

Thermal conductivity of corundum and periclase and implications for the lower mantle

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Abstract. The pressure and temperature dependent thermal conductivity k of Al_2O_3 and MgO is determined for mantle and core conditions. We model the pressure dependence by relating k to compression, either linearly or with a power law relationship; we assume k is inversely proportional to temperature as expected in nonmetals at temperatures greater than a few hundred Kelvin. In order to determine model parameters, we use experimental data collected in the laser-heated diamond-anvil cell at pressures of 58 and 125 GPa, combined with a model for heat transfer in the diamond cell. The thermal conductivities predicted by the two models differ by less than 30% and are in good agreement with results obtained from independent shock experiments. Throughout the lower mantle we find that $k \sim 5 - 12 \text{ W/mK}$, consistent with geodynamical constraints and previous theoretical estimates, and is nearly constant along an adiabat.

Introduction

The thermal conductivity of lower mantle materials affects the structure, thickness, and dynamics of boundary layers near the core-mantle boundary [Stacey and Loper, 1983; Brown, 1986; Leitch, 1995; Manga and Jeanloz, 1996b] and thus our interpretation of seismic observations of structure in the lower mantle and the D" region above the core-mantle boundary [Lay and Helmberger, 1983; Wysession *et al.*, 1994; Garnero and Helmberger, 1995; Dziewonski, 1995]. Combined with estimates of temperatures in the core based on high-pressure melting experiments [e.g., Williams *et al.*, 1987, 1991; Boehler, 1993; Saxena *et al.*, 1994], conductivity measurements place constraints on the amount of heat transferred into the base of the mantle, which can be compared with geodynamic estimates of heat flux from the core into the mantle [e.g., Sleep, 1990; Davies, 1993]. The rate of heat transfer from the core compared with heat production within the mantle has a large effect on the nature of mantle convection [Bercovici *et al.*, 1989]. In addition, the Rayleigh number, which determines the "vigor" of convection and the thermal evolution of the Earth, depends on the thermal conductivity in boundary layers.

Unfortunately, the thermal conductivity of mantle minerals is among the most difficult material properties to measure at high pressures and temperatures. Experimental measurements have only been made for pressures less than 5 GPa [e.g., Fujisawa *et al.*, 1968; Kieffer, 1976; Kieffer *et al.*, 1976; Horai and Susaki, 1989; Chai *et al.*, 1996]. For metals, in which electron transport dominates, thermal conductivity can be inferred from electrical conductivity measurements using the Wiedeman-Franz-Lorenz relation; this does not work for dielectrics, however, in which thermal conductivity is dominated by phonon transport. Although theoretical models of the pressure dependence can be derived [e.g., Roufosse and Jeanloz, 1983; Brown, 1986; Anderson, 1989, pp. 129-132; Poirier, 1991, pp. 185-192], the validity of such predictions has not been verified experimentally for pressures greater than about 5 GPa.

Here we determine the pressure and temperature dependence of the thermal conductivity of Al_2O_3 and MgO based on experiments conducted in the laser-heated diamond-anvil cell at pressures of 58 and 125 GPa. We argue that the thermal conductivity of lower-mantle minerals, such as silicate perovskite, should not differ greatly from that determined here, because the mechanism of heat conduction is the same (phonon transport), and Debye temperatures and compressions are similar for the oxides we consider. Finally, we compare our results with low-pressure measurements, values inferred from geodynamic constraints, and values

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Paper number 96JB02696.
0148-0227/97/96JB-02696\$09.00

obtained by modeling high-pressure and temperature shock experiments.

Thermal Conductivity of Dielectrics: Theoretical Considerations

In dielectric crystals, heat is carried by phonons, and finite conductivity is due to the coupling of phonon modes. The process can be modeled in terms of density fluctuations causing scattering among thermal-acoustic waves, although a proper description must be couched in terms of anharmonic interactions among the phonon modes within the crystal structure [Peierls, 1979]. A thorough discussion of thermal conductivity is provided by Berman [1976], and here we describe only the most basic ideas in order to provide a basis for the models we use to fit experimental observations.

Temperature Dependence

Thermal vibrations can be thought of as a phonon gas, and from the kinetic theory of gases,

$$k = \frac{1}{3} C \bar{v} d \quad (1)$$

where k is the thermal conductivity, C is the specific heat per unit volume, \bar{v} is the velocity of acoustic waves, and d is the mean-free path between collisions. In order to assist the reader, a notation list is included at the end of the paper.

At high temperatures, typically greater than a few hundred kelvin or about half the Debye temperature Θ_D , d is limited by the interactions between phonons,

$$d = \frac{D}{\alpha \gamma T}, \quad (2)$$

where D is the interatomic distance, α is thermal expansion, γ is the thermodynamic Grüneisen parameter, and T is temperature [Dugdale and MacDonald, 1955]. Combining (1) and (2), and using the relation $\alpha = C\gamma/K$, where K is the bulk modulus, we obtain

$$k = \frac{D \bar{v} K}{3 \gamma^2 T}. \quad (3)$$

The temperature dependence of the various terms on the right-hand side of (3) are typically small compared with $1/T$.

