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Mixing of heterogeneities in the mantle: Effect of viscosity differences

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Abstract. The effect of viscosity variations associated with large scale chemical heterogeneities on mixing in the mantle is studied for a model two-dimensional problem. Low viscosity regions are rapidly deformed, develop long tendrils, and are entrained towards regions of surface divergence. Very viscous regions are only slowly stretched; thus, geochemical reservoirs can persist relatively undisturbed for long periods of time if they are $> O(10-100)$ times more viscous than the surrounding mantle. High viscosity ratio blobs have a tendency to aggregate, leading to the formation of large scale heterogeneities from smaller ones.

Introduction

Geochemical studies indicate that the mantle is heterogeneous at length scales ranging from 10 m to > 1000 km, and consists of a number of distinct reservoirs which contribute characteristic geochemical signatures to igneous rocks [e.g. *Zindler & Hart* 1986]. Geochemical observations can constrain of the rate and nature of mass transfer within the mantle and thus constrain geodynamical models [e.g. *Kellogg* 1992]. However, a number of different dynamical models have been proposed to account for observations. One class of models includes those in which the mantle is layered, and the upper mantle is the source of depleted mid-ocean ridge basalts (MORB) whereas the lower mantle remains enriched. Mixing between geochemical reservoirs occurs due to entrainment associated with convection [e.g. *Allègre & Turcotte*, 1985], or periodic penetration of slabs and plumes between the upper and lower mantles [e.g. *Silver et al.* 1988]. A second class of models allows whole mantle convection, and the source of enriched ocean-island basalts is associated with plate tectonic processes [e.g. Davies 1990]. Numerical simulations of whole-mantle mantle convection in an isoviscous mantle indicate that nearly complete mixing occurs on time scales of $O(1)$ b.y resulting in a relatively homogeneous source region for MORB [e.g. *Hoffman & MacKenzie* 1985]. If viscosity increases in the lower mantle, then the rate of mixing is reduced in the lower mantle, although numerical calculations indicate that passive tracers are still thoroughly mixed over a time scale of a few b.y. [e.g. *Davies* 1990]. Numerical simulations [*Christensen & Hofman* 1994] and laboratory experiments [*Olson & Kincaid* 1991] have shown that density differences associated with subducted slabs, allow subducted components to accumulate above the core-mantle boundary or some other discontinuity, and could then provide a source of enrichment for mantle plumes.

In this paper we study the effects of viscosity variations associated with compositional heterogeneities on mixing in the mantle. Specifically, we consider the stretching of regions, referred to hereafter as "blobs", with a viscosity λ times greater than the surrounding mantle. We demonstrate that viscosity variations may have a large effect of the rate of stretching and mixing of large scale heterogeneities.

Model

We assume that both inside and outside the blob, fluid motions satisfy Stokes equations

$$\nabla \cdot \mathbf{T} + \rho \mathbf{g} = \mu \nabla^2 \mathbf{u} - \nabla p = \mathbf{0} \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0 \quad (1)$$

where $\mathbf{u}$ is velocity, μ is viscosity, p is pressure, and $\mathbf{T}$ is the stress tensor. Equations (1) can be recast in the form of integral equations [e.g. *Pozrikidis* 1992]

$$\frac{1}{\mu}\int_S \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{J}\,dS + \int_S \mathbf{n}\cdot\mathbf{K}\cdot\mathbf{u}\,dS = \begin{cases} \mathbf{u}(\mathbf{x}) & \mathbf{x}\in V \\ \frac{1}{2}\mathbf{u}(\mathbf{x}) & \mathbf{x}\in S, \\ 0 & \mathbf{x}\notin V \end{cases} \quad (2)$$

where S includes all surfaces bounding the volume V, $\mathbf{n}$ is a unit normal vector outward to V, $\mathbf{x}$ is a position vector, and $\mathbf{J}$ and $\mathbf{K}$ are the Green's functions for velocity and stress, respectively.

