Experimental study of non-Boussinesq Rayleigh–Bénard convection at high Rayleigh and Prandtl numbers

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A set of experiments is performed, in which a layer of fluid is heated from below and cooled from above, in order to study convection at high Rayleigh numbers (Ra) and Prandtl numbers (Pr). The working fluid, corn syrup, has a viscosity that depends strongly on temperature. Viscosity within the fluid layer varies by a factor of 6 to $1.8 \times 10^3$ in the various experiments. A total of 28 experiments are performed for $10^5 < Ra < 10^8$ and Pr sufficiently large, $10^2 < Pr < 10^6$, that the Reynolds number (Re) is less than 1; here, values of Ra and Pr are based on material properties at the average of the temperatures at the top and bottom of the fluid layer. As Ra increases above $O(10^3)$, flow changes from steady to time-dependent. As Ra increases further, large scale flow is gradually replaced by isolated rising and sinking plumes. At $Ra > O(10^5)$, there is no evidence for any large scale circulation, and flow consists only of plumes. Plumes have mushroom-shaped “heads” and continuous “tails” attached to their respective thermal boundary layers. The characteristic frequency for the formation of these plumes is consistent with a $Ra^{2/3}$ scaling. In the experiments at the largest Ra, the Nusselt number ($Nu$) is lower than expected, based on an extrapolation of the $Nu$–Ra relationship determined at lower Ra; at the highest Ra, Re→1, and the lower-than-expected Nu is attributed to inertial effects that reduce plume head speeds. © 1999 American Institute of Physics.

I. INTRODUCTION

In a plane layer of fluid heated from below and cooled from above, natural convection, called Rayleigh–Bénard convection, can arise from thermally induced density variations. If the Prandtl number (Pr) is sufficiently large, viscous forces will balance thermal buoyancy forces, and the influence of inertia can be neglected. This particular limit, very large Pr (effectively infinite), is appropriate for convection within the mantles of terrestrial planets. Convective motions in the Earth are manifested in plate tectonics, hotspot volcanism, and large scale continental deformation.

Previous experimental data for high Pr Rayleigh–Bénard convection is limited to Rayleigh numbers (Ra) less than $10^6$ (e.g., Refs. 2–9). By contrast, in the Earth, $Ra \sim 10^8$ and $Pr \sim 10^{12}$; within the terrestrial planets, $Ra$ is large as a result of the large depth of the mantle ($\sim 10^3$ km), and Pr is large as a result of the large viscosities ($\sim 10^{21}$ Pa s). At $Ra > 10^6$ and high Pr, two experimental studies have considered various aspects of transient convection during secular cooling and secular heating. Citing “the need for reliable data at high Ra to determine the asymptotic Nusselt number variation with Ra,” Goldstein et al. performed an analog experimental study using electrochemical mass transfer for $3 \times 10^6 < Ra < 5 \times 10^7$ and Schmidt numbers (analogous to Pr) of $\approx 2750$. It is beyond the scope of this paper to provide a summary of related work at low Pr, and the reader is referred to the list of review papers provided by Goldstein et al. and a recent review paper by Siggia.

Our goal here is to determine the nature of convective structures and time-dependent motions at high Ra, and at Pr sufficiently large that the Reynolds number (Re) is small ($Re < 1$). In practice we are able to achieve Ra up to $10^8$ and Pr $> 10^3$.

II. EXPERIMENTAL APPARATUS AND PROCEDURES

In our experiments we heat a layer of corn syrup from below and cool it from above, in both cases using water baths to control temperatures. The layer, or tank, of corn syrup has a square base and a depth $d$. Water from the baths circulates through hollow aluminum plates bounding the top and bottom of the tank. Water flows in opposite directions in the two plates in order to diminish spatial variations in the temperature difference across the fluid layer. The corn syrup is contained in the horizontal dimension by glass sidewalls. Temperatures within the fluid are measured with $27–30 J$-type thermocouples and are recorded by a datalogger every 1–15 s, with the sampling period decreasing as Ra increases. The entire apparatus is insulated with 5 cm thick polystyrene foam. Removable windows in the foam on the sides of the tank allow us to observe flow structures visually. While we aim to maintain isothermal upper and lower boundaries, the finite conductivity of the thin aluminum plates between the circulating water and convecting fluid layer may produce horizontal temperature variations. We do not, however, see any large scale circulation or convective patterns that might be attributed to such an “imperfect” boundary condition (see Fig. 1).
The viscosity of corn syrup is approximately an Arrhenian function of temperature, and it is necessary to use fairly large temperature differences, $T_1 - T_0$, to obtain large $Ra$. In Fig. 2 we show viscosity as a function of temperature for the four corn syrup solutions used here. Viscosities were measured using a rotational viscometer. In the experiments reported here, the viscosity at the top of the tank is between about 6.4 and $1.8 \times 10^4$ times greater than the viscosity at the bottom of the tank, and thus the flows are non-Boussinesq.

