Flow banding in volcanic rocks: a record of multiplicative magma deformation

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Abstract

Banding in obsidian from Big Glass Mountain (BGM), Medicine Lake volcano, California, and Mayor Island (MI), New Zealand, provides a record with a 1/wavenumber power-spectral density and multifractal characteristics. The samples are compositionally homogeneous, with banding defined by variable microlite content (BGM) or vesicularity (MI). In both samples, banding formation is well explained by a continuous deformational reworking of magma and by a concurrent change in crystallinity (vesicularity) that is a small, random multiple of the total amount already present. Banding formation, therefore, represents a multiplicative process. We complement our spectral and multifractal analysis with several null-hypothesis tests and propose repeated brittle deformation, concurrent development of textural heterogeneity (microlite content or vesicularity), reannealing, and viscous deformation as a viable process for the formation of flow banding in these samples. A brittle deformational component in a simple flow geometry, for example during magma ascent in the volcanic conduit, provides a suitable mechanism for the spatial redistribution of textural heterogeneity over a broad range of length scales, enhanced open-system magma degassing via a temporary network of highly permeable cracks and fractures, and the development of spatially variable microlite content (vesicularity) through variable degassing of magma located at different proximities to cracks and fractures.

Key words: volcanology, mixing, flow banding, obsidian, fragmentation, fractal

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1 Introduction

Banding is common in silicic volcanic rocks and preserves a record of deformation associated with magma ascent, eruption, and emplacement. Alternating light and dark bands ranging in width from tens of microns to decimeters are usually caused by differences in composition, crystallinity, or bubble content. The processes that lead to banding formation can be viscous magma mixing (e.g., [1–4]), repeated autobrecciation and reannealing during flow at the surface (e.g., [5,6]), welding and rheomorphism of pyroclastic fragments (e.g., [7–10]), vapor-precipitated crystallization in stretched vesicles [11], or repeated episodes of brittle deformation and reannealing during magma ascent in the volcanic conduit [12–15].

1.1 Textural heterogeneity

As predicted from theoretical considerations for chaotic fluid mixing flows (e.g., [16,17]), Perugini et al. [4] have found that mingling of compositionally different parent magmas during ascent in the volcanic conduit can produce a compositionally heterogeneous hybrid magma of alternating light and dark regions. The spatial distribution of compositional heterogeneity within this hybrid magma is scale invariant (fractal), but with the added complexity of different scaling properties over different regions of a given sample. In other words, it can be characterized as multifractal. Although textural heterogeneity caused by mixing of preexisting parent magmas has received considerable attention, the origin of compositionally homogeneous lavas with banding caused by spatially varying crystallinity or vesicularity still remains relatively unexplored. This is an important area of study, because crystal or bubble nucleation and growth are intimately linked to magma degassing [18–20], which in turn is responsible for the buildup of overpressure required for the transition from effusive to explosive eruptive behavior [14,21–26].

1.2 Brittle deformation and degassing

It has been suggested that banding formation may, at least in some cases, be associated with, and perhaps be integral to, brittle deformation and enhanced degassing during magma ascent [12–15]. Brittle deformation of magma occurs when strain rates locally exceed the ability of the melt to deform viscously (discussed in more detail in section 4), causing it to break like a brittle solid into fragments. If brittle deformation is predominantly caused by shear, it may be similar to brecciation or comminution of rocks in fault zones. Brittle deformation of vesicular magma will break bubble walls and allow volatiles
to be released into adjacent cracks and fractures created during the fragment-
tation process [12,15,27]. Volatiles may thereby escape from the magma at an enhanced rate, reducing the buildup of overpressure required for explosive eruptive behavior [14,21–26]. Banding has been observed within dissected con-
duits [12,15] and can also be found in close proximity to the volcanic edifice, indicating that banding may indeed form during magma ascent in the conduit.