Equations (2) for each fluid region, combined with appropriate boundary conditions (e.g. continuity of velocity and stress across fluid-fluid interfaces) can be applied to a given problem, and the resulting equations are solved numerically using standard collocation techniques [e.g. *Manga* 1996]. The position of fluid-fluid interfaces is determined by solving the kinematic equation, $d\mathbf{x}/dt = \mathbf{u}$ for $\mathbf{x} \in S$, using a second order Runge-Kutta method. In principle, the reformulation of the differential equations as integral equations reduces the dimensionality of the problem and thus allows the efficient study of problems with deformable fluid-fluid in-

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terfaces across which fluid properties, such as viscosity, change discontinuously. The method does not suffer from numerical diffusion problems of field based methods. However, in practice, numerical computation times can be very long; the results presented in figure 4ab took 250 and 280 CPU hours, respectively, on a Sparc 20.

Stretching in linear flows

In order to illustrate the effects of viscosity contrasts we present results for the deformation of a single blob immersed in a two-dimensional simple shear flow, $\mathbf{u}_s^\infty$, and a two-dimensional extensional (pure shear) flow, $\mathbf{u}_e^\infty$, described by

$$\mathbf{u}_s^\infty = G(y, 0) \quad \text{and} \quad \mathbf{u}_e^\infty = G(x, -y) \tag{3}$$

where x and y are horizontal and vertical positions, respectively, and G is the strain-rate. These two flows are frequently studied as limiting examples of local flows in a more complicated flow [e.g. *Stone* 1994], and are both characterized by a constant strain-rate throughout the fluid.

The integral equations for a single neutrally buoyant blob, with viscosity $\lambda\mu$, and boundary condition $\mathbf{u} \to \mathbf{u}^\infty$ as $\mathbf{x} \to \infty$, are

$$\mathbf{u}^\infty(\mathbf{x}) - (1-\lambda)\int_S \mathbf{n}\cdot\mathbf{K}\cdot\mathbf{u}\, dS = \begin{cases} \mathbf{u}(\mathbf{x}) & \mathbf{x} \notin \text{blob} \\ \frac{1+\lambda}{2}\mathbf{u}(\mathbf{x}) & \mathbf{x} \in S, \\ \lambda\mathbf{u}(\mathbf{x}) & \mathbf{x} \in \text{blob} \end{cases} \tag{4}$$

where S is the surface of the blob, $\mathbf{n}$ is the unit normal outward from the blob, and λ is the viscosity ratio.

Numerical results are presented in figure 1, with time normalized by $(1+\lambda)/G$. As previously discussed by many authors [e.g. *Olson et al.* 1984], pure shear is more "efficient" at stretching heterogeneities than simple shear. The former flow leads to exponential stretching, i.e.

$$l/a \propto e^{Gt/(1+\lambda)} \quad \text{for small } O(1) \text{ deformation} \tag{5}$$

where l/a (defined in figure 1) is a measure of the distortion of the blob and t is time. As observed in figure

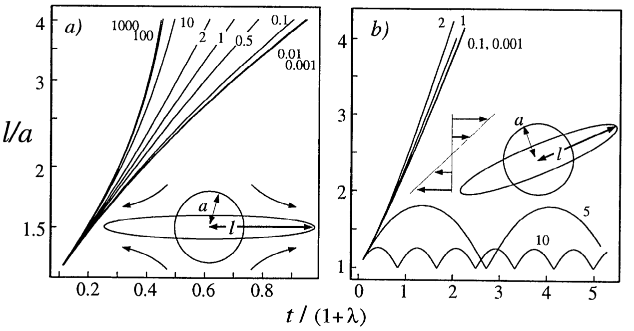

Figure 1. Stretching in *a)* an extensional flow and *b)* a simple shear flow for different viscosity ratios, λ. Time is normalized by $(1+\lambda)/G$, where G is the strain-rate.