Our problem is characterized by three dimensionless parameters, the Rayleigh number, $Ra$, the Prandtl number, $Pr$, and the viscosity contrast between fluid at the top and bottom of the tank, $\lambda$.

$$Ra = \frac{\rho g \alpha (T_1 - T_0) d^3}{\kappa \mu \theta = 0.5},$$

$$Pr = \frac{\mu \theta = 0.5}{\rho \kappa},$$

$$\lambda = \frac{\mu \theta = 0.5}{\mu \theta = 1};$$

here $\rho$, $\alpha$, $\kappa$, and $\mu$ are the fluid density, coefficient of thermal expansion, thermal diffusivity, and viscosity, respectively. In the definition of $Ra$ and $Pr$, the viscosity used is the value at $\theta = 0.5$, i.e., its value at the temperature halfway between the top and bottom temperatures. Previous experimental results have shown that with this definition of $Ra$, measured values of the Nusselt number ($Nu$) collapse to a single $Nu$–$Ra$ curve for $1 < \lambda < 10^5$. Hereafter, we will assume that this empirical definition of $Ra$ is appropriate for interpreting the experimental results.

Values for $\alpha$ and $\kappa$ for corn syrup are taken from Gianandrea and Christensen and are $4.0 \times 10^{-4} ^{\circ}C^{-1}$ and $1.1 \times 10^{-7} m^2/s$, respectively, and are assumed to be constant within the fluid layer. The ranges of $T_1 - T_0$, $\rho$, and $\mu \theta = 0.5$ in our experiments are $9.2 - 68.5 ^{\circ}C$, $1.390 - 1.431 g/cm^3$, and $0.75 - 181 Pa s$, respectively. We use tank depths of 10, 17, and 33 cm, and the corresponding aspect ratios (width to depth) of the fluid layers are 3, 2, and 1, respectively. We performed 28 experiments, using the largest aspect ratio of 3 for our smallest $Ra$, and the aspect ratio 1 for the largest $Ra$. In general, it is desirable to use the largest aspect ratio possible so that the effects of the horizontal boundaries are small; here, our aspect ratio is limited by both the weight of corn syrup we could manage, as well as the heating and cooling power of our water baths. We do not, however, expect our flows to be significantly affected by the limited aspect ratios, especially at the highest $Ra$, because the horizontal dimensions of the convective features are small relative to the tank depth. Finally, in the first set of experiments we performed (aspect ratio 3; lowest $Ra$), the corn syrup was held in a container that had a glass bottom, so that the fluid layer was separated from the aluminum plate (see Fig. 1) by a sheet of glass. In our experiments with aspect ratio 2 and 1 (at higher $Ra$) the fluid layer was in direct contact with the aluminum plates in order to reduce the magnitude of horizontal temperature variations. These never exceeded $1.5 ^{\circ}C$.

All the results presented here are based on measurements at equilibrium conditions. We identify equilibrium by requiring that both the Nusselt number ($Nu$) and the temperature in the middle of the fluid layer ($\theta_m$) are constant when averaged over sufficiently long periods of time. Each experiment runs between 1 and 12 days, with longer times being required for low $Ra$ experiments. A summary of results is presented in Table I.

