The yield strength of subliquidus basalts — experimental results

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Abstract

Yield strength is an important property of particle–fluid suspensions. In basaltic lavas that crystallize during flow emplacement, the onset of yield strength may result in threshold transitions in flow behavior and flow surface morphology. However, yield strength–crystallinity relations are poorly known, particularly in geologic suspensions, where difficulties of experimental and field measurements have limited data acquisition in the subliquidus temperature range. Here we describe two complementary experimental approaches designed to examine the effect of particle shape on the low-shear yield strength of subliquidus basalts. The first involves melting cubes of holocrystalline basalt samples with different initial textures to determine the temperature (crystallinity) at which these samples lose their cubic form. These experiments provide information on the minimum crystal volume fractions (0.20 < φ < 0.35) required to maintain the structural integrity of the cube. The second set of experiments uses suspensions of corn syrup and neutrally buoyant particles to isolate the effect of particle shape on yield strength development. From these experiments, we conclude that the shape is important in determining the volume fraction range over which suspensions exhibit a finite yield strength. As anisotropic particles may orient during flow, the effect of particle shape will be controlled by the orientation distribution of the constituent particles. We find that the so-called ‘excluded volume’ can be used to relate results of experiments on anisotropic particles to those of suspensions of spherical particles. Recent measurements of yield strength onset in basaltic melts at crystal volume fractions near 0.25 are consistent with our observations that crystal frameworks develop at low to moderate crystal volume fractions when crystals are anisotropic (e.g. plagioclase). We further suggest that conditions leading to yield strength onset at low crystallinities include rapid cooling (increased crystal anisotropy), heterogeneous nucleation (which promotes extensive crystal clustering and large cluster anisotropy) and static conditions (random crystal orientations). © 2001 Elsevier Science B.V. All rights reserved.

Keywords: lava rheology; yield strength; lava flow morphology

1. Introduction

Lava flows cool and crystallize as they move over the surface of the earth. Crystallization, in turn, causes rheological changes that affect both the velocity of the flows and their style of emplacement (Griffiths, 2000). In Hawaii, these changes are most pronounced in flows that move swiftly through open channels. Here cooling is rapid during early stages of flow, and resulting down-channel increases in flow crystallinity cause rheological changes that result in flow thickening and diminished flow velocities (e.g. Lipman and Banks, 1987; Moore, 1987; Wolfe et al., 1988; Crisp et al., 1994; Cashman et al., 1999). The rheology of sufficiently dilute crystal-melt suspensions is Newtonian, with a slightly higher shear viscosity than the particle-free fluid (Einstein, 1956). More concentrated suspensions can be non-Newtonian, exhibiting both yield
strength behavior and shear-thinning rheology (Jeffrey and Acrivos, 1976). As non-Newtonian behavior will influence flow on both a micro- and macroscopic scale, accurate flow emplacement models require a thorough understanding of the rheology of crystal-melt suspensions (e.g. Robson, 1967; Walker, 1967; Hulme, 1974; Dragoni and Tallarico, 1994; Kilburn, 1996).

The properties of particle–fluid suspensions are governed in part by two limiting particle concentrations: the volume fraction of particles at which maximum packing is achieved (φ_m) and the minimum volume fraction at which the suspension exhibits a finite yield strength (φ_y) (e.g. Krieger and Doherty, 1959; Wildemuth and Williams, 1984, 1985; Zhou et al., 1995). Particle size, shape, and orientation distributions control both of these volume fractions, although their effects are best documented for φ_m (Larson, 1999). Theoretical values of φ_m for ordered uniform spheres vary from a lower limit of 4/7 (cubic closest packing) to an upper limit of 0.74 (hexagonal closest packing). An increase in the particle size distribution can lead to an increase in packing efficiency; for example, in a bidisperse suspension φ_m can be as large as 0.85 (Yu and Standish, 1993). In contrast, an increase in the anisotropy of individual particles typically causes a decrease in packing efficiency (Onsager, 1949). A compilation of experimental studies by Blanc (1995) shows a decrease in the volume fraction of fibers (packed by vibrations) from about 0.6 for fibers with aspect ratios of 4, to 0.04 for aspect ratios of 100. Orientational order permits higher values of φ_m (Balberg, 1985), and affects the relative shear viscosity of a suspension in both the dilute (Batchelor, 1971) and semi-dilute (Shaqfeh and Fredrickson, 1990) limits. For geological applications, φ_m is usually taken as 0.6, and shear viscosity is assumed to increase with particle concentration according to the Roscoe–Einstein equation (e.g. Marsh, 1981; Pinkerton and Stevenson, 1992).