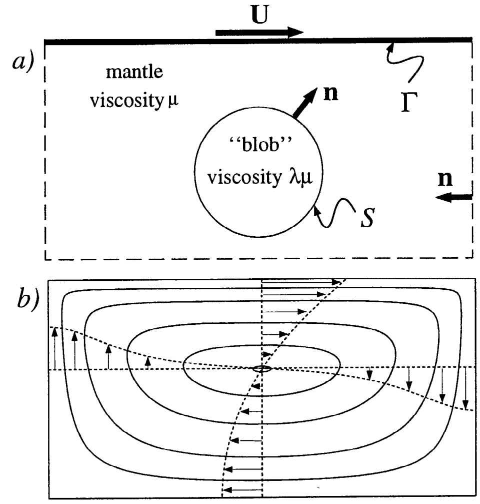

Figure 2. *a)* Geometry of the model problem. *b)* Flow driven by the moving upper plate. The 'blob' with viscosity $\lambda\mu$ will alter the flow shown in *b*. The model mantle has a free-slip lower boundary, reflecting side walls, and constant velocity surface. Box width is L.

1a, the rate of stretching depends on the aspect ratio for large distortions: *Spence et al.* [1988] showed that for aspect ratios $b_o/a_o \gg 1$ and $\lambda \gg 1$, $b/b_o \propto e^{Gt-\lambda a_o/b_o}$, where a and b are the short and long axes of the blob, respectively. Simple shear leads to linear stretching, i.e.

$$l/a \propto Gt/(1+\lambda) \quad \text{for } \lambda < 4. \tag{6}$$

For viscosity ratios $\lambda > 4$, *Taylor* [1934] first recognized that blobs will rotate more rapidly than they are stretched (simple shear consists of equal parts strain-rate, which leads to stretching, and vorticity, which leads to rotation) so that l/a will vary periodically between 1 and an $O(1)$ constant, as illustrated in figure 1b for $\lambda = 5$ and 10.

Stretching in a model mantle

Flow in the mantle is obviously more complicated that the linear flows studied in the previous section. Here we consider stretching by the steady plate-driven flow illustrated in figure 2. An appropriate form of the integral equations for the geometry illustrated in figure 2 for N neutrally buoyant blobs can be derived following *Manga & Stone* [1993]. On the boundary Γ, $\mathbf{u}_\Gamma$ and $\mathbf{n}\cdot\mathbf{T}_\Gamma$ are chosen to satisfy a free-slip lower boundary, reflecting side walls, and $\mathbf{u}_\Gamma = (U, 0)$ on the surface. Time is normalized by the convective time scale L/U where L is the width of the box.

In figure 3 we illustrate the effect of the viscosity ratio on stretching by the plate-driven flow. During one overturn time the $\lambda = 10$ blob experiences little deformation; however, the $\lambda = 0.1$ blob becomes highly deformed after only 1/4 of an overturn and a long tendril is formed and entrained towards the divergent boundary on the left. In figure 4ab we show the results of two simulations for $\lambda = 10$ with 5 and 9 blobs, respectively. Since $\lambda > 4$, stretching is due primarily to the exten-

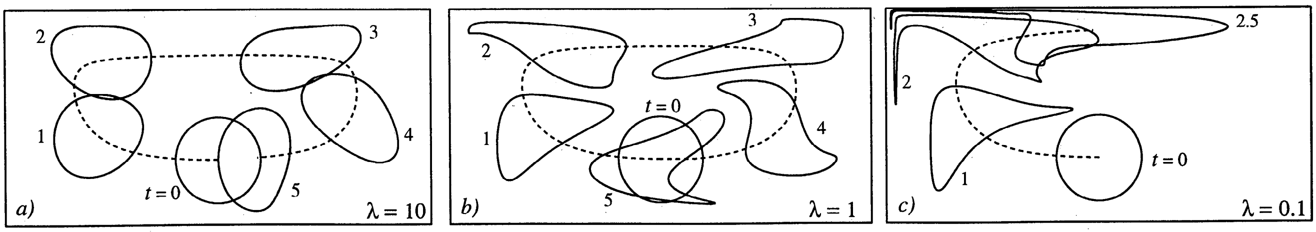

Figure 3. Translation and deformation of the 'blob' for *a)* $\lambda = 10$, *b)* $\lambda = 1$ (passive tracers) and *c)* $\lambda = 0.1$. The dashed curves are the trajectories of the center-or-mass of the blobs. Time is normalized by L/U.

