Dynamics of magma ascent in the volcanic conduit
Helge M. Gonnermann and Michael Manga

Overview

This chapter presents the various mechanisms and processes that come into play within the volcanic conduit for a broad range of effusive and dry explosive volcanic eruptions. Decompression during magma ascent causes volatiles to exsolve and form bubbles containing a supercritical fluid phase. Viscous magmas, such as rhyolite or crystal-rich magmas, do not allow bubbles to ascend buoyantly and may also hinder bubble growth. This can lead to significant gas overpressure and brittle magma fragmentation. During fragmentation in vulcanian, subplinian, and plinian eruptions, gas is released explosively into the atmosphere, carrying with it magma fragments. Alternatively, high viscosity may slow ascent to where permeable outgassing through the vesicular and perhaps fractured magma results in lava effusion to produce domes and flows. In low viscosity magmas, typically basalts, bubbles may ascend buoyantly, allowing efficient magma outgassing and relatively quiescent magma effusion. Alternatively, bubbles may coalesce and accumulate to form meter-size gas slugs that rupture at the surface during strombolian eruptions. At fast magma ascent rates, even in low viscosity magmas, melt and exsolved gas remain coupled allowing for rapid acceleration and hydrodynamic fragmentation in hawaiian eruptions.

4.1 | Introduction

In the broadest sense, volcanic eruptions are either effusive or explosive. During explosive eruptions magma fragments and eruption intensity is ultimately related to the fragmentation mechanism and associated energy expenditure (Zimanowski et al. 2003). If the cause of fragmentation is the interaction of hot magma with external water, the ensuing eruption is called phreatomagmatic. Eruptions that do not involve external water are called “dry,” in which case the abundance and fate of magmatic volatiles, predominantly H$_2$O and CO$_2$, as well as magma rheology and eruption rate are the dominant controls on eruption style. Eruption styles are often correlated with magma composition and to some extent this relationship reflects differences in tectonic setting, which also influences magmatic volatile content and magma supply rate.
4.1.1 Getting magma to the surface
Volcanic eruptions represent the episodic or continuous surface discharge of magma from a storage region. We shall refer to this storage region as the magma chamber and to the pathway of magma ascent as the conduit. For magma to erupt, the chamber pressure has to exceed the sum of frictional and magma-static pressure losses, as well as losses associated with opening of the conduit. The latter may be due to tectonic forces and/or excess magma pressure, which in turn are a consequence of magma replenishment and/or volatile exsolution in a chamber within an elastically deforming wall rock. Magma-static pressure loss is strongly dependent on magma density, which is dependent on volatile exsolution and volume expansion as pressure decreases during ascent. Feedbacks between chamber and conduit can produce complex and unsteady eruptive behavior.

4.1.2 The volcanic conduit
Eruptions at basaltic volcanoes may begin as fissure eruptions, with typical dike widths of the order of one meter (Chapter 3; Wilson and Head 1981). However, magma often erupts at the surface through structures that more closely resemble cylindrical conduits. The processes by which flow paths become focused into discrete cylindrical vents are not well understood. One possibility is that the temperature-dependent viscosity of magma results in a feedback between a cooling, viscosity increase and reduction in flow velocity, so that the flow becomes more focused over time (Wylie-Lister 1995). In addition, thermal erosion may also contribute to flow focusing. (Bruce Huppert 1989).

The conduit during silicic explosive eruptions is generally thought to be more or less cylindrical, at least at shallow depths. However, the conduit cross-sectional area can change with depth and time because of wall-rock erosion, caused by abrasion as pyroclasts collide with conduit walls, or shear stress from the erupting magma, or wall collapse (Wilson et al. 1980, Macedonio et al. 1994, Kennedy et al. 2005, Mitchell 2005).

4.2 Volatiles
Volatiles play the central role in governing the ascent and eruption of magma, as summarized in Figure 4.1. Dissolved volatiles, in particular water, have large effects on melt viscosity. Exsolved volatiles form bubbles of supercritical fluids, which we will also refer to as gas or vapor bubbles. These bubbles allow for significant magma compressibility and buoyancy, ultimately making eruption possible (Pyle and Pyle 1995, Woods and Cardoso 1997). The decrease in pressure associated with magma ascent reduces volatile solubility, leading to bubble nucleation and growth, the latter a consequence of both volatile exsolution and expansion (Sparks 1978). Volatile exsolution also promotes microlite crystallization, which in turn affects bubble growth and rheology – there is a complex feedback between decompression, exsolution, and crystallization (Tait et al., 1989). The occurrence and dynamics of explosive eruptions are mediated by the initial volatile content of the magma, and the ability of gases to escape from the ascending magma.

4.2.1 Solubility
Volatiles solubility is primarily pressure dependent, with secondary dependence on temperature, melt composition, and volatile speciation (McMillan 1994, Blank and Brooker 1994, Zhang et al., 2007). Formulations for volatile solubility can be thermodynamic or empirical. In either case, calibration is achieved using measurements of dissolved volatile concentrations in quenched melts, equilibrated with a volatile phase of known composition at fixed pressures and temperatures. The principal species we consider here are H₂O and CO₂. Solubility of CO₂ in silicate melts is proportional to partial pressure, whereas for H₂O dissociation into molecular H₂O and OH results in a square root dependence on partial pressure. An empirical formulation for combined solubility of H₂O and CO₂ in rhyolitic melts at volcanologically relevant conditions is given by (Liu et al., 2005).
DYNAMICS OF MAGMA ASCENT IN THE VOLCANIC CONDUIT

\[ C_w = 0.0012439 \ p_{w}^{3/2} + \frac{354.94 \sqrt{p_w} + 9.623 \ p_w - 1.5223 \ p_{w}^{3/2}}{T} \]

\[ + \ p_c \left( -1.084 \times 10^{-4} \sqrt{p_{w} - 1.362 \times 10^{-4} \ p_w} \right) \]

\[ (4.1) \]

\[ C_i = p_i \left[ \frac{5668 - 55.99 \ p_{w} + 0.4133 \sqrt{p_{w} + 0.002041 \ p_{w}^{3/2}}}{T} \right]. \]

\[ (4.2) \]

Here \( C_w \) is total dissolved H\(_2\)O in wt.%; \( C_i \) is dissolved CO\(_2\) in ppm and \( T \) is temperature in Kelvin. \( p_{w} \) and \( p_c \) are the partial pressures in MPa of H\(_2\)O and CO\(_2\), respectively. This formulation is also approximately applicable to other melt compositions (Zhang et al., 2007), but more accurate models are available (Dixon 1997, Newman and Lowenstern 2002, Papale et al., 2006). Equilibrium concentrations of dissolved CO\(_2\) and H\(_2\)O, based on this formulation, are shown in Figure 4.2. Notice that almost all CO\(_2\) exsolves at >100 MPa for rhyolite and at >25 MPa for basalt, whereas most H\(_2\)O dissolves at <100 MPa for rhyolite and at <25 MPa for basalt. Consequently, processes in the shallow conduit are predominantly affected by H\(_2\)O exsolution.

**4.2.2 Diffusivity**

Volatile diffusivities in silicate melts are best characterized for H\(_2\)O. A recent formulation for
H₂O diffusivity (m² s⁻¹) in rhyolite with H₂O contents > 2 wt % is (Zhang and Behrens 2000)

\[
D_{\text{ur}} = C_w \exp\left(-17.14 - \frac{10661}{T} - 1.772 \frac{p_{\text{in}}}{T}\right).
\]

(4.3)

A formulation for higher H₂O concentrations is provided by Zhang et al. (2007), who also give the following equation for H₂O diffusivity in basalt

\[
D_{\text{ub}} = C_w \exp\left(-8.56 - \frac{19110}{T}\right).
\]

(4.4)

CO₂ diffusivity, \(D_c\), is smaller than water diffusivity over a range of conditions (Watson 1994). The most reliable formulation for \(D_c\) in silicate melts is based on argon diffusivity, which is essentially identical to CO₂ at volcanologically relevant conditions (Behrens and Zhang 2001, Nowak et al., 2004)

\[
D_c = \frac{-18.239 + 17367 + 1.0964 (855.20 + 0.2712 p_{\text{in}})}{T} + \frac{555.2 + 0.2712 C_w}{T}.
\]

4.2.3 Pre-eruptive volatile content of magmas

Water

Basaltic magmas show a wide range of water contents ranging from >0.5 wt.% to 6–8 wt.% (Johnson et al., 1994, Wallace 2005), with non-arc basalts generally being considerably dryer than arc magmas, which have highly variable water contents. Silicic magmas also show a wide range of water contents up to 6 wt.%, and possibly higher (Carmichael 2002).