Yield strength (τ_y) development in suspensions is less constrained, largely because of measurement difficulties (e.g. Kerr and Lister, 1991; Nguyen and Boger, 1992). Yield strengths of crystal-melt suspensions have been measured in both the field (Shaw et al., 1968; Pinkerton and Sparks, 1978; Pinkerton and Norton, 1995) and the laboratory (Shaw, 1969; Murase and McBurney, 1973; Ryerson et al., 1988; Lejeune and Richet, 1995; Pinkerton and Norton, 1995; Philpotts and Carroll, 1996). Although the results have not been generalized into a predictive model, it is clear that the onset of yield strength is controlled by the volume fraction (φ_y) at which a ‘touching framework’ first forms, and that φ_y is not a constant. Here we examine yield strength development in viscous fluid–particle suspensions using two different but complementary experimental approaches. In the first we determine φ_y in natural basalts by melting basalt cubes until they lose their cubic form. By comparing the melting behavior of basalt samples with different initial crystal textures, we establish relationships between φ_y and crystal shape, spatial clustering and orientation distribution. A second set of experiments utilizes corn syrup and neutrally buoyant particles to isolate the effect of particle shape on yield strength development. These experiments complement a percolation-based numerical study of φ_y presented in a companion paper (Saar et al., 2001).

2. Methods

2.1. Melting experiments

Following the approach of Philpotts and Carroll (1996), we determine the minimum crystallinity required for yield strength development by melting cubes (1 cm³) of basaltic lava until the cubic form collapses. Two basalt samples from Hawai’i (one ‘a‘a and one pahoehoe) and one ‘a‘a sample from Lava Butte, OR, provide contrasting starting textures (Fig. 1) in samples with similar compositions. Prior to melting, ‘a‘a samples from both Hawaii and Lava Butte contained pyroxene and plagioclase crystals of fairly uniform size (100–200 μm). Backscattered electron (BSE) images show that pyroxene crystals are near-equant in shape, and elongate plagioclase crystals exhibit some preferred orientation (Fig. 1a and c). In contrast, the interior of a small Hawaiian pahoehoe toe has elongate plagioclase crystals surrounded by fine-scale dendritic intergrowths of plagioclase and pyroxene (Fig. 1b).

Samples are placed in a high-density graphite crucible and suspended in a furnace at temperatures ranging from 1135 to 1160°C for a minimum of 2
Fig. 1. BSE images of samples used in the melting experiments. Scale bar is 200 µm long in all images; as gray scale levels vary between images, phases are identified separately for each image. (a) ‘a’ sample from Kilauea, HI. Dark gray crystals are plagioclase, light gray are pyroxene, and white are Fe–Ti oxides. (b) Pahoehoe toe, Kilauea Volcano, HI. Black crystals are plagioclase, medium gray are pyroxene. Note fine scale dendritic intergrowths in this sample. (c) ‘a’ sample from Lava Butte, OR. Medium gray crystals are plagioclase, white are pyroxene, black areas are void spaces.

hours (Table 1). Sample temperatures are accurate to ±5°C. The bottom of each crucible has a 3/4 in. hole, which allows molten samples to drain and quench in water located at the base of the furnace. Oxygen fugacity is controlled by the graphite container, as attempts to regulate the fugacity through the introduction of CO₂ and H destroyed the crucible within 2–4 hours. The low fugacity resulting from use of a graphite buffer may affect the melting conditions at the cube surface, but we assume that the sample interior melts under conditions close to those of crystallization (QFM; e.g. Philpotts and Carroll, 1996).

Images of quenched samples are obtained using a JEOL 6300 SEM operated at 10 keV accelerating voltage, a beam current of 5 nA and a working distance of 15 mm. BSE imaging allows two-dimensional characterization of sample textures using NIH Image software.

2.2. Analog experiments

We use suspensions of corn syrup and particles to perform analog experiments for determination of yield strength. Corn syrup is diluted with water until
Table 1
Experimental conditions and results for melting of natural basalts. Temperatures are accurate to ±5°C. Volume fractions ($\phi$) of plagioclase ($\phi_{\text{plag}}$) and pyroxene ($\phi_{\text{px}}$) were determined by image analysis of BSE images, and were determined only for those samples that could be used to bracket $\phi_c$.