sional component of the flow. In the calculation with 5 blobs, the rate of stretching increases as the amount of deformation increases, but even after 1.8 b.y., the blobs are largely undeformed although the amount of strain accumulated by each blob varies significantly; in particular, part of blob number 3 becomes highly elongated. On average, the scaling described by (5) characterizes blobs in this more complicated system.

The simulation in figure 4b illustrates some consequences of the interactions between the blobs. As the blobs pass over and around each other, they deform and are observed to form pairs (for example, blobs 8 and 9 which form a pair and remain a pair). The "merging" of blobs occurs since the area of near contact increases as the blobs pass around each other, so that large stresses or long periods of time would be required to reseparate the blobs (owing to the large lubrication forces associated with the small gap). Merging is more clearly illustrated in figure 5 in which two initially circular blobs with $\lambda = 5$ in a shear flow do not simply pass around each other but merge and behave as a single larger blob.

Discussion

The results presented here indicate that blobs with viscosities only about 10 times greater than the surrounding mantle are slowly stretched and mixed and thus may persist relatively undeformed for billions of years. As noted in previous studies [e.g. *Olson et al.* 1984; *Gurnis* 1986] and illustrated in figure 1, pure shear is much more efficient than simple shear at stretching heterogeneities. In fact, for viscosity ratios $\lambda > 4$, simple shear cannot stretch heterogeneities [e.g. *Taylor* 1934]. In contrast to high viscosity blobs, regions with $\lambda \ll 1$, e.g. figure 3 for $\lambda = 0.1$, are very rapidly stretched, are entrained into regions of divergence (e.g mid-ocean ridges), and are likely to be rapidly mixed into the rest of the mantle. High viscosity blobs also tend to aggregate, a mechanism which counteracts mixing, and could possibly lead to the formation of large scale heterogeneities from smaller ones. Although we have been limited to considering a two-dimensional geometry, the results in figure 1 are quantitatively similar to three-dimensional results. The dynamics in figures 4b and 5 are due to the close approach the blobs, and thus should occur in three dimensions.

It has been observed that time-dependent convection is much more efficient than steady flows, such as the one studied here, at stretching heterogeneities [e.g. *Kellogg* 1992; *Ottino* 1989], although it is also a general feature of chaotic fluid advection that "islands" of unmixed fluid remain [e.g. *Metcalfe et al.* 1995]. We can estimate the rate of stretching based on the approximate relationships, equations (5-6), describing the results in figure 1. For small, $O(1)$, deformations $l/a \approx e^{Gt/(1+\lambda)}$. Assuming, the blob always remains in an extensional flow with strain rate $G = 10^{-15}\mathrm{s}^{-1}$, then blobs with viscosity ratios $\lambda > 100$ will experi-