Carbon dioxide

CO₂ content of magmas is more difficult to constrain, because of its low solubility at shallow depths (Fig. 4.2). Measurements of H₂O and CO₂ in melt inclusions, in conjunction with solubility relations, suggest that arc magmas contain several wt.% of CO₂, much of which exsolved pre-eruptively, for example during vapor-saturated fractional crystallization (Papale 2005; Wallace 2005). Thus, arc magmas may already contain, or have lost, significant amounts of exsolved CO₂
prior to eruption. CO₂ of undegassed mid-ocean ridge basalts and ocean island basalts appears to be about 1 wt.% or less (Hauri 2002). Because CO₂ exsolves at greater pressures than H₂O, relative proportions of CO₂ and H₂O in erupted gases, glasses and, melt inclusions can provide constraints on depths of magma degassing. Because typical CO₂ contents of silicic magmas are considerably lower than H₂O, the latter dominates eruption dynamics.

**Sulfur**

H₂S and SO₂ are, after water and carbon dioxide, the most abundant magmatic volatiles. S concentrations are, at a minimum, several thousand ppm in arc and back-arc magmas (Wallace 2005). Volcanic sulfur emissions are spectrographically readily detectable and, therefore, play an important role in monitoring of active volcanoes via remote sensing. Emissions from active volcanoes, especially during voluminous eruptions and are the subject of this section.

### 4.3 Bubbles

The “birth,” “life,” and “death” of bubbles are key to understanding the dynamics of volcanic eruptions and are the subject of this section.

#### 4.3.1 Nucleation

Pressure decreases as magma ascends. If the concentration of dissolved volatiles exceeds the equilibrium solubility at a given pressure, the melt is supersaturated, a requirement for bubble nucleation. Supersaturation, $\varepsilon p_s$, is defined as the difference between actual pressure and the pressure at which the concentration of dissolved volatiles would be in equilibrium with the co-existing vapor phase. The supersaturation required for nucleation corresponds to the energy that must be supplied to increase the surface area between two fluids. It is a consequence of surface tension, $\gamma$, a measure of the attractive molecular forces that produce a jump in pressure across a curved interface between two fluids. This so-called Laplace or capillary pressure is $2\gamma/R$ for a spherical bubble of radius $R$.

Supersaturation is typically produced when fast magma ascent allows insufficient time for volatile diffusion into existing bubbles, which occurs when $\tau_{\text{dif}}/\tau_{\text{dec}} \gg 1$ (Toramaru, 1989). Here, $\tau_{\text{dec}} = p_m/p_m$ is the characteristic decompression time, $\tau_{\text{dif}} = (S - R^2)/D$ is the characteristic time for volatile diffusion, $S$ is the radial distance from bubble center to the midpoint between adjacent bubbles, $D$ is volatile diffusivity in the melt, $p_m$ is pressure of the melt, and $\dot{p}_m$ is the decompression rate.

Bubble nucleation can be heterogeneous or homogeneous. In heterogeneous nucleation, crystals provide substrates that facilitate nucleation because of the lower interfacial energy between solid and vapor than between melt and vapor. Typically, $\varepsilon p_s$ is a few MPa for heterogeneous nucleation (Hurwitz and Navon, 1994, Gardner and Denis, 2004, Gardner, 2007) and ~10 to 100 MPa for homogeneous nucleation, in which nucleation sites are lacking (Mangan and Sisson, 2000).

Classical nucleation theory (Hirth et al., 1970) predicts a very strong dependence of the nucleation rate $J$ (number of bubbles per unit volume per unit time) on $\varepsilon p_s$ and $\gamma$

$$J \propto \exp \left[ -\frac{16\pi \gamma^2 \omega}{3k_B T \Delta p_s^2} \right]. \quad (4.5)$$

Here $k_B$ is the Boltzmann constant and $\gamma$ is typically 0.05 to 0.3 N m⁻¹, with a dependence on...
both temperature and composition of the melt (Bagdassarov et al., 2000, Mangan and Sisson, 2005). $\psi$ is a geometrical factor that ranges between 0 and 1, depends on $\theta$, the contact angle between bubble and crystal, and is given by

$$\psi = \frac{(2 - \cos\theta)(1 + \cos\theta)^2}{4}. \quad (4.6)$$

$J$ determines the number density of bubbles per unit volume of melt, $N_i$ and if bubbles nucleate at large $\varepsilon_p$, they do so at a high rate.

### 4.3.2 Growth

As ambient pressure, $p_m$, decreases during magma ascent, bubbles may grow if volatiles exsolve and the vapor expands. The momentum balance for a growing bubble is obtained by neglecting inertial terms in the Rayleigh–Plesset Equation (Scriven, 1959). This is justified because large melt viscosities and short length scales result in Reynolds numbers for bubble growth $\ll 1$. The resulting equation for a bubble surrounded by a shell of melt with constant viscosity is (Proussevitch et al., 1993)

$$p_g - p_m = \frac{2\gamma}{R} + 4\eta_0v_k \left(\frac{1 - R^2}{R^2S^2}\right). \quad (4.7)$$

Here, $p_g$ is the pressure inside the bubble, $\eta_0$ is the Newtonian melt viscosity, and $v_k$ is the radial velocity of melt at radius $r$. An idealized uniform packing geometry is assumed, so that each bubble can be represented as a sphere of radius $R$ surrounded by a spherical melt shell of thickness $S - R$. In this formulation the time-dependent $p_m$ couples bubble growth with magma ascent. Mass conservation of volatiles requires that

$$d \rho_g R^2 = 4R^2 \rho_m \sum_i D_i \left(\frac{\partial C_i}{\partial r}\right)_{r=R} dt. \quad (4.8)$$

where $r$ is radial distance from the center of the bubble, $t$ is time, $\rho_g$ is the pressure-dependent vapor density, $\rho_m$ is melt density, $D_i$ is the concentration dependent diffusivity of dissolved volatile species $i$, and $C_i$ is the mass fraction of dissolved volatile $i$. Volatile diffusion is governed by

$$\frac{\partial C_i}{\partial t} + v_r \frac{\partial C_i}{\partial r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C_i}{\partial r}\right). \quad (4.9)$$

with $v_r$ as the radial velocity of melt at radius $r$. The boundary condition at $r = S$ is $\partial C_i/\partial r = 0$. At $r = R$ the prescribed concentration is obtained from a suitable solubility model with the additional assumption that dissolved volatiles at the melt–vapor interface are locally in equilibrium with the vapor at $p_g$.

Under most conditions bubble growth is either limited by (1) the rate of viscous flow of the melt to accommodate the ensuing volume increase, (2) diffusion rate of volatiles to the melt–vapor interface where they can exsolve, or (3) the change in solubility caused by decompression. For each growth limit it is possible to simplify the solution of the governing equations or to derive analytical growth laws.

### Modeling of bubble growth

Explicit modeling of diffusive bubble growth during magma ascent was first presented by Sparks (1978), based on the equations of Scriven (1959) and Rosner and Epstein (1972). Subsequently, Proussevitch et al. (1993) applied a model that included the diffusion of volatiles to bubble growth in magmas. This model was enhanced by Sahagian and Proussevitch (1996) and Proussevitch and Sahagian (1998) to include the thermal effects associated with the heat of volatile exsolution and the work done against the surrounding melt by bubble expansion. Figure 4.3 shows the results for the diffusive bubble growth in a rhyolite containing both $\text{H}_2\text{O}$ and $\text{CO}_2$.