<table>
<thead>
<tr>
<th>Flow type</th>
<th>Temperature (°C)</th>
<th>Duration (h)</th>
<th>Final shape</th>
<th>$\phi_{\text{plag}}$</th>
<th>$\phi_{\text{px}}$</th>
</tr>
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<tr>
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<td>31</td>
<td>Cubic</td>
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<td>Cubic w/melt</td>
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<tr>
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<td>0.34</td>
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<td>All melt</td>
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<td>0.16</td>
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<tr>
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<td>5</td>
<td>All melt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>'a'a (Lava Butte)</td>
<td>1150</td>
<td>3</td>
<td>Cubic w/melt</td>
<td>0.31</td>
<td>0.04</td>
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</table>

The mixture viscosity is 50–100 Pa s at room temperatures (similar to the viscosity of near-liquidus Hawaiian basalt), with a density of 1440 kg/m$^3$. Two different populations of particles are used, spherical (poppy seeds) and prismatic (wax). While neither perfectly spherical (Fig. 2A) nor perfectly neutrally buoyant (density $\rho = 1330$ kg/m$^3$), poppy seeds provide a sufficiently close approximation to both characteristics to determine the behavior of suspensions of spherical particles. We create prismatic neutrally buoyant particles by mixing melted paraffin wax with kaolin, shaving the cooled and solidified mixture, and sieving the shavings to obtain a uniform grain size. The particles average 1 mm in length, and have density of 1440 kg/m$^3$ and average aspect ratio (length/width) of 3.4 (Fig. 2B). The particle size is thus much smaller than the typical final flow thickness of ~10 mm.

The experimental apparatus used is shown in Fig. 3. Water pumped from a graduated cylinder into a sealed container compresses a bag containing the corn syrup suspension and extrudes it at a constant rate (21 g/min) into a clear tank. The tank is covered (to prevent dehydration of the corn syrup), and temperature and effusion rate are held constant for all experiments, while liquid:particle ratios are varied. To ensure that the flow is controlled by the rheology of the suspension and not by a particle-free fluid layer at the flow base, experiments with suspensions of prismatic particles use both a smooth and a textured base. After the pump is shut off, the extruded suspension is allowed to spread. Flow height and radius are recorded throughout extrusion. Three examples of these measurements are shown in Fig. 4; a complete description of the results can be found in Hoover (1999).

For Newtonian fluids with constant physical properties and a flow radius ($R$) that greatly exceeds flow height ($H$), the flow radius is expected to increase with time ($t$) as $R \propto t^{1/2}$ during active extrusion (constant discharge), and as $R \propto t^{1/8}$ after extrusion ceases (constant volume) (Huppert, 1982). Both spreading regimes can be observed in our experiments with particle-free corn syrup (Fig. 4). A balance of surface tension and gravitational forces indicates that surface tension will dominate only when the corn syrup is less than 2 mm thick, assuming a surface tension of 0.08 N/m (Borhan and Pallinti, 1999). Continued spreading of the particle-free corn syrup throughout the duration of the experiment confirms the absence of surface tension influence for the flow dimensions of our experiments.

If the fluid has a yield strength, spreading will stop once the gravitational force driving flow equals the yield strength, as can be seen in the spreading behavior of the particle–fluid suspensions (Fig. 4). Particle-bearing experiments show $R \propto t^{1/8}$ for several hundred seconds after active extrusion ceased, after which the
radius remains constant. Although spreading thus appears to cease well before the end of each 2- to 3-hour experiment, yield strengths inferred from measurements of R/H should be considered maxima. In the analog experiments, particle size does not appear to influence final flow thickness, as illustrated by a comparison of $\phi_c$ and $\tau_y$ determined from experiments using particles of 0.5 and 1 mm (Hoover, 1999).

3. Results

3.1. Melting experiments

Starting materials for the melting experiments were chosen to compare the relative strengths of networks formed under different conditions of flow emplace-
samples is similar (Hoover, 1999). Also potentially important are variations in particle shape between different crystalline phases (Philpotts et al., 1998; Saar et al., 2001). For this reason, we report two values of $\phi_c$ — one based on the total crystallinity ($\phi_c$) and one based on the crystallinity of anisotropic plagioclase crystals ($\phi_c^{\text{pl}}$). 'A'a samples collapse at a lower temperature than the pahoehoe sample. The Hawaiian 'a'a sample shows a large decrease in crystallinity between 1140 and 1145°C, the temperatures bracketing structural collapse. Resulting critical crystallinities are $0.57 > \phi_c > 0.34$, and $0.23 < \phi_c^{\text{pl}} < 0.18$ (Table 1). The plagioclase-dominated Lava Butte 'a'a sample provides tighter constraints on the minimum crystallinity of network formation, as the 1142°C melting experiment ($\phi = 0.35$) is very close to collapse (Fig. 5c). Collapse occurs by 1150°C ($\phi = 0.29$), yielding $0.35 > \phi_c > 0.29$ and $0.31 < \phi_c^{\text{pl}} < 0.26$. In contrast, the initially dendritic pahoehoe sample maintains its cubic form to a temperature between 1155 and 1160°C. Bracketing crystallinities for this sample extend both $\phi_c$ and $\phi_c^{\text{pl}}$ to lower limits than observed in the 'a'a, with $0.35 > \phi_c > 0.18$ and $0.18 < \phi_c^{\text{pl}} < 0.08$ (Fig. 6c and d).