For most dielectrics k is in fact proportional to $1/T$ at high temperatures, in good agreement with (3) [e.g., Touloukian and Ho, 1971], although experimental techniques for determining thermal conductivity measure an effective conductivity that can be influenced by radiative transfer at temperatures above 1000–1500 K. At temperatures lower than about half the Debye temperature, k decreases more rapidly than $1/T$ with increasing temperature, with $k \propto \exp[c\Theta_D/T]$ where c is a constant; at still lower temperatures $k \propto T^3$ [Berman, 1976, pp. 56–58].

Measured and calculated temperature distributions in a laser-heated perovskite sample inside the diamond cell at 56 GPa are consistent with $k \propto 1/T$ [Li et al., 1996]. We assume

$$k_{0T} = a/T \quad \text{at } P = 0, \quad (4)$$

where we use the subscript $0T$ to denote the value at $P = 0$ and temperature T .

Pressure Dependence

The relative change in thermal conductivity due to compression can be evaluated from (3), assuming $\partial \ln \bar{v} / \partial \ln \rho \sim \gamma$ [Anderson, 1989, p. 131],

$$\left(\frac{\partial \ln k}{\partial \ln \rho} \right)_T \sim -\frac{4}{3} + \gamma + \frac{\partial \ln K}{\partial \ln \rho} - 2 \frac{\partial \ln \gamma}{\partial \ln \rho} \sim 7. \quad (5)$$

The value of 7 is based on “typical” values. For example, $(\partial \ln K / \partial \ln \rho)_T = 3.80$ and 4.34 [Anderson, 1989, p. 117] and $\gamma = 1.54$ and 1.32 [Anderson and Isaak, 1995] for MgO and Al_2O_3 , respectively; $(\partial \ln \gamma / \partial \ln \rho)_T \sim -1$ [Anderson, 1989, p. 118]. Given the range of possible values for terms on the right-hand side of (5), $\partial \ln k / \partial \ln \rho$ can range from about 4 to 9. Assuming (i) the phonon-scattering processes responsible for thermal conduction do not change as a function of pressure, and (ii) the relative dispersion of acoustic modes does not change much with compression (i.e., modeling the average of the shear and transverse modes via the bulk velocity is a reasonable approximation for evaluating the effects of compression on phonon scattering), (5) should hold throughout the mantle.

Here we will assume two different models for the pressure dependence (modeled through density changes) of thermal conductivity. First, we consider a linearized model [e.g., Roufosse and Jeanloz, 1983], $\delta k/k \propto \delta \rho/\rho$, that is

$$\frac{k - k_{0T}}{k_{0T}} = \phi \frac{\rho - \rho_{0T}}{\rho_{0T}}, \quad (6)$$

where we again use the subscript $0T$ to denote values at $P = 0$ and temperature T . Experimental results for a wide range of materials at pressures up to 5 GPa, including geologically relevant materials such as olivine, show that the effect of compression on thermal conductivity is well described by the linear model with $\phi \sim 7$ [e.g., Roufosse and Jeanloz, 1983, Figure 1].

At lower-mantle pressures, the density change due to compression will be sufficiently large that the linearized model may become a poor approximation. Thus, we also consider a power-law model which follows from equation (5),

$$\frac{k}{k_{0T}} = \left(\frac{\rho}{\rho_{0T}} \right)^\psi. \quad (7)$$

From (5), we expect the coefficients to be $\phi \sim 7$ and $\psi \sim 7$ for the linear and power law models, respectively.

In order to convert (6) and (7), which relate thermal conductivity and compression, to an analytic expression

involving pressure, we use the Murnaghan equation of state

$$\frac{\rho}{\rho_0} = \left(1 + \frac{K'_0}{K_0} P\right)^{1/K'_0}, \quad (8)$$

where K_0 is the bulk modulus, $K'_0 = dK_0/dP$, and the subscript 0 denotes conditions of $P = 0$ and room temperature. For compressions corresponding to the core-mantle boundary, the pressures predicted by the Murnaghan and second-order Birch-Murnaghan equations differ by 3% [e.g., Poirier, 1991, p. 65]. The effect of temperature on changes of thermal conductivity due to thermal expansion is small compared with the $1/T$ behavior discussed above and can be reasonably neglected; this approximation corresponds to replacing $\rho_0 T$ in (6) and (7) with ρ_0 .

Thus, our pressure- and temperature-dependent models of thermal conductivity are given by

$$k = \frac{a}{T} \left[\phi \left(1 + \frac{K'_0}{K_0} P\right)^{1/K'_0} - \phi + 1 \right] \quad (9)$$

and

$$k = \frac{a}{T} \left(1 + \frac{K'_0}{K_0} P\right)^{\psi/K'_0} \quad (10)$$

for the linear and power law models, respectively.

For small compressions, $P \ll K_0/K'_0 \sim 50$ GPa, k increases linearly with pressure; for example, the linear model predicts

$$k \sim \frac{a}{T} \left(1 + \phi \frac{P}{K_0}\right). \quad (11)$$

The measured thermal conductivity of MgO at $P < 5$ GPa is consistent with a linear pressure dependence [Yukutake and Shimada, 1978].