1.3 Viscous vs. brittle deformation

Microlites (crystals < 30 µm) may form in response to magmatic degassing (e.g., [18]) on eruptive timescales [19,20]. Accordingly, Castro and Mercer [28] find that differences in the degree of volatile loss could account for some variability in microlite content in obsidian. However, they ultimately con-
clude that varying microlite content is more likely to record different ascent rates and residence histories in the volcanic conduit, rather than variations in magma degassing. They suggest that slower magma ascent rates near the conduit walls permit the development of higher crystallinity compared to that of faster ascending magma further in from the conduit walls. Similarly, it has been suggested that varying vesicularity may be caused by variable ascent rates [29], or by a spatially variable temperature distribution [30]. Regardless, the formation of banding, on scales as small as tens of microns, requires an efficient and extensive spatial redistribution through some process of magma deformation. If deformation is purely viscous, then repeated stretching and folding of magma is required. For a highly viscous silicic magma, this requires a considerable complexity of the flow geometry, in which nearby fluid ele-
ments can diverge strongly from each other (e.g., [31] and references therein). If, however, viscoelastic rheology of the magma is taken into consideration, then the effectiveness of mixing in a relatively simple flow may be consider-
ably increased [32]. For magma, viscoelastic behavior occurs near the glass transition (discussed in more detail in Section 4) and involves both viscous and brittle deformation. Potentially analogous behavior has been observed experimentally in shear flows of polymers (e.g., [33] and references therein). The possibility of brittle deformation is attractive, because obsidians and other effusive silicic lavas are thought to have lost most of their volatiles through open-system degassing during ascent (e.g., [12,21,24–26,34]), and as discussed previously, brittle deformation may potentially provide an effective mecha-
nism for enhanced degassing. Regardless of brittle or viscous deformation, it is important to understand whether the deformational process that results in banding is subsequent to or concurrent (and integral) with the actual de-
velopment of variable crystallinity and vesicularity. At this point, we wish to emphasize that we distinguish the mechanical deformation process that results in banding formation from the development of textural heterogeneity, such as the apparent spatial differences in nucleation and growth of microlites or vesi-
cles. The former will hereafter be referred to as the “deformational process” of banding formation, and we will refer to the latter as the “development of textural heterogeneity.”

1.4 Hypothesis and outline

We test whether banding in obsidian bears evidence of a multiplicative process (discussed in more detail below) of banding formation, whereby deformation and the development of textural heterogeneity are concurrent and integral to one another; and whether a statistical analysis of banding permits discrimination between such a multiplicative process, as opposed to the mixing of pre-existing heterogeneity. We focus on obsidian because the optical transparency allows measurements of structures to be made over a broad range of scales. We report results from a quantitative petrographic analysis of obsidian with evidence of a genetic relation between brittle magma deformation and banding formation. This obsidian was collected from Big Glass Mountain (BGM), a large obsidian flow at Medicine Lake Volcano, California. We compare results with obsidian from Mayor Island (MI), New Zealand. The MI obsidian sample analyzed here is thought to have formed by rheomorphism of fountain-fed spatter [7,35,36] and therefore provides a control sample of known origin involving magma breakup, reannealing, and subsequent viscous deformation [37]. A petrographic description of both samples is provided in Section 2.

Our working hypothesis is that of repeated brittle deformation, concurrent development of textural heterogeneity, reannealing, and viscous deformation into flow-aligned bands (Figure 1); in other words, a multiplicative process. We conduct a powerspectral and a multifractal analysis of both samples (Section 3). Powerspectral analysis permits the identification of a scale-invariant record, whereas multifractal analysis potentially allows the identification of a signal that preserves a record of a multiplicative process. If magma is subjected to a multiplicative process, it will undergo a change in some attribute during each step of this process. In the context of brittle deformation and the two samples analyzed here, this change corresponds to either an increase in crystallinity possibly associated with magma brecciation or enhanced degassing of fragments (BGM), or a change in vesicularity resulting from different degrees of degassing associated with magma fragmentation (MI). Throughout the entire process (i.e., during all steps), the probability distribution of the fractional change in crystallinity or vesicularity is assumed to remain constant, as it represents a characteristic scaling of the underlying physics. However, the spatial distribution will be nonuniform and vary randomly from step to step. Because of the repeated reworking of the magma, the change in crystallinity or vesicularity from one step to the next is a small random multiple of the total amount of crystallinity or vesicularity already present. This is what is meant
by a multiplicative process, and it will be elaborated on further in Section 3.2. It differs from a nonmultiplicative process, in which development of textural heterogeneity takes place before (and decoupled from) the actual formation of banding. Our analysis confirms that banding in the BGM and MI samples is multifractal and likely preserves a record of a multiplicative process. To verify the reliability of our result, we use a series of null hypothesis tests, which confirm that our hypothesis of a multiplicative formational processes is sound (Section 3.3). Because we suggest brittle deformation of magma as a likely physical mechanism for this multiplicative process, we provide a discussion of the underlying physics in Section 4 before concluding our report in Section 5.