Qualitative descriptions of pre-collapse sample textures are augmented by measurement of twodimensional plagioclase shape (minor and major axes of the best-fit ellipse) and orientation, with the
goal of assessing the extent to which the anisotropic plagioclase crystals play a role in network formation (e.g. Philpotts et al., 1998). For crystals of uniform shape, the mode of the frequency distribution of minor/major axis ratios is the average short/intermediate axis ratio of the three-dimensional crystal population (Higgins, 1994). The average long/short axis ratio may be estimated from the minimum minor/major axis category (e.g. Hammer et al., 1999). Axis ratio histograms show modes of 0.3–0.5 for all samples (Fig. 7a, c and e), indicating that the crystals are tabular, with intermediate:short axis ratios of 2–3:1. The small number of crystals with axis ratios <0.2 indicates that long:short axis ratios are between 5:1 and 10:1. Near-equant crystals are relatively infrequent. Crystal orientations (Fig. 7b, d and f; measured as the angle between the apparent long axis direction and the horizontal) show minimal alignment of plagioclase crystals except in the Hawaiian 'a’a, where crystals are moderately aligned at an angle approximately perpendicular to horizontal in Fig. 6a. However, the variation among samples is not large, and it appears that the orientation of individual crystals cannot explain the differences in $\phi^a$ observed in these samples (although this conclusion is qualified by the lack of three-dimensional data).

In summary, while complete three-dimensional characterization of our melting experiments is not
Fig. 6. BSE images of melted samples. Scale bar is 100 µm in all images. Plagioclase crystals are dark gray, pyroxene crystals are medium gray (Hawaiian samples) or white (Lava Butte samples), glass is light gray. (a) Hawaiian 'a`a at 1140°C. (b) Hawaiian 'a`a at 1145°C. (c) Hawaiian pahoehoe at 1155°C. (d) Hawaiian pahoehoe at 1160°C. (e) Lava Butte 'a`a at 1142°C. (f) Lava Butte 'a`a at 1150°C.
possible, qualitative observations, together with two-dimensional measurements of BSE images, suggest the following minimum requirements for yield strength development in basaltic lavas. In all samples, we find that the presence of anisotropic crystals or crystal clusters allows yield strength development at particle volume fractions less than 0.35. The pahoehoe toe shows the lowest bracketing values of \( \phi_c \), with \( 0.35 > \phi_c > 0.18 \) (Table 1). At \( \phi = 0.35 \), the sample contains connected clusters of plagioclase and pyroxene crystals, which gives the sample a more pronounced textural heterogeneity than indicated by measurement of the shape or orientation characteristics of the individual crystals. The dissolution of most crystal clusters in the collapsed pahoehoe samples suggests that these clusters (plagioclase + pyroxene) provide structural integrity in the pre-collapse sample. The Lava Butte sample contains primarily plagioclase, and thus bracketing conditions for framework formation are best approximated by the

Fig. 7. Axis ratio (minor/major axis lengths) and long axis orientation histograms of plagioclase crystals in pre-collapse melting experiments. (a) and (b) Hawaiian 'a'a at 1140°C. (c) and (d) Hawaiian pahoehoe at 1155°C. (e) and (f) Lava Butte 'a'a at 1142°C.
limiting plagioclase contents of 0.31 < \phi_c < 0.26. Yield strength development in the Hawaiian 'a'a sample is less well constrained, as is the extent to which plagioclase alone controls yield strength, but \phi_c is probably comparable to that of the Lava Butte sample. \phi_c of <0.35 are similar to those measured by Philpotts and Carroll (1996) and Philpotts and Dickson (2000) in slowly cooled samples from the interior of flood basalt lava flows. However, Philpotts et al. (1998) suggest that plagioclase alone is responsible for framework formation in these samples. If plagioclase represents approximately half of the crystal population, then the resulting \phi_c \leq 0.14 in the flood basalt samples is low relative to our measurements.

3.2. Analog experiments

Suspensions used in the analog experiments contained from 0 to 0.50 volume fraction particles (\phi). As \phi increases, the final flow thickness increases and the surface texture changes from smooth to rough. The particle concentration at which these changes occur is different for suspensions composed of spherical particles than for those containing prisms (Figs. 8 and 9). Suspensions of spherical particles retain a smooth surface to concentrations of about 0.40, and increase substantially in final thickness at particle concentrations of about 0.50. In contrast, suspensions of prismatic particles develop a rough surface texture at volume fractions of about 0.18, and the final flow thickness increases substantially at particle concentrations of about 0.23. Prismatic suspensions containing low particle volume fractions also show distinct alignment of particles in the direction of flow.