Thermal Conductivity Determined From Diamond Cell Experiments

We calculate the thermal conductivity of Al_2O_3 and MgO from data collected by Saxena *et al.* [1994] during a study of the melting of iron in the laser-heated diamond-anvil cell. In their experiment an iron foil is sandwiched between layers of MgO and Al_2O_3 , as illustrated in Figure 1a, and the foil is heated by a laser. Since the thermal conductivity of metals is greater than that of most dielectrics, typically by about 1 order of magnitude, the temperature is nearly constant across the thickness of the metal foil. Due to the large size and high thermal conductivity of the diamond anvils, the diamonds are effective heat sinks and the diamond-dielectric interface is close to room temperature [Bodea and Jeanloz, 1989]. The resulting temperature distribution across the thickness of the sample is sketched in Figure 1b.

Saxena *et al.* [1994] measured the relationship between the power of the laser used to heat the iron foil,

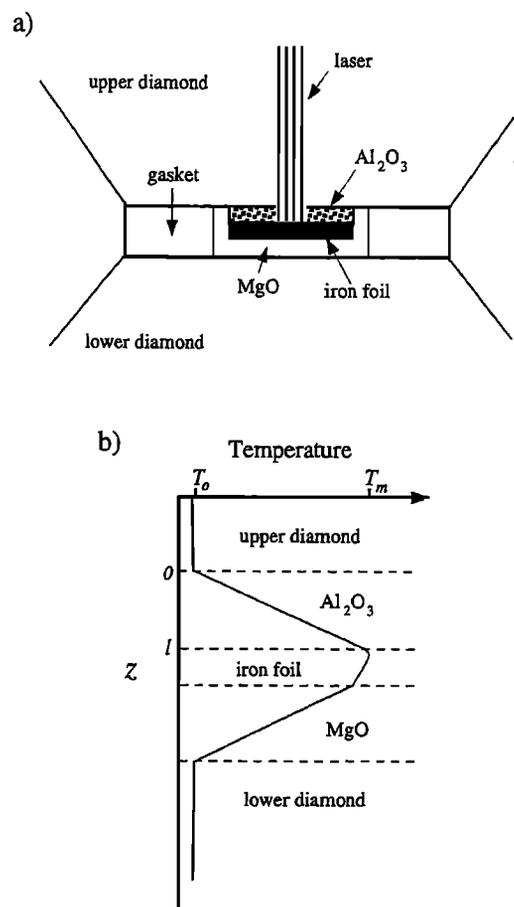


Figure 1. (a) Schematic illustration of the experimental configuration used by Saxena *et al.* [1994] in the diamond-anvil cell; an iron foil is surrounded by Al_2O_3 and MgO and heated by a laser. (b) Sketch of the temperature distribution across the thickness of the sample.

and the temperature of the foil. These two quantities are related by the thermal conductivity of the insulating Al_2O_3 and MgO: the laser energy is absorbed by the metal foil and is removed by heat transfer through the Al_2O_3 and MgO.

Heat Conduction Model

If heat transfer is dominated by axial conduction through the insulating dielectric layers (see appendix), then the temperature distribution within these layers satisfies the one-dimensional steady heat conduction equation

$$\frac{d}{dz} k(T) \frac{dT}{dz} = 0, \quad (12)$$

with boundary conditions

$$T = T_0 \text{ at } z = l \text{ and } T = T_m \text{ at } z = 0, \quad (13)$$

where l is the thickness of the insulating layer on each side of the foil, T_m is the temperature of the metal foil, and T_0 is the ambient temperature. The results of a similar one-dimensional analysis for laser-heated dielectrics

[Manga and Jeanloz, 1996a] are in good agreement with the fully three-dimensional calculations of Bodea and Jeanloz [1989], which account for heat transfer in the sample, diamonds, and gasket.

Assuming k is inversely proportional to T , the temperature distribution within the dielectric layers is

$$T(z) = T_0 \left(\frac{T_m}{T_0} \right)^{z/l}. \quad (14)$$

Application of the Heat Transfer Model to Experimental Results

The laser in TEM₀₀ mode has an axisymmetric Gaussian distribution, that is, the energy density in the beam is $\propto \exp[-(r/R)^2]$, where r is the radial coordinate and R is the beam radius. Thus the absorbed energy flux at the center of the hotspot is

$$Q_{\text{laser}} = \frac{\Lambda\beta(T)}{\pi R^2} \quad (15)$$

where Λ is the laser power (total output) and β is the temperature-dependent (and reflection-corrected) absorption of the iron. For simplicity, we ignore losses in the optical components and diamonds due to reflection and absorption; these losses are small and are folded into an effective β .