### Viscous limit

In the viscous limit $\tau_{vis}/\tau_{dec} \gg 1$ and bubble growth is retarded, following an exponential growth law (Lensky et al., 2004). Here $\tau_{vis} = \eta_0(p_g - p_m)$ is the characteristic viscous time-scale. The viscous limit to bubble growth is the consequence of fast decompression rates relative to the viscous deformation of melt by the growing bubbles, and is most significant at melt viscosities in excess of $10^6\text{Pa}s$ (Sparks et al., 1994), which in the absence of significant
amounts of crystals, is achieved for silicic melts at shallow depths and low H₂O content. Because of retarded bubble growth, \( p_g \) decreases at a slower rate than \( p_m \), resulting in the build-up of overpressure with \( p_g - p_m \gg 2\gamma/R \) (Fig. 4.3). At the same time, a high \( p_g \) inhibits volatile exsolution, causing supersaturated conditions. This is referred to as “viscosity quench” (Thomas et al., 1994) and increases the potential for accelerated bubble nucleation and/or growth.

**Diffusive limit**

During decompression, the volatile concentration at the melt–vapor interface, based on the assumption of local equilibrium, decreases. This creates a concentration gradient for volatiles to diffuse to this interface and exsolve. If \( \tau_{\text{dec}} \ll \tau_{\text{diff}} \), volatile concentrations will remain close to equilibrium throughout the melt. On the other hand, if \( \tau_{\text{diff}}/\tau_{\text{dec}} \gg 1 \) (i.e., at high decompression rates) the melt becomes supersaturated, which is a necessary condition for bubble nucleation (Lensky et al., 2004). Furthermore, in the diffusive limit, volatiles with different diffusivities may fractionate (Gonnermann and Manga, 2005b; Gonnermann and Mukhopadhyay, 2007).

**Solubility limit**

During solubility-limited growth, bubbles are close to mechanical and chemical equilibrium, that is, \( \tau_{\text{diff}}/\tau_{\text{dec}} \ll 1 \) and \( \tau_{\text{vis}}/\tau_{\text{dec}} \ll 1 \) (Lensky et al., 2004) so that both overpressure and supersaturation are small. Dissolved volatiles are near their equilibrium concentrations throughout the melt and bubble radius \( R \) can be directly calculated from mass balance, equilibrium solubility, and an equation of state. Conditions favorable to solubility-limited bubble growth are low melt viscosity and/or low magma ascent rates.

**4.3.3 Coalescence**

Coalescence takes place when bubbles come into increasingly close proximity and the melt film between bubbles ruptures. This may be the consequence of (1) gravitational or capillary drainage of interstitial liquid (Proussevitch et al., 1993b); (2) coalescence-induced coalescence, whereby the deformation of coalesced bubbles by capillary forces induces a flow field...
that brings nearby bubbles into proximity (Martula et al., 2000); (3) bubble growth leading to stretching and thinning of the liquid film that separates individual bubbles (Borrell and Leal, 2008); and (4) advection and collision of bubbles by buoyancy or by magma flow (Manga and Stone, 1994).

Gravitational and capillary film drainage, as well as bubble collisions are probably most important in low viscosity magmas. A positive feedback between bubble coalescence, which increases bubble size, and bubble mobility due to buoyancy is expected (Section 4.3.5). Whereas capillary drainage is dominant at bubble radii <3 cm, gravitational drainage becomes dominant at radii >3 cm (Proussevitch et al., 1993). A characteristic velocity for capillary drainage can be obtained by balancing the capillary pressure gradient, $\gamma/R^2$, with viscous resistance to flow, $\eta_0 v_{f}/R^2$, where $v_f$ is the velocity in the film. Because the bubble–melt surface is assumed to be a free-slip surface, the characteristic length scale is the bubble radius $R$ rather than film thickness. The resultant scaling $v_f \sim \gamma/\eta_0$ highlights the importance of melt viscosity. Whether and at what rate bubble collisions result in coalescence depends on the Weber number, $We = 2\rho v^2 R / \gamma f$, where $v$ is the velocity at which two bubbles approach one another.

In high viscosity magmas bubbles remain essentially “frozen” in the melt and coalescence is principally a consequence of bubble growth. In this case, coalescence appears to create a permeable network of bubbles due to relatively persistent holes in the ruptured melt films (Klug and Cashman, 1996). Because the interfacial tension between melt and crystals tends to be lower than between gas and crystals, coalescence in the presence of crystals is expected to occur at greater film thickness than in the absence of crystals (Proussevitch et al., 1993b).

### 4.3.4 Breakup

The breakup of bubbles into several smaller bubbles originates with their deformation by some combination of viscous stresses and inertial forces. In silicic magmas, the bubble Reynolds number, $Re_b = \varepsilon R^2 \rho/\eta_0$, will always be small and inertia can be neglected. Here $\varepsilon$ is the strain rate in the melt caused by buoyant rise of the bubble or by magma flow. For small $Re_b$, deformation scales with the Capillary number, $Ca = \eta_0 \varepsilon R / \gamma$, which characterizes the relative importance of viscous stresses that tend to deform bubbles, and surface tension stresses that act to keep bubbles spherical. If $Ca$ exceeds some critical value, $Ca_{cr}$, bubble elongation by the flow becomes large enough that bubbles will break up. The value of the $Ca_{cr}$ depends on the steadiness of the flow, melt viscosity, and flow type. For the viscosities of silicate melts, $Ca_{cr}$ should range from $\sim 1$ to $>10^3$. In both types of flows, bubbles will become highly elongated before breakup occurs. Capillary numbers will generally be large enough in conduits for large deformation and breakup to occur. However, to achieve large deformation also requires large strains which may be limited to the sides of conduits. It is thought that tube pumice is a manifestation of these conditions (Martí et al., 1999).

In basaltic magmas, inertial forces can no longer be neglected once bubbles become larger than a few centimeters. In this limit, velocity differences across bubbles can lead to breakup. The relative importance of inertial forces and surface tension forces is characterized by $We$, based on the velocity difference across the melt that surrounds the bubble. For large $Re_b$, breakup occurs if $We$ exceeds a critical value, $We_{cr}$, which implies that there might be a maximum stable bubble size (Hinze, 1955). Although the actual value of $We_{cr}$ depends on the origin of the velocity differences, it is generally between 1 and 5 for a wide range of flow conditions.

### 4.3.5 Bubble mobility

Bubbles are buoyant and their rise speed depends on size, volume fraction, and viscosity. If buoyant bubble rise is much slower than magma ascent, bubbles are dispersed throughout the continuous melt phase and move passively with the flow. Such flows are called “dispersed” and in the asymptotic limit of infinitesimally small dispersed bubbles the flow is termed “homogeneous.” If the velocity of buoyant bubble rise is similar to or greater than magma ascent velocities, bubbles are decoupled from the liquid phase
and the flow is called “separated” (Brennen, 2005). During separated flow in laboratory experiments the topology of the gas and liquid phases is associated with different flow regimes (Fig. 4.4), that depend primarily on the relative volumetric flow rates of liquid and gas phase (Wallis, 1969). As this flux ratio increases to values of ~1, the flow transitions from “bubbly flow,” where bubbles are randomly dispersed, to “slug flow,” where coalesced bubbles form rapidly ascending gas slugs of diameter comparable to the conduit diameter. At flux ratios above ~10 the flow transitions to “annular flow,” where the liquid phase forms an annular ring surrounding a cylindrical core of rapidly ascending gas. However, to what extent these different flow regimes are applicable to volcanic eruptions remains controversial (Chapter 6).

The terminal velocity of a single bubble containing vapor of viscosity \( \eta_g \) and density \( \rho_g \), rising in an infinite liquid of viscosity \( \eta_l \) and density \( \rho_m \) is (Batchelor, 1967)

\[
U_i = -\frac{R^2 g (\rho_m - \rho_g)}{3 \eta_l} \frac{\eta_l + \eta_g}{\eta_l + \frac{3}{2} \eta_g},
\]

(4.10)

Therefore, bubbles within a silicic melt have negligible mobility, even at very low eruption rates, whereas significant bubble mobility can be expected for mafic melts. At finite volume fractions of bubbles, \( \phi_b \), hydrodynamic interactions reduce \( U_i \) by a factor \( h(\phi_b) \), the hindering function. Various formulations exist for \( h(\phi_b) \) and a commonly adopted one is the Richardson–Zaki equation (Richardson and Zaki, 1954)

\[
h(\phi_b) = (1 - \phi_b)^n.
\]

(4.11)

Here \( n \) is an empirical coefficient for the appropriate range of bubble size and flow conditions, typically with values around 2.5 (Zenit et al., 2001). At flow regimes other than bubbly flow a different formulation for the relative velocity between gas and liquid phase has to be used (Dobran, 2001).