We determine the yield strength (\tau_y) of each suspension from the final height and radius of the flow (Table 2) by assuming that flow ceases when the gravitational (buoyancy) force is balanced by yield strength (e.g. Griffiths and Fink, 1997), such that

\[ \tau_y = C' g \Delta \rho H^2 / R, \]

where \Delta \rho is the density difference driving spreading, \( g \) is acceleration due to gravity, and \( C' \) is a constant. Following Blake (1990), we assume \( C' = 0.32 \). Calculated yield strengths are shown as a function of particle concentration in Fig. 10. In all cases, yield strength increases rapidly for \( \phi > \phi_c \). Curves for suspensions of spherical and prismatic particles are similar in form to simple exponential curves (shown as solid and dashed curves in Fig. 10). Differences between \( \tau_y \) for prismatic suspensions extruded on smooth and textured bases are probably within the measurement uncertainties.

4. Discussion

The experiments described here provide constraints on the onset of yield strength in both natural basalts and in simpler analog suspensions. The two types of experiments are complementary. Natural samples provide realistic crystal architectures, with both interpenetrating and isolated crystals of varying sizes, shapes, and compositions. However, the textures are difficult to quantify and the experiments are static (do not involve flow). Analog suspensions are well characterized, with particles of controlled sizes and shapes and a simple flow regime, but have isolated (rather than interpenetrating) particles.

Here we use the melting experiments to estimate \( \phi_c \) in natural systems. Next, we quantify the effect of variable particle shape on \( \phi_c \) using the results of the analog experiments, and introduce an invariant function based on the so-called 'excluded volume'. We show that our estimates of \( \phi_c \) for different particle shapes are consistent with those of field (e.g. Pinkerton and Norton, 1995) and laboratory (e.g. Lejeune and Richet, 1995; Pinkerton and Norton, 1995; Philpotts and Carroll, 1996) measurements, and also with predictions of percolation theory (e.g. Garboczi et al., 1995; Saar et al., 2001). Finally, we speculate about the possible effects of different lava flow emplacement regimes on the development of variable textures and resulting yield strength behaviors.

4.1. Onset of yield strength in crystallizing basalt: melting experiments

The melting experiments provide qualitative observations on the development of crystal frameworks in basaltic lavas. 'a'a samples, with uniform populations of plagioclase and pyroxene crystals, lose structural integrity at particle volume fractions of ~0.3. Before structural collapse, the Hawaiian 'a'a showed extensive interconnection of plagioclase and pyroxene crystals, while after collapse the sample consisted of isolated (in two-dimensional images; Fig. 6a and b)
plagioclase and pyroxene crystals. Loss of a crystal network as the cause of collapse is less obvious in the plagioclase-dominated Lava Butte samples (Fig. 6e and f). However, the prevalence of highly anisotropic plagioclase crystals is likely to result in intricate three-dimensional networks that may not be visible in two-dimensional images (e.g. Philpotts et al., 1998). Interconnected crystal networks also appear to provide yield strength in the Hawaiian pahoehoe, with an interconnected plagioclase + pyroxene network present in the highest temperature cubic sample (1155°C; Fig. 6c) and absent in the droplet-forming sample (1160°C; Fig. 6d). Although the precise crystallinity of network formation is not well established, bracketing crystallinities of 0.18 and 0.35 indicate yield strength behavior at relatively low crystal volume fractions.

The melting experiments illustrate the importance of particle shape and clustering in determining the onset of yield strength. Both 'a'a samples lose structural coherence at crystal volume fractions of <0.30–0.35, while the pahoehoe sample retains structural rigidity to somewhat lower crystallinities. These critical crystallinities are lower than observed (Lejeune and Richet, 1995) or assumed (Kerr and Lister, 1991) for magmatic suspensions containing spherical particles (where \( \theta_c = 0.4–0.5 \)) and thus have been considered unusual when observed in basaltic samples (e.g. Philpotts and Dickson, 2000). Based on our observations, we suspect that limiting crystal volume fractions of less than 0.4 are common in melts that contain randomly oriented anisotropic crystals. As crystal shape, spatial heterogeneity, and orientation distribution are controlled by cooling and flow regimes that accompany crystallization (e.g. Kouchi et al., 1986; Sato, 1995; Manga, 1998), our results suggest that \( \theta_c - \tau_c \) relations in lavas should reflect both the cooling and strain history. Finally, these experiments illustrate the importance of both textural characterization (particularly crystal shape, orientation, and spatial distribution) of natural and experimental samples, and the importance of integrating sample cooling and flow histories into rheological models.