The heat flux conducted through the insulating layers is

$$Q_{\text{conducted}} = k \frac{dT}{dz}. \quad (16)$$

Due to the geometry of the experiment which involves both oxides, we are only able to calculate an averaged

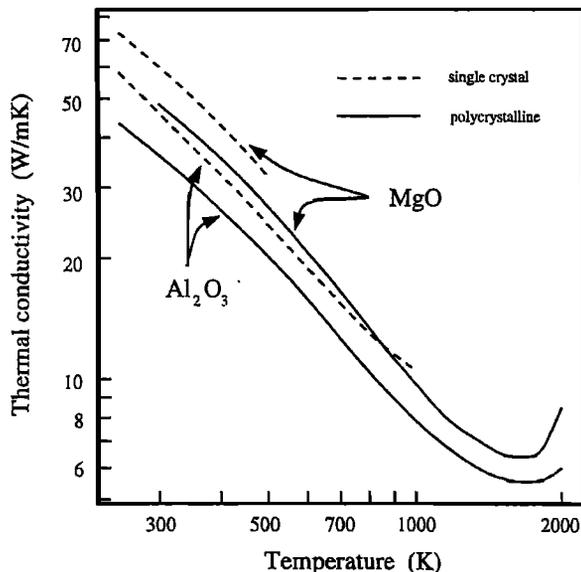


Figure 2. Thermal conductivities of single and polycrystalline Al₂O₃ and MgO at room pressure [Touloukian and Ho, 1971]. The change in slope at temperatures greater than about 1500 K is due to a radiative contribution to the measured (effective) thermal conductivity.

Table 1. Bulk Modulus of the Model and Lower Mantle Minerals

Material	K_0 , GPa	dK_0/dP at $P = 0$
Model	200	4
Al ₂ O ₃	252 ^a	4.3 ^b
MgO	162 ^a	4.0 ^c
Perovskite ^d	266 ± 6	3.9 ± 0.4
Lower mantle ^e	212-223	3.8-4.1

^a Anderson and Isaak [1995].

^b d'Amour et al. [1978].

^c Drickamer et al. [1966].

^d Knittle and Jeanloz [1987].

^e Anderson [1989], p. 119.

thermal conductivity for both Al₂O₃ and MgO. At least at room pressure, the thermal conductivities of these two materials differ by only about 25% (Figure 2).

We determine a in (4), (9), and (10) from thermal conductivity measurements at $P = 0$ for $100 < T < 1500$ K [Touloukian and Ho, 1971] (see Figure 2). Measurements for $T > 1500$ K are influenced by radiative transfer, and thus do not reflect the true lattice conductivity [McQuarrie, 1954].

In order to determine ϕ and ψ we compare the laser power required to achieve a given foil temperature T_m at two different pressures, P_1 and P_2 . We assume $K'_0 = 4.0$ and $K_0 = 200$ GPa, which are in the range of values for Al₂O₃, MgO, perovskite, and the lower mantle (Table 1), and further assume l does not change between 50 and 130 GPa. We also assume that the absorption β is only a function of temperature. We use data from Saxena et al. [1994] at 1500 K (see Figure 3) because laser-heated samples are generally "better behaved" at temperatures well below the melting temperature (this is evident in the smooth relationship between temperature and power in Figure 3); at low temperatures, dynamic recrystallization and processes associated with kinetics are less likely to influence the spatial and time dependence of optical properties [Lazor and Saxena, 1996; Saxena and Dubrovinsky, 1996]. In fact, using the models developed here, which assume that absorption is only temperature dependent, the slopes of the temperature versus power curves at $P = 58$ and 125 GPa are compatible with the same temperature dependence at both pressures.

Equating the absorbed energy, equation (15), and the conducted heat, equation (16), we obtain for the power law model

$$\psi = \frac{K'_0 \ln(\Lambda_2/\Lambda_1)}{\ln[(1 + P_2 K'_0/K_0)/(1 + P_1 K'_0/K_0)]}, \quad (17)$$

where Λ_i denotes the laser power at pressure P_i . An analogous expression for ϕ can be obtained for the linear

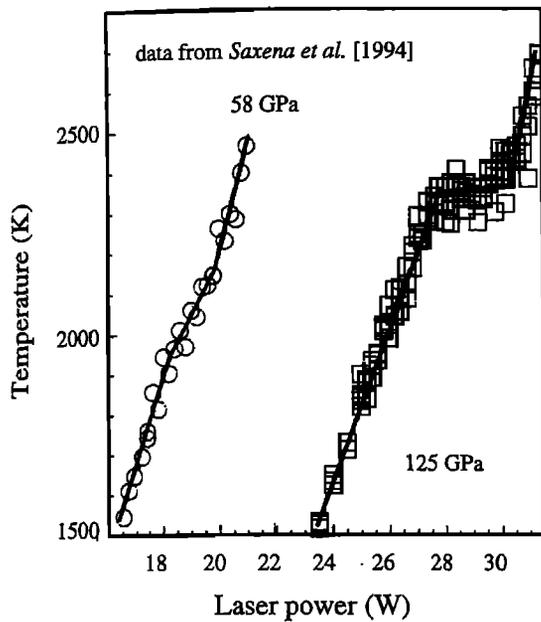


Figure 3. The measured relationship between the iron foil temperature and laser power, at pressures of 58 and 125 GPa (Reprinted with permission from *Saxena et al.* [1994]. Copyright 1994 American Association for the Advancement of Science). A change in the slope of, or offsets in, these curves is thought to indicate the occurrence of a phase change.