4.3.6 Bubbles and pressure loss

In general, mass discharge rate depends on the total pressure drop between magma chamber and vent. Within the conduit the pressure gradient is to first order the sum of magma-static pressure loss, \( \frac{dp}{dz} \), and frictional pressure loss, \( \frac{dp}{dz} \). Magma-static pressure depends on magma density and, hence, vesicularity as

\[
\frac{dp}{dz} = \rho g (1 - \phi_b) \rho_m g.
\]

(4.12)

For dispersed bubbly flow the frictional pressure loss can be approximated on the basis of established single-phase flow correlations using the appropriate mixture viscosity (Ghiaasiaan, 2008)

\[
\frac{dp}{dz} = \rho u^2 f a.
\]

(4.13)

where \( a \) is conduit radius, \( u \) is magma velocity, and \( f \) is the friction factor given by

\[
f = \frac{16}{Re} + f_0,
\]

(4.14)

Here \( Re = 2\mu u a/\eta \), where \( \eta \) is the magma viscosity and \( f_0 \) has values of about 0.002–0.02 (Mastin...
and Ghiorso, 2000). For highly viscous magmas the flow below the fragmentation level will, under most conditions, be laminar (Re < 10³) and \( f \approx 16/Re \). Above the fragmentation level the flow will be turbulent and \( f \approx f_0 \). For basaltic magma, frictional pressure loss over a wide range of ascent velocities is less than magma-static pressure loss and the flow dynamics will be more sensitive to changes in the density of the ascending magma than for eruptions where frictional pressure losses are larger. Consequently, under separated flow conditions there is potential for feedbacks between discharge rate, gas-to-melt flux ratios, and flow regime (Seyfried and Freundt, 2000; Guet and Ooms, 2006).

### 4.3.7 Permeable outgassing

The degassing history during ascent of an individual magma parcel may be complex and in many cases gas may separate from rising parcels of magma. We refer to this as outgassing, which may be a consequence of buoyant bubble rise, magma fragmentation, or permeable gas flow. Chapters 6 and 8 discuss outgassing associated with separated flow and high gas-to-melt flux ratios during Strombolian and perhaps Hawaiian eruptions. In some silicic eruptions high melt-to-gas flux ratios are also suggestive of decoupled gas flow (Edmonds et al., 2003). However, high viscosities and negligible bubble mobility require that outgassing is associated with magma permeability, presumably a consequence of coalescing bubbles that form a permeable network (Eichelberger et al., 1986; Klug and Cashman, 1996) and perhaps cracks and fractures produced by brittle magma deformation (Gonnermann and Manga, 2003; Gonnermann and Manga, 2005a; Tuffen et al., 2003). It has been suggested that if conduit walls are permeable, which is controversial (Boudon et al., 1998), then volatiles may also escape laterally from ascending magma into the conduit walls and permeable outgassing has the potential to modulate eruptive behavior (Jaupart and Allegre, 1991; Woods and Koyaguchi, 1994; Eichelberger, 1995).

Vesicularity-permeability measurements for volcanic rocks, and by inference magmas, are usually analyzed using the Kozeny–Carman equation (Carman, 1956) where permeability, \( k \), is given as

\[
k(\phi_b) = \chi (\phi_b - \phi_{bp})^\beta.
\]

(4.15)

Here \( \chi \) is an empirical constant and \( \phi_{bp} \) is the volume fraction of bubbles that corresponds to percolation threshold (Saar and Manga, 1999), with typical values of \( 2 \beta \geq 4 \). An uncontrovertial permeability model for vesicular magma remains elusive (Takeuchi et al., 2005; Wright et al., 2009). Furthermore, it appears that the nature of magma deformation plays a critical role in creating permeability (Okumura et al., 2008; Okumura et al., 2010).

Permeable outgassing can be modeled as flow through a porous medium by combining the continuity equation for the exsolved volatile phase with Darcy’s equation or, if the velocity of the gas phase becomes too large for inertial effects to be neglected, Forcheimer’s equation (Rust and Cashman, 2004).

### 4.4 Crystal nucleation and growth

Crystals nucleate due to undercooling, which is predominantly a consequence of \( \text{H}_2\text{O} \) exsolution. Nucleation rates determine the crystal number density and size distribution, which provide a record of the magma’s ascent history. During crystallization the volatile content of the residual melt phase increases because volatiles preferentially partition into the melt phase. This in turn affects bubble nucleation and growth, as well as melt viscosity.

Undercooling, the thermodynamic driving force for crystallization, is the chemical potential of \( \text{H}_2\text{O} \) in the melt and can be expressed as \( \Delta T \), the difference between actual temperature, \( T \), and the liquidus temperature, \( T_l \). Because the latter depends on composition and volatile content, bubble nucleation and growth will increase \( T_l \) and can result in sufficient undercooling for crystallization (Hammer, 2004). The rate of homogeneous nucleation of critical nuclei, \( I \) \( (\text{m}^{-3} \cdot \text{s}^{-1}) \), is given by classical nucleation theory as
\[ I = I_0 \exp \left( -\frac{\Delta G' + \Delta G_0}{k_B T} \right). \]  

(4.16)

Here \( I_0 \) is a reference nucleation rate, \( k_B \) is the Boltzmann constant, and \( \Delta G_0 \) is the activation energy for diffusion (James, 1985). \( \Delta G' \) is the free energy required to form a spherical nucleus of critical size and depends on the free energy, \( \sigma \), associated with the crystal-liquid interface. Therefore \( \sigma \), nucleation rate increases with decreasing \( \sigma \) (Mueller et al., 2000). The rate of heterogeneous nucleation also follows Eq. (4.16), but with a modified \( \sigma \) to account for the lower interfacial energies (Spohn et al., 1988). Because \( \sigma \) is relatively poorly constrained, estimates of crystal nucleation rates are typically associated with large uncertainties. Moreover, the validity of classical nucleation theory and its assumptions that (1) the interface between nucleus and melt is sharp; (2) the critical nucleus has the thermodynamic properties of the bulk solid; and (3) \( \sigma \) can be treated as a macroscopic property equal to the value for a planar interface, remain controversial.

Once a crystal nucleates its growth rate, \( Y \), is given by (Spohn et al., 1988)

\[ Y = Y_0 \left[ 1 - \exp \left( \frac{\Delta H \Delta T}{k_B T^2} \right) \right] \exp \left( -\frac{\Delta G_0}{k_B T} \right). \]

(4.17)

where \( Y_0 \) is a reference growth rate and \( \Delta H \) is the change in enthalpy between melt and crystalline phases. If empirical parameters such as \( T_i \) and \( \sigma \) are known, the above equations can be used to model magma crystallization. Alternatively, parameterizations for \( I \) and \( Y \) can be used to explore the dependence of eruption behavior on syneruptive crystallization (Melnik and Sparks, 1999). The prediction of crystal size distributions is possible through the use of the Avrami equation (Marsh, 1998)

\[ \phi_x = 1 - \exp \left(-k_v Y^3 t^4 I \right). \]

(4.18)

Here a given crystal volume fraction, \( \phi_x \), is assumed to correspond to thermodynamic equilibrium after crystallization over time, \( t \), and with \( k_v \) as a volumetric factor.

### 4.5 Magma rheology

Silicate melts form a disordered network of interconnected SiO\(_4\) tetrahedra where the self-diffusive motion of atoms, called structural relaxation, results in a continuous unstructured rearrangement of the molecular structure. In the presence of an applied stress, this molecular rearrangement results in a directional motion of SiO\(_4\) tetrahedra relative to one another and macroscopically manifests itself as viscous flow (Moynihan, 1995).

The intrinsic viscosity of silicate melts varies over orders of magnitude, even within a single volcanic eruption. It depends on the degree of polymerization, a function of chemical composition and volatile content. Within realistic ranges of compositional variability, eruptive temperature, and volatile content, the viscosity of basaltic melts may vary between about 10 and 10\(^4\) Pa s, and between about 10\(^4\) and 10\(^{12}\) Pa s or more for silicic melts (Fig. 4.5). Changes in viscosity during individual eruptions are especially pronounced at low pressures (depths <1 km), where most of the H\(_2\)O exsolves (Figs. 4.2 and 4.6). Magma rheology is discussed further in the context of lava flow emplacement in Chapter 5.