2 Samples

Banding in silicic volcanic rocks is generally defined by alternating “layers” of varying color or texture. In their shortest dimension (i.e., cross section), these layers have the appearance of parallel bands that range in thickness from microns to decimeters, with lengths of up to several meters in their longest dimensions. In the third dimension, these layers can be somewhat distorted or folded [37].

2.1 BGM obsidian

BGM is a 900-year-old rhyolite-to-dacite obsidian flow [38–41]. The BGM eruptive center, the youngest part of the Medicine Lake composite shield volcano [38], provides an example of the effusive emplacement of a cubic kilometer-sized volume of silicic melt (Figure 2). The effusive eruptive phase of BGM was preceded by an explosive phase that resulted in the deposition of tephra over an area of approximately 320 km$^2$ [42]. Thus, BGM provides an excellent example of a sizable silicic eruption with a transition from explosive to effusive behavior. The youngest eruptive sequence of the BGM eruptive center includes obsidian that is ubiquitously banded (Figure 3), and some samples bear evidence for a genetic relation between fragmentation and banding (Figure 4). Banded BGM obsidian, therefore, is well suited for testing our hypothesis of banding formation through a multiplicative process that includes brittle magma deformation, and hence, the possibility of enhanced magma degassing associated with the transition from explosive to effusive eruptive behavior.

Banding is predominantly caused by varying concentrations of pyroxene microlites (Figure 5), with a higher microlite content resulting in a darker color. At times, dark bands also contain increased concentrations of oxides (e.g., the
darkest band in Figure 5). However, overall there appears to be no compositional difference or difference in water content (J. Castro, personal communication) between individual bands of BGM obsidian. Some BGM samples show a continuous gradation from angular, reannealed fragments (right third of Figure 4), to deformed fragments (middle of Figure 4), to well-developed banding (left third of Figure 4), indicating that brittle deformation, reannealing, and subsequent deformation can result in banding.

2.2 MI obsidian

MI is the visible portion of a 700-m-high and 15-km-wide Quaternary shield volcano. Despite its uniform peralkaline magma composition, MI's history includes virtually the full range of known eruption styles over a wide range of eruption sizes [35,36], including Hawaiian fire-fountaining and spatter-fed lava flows. Hawaiian eruptions involve rapid ascent of relatively low viscosity magma disrupted by gas bubbles. The immediate product is usually a pumice cone formed of magma fragments. These fragments may reanneal to form a spatter-fed lava flow, such as the 8-ka obsidian flow from which our sample was collected [7,36]. Banding in the MI obsidian sample under consideration is predominantly a result of varying bubble content (Figures 6 & 7), with higher vesicularity corresponding to lighter bands. Rust et al. [37] infer from bubble orientation that flow deformation was predominantly two-dimensional pure shear. Given a presumably vigorous nature of magma ascent, it is reasonable to assume that variations in vesicularity may represent a record of varying degrees of repeated magma breakup and degassing during ascent. The MI sample provides a sample of banded obsidian in which banding formation is known to have involved magma breakup and reannealing. It is therefore ideally suited for comparison with the BGM sample.

2.3 Banding data

Our subsequent analysis is based on digital photomicrographs of polished obsidian samples. Because of the degree of magnification, each image actually represents a composite of a number of high-resolution digital photographs. Our goal is to test banding from each sample for evidence of a multiplicative formational process. Our analysis, however, should be viewed as exploratory, because the ability to make detailed quantitative estimates of governing parameters is limited by the nonunique nature of multifractal analysis [43]. Nevertheless, through the use of null-hypothesis tests, we can establish the general nature of underlying processes that are most likely to result in the formation of banding in the obsidian samples under consideration.
The digitized sample of BGM obsidian has a length of 200 mm and a resolution of 2 µm/pixel (Figure 8a). Equivalently, we obtain a digital record with 5 µm/pixel resolution from a 55-mm-long sample of MI obsidian, collected by K. V. Cashman, University of Oregon (Figure 8b). The cumulative frequency distribution of resulting measures (amplitudes) for each sample is shown in Figure 9. Each digitized sample will hereafter also be referred to as the “record.”

We normalize gray-scale color index, $C_i$, for each pixel, $i$, as

$$
\phi_i = \left[ C_i - \min(C) \right] \left[ \sum_{i=1}^{N} C_i - N \min(C) \right]^{-1} \text{ where } \sum_{i=1}^{N} \phi_i = 1.
$$

(1)

Here $0 \leq C_i \leq 255$, and $p_i$ is the normalized gray-scale color index, hereafter also referred to as the “measure.” We do not obtain a precise calibration of gray-scale color index to textural heterogeneity in our samples because, unlike Perugini et al. [4], our bands are not defined by compositional variations, and it is not our goal to draw detailed quantitative estimates of process parameters from our analysis.