model. Using data from *Saxena et al.* [1994] at 1500 K (see Figure 3) gives

$$\phi = 7.0 \pm 1.0 \quad (18)$$

and

$$\psi = 3.0 \pm 0.7. \quad (19)$$

The uncertainties in (18) and (19) are dominated by the uncertainties in the pressures reported by *Saxena et al.* [1994], up to 6% for $P < 100$ GPa and 15% for $P > 100$ GPa, and include a smaller contribution associated with the scatter of the data in Figure 3. The uncertainties in (18) and (19) reflect the uncertainties in the predicted thermal conductivities, since changes in K_0 and K'_0 result in corresponding changes in ϕ and ψ , and have little effect on predicted values of k .

In Table 2, we compare the pressure dependence of k at $P = 0$ of the Debye-Grüneisen model [*Roufosse and Jeanloz*, 1983], room temperature experimental results up to 4 GPa for MgO [*Yukutake and Shimada*, 1978], and the results determined here based on measurements at ultrahigh pressures of $P = 58$ and 125 GPa. The present results are similar to the Debye-Grüneisen model, but underestimate dk/dP found in low pressure experiments by a factor of 1.5 to 3. However, the pressure dependence found here is similar to that of other materials including NaCl for $P < 4$ GPa [*Yukutake and Shimada* 1978], olivine for $P < 5$ GPa [*Chai et al.*, 1996], and β -oxygen for $6 < P < 9$ GPa and $T > \Theta_D$ [*Abramson et al.*, 1996] which have $k^{-1}dk/dP$ of about $6/K_0$, $5/K_0$ and $2/K_0$, respectively.

Table 2. Pressure Dependence of Thermal Conductivity

Model	$\frac{1}{k} dk/dP$ at $P = 0$
Present study, ^a linear model	$(7.0 \pm 1.0)/K_0$
Present study, ^a power law model	$(3.0 \pm 0.7)/K_0$
Debye-Grüneisen model ^b	$\sim 7/K_0$
Measurements for MgO to 4 GPa ^c	$11/K_0$

^a Weighted average for Al_2O_3 and MgO.

^b *Roufosse and Jeanloz* [1983].

^c At room pressure, from *Yukutake and Shimada* [1978], assuming $K_0 = 162$ GPa.

Discussion

The thermal conductivity of Al_2O_3 and MgO obtained from our analysis of experiments performed at high pressures and temperatures is summarized in Figure 4. For the range of temperatures which might exist within the Earth, we find values of $k \sim 5 - 12$ W/mK at lower-mantle conditions, with k changing very little, less than about $\sim 30\%$, along an adiabat. Despite the

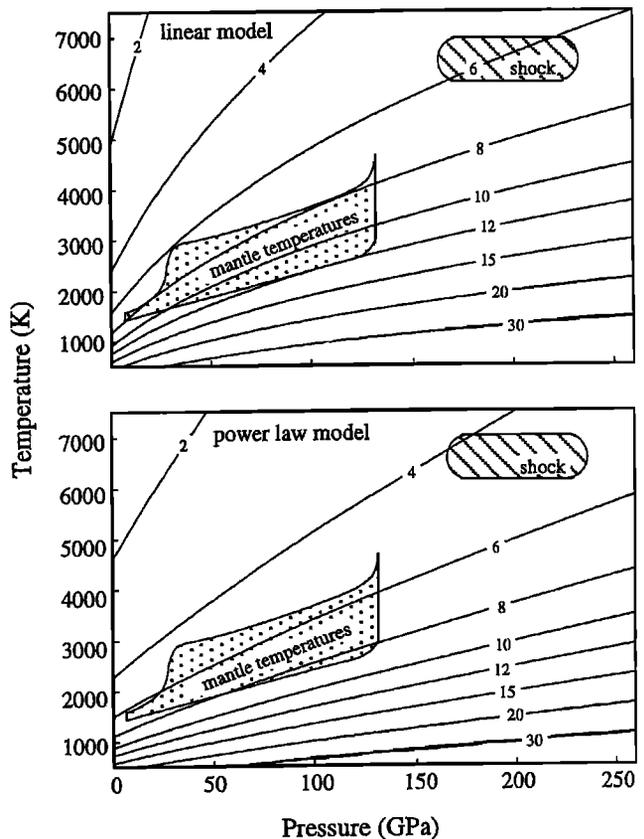


Figure 4. Temperature and pressure dependence of thermal conductivity in watts per meter-kelvin for the (top) linear and (bottom) power law models. Typical mantle temperatures are denoted by the stippled region [*Jeanloz and Morris*, 1986], and typical conditions in shock experiments are shown in the hatched regions.

difference of a factor of 2 in dk/dP at $P = 0$ for the two models studied here, predicted thermal conductivities in the lower mantle differ by less than 4 W/mK (less than 30%). In fact, fits of other models to the diamond-cell data, including models in which k has a linear, quadratic, or power law dependence on P (rather than on compression), also lead to conductivities which differ by less than 30% from those shown in Figure 4. Thus the largest uncertainties in our model are probably not associated with the form of the model itself but with the validity of our interpretation of the experimental measurements. In particular, the most critical assumption is that the absorption of the iron foil at a given temperature does not change significantly between pressures of 58 and 125 GPa.

