measured in the x direction in response to this gradient (there is no gradient in the x direction). What is the magnitude of the flux q_x in response to $\partial h/\partial y$?

This question should serve as a reminder that flow will not necessarily only be from high to low head.

4. This question is adapted from question 1.4 in Ingebritsen and Sanford.

A variety of assumptions or approximations are made to the groundwater flow equation. Listed below are 6 assumptions and 7 equations. Identify the equations that implictly make each assumption.

Assumptions

- homogeneous medium
- isotropic medium
- two-dimensional flow
- one-dimensional flow
- steady-state conditions
- constant fluid density

Equations

$$s_s \frac{\partial h}{\partial t} = \left[\frac{\partial (K_x \partial h/\partial x)}{\partial x} + \frac{\partial (K_y \partial h/\partial y)}{\partial y} + \frac{\partial (K_z \partial h/\partial z)}{\partial z} \right] \tag{1}$$

$$s_s \frac{\partial h}{\partial t} = K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} \tag{2}$$

$$s_s \frac{\partial h}{\partial t} = K \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right] \tag{3}$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{4}$$

$$\frac{\partial h}{\partial t} = \frac{T}{s} \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right] \tag{5}$$

$$\frac{s}{T} \frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} \tag{6}$$

$$\frac{\partial \phi \rho_f}{\partial t} = \nabla \cdot \left[\frac{\mathbf{k} \rho_f}{\mu_f} \cdot (\nabla P + \rho_f g \nabla z) \right] \tag{7}$$

5. A series of piezometers are installed at various depths in a sandy aquifer near Ottawa (Canada) to determine groundwater flow rates. Piezometric measurements indicate that flow is vertically downward. The following measurements were obtained in the summer of 1996 (this is real data from Clark and Fritz). TU is a unit of measure (tritium units) commonly used to report tritium concentrations.

depth	^{3}H (TU)
surface	19
0.5	20.2
1.0	21.8
2.5	21.3
4.0	19.9
6.0	29.9
8.0	47.1
10.0	113.8
12.0	40.8
14.0	1.9
16.0	<0.8

To help answer this question you will need to know how tritium concentration in precipitation changed over time. A figure will be provided in class (or search the web).

a) Explain the observed distribution of tritium in this aquifer

b) Calculate the groundwater velocity through this vertical profile (did you calculate the Darcy or pore velocity?)

c) What are the initial tritium concentrations (tritium half-life is 12.43 years). How do these values compare with measurements at Ottawa (see handout)? Can you explain any discrepancies?

6. A typical large hot spring the Oregon Cascades discharges about 300 L/s of water at a temperature of 70 degrees Celcius. Assuming a heat capacity of 4 kJ/kg, what is the rate of heat transport and discharge by the hot spring? State (and justify) any assumptions you need to make for this calculation.

A typical person uses about 10^{-3} MW (megawatts). Assuming an energy extraction efficiency of 30% (30% of the discharged heat is converted to usable energy), how many people can be supported from the energy produced by a single hot spring?