#### 4.5.1 The effect of dissolved volatiles and temperature

Dissolved water dissociates into molecular water and hydroxyl ions, thereby depolymerizing the melt. In rhyolitic melts the effect of water content can be tremendous and exceeds the effect of temperature (Fig. 4.5), but it is less pronounced for mafic magmas (Giordano and Dingwell, 2003). In contrast, viscosity is less affected by CO\(_2\) (Bourgue and Richet, 2001). Recent viscosity formulations applicable over a range of compositions, temperature, and water contents are discussed in Hui and Zhang (2007), Zhang et al. (2007), and Giordano et al. (2008).

#### 4.5.2 The effect of deformation rate

If the applied rate of deformation exceeds a certain threshold, the induced molecular motions of the melt are no longer compensated by random reordering of the melt structure. This is
manifested in shear thinning, a decrease in viscosity from its “relaxed” or Newtonian value and will result in brittle failure if deformation rates exceed the onset of shear-thinning by approximately two orders of magnitude (Webb, 1997, and references therein). The strain rate at which shear thinning is first observed can be defined in terms of the viscous relaxation time, \( \tau_r = \eta_0 / G_\infty \), as

\[
\dot{\varepsilon}_a \sim 10^{-3} \frac{G_\infty}{\eta_0}
\]

Here \( G_\infty \sim 10^{10} \) Pa is the shear modulus and \( \eta_0 \) is the Newtonian melt viscosity, measured at \( \dot{\varepsilon} \ll \dot{\varepsilon}_a \).

4.5.3 The effect of crystals
Crystals increase magma viscosity (Lejeune and Richet, 1995; Stevenson et al., 1996; Arbaret et al. 2007; Caricchi et al., 2007; Lavallee et al., 2007). For small crystal volume fractions, \( \phi_x < 10 - 30\% \), and low strain rates, the viscosity of the crystal-lin magma relative to the crystal-free melt, \( \eta_r = \eta / \eta_0 \), can be approximated as

\[
\eta_r = (1 - \phi_x / \phi_{acr})^\alpha.
\]

Here \( \phi_{acr} \) represents a critical volume fraction at which crystals start to impede the ability of the suspension to flow and \( \alpha \approx 2.5 \) (Llewellin and Manga, 2005). At \( \phi_x \) greater than some value, typically in the range of 10-50\%, crystals may come into contact creating a framework that provides a finite yield strength. Shear thinning and shear localization may occur as melt and crystals redistribute themselves to decrease the flow disturbance. It most likely occurs in a narrow zone near the conduit wall and may produce a plug flow that can manifest itself at the surface as volcanic spine. A viscosity formulation that is calibrated to a broad range of crystal
DYNAMICS OF MAGMA ASCENT IN THE VOLCANIC CONDUIT

content and strain rates (Fig. 4.7) is provided by Caricchi et al. (2007)

\[
\eta = \eta_0 \left[ \frac{1 + \left( \frac{\phi_b}{\phi_{bc}} \right)^3}{1 - \alpha \text{erf} \left( \frac{\sqrt{\pi} \phi_b}{2 \alpha \phi_{bc}} \left[ 1 + \left( \frac{\phi_b}{\phi_{bc}} \right)^2 \right] \right)} \right]^{-\delta_{bc}}.
\]

(4.21)

Here \( B = 2.5 \) and the remaining empirical parameters \( \delta, \alpha, \) and \( \beta \) are provided in Figure 4.7.

4.5.4 The effect of bubbles

The effect of bubbles on viscosity depends on whether the bubbles are able to deform and is a function of \( Ca = \eta_{0}^{5}R^{2}/\gamma \). If \( Ca \ll 1 \), bubbles remain nearly spherical and the viscosity is greater than that of the melt (Taylor, 1932). In contrast, for \( Ca > 0(1) \), bubbles can become elongated (Rust et al., 2003) and the viscosity decreases (Chapter 5; Bagdassarov and Dingwell, 1992; Manga et al., 1998; Lejeune et al., 1999; Stein and Spera, 2002). An expression for this dependence on \( Ca \) is given by Pal (2003) as

\[
\eta = \left( \frac{1 - 12 \eta_{0}^{5}Ca^{2}/5}{1 - 12 \eta_{0}^{5}Ca^{2}/5} \right)^{-\frac{4}{5}} \left( 1 - \phi_{b} \right)^{-\phi_{bc}},
\]

(4.22)

where \( \phi_{bc} \) is an empirical parameter that is conceptually analogous to \( \phi_{xcr} \). The strain-rate dependent rheology of melt with suspended bubbles and crystals is derived from a linear superposition of the individual dependencies (Thies, 2002). As illustrated in Figure 4.8, during laminar flow at a given pressure gradient, the presence of bubbles can increase or decrease mass discharge rate by almost one order of magnitude. Chapter 5 further considers the combined effects of bubbles and crystals on magma rheology (Section 5.2.2).

4.6 Magma fragmentation

During all explosive eruptions pyroclasts are ejected from the volcanic vent. Here we focus on “dry” magma fragmentation, which does...
not involve any interaction with non-magmatic water. The type and efficiency of the fragmentation process determines pyroclast size and how much magmatic gas is released per unit mass of magma. This, in turn, has implications for the volcanic jet, column, and plume that are produced when the gas-pyroclast mixture exits the volcanic vent (Chapters 8 and 9). Fragmentation is thought to occur when specific conditions are reached within the conduit. The proposed criteria for fragmentation include: (1) a critical bubble volume fraction; (2) a stress criterion; (3) a strain-rate criterion; (4) a potential energy criterion; and (5) an inertial criterion.

The critical volume fraction criterion is thought to arise from some form of instability within the thin bubble walls, once \( \phi_b \approx 0.75 \) is reached (Verhoogen, 1951; Sparks, 1978). However, magma fragments have vesicularities that range from 0 (obsidian) to >98% (reticulite) implying that a fragmentation criterion governed by a critical volume fraction cannot be generally applicable (though very high vesicularities may reflect post-fragmentation growth of bubbles).

The stress criterion is based on the view that fragmentation in high-viscosity magmas takes place when volatile overpressure, \( \Delta p_f \), exceeds the tensile strength of the melt and ruptures bubble walls (Alidibirov, 1994; Zhang, 1999). Spieler et al. (2004) provide a formula with good fit to a broad range of experimental data

\[
\Delta p_f = 10^6 \text{Pa} / \phi_b
\]

that has been modified by Mueller et al. (2008) to account for permeable gas flow.

The strain-rate criterion is based on the observation that silicate melts can fragment if deformation rates exceed the structural relaxation rate of the melt at \( \varepsilon \sim 10^3 \text{ s}^{-1} \) (Section 4.5.2). It is thought to be the consequence of a rapid decompression that forces rapid bubble growth (Papale, 1999). Ittai et al. (2010) have argued that both the stress criterion and the strain-rate criterion are equivalent, because of their mutual dependency on shear modulus and magma rheology.

Namiki and Manga (2005) suggested a potential energy criterion, based on the observation that during rapid decompression \( \phi_b \) and \( \Delta p_f \) determine the expansion velocity of a bubbly liquid. Potential energy depends on both \( \phi_b \) and \( \Delta p_f \) (Mastin, 1995), with fragmentation taking place above some threshold.

Hydrodynamic or inertial fragmentation should be predominantly associated with mafic magma, where rapid decompression results in rapid bubble growth, inertial stretching, and hydrodynamic breakup (Shimozuru, 1994; Zimanowski et al., 1997; Villermaux, 2007). A criterion for inertial fragmentation is based on the Reynolds number, \( R_e \), and Weber number, \( W_e \), of the expanding magma (Namiki and Manga, 2008)

\[
R_e = \frac{\rho (1 - \phi_b) v^2 L}{3 \eta_0} > O(1)
\]

and

\[
W_e = \frac{\rho (1 - \phi_b) v^2 L}{\gamma} \gg 1.
\]

Here \( L \) is a characteristic length scale and \( v \) is the expansion velocity of the vesicular magma, which can be obtained from the bubble growth rate.