Thermal Conductivity of the Lower Mantle

Other lower-mantle minerals, such as perovskite, are expected to have thermal conductivities similar to those obtained here, because K_0 and K'_0 of silicate perovskites are similar to those of Al_2O_3 and MgO , and all three minerals have dense and compact structures. In addition, there is an empirical correlation between debye temperature Θ_D and thermal conductivity [e.g., *Horai and Simmons*, 1970] with k increasing as Θ_D increases. The elastic debye temperatures at 300 K of silicate perovskite, Al_2O_3 and MgO are 1094 K, 1034 K, and 945 K, respectively [*Anderson and Isaak*, 1995]. It should be cautioned, however, that although Θ_D of Al_2O_3 is greater than that of MgO its thermal conductivity is smaller. In summary, there is no obvious reason or evidence that k of perovskite should differ greatly from that determined here.

Osako and Ito [1991] and *Akaogi and Ito* [1993] measured the thermal diffusivity and specific heat, respectively, of MgSiO_3 perovskite at room pressure and $160 < T < 340$ K. Their measurements imply a constant thermal conductivity over the studied temperature range, even though, as discussed earlier, the temperature dependence is greatest at low temperatures. The absence of a measured temperature dependence can be explained, however. At temperatures below a few hundred kelvin, the mean-free path (d) of phonons can be several millimeters in dielectrics, so that if the sample has dimensions comparable to d , the thermal conductivity approaches a constant which depends on the geometry of the sample [*Berman*, 1976, pp. 13-14]. In fact, the studied perovskite sample had dimensions ~ 1 mm so that the measured conductivity represents an extrinsic value. The phonon mean-free path decreases with increasing temperature (due to increased anharmonic coupling), so the deviation of the measurements of Ito and collaborators from theoretical predictions highlights the importance of making measurements at the high temperatures relevant to the Earth's interior.

It is sometimes suggested that the actual effective thermal conductivity in the mantle may be higher than the lattice conductivity due to a contribution from ra-

diative transport. The potential importance of radiative transport is illustrated by its effect on measured conductivities at room pressures for temperatures as low as 1500 K (see Figure 2). Based on experimental measurements of the optical properties of olivine, *Shankland et al.* [1979] calculated an effective thermal conductivity due to radiation of ~ 2 W/mK for $T > 800$ K, which is small compared with the lattice conductivity determined here. In addition, the overlap of absorption bands due to iron within the minerals can greatly increase optical absorption and effectively block radiative transfer at lower-mantle conditions [*Mao*, 1976].

A thermal conductivity of 5 – 12 W/mK is similar to commonly assumed values for lower-mantle conductivity [*Jeanloz and Richter*, 1979; *Brown*, 1986; *Stacey*, 1992]. *Leitch* [1995] has shown that the detailed dynamics of D" affect the heat flux from the core into the mantle and the fraction of the heat flux carried by plumes; *Leitch* [1995] concluded that for a temperature difference of 1350 K across D", k is likely to be less than 8 W/mK, a value consistent with our results for k at the base of the mantle (see Figure 4). Thus, the conductivity found here is compatible with both a thermal origin for the reduced velocity gradients in D" [e.g., *Stacey and Loper*, 1983; *Leitch*, 1995] and estimated values of the heat flux from the core based on the heat flux carried by mantle plumes [e.g., *Sleep*, 1990; *Davies*, 1993].

Comparison With Conductivities Inferred From Shock Experiments

Single-crystal Al_2O_3 is used as a window to contain samples in shock experiments. For metallic samples the relationship between a measured "interface temperature" and the actual Hugoniot temperature depends on the thermal conductivity of the window material under shock conditions [e.g., *Bass et al.*, 1987]. The results derived above predict $k \sim 5$ W/mK for the conditions typical of shock experiments, as illustrated in Figure 4. In shock experiments with iron the iron Hugoniot temperature is $\sim 5500 - 7500$ K at $P \sim 230 - 320$ GPa [e.g. *Gallagher et al.*, 1994].

The values of k determined here are similar to early estimates used in reducing Hugoniot temperature measurements [e.g., *Bass et al.*, 1987; *Tan and Ahrens*, 1990] but differ significantly from $k < 0.84$ W/mK and $k \sim 0.02$ W/mK determined by more recent analyses [*Gallagher et al.*, 1994; K.G. Gallagher and T.J. Ahrens, Ultra high pressure thermal conductivity measurements of greceite and corundum, unpublished paper, 1996]. The procedure for determining k from shock experiments involves comparing the measured relaxation of the interface temperature with that predicted by a thermal diffusion model. These lower values of k would imply a decrease by about 700 K in the Hugoniot temperature of iron relative to the earlier work of *Bass et al.* [*Gallagher et al.*, 1994].