Heat transport is characterized in the standard way by determining $Nu$. In our experiments we chose to fix boundary temperatures rather than to specify the heat input to the system. Our estimate of $Nu$ is thus based on the measured
near-surface temperature gradient obtained from a set of 10–12 thermocouples located at a depth of 3 mm or 5 mm below the upper surface. Although Giannandrea and Christensen\(^9\) observed that "wires could trigger down-streams in their surroundings," in our case, the probes are located in the quiescent and most viscous region of the tank and are isolated from the actively convecting region. To test our procedures and the reliability of \(\text{Nu}\) obtained this way, in Fig. 3 we compare our measured \(\text{Nu}\) with previous experimental measurements of Giannandrea and Christensen\(^9\) at low Ra. We find excellent agreement, though we note that the literature contains variations of about 5%–10% for \(\text{Nu}\) which are usually attributed to uncertainties in the thermal conductivity and other properties.\(^5,7,9\) We estimate that the uncertainty in our values of Ra is about 15%, reflecting the \(\approx 5\%\) variation of thermal diffusivity and thermal expansivity reported for corn syrup solutions\(^7,9,10\) and the uncertainty in our measured viscosity of 5%. Also, the data shown in Fig. 3 involve viscosity ratios covering more than four orders of magnitude and demonstrate that the single curve relating \(\text{Nu}\) and Ra based on the viscosity at \(\theta = 0.5\) works very well.

Uncertainties in our reported \(\text{Nu}\) are based on the standard deviations of the temperature measurements used to obtain \(\text{Nu}\) and are thus not "real" errors. For example, in steady flows, the local heat flux varies over the surface of the tank, and in unsteady flows, the Nusselt number changes in time as well. Here, \(\text{Nu}\) is averaged over space and time.

### III. RESULTS AND DISCUSSION

Here we summarize our experimental measurements and attempt to provide an interpretation of the relationship between parameters and measurements. Specifically, we consider the distribution of temperature variations in space (Sec.
III (A)) and time (Secs. III B and C), and the relationship between Nu and Ra (Sec. III D). Results and parameters for the 28 experiments are listed in Table I.

First, however, we describe qualitative observations. Figure 4, is a shadowgraph showing mushroom-shaped plumes, with "tails" that are connected to thermal boundary layers at the top or bottom of the tank. Following previous terminology,18 we refer to the mushroom-shaped regions as plume heads. The plumes rise and fall nearly vertically, suggesting that any large scale flow is weak or nonexistent. We never observe plume heads forming discrete, detached thermals as suggested by Hansen et al.19 More recently, Tromp et al. found that the formation of detached thermals is a consequence of the two-dimensional geometry used in the earlier calculations,19 and that detached thermals did not form at similar Ra in three-dimensional calculations.

**A. Vertical temperature distribution**

In Fig. 5 we show one example of a vertical temperature profile obtained from a thermocouple that could be moved in the vertical direction. The filled symbols show the long term mean temperatures at their respective depths based on the array of stationary thermocouples. As noted in previous studies,7,9 the temperature in the middle of the tank is not 1/2 as symmetry would require it to be for Boussinesq convection.

The middle temperature \( \theta_m \) is related to the relative thicknesses of the top and bottom thermal boundary layers because the heat conducted into the tank through the lower thermal boundary layer must equal the heat conducted through the top thermal boundary layer, i.e.,

\[
\frac{\theta_m}{\delta_0} = \frac{1 - \theta_m}{\delta_1},
\]

or

\[
\theta_m = \frac{\delta_0}{\delta_0 + \delta_1}.
\]

We expect that the relative thickness of the boundary layers (\( \delta_0 \) at the top, and \( \delta_1 \) at the bottom) to be given by

\[
\frac{\delta_0}{\delta_1} = \left( \frac{u_{\delta_0}}{u_{\delta_1}} \right)^{-1/3} \approx \left( \frac{\mu_{\delta_0}}{\mu_{\delta_1}} \right)^{1/3},
\]

where \( u_{\delta_i} \) and \( \mu_{\delta_i} \) are representative velocities and viscosities in boundary layer \( i \). In order to simplify the scaling, we will approximate the Arrhenian temperature-dependence of viscosity with a negative exponential function,

\[
\mu = \mu_0 e^{-\gamma \theta},
\]

a form that is in reasonable agreement with the viscosity data in Fig. 2. Assuming that the temperature difference across the active convecting region

\[
\theta_{\delta_1} - \theta_{\delta_0} \approx 1/2,
\]

we obtain a relationship between the middle temperature \( \theta_m \) and the viscosity ratio \( \lambda \),

\[
\theta_m = \frac{1}{1 + \lambda} \ln \lambda.
\]