### 4.7 Modeling of magma ascent

The past decade has seen much progress in modeling magma ascent (Sahagian, 2005). In contrast to the modeling of effusive eruptions, explosive eruptions require a criterion to estimate the depth of magma fragmentation, and merging of a model for the bubbly flow region below the fragmentation level with one for the gas-pyroclast region above. Flow in the gas-pyroclast flow region is compressible and is typically modeled as one-dimensional with a parameterization for wall friction and granular stresses during highly turbulent flow (Wilson and Head, 1981; Dobran, 1992; Koyaguchi, 2005).

Conduit models may be steady or time-dependent. The steady approximation is
reasonable when the timescales over which changes in eruption rate and geometry exceed ascent times (Slezin, 2003). Time-dependent behavior becomes important when ascent times are similar to the timescale over which boundary conditions change. This can be a consequence of coupling between magma flow and (1) magma storage; (2) conduit wall elasticity; (3) magma compressibility; (4) magma outgassing; (5) viscous dissipation; and (6) magma rheology. A departure from one-dimensional models is necessary if lateral variations in properties are important, for example due to shear-rate dependent viscosity, shear localization, viscous dissipation, or permeable outgassing into conduit walls.

Dispersed two-phase flow below the fragmentation level is typically treated as homogeneous, using an appropriate magma viscosity. Above the fragmentation level, the homogenous flow assumption is sufficient if fragments are $\Sigma 10^{-4}$ m in diameter (Papale, 2001). However, treating the flow as separated for larger fragments results in a more accurate characterization of ascent velocity (Dufek and Bergantz, 2005). In the case of separated flow above or below the fragmentation level there are two classes of models, those that explicitly incorporate interfacial momentum exchanges and those that do not (Dobran, 2001). Separated flow models with no interfacial momentum exchange treat the flow as single-phase, but incorporate closure laws for the gas volume fraction and frictional pressure loss of the two-phase mixture (Wilson and Head, 1981). Separated models with interface exchange allow for different gas and liquid (pyroclast) velocities and flow directions. However, a priori assumptions about the flow regime are required for these closure laws.

4.7.1 Steady homogeneous flow in one dimension

Below the fragmentation level homogeneous flow models advect bubbles passively with the melt. That is $U_1 \ll U_m$ and $u = U_m = U_p$, where $U_b$ is the bubble velocity. Together with the assumption of constant mass flux throughout the conduit, $Q$, the equation of mass conservation becomes

$$\frac{dQ}{dz} = \frac{d}{dz}(\rho u A) = 0. \quad (4.26)$$

Here $A$ is conduit cross-sectional area and $\rho$ is the bulk density of bubbly magma, given by $\phi_b \rho_s + (1 - \phi_b) \rho_m$. The value of $\rho_s$ may be obtained from the ideal gas law, but use of another equation of state may be more appropriate at high pressures (Kerrick and Jacobs, 1981). $\rho_m$ can be obtained from empirical formulations (Lange, 1994), but in many cases the melt phase is assumed to be incompressible. The bubble volume fraction $\phi_b$ depends on $\rho_s$ and the amount of exsolved volatiles. If it is assumed that bubble growth is limited by solubility, the amount of exsolved volatiles per unit mass of melt is equal to the initial volatile concentration minus the equilibrium solubility at the given pressure. However, in many eruptions bubble growth is viscously and/or diffusively limited, requiring a concurrent calculation for bubble growth. If permeable outgassing is important, Eq. (4.26) may include a sink term based on a suitable porosity-permeability relation and Darcy’s law or Forcheimer’s equation for radial gas flow to the conduit walls and/or upward gas flow to the vent (Jaupart and Allegre, 1991).

Equation (4.26) states that mass flux is constant and volume expansion of the magma by bubble growth must be balanced by acceleration and/or an increase in conduit cross-sectional area. Assuming a vertical conduit of cylindrical shape, conservation of momentum is governed by (Mastin and Ghiorso, 2000; Dobran, 2001)

$$pu \frac{du}{dz} = \frac{dp}{dz} - \left( \frac{dp}{dz} \right)_\rho - \left( \frac{dp}{dz} \right)_\eta = \frac{dp}{dz} - \rho g - pu^2 \frac{f}{a}. \quad (4.27)$$

Expanding Eq. (4.26) and substituting for $du/dz$ in Eq. (4.27) gives

$$-\frac{dp}{dz} = \rho g + pu^2 \frac{f}{a} - \frac{pu^2}{A} \frac{dA}{dz} - u^2 \frac{d\rho}{dz}. \quad (4.28)$$

Equation (4.28) indicates that the change in magma pressure with respect to height depends, from left to right, on magma-static
pressure loss, frictional pressure loss, change in conduit diameter, and change in magma density. Assuming isentropic conditions (denoted by subscript \( S \)), that is an infinitesimal reversible pressure change and negligible time for heat transfer, then

\[
\frac{dp}{dz} = \left( \frac{dp}{dp_s} \right)_S \frac{dp}{dz} = \frac{1}{c^2} \frac{dp}{dz}.
\]

(4.29)

Here \( c \) is the speed of sound in the magma and using the Mach number, \( M = \frac{u^2}{c^2} \), Eq. (4.28) becomes

\[
-\frac{dp}{dz} (1 - M^2) = \rho g + \rho u^2 f - \rho u^2 \frac{dA}{dz}.
\]

(4.30)

Equations (4.26) and (4.30) are applicable to homogeneous flow above or below the fragmentation level and can be solved with standard numerical methods (Press et al., 1992). There are two solution approaches: (1) set \( \frac{dp}{dz} \) to be constant, for example lithostatic, and then \( \frac{dA}{dz} \) and \( u \) can be calculated, or (2) if \( A \) is known, then conduit entrance and exit pressures are specified and \( \frac{dp}{dz} \) and \( u \) can be calculated (Wilson and Head, 1981; Giberti and Wilson, 1990; Dobran, 1992; Mastin and Ghiorso, 2000).

For \( M < 0.3 \) the effect of magma compressibility on the flow can be neglected, whereas for \( M > 0.3 \) magma compressibility contributes to the change in pressure of the ascending magma. If \( p = 1 \) atm at the vent exit, then the right-hand side of Eq. (4.28) has to go to zero to avoid a singular solution as \( M \) approaches 1. Therefore, the conduit flares upward with

\[
\frac{da}{dz} = \frac{1}{2} \left( g \frac{a}{u^2} + f \right) > 0.
\]

(4.31)

This result can be derived by setting the right-hand side of Eq. (4.28) to zero with \( A = \pi a^2 \) (Mastin and Ghiorso, 2000). If Eq. (4.31) is satisfied at \( M < 1 \) and the conduit above continues to flare, the magma will decelerate. On the other hand if \( M = 1 \), the magma will accelerate to supersonic velocities and \( p \) will continue to decrease, potentially to values below 1 atm. If that is the case, a shock wave will adjust the flow back to 1 atm at the vent exit. If the exit pressure exceeds 1 atm, the flow is “choked,” that is \( M = 1 \) and the exit velocity is equal to \( c \), the speed of sound in the gas-pyroclast mixture.

In many cases the assumption of isothermal conditions is reasonable; if not, an equation for conservation of energy has to be added (Mastin and Ghiorso, 2000; Dobran, 2001)

\[
dH + udu + gdz = 0.
\]

(4.32)

Here \( H \) is the specific enthalpy of the melt-gas mixture from which the magma temperature can be calculated.

### 4.7.2 Steady separated flow in one dimension

As discussed in Section 4.3.5, the flow of the separated melt and gas phases below the fragmentation level may occur under different regimes. Criteria that determine the transitions between individual flow regimes, as well as formulations for interphase drag forces, \( F_{mg} \), need to be specified. Existing models do not predict the occurrence of flow-regime transitions, but rather rely on a-priori assumptions about the occurrence of flow transitions, equivalent to the flow transition associated with magma fragmentation. For example, the model of Dobran (1992) is based on the assumption of a constant mass flow rate through the conduit

\[
\frac{d}{dz} (Q_s + Q_{gd}) = 0
\]

(4.33)

and

\[
\frac{d}{dz} (Q_m - Q_{gd}) = 0.
\]

(4.34)

Here subscript \( g \) denotes the exsolved volatile phase, \( gd \) the dissolved volatile phase, and \( m \) the melt phase. As in the homogeneous flow model it is possible to account for permeable gas loss through an additional flux term on the right-hand side of Eq. (4.36). The momentum equations for the exsolved volatile and the melt phases are, respectively
\[ \rho_s u_s \phi_s \frac{du_s}{dz} = -\phi_b \frac{dp}{dz} - F_{wg} - F_{wg} - \rho \Phi g \quad (4.35) \]

and

\[ \rho_m u_m (1 - \phi) \frac{du_m}{dz} = -(1 - \phi) \frac{dp}{dz} - F_{wg} - F_{wm} \quad (4.36) \]

Here \( F_{wg} \) and \( F_{wm} \) are the wall drag forces for the phase that is assumed to be in contact with the conduit wall. For example, Dobran (1992) used \( F_{wm} = (dp/dz) \), \( F_{wg} = 0 \) below the fragmentation level and \( F_{wm} = 0 \), \( F_{wg} = (dp/dz) \) above.