Gallagher et al. [1994] argued that thermal conduction can be affected by the presence of defects, so the

conductivity inferred from shock wave experiments is not necessarily the same as that obtained under conditions more closely approximating thermodynamic equilibrium. In this sense, values obtained from static experiments may be more appropriate for applications to the Earth's mantle and core.

It is important to note, however, that the exceptionally low inferred thermal conductivity is based on a very low value for the thermal conductivity of iron used in the *Gallagher et al.* [1994] analysis. In metals, both electrical and thermal conduction are dominated by electrons, and these processes are related by the Wiedeman-Franz-Lorenz (WFL) law,

$$k = L_0 \sigma T, \quad (20)$$

where σ is the electrical conductivity, and $L_0 = 2.45 \times 10^{-8} \text{ W}\Omega/\text{K}^2$ is the Lorenz number. Although the pressure dependence of the Lorenz number is not well determined, there are good experimental and theoretical reasons for (20) to hold in general [*Ziman*, 1960; *Shimoji*, 1977]. In metals, k is expected to be independent of temperature at sufficiently high temperatures (greater than a few hundred kelvin) so it is important that the value of T used in the WFL law is that at which the electrical conductivity measurement was made. *Gallagher* and collaborators use $k \sim 2 \text{ W/mK}$ for iron. This value is significantly lower than that of 112 W/mK at 140 GPa based on electrical conductivity measurements reported by *Keeler and Mitchell* [1969] and *Matassov* [1977] and using the Hugoniot temperature from *McQueen et al.* [1970]. In fact, the thermal conductivity of iron obtained from electrical conductivity measurements increases with increasing pressure (Figure 5), and thus may be even higher than 100 W/mK at shock conditions. Upon melting, k for metals typically decreases by about a factor of 2 [*Iida and Guthrie*, 1988], although

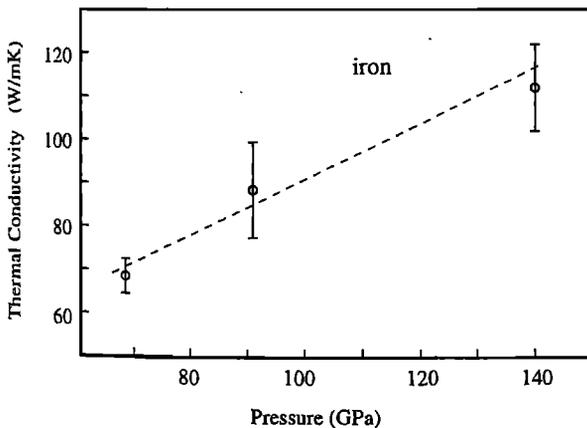


Figure 5. Thermal conductivity of iron obtained from electrical conductivity measurements of *Keeler and Mitchell* [1969] using the Wiedeman-Franz-Lorenz law, equation (20). Error bars are based on uncertainties in the electrical conductivity measurements.

k decreases only about 20% for iron at room pressure [*Touloukian and Ho*, 1971].

Assuming a thermal conductivity for iron of 100 W/mK , and noting the relaxation of the interface temperature scales with $\kappa^{1/2} \propto k^{1/2}$, where κ is the thermal diffusivity, then the ratio of thermal conductivities of ~ 100 inferred by *Gallagher* and collaborators would be reduced to $\sim 10 - 30$ (this ratio is obtained by comparing model calculations with the measured relaxation of the interface temperature). Thus k of the Al_2O_3 window material is $\sim 3 - 10 \text{ W/mK}$, similar to that predicted by both models studied here.

This last conclusion is independently supported by the recent shock wave measurements of *Tang et al.* [1996]. The thermal conductivities obtained from shock wave and laser-heated diamond cell experiments are therefore in good agreement with each other and provide reliable constraints on the values pertaining to the Earth's deep interior.

Summary

Experiments using the laser-heated diamond cell, combined with heat transfer models for the apparatus, can be used to determine thermophysical material properties such as thermal conductivity at high pressures and temperatures. Here we derived an averaged thermal conductivity of MgO and Al_2O_3 using measurements of laser power and sample temperature collected by *Saxena et al.* [1994] during a melting study of iron. The thermal conductivity of these oxides at lower-mantle conditions is $\sim 5 - 12 \text{ W/mK}$, a value consistent with previous theoretical estimates and geodynamic constraints for the lower mantle.