Fig. 8. Final flow forms for analog experiments using poppy seeds. Volume fraction of solid particles (\( \phi \)) shown for each image. Circular grid underlying the flows has a spacing of 1 cm.
4.2. Effect of particle shape and concentration: analog experiments

The effect of shape on the formation of particle networks is most easily examined in the analog experiments, where particle shape and size are uniform, and flow history is known. Two important observations can be made about the results of our analog experiments (Fig. 10). First, each set of experiments shows an approximately exponential relationship between yield strength and particle concentration. Second, the onset of measurable yield strength occurs at lower particle concentrations in suspensions of prisms than in suspensions of spheres. Each of these observations will be discussed separately.

4.2.1. Yield strength models

The presence of a yield strength requires that a touching framework of crystals exists across the entire suspension, and that the framework is able to support a small but non-zero stress (Kerr and Lister, 1991).

Although the existence of yield strength has been questioned (e.g. Barnes and Walters, 1985; Barnes 1999), Blanc and van Damme (1995) note that “theoretically, nothing is against the very concept of yield stress … and that, in any case, it remains a useful operational concept if the stresses do not vanish at the smallest shear rates that one is able to apply or, equivalently, at observation times compatible with the experiment”. Empirical relationships describing yield strength in particle suspensions are reviewed in Pinkerton and Stevenson (1992); Zhou et al. (1995). One convenient model is described by

$$\tau_y = A \left( \frac{\phi/\phi_c - 1}{1 - \phi/\phi_m} \right)^{1/p}$$

(Wildemuth and Williams, 1984, 1985; Zhou et al., 1995), where $\phi_c$ is the minimum particle concentration at which the suspension can sustain some (low) external stress (the zero-shear yield strength), and $\phi_m$ is volume concentration at which the yield strength
Table 2
Height, radius, and calculated yield strength for all analog experimental data presented in Fig. 10

<table>
<thead>
<tr>
<th>% particles</th>
<th>H (cm)</th>
<th>R (cm)</th>
<th>( \tau_y ) (Pa)</th>
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</table>

Fig. 10. Yield strength measurements from analog experiments. Open diamonds — kaolin/wax prisms extruded on smooth (non-textured) base. Solid diamonds — kaolin/wax prisms extruded on textured base. Solid squares — poppy seeds extruded on smooth (non-textured) base. Solid and dashed lines are exponential fits to the yield strength data. Dotted lines are fits to the data using Eq. (2), with \( p = 1 \).

and \( A = 5.3 \). \( p \) is thus within the range \( (0.5–2.0) \) observed in concentrated suspensions (Zhou et al., 1995). As discussed previously, \( A \) depends on a number of properties and interparticle interactions, and its absolute value is not meaningful for extrapolation to geological materials. Turning to the suspension of prisms, we obtain the same value of \( p \) for \( \phi_c = 0.09 \). The yield strength data is now well described by \( \phi_m = 0.28 \) and \( A = 6.9 \). Comparing these two fits, we note that not only are estimates of \( A \) similar, but also that \( \phi_c \) and \( \phi_m \) differ by a factor of 2–3 in both cases. This similarity suggests that a simple scaling relationship may relate the two curves, although this comparison is qualified by that observation that the fit parameters are quite sensitive to the choice of \( \phi_c \).

4.2.2. Effect of particle shape on \( \phi_c \)

The observed reduction of both \( \phi_c \) and \( \phi_m \) for anisotropic particles is well known, particularly for fiber suspensions (see review by Blanc, 1995). Here we seek to generalize our experimental results, and to relate our results to theoretical models, through the use of an invariant function to relate \( \tau_y \), \( \phi_c \) and \( \phi_m \). In Saar et al. (2001) we provide a more comprehensive discussion of these ideas and scaling relationships.
One way to view $\phi_c$ is as a percolation threshold that defines the initial development of a connected pathway of crystals across the suspending fluid. The percolation threshold, $p_c$, for overlapping (soft-core) spheres is 0.29 (Pike and Seager, 1974) and is reduced to 0.18 for non-overlapping (hard-core) spheres (Powell, 1979). For comparison, in the analog experiments, where the dimpled shape of the poppy seeds may allow partial overlap, $\phi_c \sim 0.25$. The percolation threshold decreases with increasing particle anisotropy for randomly oriented particles (Balberg 1985; Garboczi et al., 1995; Saar et al., 2001), consistent with the observed decrease of $\phi_c$ for prisms relative to spheres in the analog experiments.