Solomatov derived scaling relationships for the boundary layer thicknesses \( \delta_0 \) and \( \delta_1 \) in temperature-dependent viscosity convection with free-slip boundaries. In the limit of very large viscosity contrasts, \( \lambda > O(10^4) \) (Refs. 21, 22), an effectively stagnant lid develops even for the case of a free-slip upper surface. In this so-called "stagnant lid" regime, advective heat transport by the cold boundary layer is negligible compared to the transport by the more vigorous convection beneath the boundary layer. Solomatov finds that for a temperature-dependence of viscosity described by Eq. (8)

\[
\theta_{\delta} = \ln \lambda
\]

Two-dimensional20,22 and three-dimensional20 numerical calculations for convection with a free-slip surface and sufficiently large \( \lambda \) agree with Eq. (11). Solomatov also proposed a transitional regime for smaller \( \lambda \) in which dissipation in the cold boundary layer becomes comparable to dissipation in the actively convecting region. In this so-called "transitional" regime
In Fig. 6 we show the relationship between $\mu_m$ and $l$. $\mu_m$ is based on the average temperature recorded by six thermocouples in the middle of the tank. The approximate scalings given by Eqs. (10) – (12) are also shown. The values of $l$ in the lab experiments fall in the “transitional” regime, and in general, the experimental data follow Eq. (12), despite the simple approximations it involves, also captures the general trend in the data, and suggests that Eq. (9) might be a reasonable approximation for the temperature difference across the actively convecting region.

B. Characteristic period

We now examine the temporal variability of temperature for characteristic periods and frequencies. In Fig. 7(a) we show two examples of temperature records in the middle of the tank for experiments with low and high Ra. We will identify the characteristic period by computing the autocorrelation function of temperatures recorded in the middle of the tank e.g., Eq. (1.2.5) in Ref. 23.

The autocorrelation, as a function of the dimensionless time lag is shown in Fig. 7(b). Time is normalized by the thermal diffusion time scale $d^2/\kappa$. The first peak in the autocorrelation corresponds to the characteristic period, and, of course, peaks are repeated at time lags that are integer multiples of the characteristic period. The bars in Fig. 7(b) show the 95% confidence limits.

Howard24 suggested that the plumes or thermals we observe (Fig. 4) form through the breakup of thermal boundary layers. The boundary layer thickness ($\delta^*$ = $d/d$) increases because of thermal diffusion and thus grows as $t^{1/2}$. The thermal boundary layer becomes unstable when the local Rayleigh number (Ra) exceeds the critical value (Ra_c), i.e.,

$$R_{ai} = Ra_0 \delta^3 > Ra_c \approx 10^3.$$  

We thus obtain a relationship between the period ($t^*$) and Ra,

$$t^* \propto Ra^{-2/3},$$  

and $t^*$ should be independent of Pr (e.g., Ref. 25).

In Fig. 8 we plot our measured $t^*$ against Ra, along with a slope of $-2/3$ for $3.4 \times 10^5 < Ra < 1.2 \times 10^8$. A least squares fit to all the data gives a slope of $-0.61$. Previous high Pr studies found slopes similar to $-2/3$ for $Ra < 10^6$ (e.g., Refs. 2, 4, 25). We are able to identify characteristic periods from only some of the thermocouples in the tank;

![FIG. 6. Mean middle temperature $\mu_m$ as a function of the viscosity ratio $l$. Curve are predictions of Eqs. (10) – (12).](image)

![FIG. 7. (a) Time series of temperature measurements for experiments at $Ra=3.4 \times 10^5$ (Pr = $1.3 \times 10^5$, $\lambda = 36$, aspect ratio 2) and $9.8 \times 10^6$ (Pr = 4.6 $\times 10^4$, $\lambda = 40$, aspect ratio 1). Time 0 is arbitrary. (b) Autocorrelation as a function of time lag; characteristic periods are the first peaks.](image)

![FIG. 8. Characteristic period ($t^*$) as a function of Ra. The slope of $-2/3$ is the prediction of Howard (Ref. 24), Eq. (14). The least-squares best fit slope is $-0.61$.](image)
this suggests that there are preferred (i.e., not random) spatial locations for the formation of plumes. The preferred locations in each experiment are not the same.