### 4.7.3 Two-dimensional flow

If magma flow is radially non-uniform, a two-dimensional modeling approach can provide additional insight. Cases where two-dimensional modeling has been important involve non-Newtonian magma rheology and shear heating, as well as permeable outgassing and heat loss through the conduit walls.

The direct approach is to solve the Navier-Stokes equations in two cartesian or axisymmetric cylindrical coordinates. For axisymmetric homogeneous flow, conservation of momentum is given by (Bird et al., 1960)

\[ \rho \frac{\partial u_r}{\partial t} + \rho u_r \frac{\partial u_r}{\partial r} + \rho u_z \frac{\partial u_z}{\partial z} = -\frac{\partial p}{\partial r} - u_r \frac{\partial (r \tau_{rz})}{\partial r} - \frac{\partial \tau_{rz}}{\partial z} \]

(4.37)

and

\[ \rho \frac{\partial u_z}{\partial t} + \rho u_r \frac{\partial u_z}{\partial r} + \rho u_z \frac{\partial u_z}{\partial z} = -\rho g - \frac{\partial p}{\partial z} - u_z \frac{\partial (r \tau_{rz})}{\partial r} - \frac{\partial \tau_{rz}}{\partial z}, \]

(4.38)

with stress tensors

\[ \tau_{rz} = -2\eta \frac{\partial u_r}{\partial r} - \lambda \left( \frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{\partial u_z}{\partial z} \right), \]

(4.39)

\[ \tau_{rz} = -2\eta \frac{\partial u_z}{\partial z} - \lambda \left( \frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{\partial u_z}{\partial z} \right), \]

(4.40)

Here \( \lambda = \frac{2}{3} \eta - K \), where \( \eta \) is the viscosity of the magma. \( K \) is called the bulk viscosity, which accounts for the compressibility of the two-phase magma mixture (Massol et al., 2001) and is often assumed negligible. Various simplifications of Eqs. (4.37) and (4.38) can be employed. If the flow is laminar \( (Re < 10^3) \), which is usually the case below the fragmentation level, the second and third terms on the left-hand side can be neglected. Furthermore, if the flow is steady the time derivatives are zero. Assuming isothermal conditions, the momentum equations are complemented by an equation for the conservation of mass

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u_r) + \frac{\partial (\rho u_z)}{\partial z} = 0. \]

(4.42)

Examples of recent two-dimensional conduit models are given by Collier and Neuberg (2006); Costa et al. (2007a,b); Hale (2007); Hale and Muehlhaus (2007).

If vertical flow is a reasonable approximation \( (u = u_z, u_r = 0) \), Eqs. (4.37) and (4.38) become

\[ \frac{\partial}{\partial r} (r \tau_{rz}) = -r \frac{\partial p}{\partial z}, \]

(4.43)

and upon integration

\[ \tau_{rz} = -\eta \frac{\partial u_r}{\partial r} - \frac{r}{2} \frac{\partial p}{\partial z}. \]

(4.44)

The relative ease with which Eq. (4.44) can be solved makes it easier to model more complex magma rheologies (Costa and Macedonio, 2005; Mastin, 2005; Vedeneeva et al., 2005), and to include subgrid scale calculations of bubble growth, where the model for bubble growth is discretized at a smaller scale than the conduit model itself (Fig. 4.9; Gonnemann and Manga, 2003, 2007).

To investigate the role of shear heating or magma cooling near the conduit walls, an energy equation has to be included. Neglecting thermal diffusion in the vertical direction, the energy equation is (Bird et al., 1960)
\[ \frac{\partial T}{\partial t} = \frac{\kappa}{\rho c_p} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - \frac{1}{\rho c_p} \left( \tau_{rr} \frac{\partial u_r}{\partial r} \right), \]  

(4.45)

where \( c_p \) is the specific heat and \( \kappa \) is the thermal conductivity of the magma. The first term on the right-hand side accounts for diffusion of heat, and the last term represents viscous heating.

4.7.4 Coupling the conduit and the magma chamber

The boundary conditions for the momentum equations at the bottom of the conduit can be a specified magma flux or specified pressure. Either may be transient because of changes in chamber pressure, in turn a consequence of magma withdrawal or replenishment, as well as compressibility of the vesicular magma and surrounding rock. A simple equation for chamber pressure is obtained by combining conservation of mass with an equation of state (Denlinger and Hoblitt, 1999; Wylie et al., 1999)

\[ \frac{dp}{dt} = \frac{1}{\rho \beta_e} V (Q_{in} - Q_{out}). \]  

(4.46)

Here \( V \) is the volume of the magma chamber or conduit, \( Q_{in} \) is mass recharge rate, \( Q_{out} \) is mass discharge rate, and \( \beta \) is the effective compressibility. The latter incorporates both magma compressibility and wall-rock elasticity, and is usually much smaller than the compressibility of vesicular magma (Huppert and Woods, 2002; Woods et al., 2006). Typically it is the feedbacks between magma chamber and conduit that result in complex and unsteady eruptive
behavior (Melnik and Sparks, 1999; Macedonio et al., 2005).

4.8 What conduit models have taught us

Figure 4.10 shows the relationship between magma composition (in essence a proxy for viscosity), eruption rate (or equivalently ascent rate), and eruption style. The boundaries shown are not “hard” in that uncertainties in thresholds for fragmentation, unsteady flow, variation in volatile content, conduit geometry, two-dimensional flow, and crystallization are not addressed. Overall these boundaries are meant to provide insight into the conditions under which certain processes are expected to dominate and to illustrate their relationships with eruption style.

4.8.1 Subplinian and plinian eruptions

Subplinian and plinian eruptions are characterized by brittle magma fragmentation at mass discharge rates on the order of $10^6$ kg s$^{-1}$ (subplinian) and $10^6$–$10^8$ kg s$^{-1}$ (plinian), sustained over several hours (Chapter 8). Syneruptive volatile exsolution and crystallization can lead to viscosity-limited bubble growth, large frictional pressure losses, and large decompression rates (Papale and Dobran, 1993, 1994), resulting in gas overpressure, supersaturation, and bubble nucleation (Massol and Koyaguchi, 2005), as well as fragmentation (Papale, 1999).

Whether fragmentation takes place depends on the conditions of magma ascent. However, fragmentation also feeds back on eruption dynamics. For example, the presence of pumice fragments after fragmentation, as opposed to complete fragmentation to ash, means that not all compressed gas contained in bubbles may be released, resulting in ascent velocities and pressures that are lower than if all gas were released. Incomplete outgassing of coarse fragments also increases the density of the erupting gas-pyroclast mixture, with consequences for the dynamics of the eruption column (Chapter 8, Papale, 2001). Furthermore, the granular stress
induced by >~1 cm-size pyroclasts decreases exit velocities and enhances the lateral inhomogeneity in clast-size distribution throughout the conduit (Dufek and Bergantz, 2005).

The strong dependence of viscosity on temperature in silicic magmas is of considerable importance for shear heating near the conduit walls (Rosi et al., 2004; Polacci et al., 2005). The resulting viscosity reduction will change the flow profile toward a more plug-like flow and reduce frictional pressure loss. Consequently, the discharge rate increases for a given pressure drop between chamber and surface, and hence affects the depth of fragmentation and allows for multiple stable discharge rates (Costa and Macedonio, 2002; Mastin, 2005; Vedeneeva et al., 2005; Costa et al., 2007).