Appendix: Heat Transfer in the Laser-Heated Diamond Anvil Cell

We first demonstrate that radiative transfer within the diamond cell is negligible when compared with conductive heat transport (radiative transfer is often a problem in direct measurements of k at high temperature). Consider a blackbody metal foil at 4000 K , with emissivity $\epsilon = 1$. The radiative energy flux into a dielectric material with index of refraction n is

$$Q_{\text{radiative}} = n^2 \epsilon \sigma T^4 = 3.3 \times 10^7 \text{ W/m}^2, \quad (\text{A1})$$

where σ is the Stefan-Boltzmann constant, and we assume $n = 1.5$. In comparison, the conductive heat transfer through the insulating material is

$$Q_{\text{conductive}} = k \nabla T = 4 \times 10^8 \text{ W/m}^2, \quad (\text{A2})$$

assuming a thickness of $10 \mu\text{m}$ for the insulating layer and a (low value of) thermal conductivity of 1 W/mK . From (A1) and (A2) it is evident that conductive heat transfer dominates due to the small length scales in the diamond cell. Indeed, a more detailed analysis, which accounts for the temperature dependence of thermal

conductivity, demonstrates that conductive heat transfer dominates to temperatures greater than 10,000 K.

Laser energy is absorbed by the metal foil, because the insulating dielectrics MgO and Al₂O₃ are optically thin, whereas the metal is opaque. Heat is conducted both axially through the insulating layers and radially through the metal foil. Here we present scaling arguments which allow us to estimate the relative importance of axial and radial conduction.

The total heat flow conducted radially through a sample from a hotspot with radial dimension H and thickness h can be estimated as

$$Q_{\text{radial}} \sim 2\pi h H k_m (T_m - T_0)/H \quad (\text{A3})$$

where T_m is the temperature of the metal foil at the center of the hotspot, T_0 is the ambient temperature, and k_m is the thermal conductivity of the metal. The heat conducted axially from the hotspot through the insulating dielectric layers with thickness l is

$$Q_{\text{axial}} \sim 2\pi H^2 k_d (T_m - T_0)/l, \quad (\text{A4})$$

where πH^2 is approximately the area of the hotspot and k_d is the thermal conductivity of the dielectric. Equations (A3) and (A4) are obviously only estimates; clearly, the relative importance of axial and radial conduction will vary with radial position and will in turn affect the temperature distribution within the hotspot.

We can identify two limits, corresponding to the cases in which either radial or axial heat conduction dominates. If

$$\frac{Q_{\text{radial}}}{Q_{\text{axial}}} \sim \frac{k_m l h}{k_d H^2} \ll 1 \quad (\text{A5})$$

then axial conduction dominates. Since this condition is more likely to hold when the film thicknesses are small, that is, $(l, h) \ll H$, we refer to this condition as the thin-film limit. If

$$\frac{Q_{\text{radial}}}{Q_{\text{axial}}} \gg 1 \quad (\text{A6})$$

then radial conduction dominates, and we refer to this condition as the thick-film limit. Because the thermal conductivity of metals is typically much greater than that of dielectrics, the thick-film limit applies for $l, h \sim H$. Because the thickness of metal foils in the diamond cell is usually only a few micrometers, however, the thin-film limit is often the better approximation and it is the one used in the present study.

The preceding analysis assumes that the radial extent of the metal foil is large relative to the diameter of the heat source (focused laser beam), so that the edge of the foil is at the ambient temperature (typically 300 K). However, if the radial extent of the foil is not much greater than the beam radius, or the foil is surrounded on all sides by an insulating dielectric (as in the experiment by *Saxena et al.* [1994], see Figure 1a), radial temperature gradients can be significantly reduced. In this case the temperature gradient across

the hotspot in (A3) is reduced, and heat transfer by conduction will occur primarily in the axial direction. Temperature measurements for metal foils of limited width confirm that radial temperature gradients can be small [e.g., *Boehler et al.*, 1990; *Shen and Lazor*, 1995; *Saxena and Dubrovinsky*, 1996].

Notation

a	constant describing thermal conductivity at room pressure, equations (4), (9), and (10).
A	power/unit volume absorbed by the sample from the laser.
C	specific heat per unit volume.
d	mean-free path of phonons.
D	interatomic distance.
h	thickness of metal foil.
H	characteristic radial dimension of the laser-heated hotspot.
k	thermal conductivity.
k_{0T}	thermal conductivity at $P = 0$ and temperature T .
K	bulk modulus.
K_0	bulk modulus at $P = 0$ and room temperature.
K'_0	dK_0/dP at $P = 0$ and room temperature.
l	thickness of the dielectric layer insulating the metal foil.
Q	heat flux.
r	radial position.
R	laser beam radius.
t	time.
T	temperature.
T_0	ambient temperature.
T_m	temperature of the metal foil.
\bar{v}	mean velocity of acoustic waves.
z	axial position.
α	coefficient of thermal expansion.
β	absorption coefficient.
ϵ	emissivity.
γ	thermodynamic Grüneisen parameter.
Λ	laser power.
ϕ	coefficient in the linear model, equation (9).
ψ	coefficient in the power law model, equation (10).
ρ	density.
ρ_0	density at $P = 0$.
ρ_{0T}	density at $P = 0$ and temperature T .
σ	Stefan-Boltzmann constant.
Θ_D	Debye temperature.

Acknowledgments. This work was supported by NSF and the Miller Institute for Basic Research in Science. We thank the associate editor, A.M. Leitch and J.J. Ita for comments and suggestions.

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(Received April 11, 1996; revised August 29, 1996; accepted September 3, 1996.)