C. Distribution of temperature variations

In Fig. 9 we show time series of temperature measurements of three or four thermocouples located in the middle of the tank for three experiments. Time is again normalized by the diffusive time scale $d^2/\kappa$. At the lowest $Ra$ [Fig. 9(a)], temperature varies “slowly” in time, the amplitude of temperature variations is large, and flow is obviously unsteady. By contrast, at the highest $Ra$ [Fig. 9(c)], the temperatures at all four thermocouples fluctuate about a constant mean temperature. In the low $Ra$ experiment we attribute the observed temperature variations to the unsteady nature of large scale convective patterns; in the high $Ra$ experiment we attribute the short period fluctuations to rising and falling plumes, and the absence of long period temperature variations indicates the absence of a large scale flow. At intermediate $Ra$ [Fig. 9(b)], we observe both long and short period temperature variations. For the purposes of classifying the observed convective behavior, we will refer to flows such as that in Fig. 9(c) as “plume-dominated.” Flows that have only long period temperature variations will be called simply “unsteady,” and flows that appear to have both long period and short period variations will be called “transitional” (referring to transitional between unsteady and the plume-dominated). The style of convection we observe is summarized in the regime diagram shown in Fig. 10, which also shows the parameter space covered by previous studies and characteristic of commonly studied fluids (we ignore $\lambda$ in Fig. 10).

Another way of characterizing the data shown in Fig. 9 is to determine the probability distribution function (PDF) for temperature (see Fig. 11). PDFs have been used, for example, to identify the transition to hard turbulence in Helium experiments. The transition corresponds to a change from a Gaussian to exponential distribution. At the highest $Ra$ shown in Fig. 11, the PDF only appears triangular due to scale compression. The PDF for plume-dominated flows consists of a peak (at $\theta_m$) with a superimposed curve (see inset of Fig. 11). The suggestion by Hansen et al. that at $Ra>10^7$ the mode of heat transfer in infinite Pr fluids is...
with the best-fit relationship of Richter et al. for the Nusselt number (\(\text{Nu}^{\text{exp}}\)) at high \(\text{Ra}^{\text{exp}}\). For comparison, in Fig. 12 we plot \(\text{Nu}^\sim\) of \(\text{Nu}^{\text{exp}}\) at high \(\text{Ra}^{\text{exp}}\). We find systematic deviations of our measurements and the calculations reflect our finite \(\text{Pr}\). We- refer to infinite \(\text{Pr}\), suggesting that the discrepancies between our experiments are probably very different. Given that boundary layer analyses are nontrivial and do not explain the experimental data, we will therefore focus on trying to account for the general form of the discrepancies between measured and expected \(\text{Nu}\).

D. Nu–Ra relationship

Finally, we consider the relationship between the Nusselt and Rayleigh numbers, assumed to be of the form

\[
\text{Nu} \sim \text{Ra}^\beta. \tag{15}
\]

The scaling law relating these two quantities has been the focus of many theoretical, experimental, and numerical studies, because it relates heat transport to physical properties of the convecting system. In high \(\text{Pr}\) convection (low \(\text{Re}\)), \(\beta = 0.28\) (e.g., Refs. 5, 7, 9).

Previous studies\(^ {7,28}\) have found that \(\text{Nu}\) is more closely related to \(\text{Ra}/\text{Ra}^c\), where \(\text{Ra}^c\) is the critical value for the onset of convection.\(^ {29}\) In Fig. 12 we plot \(\text{Nu}\) against \(\text{Ra}/\text{Ra}^c\) along with the best-fit relationship of Richter et al.\(^ {7}\):

\[
\text{Nu}^{\text{exp}} = 1.46(\text{Ra}/\text{Ra}^c)^{0.281}, \tag{16}
\]