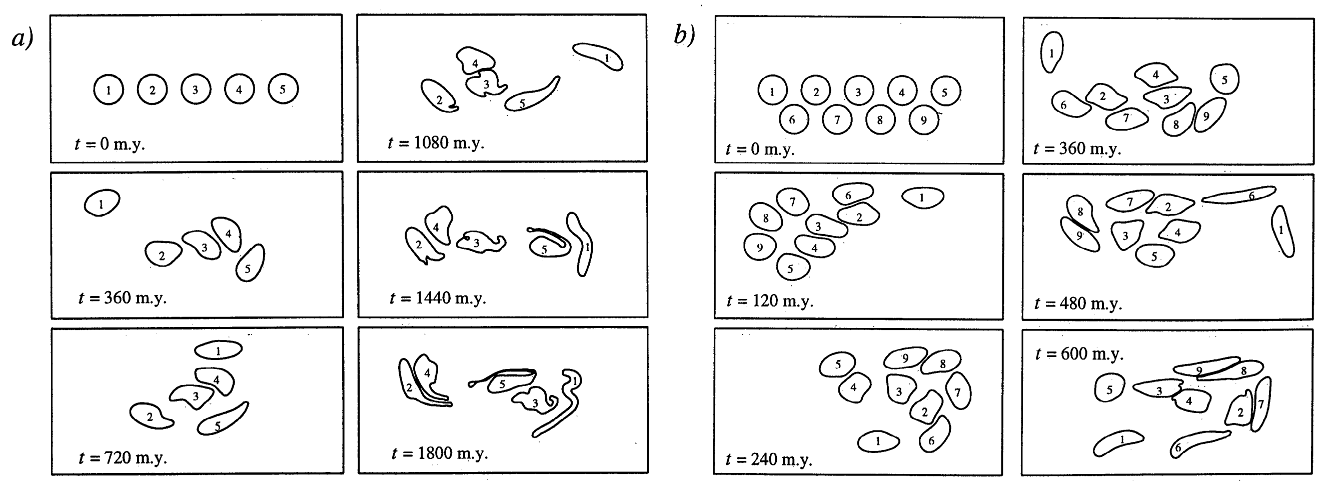

Figure 4. Stretching of blobs in simulations with *a)* five and *b)* nine blobs for $\lambda = 10$. Times are based on a mantle thickness of 3000 km and a plate velocity of 10 cm/year.

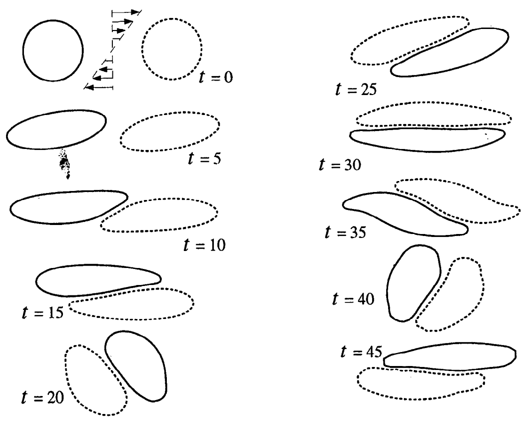

Figure 5. Interaction of two blobs with $\lambda = 5$ in a simple shear flow. The blob which begins on the left is shown with solid curves. Time is normalized by $1/G$, where G is the strain-rate.

ence only an $O(1)$ distortion over the course of earth history. However, *Spence et al.* [1988] noted that heterogeneities with large aspect ratios are more rapidly stretched, so that subducted slabs with large aspect ratios are rapidly stretched even though they may be much more viscous than the surrounding mantle. If the lower mantle is more viscous than the upper mantle, the strain-rate will be smaller in the lower mantle, thus smaller λ are required to hinder stretching.

Large compositionally distinct blobs may form due to the early differentiation of the earth or may represent idealized forms of subducted slabs which become thickened as they enter the lower mantle [e.g. *Gurnis & Hager* 1988]. Differentiation early in earth history would produce a radial structure which may be continuously or discontinuously layered [e.g. *Agee* 1993]. The onset of convection would destroy the radial structure and may form compositionally distinct blobs. Compositional variations can affect the effective viscosity of mantle materials. For example, the large difference in the melting temperature of MgO and perovskite [e.g. *Zerr & Boehler* 1994] implies a large (many orders of magnitude) difference in the effective viscosity of these two materials. Thus, bulk composition variations will be associated with viscosity variations (although, there is no evidence that variations of major element composition are associated with the various geochemical reservoirs). Similarly, garnet rich slabs may have a high effective viscosity and would thus be slowly stretched and mixed.

Summary

Blobs of very viscous mantle can persist relatively undeformed for long periods of time, even in a mantle undergoing 'whole mantle' convection, and could thus represent regions of the mantle which are largely unsampled geochemically. In addition, high viscosity ratio blobs have a tendency to aggregate.

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