One of the relevant results of percolation theory studies (e.g. Pike and Seager, 1974; Balberg, 1985) and numerical simulations (e.g. Garboczi et al., 1995; Saar et al., 2001) is that $p_c$ (and by analogy $\phi_c$) scales with the so-called excluded volume, $V_e$. The excluded volume is the volume containing a particle in which the center of a similar particle cannot be placed without overlap of the particles. In a suspension of non-overlapping spheres, for example, $V_e$ is eight times the volume of each sphere.

In detail, for randomly positioned and parallel-aligned particles, $n_c\langle V_{ex}\rangle$ is independent of shape, and also independent of the size distribution for soft-core particles (e.g. Haan and Zwanig, 1977; Balberg et al., 1984a). Here $n$ is the number of particles per unit volume (the subscript $c$ denotes its value at the percolation threshold), and $\langle V_{ex}\rangle$ is the mean excluded volume of particles in the suspension. For randomly oriented particles, $n_c\langle V_{ex}\rangle$ is no longer a true invariant, and varies by a factor of about two for all particles shapes (Balberg, 1985). The variability of $n_c\langle V_{ex}\rangle$ is small, however, compared to the orders-of-magnitude variability of $p_c$ (Garboczi et al., 1995; Saar et al., 2001).

If $\tau_j$ also scales with $n\langle V_{ex}\rangle$ then we should be able to relate our two sets of $\tau_j(\phi)$ measurements. To compare results for non-spherical particles with those for spheres, we can define an ‘a’ ‘effective’ volume fraction as

$$\phi_{\text{eff}} = \frac{\langle V_{ex} \rangle}{8V} \phi,$$

where $V$ is the particle volume, $\phi$ is the actual volume fraction of the non-spherical particles and $\phi_{\text{eff}}$ is the volume fraction of an equivalent object (here a sphere with $V_{ex} = 8V$). If we assume that the laths in the analog experiments are randomly oriented rods with a hemispherical cap of length $L + W$ and width $W$, Balberg et al. (1984b) showed that

$$\langle V_{ex} \rangle = \frac{4\pi}{3}W^3 + 2\pi W^2 L + (\pi/2)WL^2.$$

Note that for $L \gg W$, $\langle V_{ex} \rangle / V$ is proportional to the aspect ratio $L/W$, rather than to the volume of the individual particles. Using Eqs. (3) and (4), and $L = 2.4W$ (Hoover, 1999), we calculate that $\phi_{\text{eff}} = 1.5\phi$, and use this to scale measured $\tau_j(\phi)$ relations for prismatic suspensions (Fig. 11). While this scaling moves the curve for prismatic suspensions toward that of spheres, it does not superimpose the curves (i.e. $\langle V_{ex} \rangle$ does not appear to be a true invariant). This discrepancy is probably the result of the reduction of $n_c\langle V_{ex}\rangle$ by a factor of two for anisotropic particles with non-parallel orientations (Balberg 1985). We can superimpose the curves shown in Fig. 10 using $\phi_{\text{eff}} = 2.2\phi$ (Fig. 11). As this constant is only 1.5 times greater than that predicted, it falls within the anticipated factor-of-two range. Furthermore, as noted in Section 4.2.1, $\phi_{\text{m}} \approx 2\phi_c$, an observation that suggests that the linear scaling implied by Eq. (3) may be a reasonable approximation over a wide range of $\phi$.

In summary, the dependence of $\tau_j$ on $\phi$ observed in our analog suspensions indicates that particle shape is an important parameter in determining the volume fraction range over which a suspension exhibits a finite yield strength. In real crystal-melt suspensions, the exact effect of shape on $\tau_j$ is complicated by both particle interpenetration and by particle orientation distribution. The importance of the latter parameter suggests that the flow regime (and resulting particle ordering) of a given suspension may play an important role in defining the actual values of $\phi_c$ and $\phi_{\text{m}}$ applicable to a specific lava flow.

### 4.3. Implications for basaltic lava flows

This study was motivated by a relationship observed between lava flow surface morphology and flow crystallinity, and particularly by the hypothesis that threshold transitions in flow surface morphologies may be explained by the onset of a yield strength sufficient to inhibit continuous fluid deformation (Cashman et al., 1999). Results presented here support this hypothesis, and provide a framework for relating
the rheology of basaltic lava flows to flow emplacement histories.