where the subscript \(\text{exp}\) indicates that \(\text{Nu}^{\text{exp}}\) is the “expected value” of \(\text{Nu}\). We find systematic deviations of our measured \(\text{Nu}\) from \(\text{Nu}^{\text{exp}}\) at high \(\text{Ra}\). For comparison, in Fig. 12 we also show calculated \(\text{Nu}\) by Tackley in which he simulated our experimental geometry and viscosities for four specific experiments (experiments 19, 23, 25, and 27 listed in Table I). The numerical calculations of Tackley\(^ {30}\) assume an infinite \(\text{Pr}\), suggesting that the discrepancies between our measurements and the calculations reflect our finite \(\text{Pr}\). Werarame and Manga\(^ {31}\) previously attributed the change in slope of the \(\text{Nu}–\text{Ra}\) relationship shown in Fig. 12 to the change in convective style associated with the transition to plume-dominated flows. However, the numerical calculations of Tackley indicate that our measured \(\text{Nu}\) is indeed lower than the expected value if \(\text{Pr} \to \infty\). Moreover, the numerical calculations suggest that despite changes in convective style from steady to unsteady to plume-dominated (as illustrated in Fig. 10), \(\beta = 0.28\) continues to relate \(\text{Nu}\) and \(\text{Ra}\) at high \(\text{Ra}\), provided \(\text{Pr}\) is sufficiently large. In addition, our definition of \(\text{Ra}\) in Eq. (2) appears to continue to be appropriate for temperature-dependent convection at high \(\text{Ra}\).

Before attempting to provide an explanation for the lower-than-expected Nusselt numbers, we note that existing boundary layer theories (e.g., Refs. 32–34) suggest powers of 1/5 for convection between rigid boundaries, and 1/3 for stress-free boundaries.\(^ {21}\) By contrast, the power found in experiments\(^ {7,9,28}\) is close to 0.28, and is thus significantly different. Two different scaling analyses obtain \(\beta = 2/7\) for turbulent convection,\(^ {26,35}\) however, the mechanisms through which heat and momentum are transported in our low \(\text{Re}\) experiments are probably very different. Given that boundary layer analyses are nontrivial and do not explain the experimental data, we will therefore focus on trying to account for the general form of the discrepancies between measured and expected \(\text{Nu}\).

At high \(\text{Ra}\), our shadowgraphs (e.g., Fig. 4) and temperature measurements (e.g., Fig. 9(c)) indicate that flow consists of rising and sinking plumes, and these presumably carry most of the heat. Consider an analog system that consists of rising and sinking plumes, and these presumably carry most of the heat. Consider an analog system that consists of rising and sinking plumes, and these presumably carry most of the heat. Consider an analog system that consists of rising and sinking plumes, and these presumably carry most of the heat. Consider an analog system that consists of rising and sinking plumes, and these presumably carry most of the heat. Consider an analog system that consists of rising and sinking plumes, and these presumably carry most of the heat. Consider an analog system that consists of rising and sinking plumes, and these presumably carry most of the heat. 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decreasing trend of Nu with increasing Re, qualitatively similar in form to that expected from the increased drag due to finite Re numbers. The greater magnitude of the reduction of Nu might be due to inertia also reducing the size of plume heads (advective heat transport by a plume head scales with $a^3$). The frequency-scaling of plume formation (Fig. 8), however, does not appear to be affected as Re→1.

IV. SUMMARY AND CONCLUDING REMARKS

In the present experimental study, we have extended the range of Ra studied at high Pr by two orders of magnitude. The corresponding Re ranges from $\ll 1$ to $O(1)$. At $Ra/O(10^5)$ we find that flow is dominated by isolated rising and falling plumes with plume head radii much smaller than the tank depth; these appear to move vertically and there is no evidence for the existence of a large scale flow. The characteristic frequency for the formation of plumes appears to scale with Ra$^{2/3}$, as suggested by Howard. We also find that Nu at high Ra is lower than expected based on an extrapolation of low Ra experimental data; at the highest Ra, Re approaches 1 and we suggest that the reduced Nu is the result of inertia reducing the speed of ascending and descending thermals, and thus the rate of advective heat transport.

In applying results from studies such as the one presented here to the Earth and other planets, we recognize that many important features of the Earth are not simulated in our experiments. In particular, the presence of mobile surface plates, internal heating, and depth-dependent viscosity variations associated with pressure rather than temperature, are thought to play a key role in governing the pattern and character of convection in the Earth. Nevertheless, our experiments illustrate some of the physical processes that occur in high Ra and high Pr convection. These experiments also address the limit of finite Pr in which inertia begins to play a role, a limit that will apply to other geological systems such as magma chambers.

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