Eruption intensity may correlate with pre-eruptive H\textsubscript{2}O content and magma composition (Papale et al., 1998; Polacci et al., 2004; Starostin et al., 2005). However, changes in CO\textsubscript{2} content can have the opposite effect of H\textsubscript{2}O (Papale and Polacci, 1999; Ongaro et al., 2006). The transition from explosive to effusive eruptions in silicic magmas is thought to occur as magma ascent rates decrease, which allows more time for permeable outgassing and bubbles grow at near equilibrium conditions (Jaupart and Allegre, 1991; Woods and Koyaguchi, 1994; Gonnermann and Manga, 2007).

4.8.2 Strombolian eruptions

Typical strombolian eruptions (Chapter 6) consist of prolonged periods of impulsive explosions that typically eject 0.01-10 m\textsuperscript{3} of pyroclastic material and volcanic gases at about 100 m s\textsuperscript{-1}. Mass balance considerations imply that much of the gas is derived from a larger volume of magma than is erupted, a consequence of slow magma ascent rates (<0.01–0.1 m s\textsuperscript{-1}). This permits significant bubble coalescence to form gas slugs and/or accumulations of bubbles, which decouple from the melt and rise to the surface where they burst. Aside from thermal modeling (Giberti et al., 1992), some numerical conduit models that have dealt directly with details of strombolian eruptions are Wilson and Head (1981); Parfitt and Wilson (1995); James et al. (2008); O’Brien and Bean (2008); D’Auria and Martini (2009); James et al. (2009); Pioli et al. (2009).

4.8.3 Hawaiian eruptions

Hawaiian eruptions are characterized by gas-rich fountains of basaltic magma sustained for hours to days at mass discharge rates of the order of 10\textsuperscript{4} to >10\textsuperscript{6} kg s\textsuperscript{-1} (Houghton and Gonnermann, 2008). Fountains reach heights that exceed hundreds of meters producing ash and pyroclasts, presumably through hydrodynamic fragmentation (Namiki and Manga, 2008). Microtextural studies of pyroclasts indicate less open-system gas loss than for strombolian clasts (Polacci et al., 2006).

Conduit models by Wilson and Head (1981) predict that magma ascent in hawaiian eruptions is homogeneous, with dispersed bubbles, so that rapid decompression and shallow H\textsubscript{2}O exsolution lead to expansion-driven acceleration and the formation of a sustained magma fountain above the vent (Parfitt et al., 1995; Parfitt, 2004). The transition from hawaiian to strombolian behavior is thought to occur at magma rise speeds of less than about 1 m s\textsuperscript{-1}, with a correlation between required rise speed and initial magmatic volatile content (Parfitt and Wilson, 1995). Alternatively, based on analog laboratory experiments it has been suggested that bubbles accumulate to form a magmatic foam, whose collapse produces an annular gas-melt flow within the conduit (Chapter 6; Jaupart and Vergniolle, 1988).

4.8.4 Effusive eruptions

Large volumes of basaltic magma often erupt effusively over prolonged periods of time, at rates up to 10\textsuperscript{4} kg s\textsuperscript{-1} (Wolfe et al., 1987). Silicic magmas of high viscosity may erupt effusively to form lava domes and coulees with time-averaged eruption rates ranging from 10\textsuperscript{-1} to 10\textsuperscript{4} kg s\textsuperscript{-1} (Pyle, 2000). The resulting low ascent rates (\textgreek{Σ} 0.01 m s\textsuperscript{-1}) are thought to facilitate permeable outgassing (Eichelberger et al., 1986; Jaupart and Allegre, 1991; Woods and Koyaguchi, 1994; Massol and Jaupart, 1999). Sparks (1997) attributed the occurrence of time-dependent behavior during dome-forming eruptions to feedbacks associated with volatile-dependent magma viscosity, magma degassing, outgassing, and crystallization. Conduit models suggest that large
changes in discharge rate at relatively small changes in boundary conditions are the consequence of such feedbacks (Melnik and Sparks, 1999, 2005). They also demonstrate how non-linear feedbacks produce different steady discharge rates for the same boundary conditions but different initial conditions, perhaps explaining how transitions between effusive and explosive behavior occur (Melnik et al., 2005; Clarke et al., 2007).

For long-lived dome-forming eruptions, magma recharge is often treated as constant over part of the eruption. However, transient eruptive behavior can ensue as a direct consequence of compressibility of the vesicular magma and/or surrounding wall rock (Meliaux and Jaupart, 1995; Denlinger, 1997; Huppert and Woods, 2002; Costa et al., 2007a). Feedbacks may be such that pressure build-up drives the system into a state where discharge rate exceeds recharge rate. With time, pressure decreases until the cycle repeats or is dynamically damped toward a stable state (Barmin et al., 2002). High shear stress at the conduit walls, in conjunction with shear-thinning rheology, may result in shear localization and brittle deformation and this may in turn enhance permeability and outgassing. Conduit models have also explored how brittle deformation may be associated with stick-slip behavior at the conduit walls and shallow seismicity, especially for magmas with high crystal content (Denlinger and Hoblitt, 1999; Goto, 1999; Collier and Neuberg, 2006; Iverson et al., 2006; Hale, 2007).

### 4.9 Summary

Conduit models of magma ascent help us understand which mechanisms and processes are important for a broad range of effusive and dry explosive volcanic eruptions (Fig. 4.10). Some important open questions are:

1. What causes the observed unsteady behavior during sustained explosive eruptions? This will require models capable of exploring time-dependent dynamical feedbacks within the conduit, coupling between conduit and subaerial flow, and incorporation of bubble nucleation.

2. The dynamics of magma flow within the volcanic conduit during strombolian and hawaiian eruptions remains controversial. The main challenge here is the modeling of the separated two-phase flow, where the topology of the flow can change drastically and rapidly in both space and time.

3. Magma flow in the volcanic conduit is affected by processes that occur over a wide range of length scales, from the fluid dynamics of magma ascent over kilometers to crystal/bubble nucleation and growth, as well as fragmentation over millimeter to micrometer scales. Accounting for these different scales requires the development of subgrid-type models capable of coupling explicit modeling of the various small-scale processes with fluid dynamical modeling of magma flow.

4. Eruption triggering is perhaps of foremost importance in terms of hazard mitigation. Integration of geodetic and remote-sensing observations with models of magma flow, supply, and storage is thus highly desirable. A major challenge is the incorporation of geological (tectonic and structural) complexities, which may require a departure from idealized one- or two-dimensional model geometries, as well as coupling between tectonically produced stresses and stresses within the magmatic system.

### 4.10 Notation

- $a$: conduit radius (m)
- $A$: cross-sectional area of conduit ($m^2$)
- $B$: empirical constant
- $c$: speed of sound in magma ($m/s$)
- $c_p$: specific heat ($J/kg\cdot K$)
- $C_o$: concentration of dissolved CO$_2$ (ppm)
- $C_i$: mass fraction of dissolved volatile $i$
- $C_w$: concentration of dissolved H$_2$O (wt.%)
- $D$: diffusivity of dissolved volatile in melt ($m^2/s$)
- $D_{CO_2}$: diffusivity of CO$_2$ dissolved in melt ($m^2/s$)
Acknowledgments

The authors thank B. F. Houghton for stimulating discussions, as well as J. E. Hammer, L. Karlstrom, and M. Rudolph for constructive comments on the manuscript. Both authors were supported by NSF while preparing this review.

References


**Exercises**

Consider two magmas, a basalt (η = 1 Pa s, ρ = 2800 kg m⁻³) and rhyolite (η = 10⁶ Pa s, ρ = 2600 kg m⁻³), rising in a conduit. Assume a conduit radius of 10 m and 1 m for the rhyolite and basalt, respectively. Assume a pressure gradient driving magma ascent of 500 Pa m⁻¹. For both cases:

1. Calculate the velocity as a function of radial position in the conduit.
2. Evaluate the strain rates at the conduit walls. Are the strain rates large enough that the melt might be shear-thinning or undergo structural failure?
3. Calculate the capillary number for a 1 cm radius bubble. Will the bubbles become deformed by the ascending magma?
4. Calculate the rise speed of a 1 cm bubble relative to the surrounding melt. Given this speed, what flow regime is likely to characterize the magma (see Fig. 4.4).