Abrupt changes in surface morphology commonly accompany the down-slope flow of lava. Most obvious is the transition from smooth (pahoehoe) to rough (‘a‘a) surfaces that occurs during flow through open channels (e.g. Peterson and Tilling, 1980). This transition appears to coincide with a pronounced increase in groundmass crystallinity (Cashman et al., 1999; Polacci et al., 1999), and thus may result from the onset of a yield strength sufficient to resist shear stresses imposed by the flow. Measurements of the crystal content of quenched flow surfaces may thus be used to estimate limiting crystallinities for different flow surface types (Folley, 1999). Smooth pahoehoe may form when φ ≤ 0.25, while ‘a‘a may form when φ ≥ 0.35 (Fig. 12). These measurements thus allow a conservative inference of 0.25 < φ_c < 0.35, which coincides with estimates provided by our melting experiments. If transitional flow surfaces reflect small but finite values of yield strength, then measured crystallinities indicate φ_c as low as 0.15 in some samples.

Low estimated values of φ_c most likely reflect both highly anisotropic crystal shapes and low degrees of orientational order. Scaling of the analog experiments illustrates the potential effect of increasing crystal anisotropy on φ_c for randomly oriented crystals (Fig. 13). Here φ_eff is calculated for different particle aspect ratios using Eqs. (3) and (4). We use effective values of φ_c = 0.3 and φ_m = 0.6 (the maximum crystallinity observed in flow samples; Fig. 12) to estimate the φ range over which suspensions of different particle aspect ratios may exhibit yield strength behavior. This simple model indicates that for a constant φ_eff = 0.3, actual values of φ_c will vary from 0.30 to 0.09 as the particle axis ratio varies from 1:1 to 10:1. For the same range in particle shapes, φ_m varies from 0.60 to 0.19 for a constant φ_eff = 0.6. For ‘typical’ particle aspect ratios of 3:1–5:1, 0.16 ≤ φ_c ≤ 0.22, and 0.32 ≤ φ_m ≤ 0.44, with the effect of each incremental increase in aspect ratio diminishing as the aspect ratio increases. Extension of such a diagram to other crystal shape orientations requires computer simulations, and is discussed in Saar et al. (2001).

These observations lead to some general predictions about yield strength development in basaltic lavas. First, rapid cooling is more likely to generate anisotropic crystal shapes than slow cooling (e.g. Lofgren, 1980); rapid cooling may also promote heterogeneous nucleation, and thus highly anisotropic crystal clusters (as seen in the dendritic pahoehoe toe sample, Fig. 6c). Both of these textural characteristics should lead to yield strength onset at relatively low crystallinities. However, sustained rapid cooling requires sufficient stirring to rupture surface crusts (e.g. Crisp and Baloga, 1994); the hydrodynamic effects of stirring in high velocity flow will serve to both break-up crystal clusters (increasing the apparent nucleation rate, e.g. Kouchi et al., 1986) and increase crystal alignment. Both processes will tend to increase φ_c. Thus we expect that rapid cooling in a near-static environment will produce the lowest values of φ_c, while high flow rates, with resulting high degrees of orientational order, should produce higher values of φ_c.

5. Conclusions

We have used both melting and analog experiments to examine the roles of crystal shape, spatial distribution and orientation on yield strength development in crystal-melt suspensions. Most important is our conclusion that crystal shape is an important
parameter in determining the crystallinity over which a crystal-melt suspension will exhibit yield strength behavior. Flow dynamics may also be important, to the extent that the dynamics of flow controls the orientation distribution of anisotropic crystals, and changes the suspension architecture through breakage of weak crystal–crystal bonds. The use of the excluded volume $V_{ex}$ to scale the results of our analog experiments was not entirely successful. Nevertheless, scaling by a simple factor related to $V_{ex}$ seems to be a reasonable approach that may only require better constraints on variations of $\langle V_{ex} \rangle$ with particle orientation distributions. Finally, our measurements support earlier work indicating the onset of yield strength behavior in natural basalts at moderate crystallinities (e.g. Shaw, 1969; Murase and McBriney, 1973; Pinkerton and Norton, 1995; Philpotts and Carroll, 1996; Cashman et al., 1999). These results are consistent with predictions of percolation theory for the minimum particle volume fraction at which a touching framework is first created (e.g. Garboczi et al., 1995; Saar et al., 2001). Based on these data, together with the common occurrence of anisotropic plagioclase crystals in basaltic lavas, we find it reasonable that basaltic lavas exhibit a finite yield strength at moderate (0.15–0.4) crystal volume fractions. Conditions that promote low values of $\phi_{c}$ include rapid cooling (increased crystal anisotropy), heterogeneous nucleation (increased spatial clustering) and static conditions (random crystal orientations). These results help to explain discrepancies among field measurements, experimental measurements, theoretical calculations and provide a basis for linking emplacement conditions of basaltic lava flows with appropriate rheological models.

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