3 Analysis of banding

The main goal of our analysis is to assess whether banding in the BGM and MI obsidian samples preserves a record from which we can gain insight into the general nature of the underlying formational process, and hence about magma ascent processes inaccessible to direct observation. The wave-number power spectrum, $S(k)$, is one of the most often used tools for studying complex spatial patterns, because of its compact quantitative description of the presence of many spatial scales [44]. Multifractal analysis of signals, in contrast is proven to be successful in discerning formational processes of a multiplicative nature [45].

3.1 Power spectrum

As spectral estimator we use the multitaper method (e.g., [46] and references therein), which is well suited for detailed spectra with a large dynamic range. Both BGM and MI spectra (Figure 10) are characterized by a 1/wavenumber scaling of spectral power, $S(k) \sim k^{-1}$, indicating the presence of scale invariance.
3.2 Multifractal analysis

Monofractal records are often characterized by \( k^{-1} \) power spectra. They have the same scaling properties throughout the entire signal and can be characterized by a single fractal dimension. Records produced by multiplicative processes, in contrast are usually found to be multifractal [47,48]. Instead of being characterized by a single fractal dimension, they have locally varying scaling properties that contain valuable information regarding the formational process (e.g., [48]). For example, the energy-cascading process embodied by fluid turbulence generates a multifractal timeseries of velocity fluctuations that is mathematically well represented by a multiplicative process [48]. Multifractal analysis has also been successful in discriminating between healthy and heart failure patients on the basis of a time series of human heartbeat intervals [45]. In the latter case, other methods such as spectral analysis are insufficient in discriminating between the two “processes” (healthy vs. heart failure). We therefore use a multifractal analysis to test whether banding in BGM and MI obsidian is multifractal and, hence, possibly preserves a record of a multiplicative formational process.

3.2.1 Method

Multifractal records can be characterized by the singularity spectrum, \( f(\alpha) \), of some measure, \( \phi(x) \), which in our case represents the normalized color index as a proxy for microcrystallinity (BGM) or vesicularity (MI). It is possible to evaluate the presence of different scaling laws throughout the record via the integrated measure

\[
\mu_i = \int_{i-L/2}^{i+L/2} \phi \, dx, \text{ where } \sum_{i=1}^{N} \mu_i = 1.
\]

(2)

Here \( L \) is the length of a record segment centered at position \( i \), with the entire record of length \( X \) subdivided into \( N = X/L \) segments. The \( f(\alpha) \) singularity spectrum provides a mathematical description of the spatial distribution of \( \phi \) over the entire record, with the singularity strength, \( \alpha \), defined by

\[
\mu_i(L) \sim L^\alpha.
\]

(3)

Counting the number of segments, \( N(\alpha) \), where \( \mu_i(L) \) has a singularity strength, \( \alpha \), allows the loose definition of \( f(\alpha) \) as the fractal (Hausdorff) dimension of the part of the record with singularity strength of \( \alpha \) [47,49]

\[
N(\alpha) \sim L^{-f(\alpha)}.
\]

(4)
For a given segmentation length, or an equivalently box-counting dimension, \( L \), the value of \( \alpha \) will vary with position \( i \). As a consequence, \( f(\alpha) \) characterizes the spatial complexity of the actual record. If the record is uniformly fractal (monofractal) or not fractal at all, then there will be only one value of \( \alpha \), reducing \( f(\alpha) \) to a single point. If the record is complexly (though not randomly) structured so that its fractal dimension changes with position, then there exists a range of \( \alpha \) with different Hausdorff dimensions, \( f(\alpha) \). The more complexly structured the record is, the broader the spectrum will be.

The spectrum \( f(\alpha) \) can be obtained by considering different moments, \( \mu^q_i \), of the integrated measure. Here the parameter \( \phi \) provides a “microscope” for exploring different regions of the banding structure. As \( \phi \) is varied, different subsets of the record, which are associated with different scaling exponents, \( \alpha \), become dominant. The singularity spectrum can then be obtained from

\[
f(q) = \lim_{L \to 0} \frac{\sum_i \mu^q_i(L) \log[\mu^q_i(q, L)]}{\log L}
\]

and

\[
\alpha(q) = \lim_{L \to 0} \frac{\sum_i \mu^q_i(L) \log[\mu_i(L)]}{\log L}
\]

In practice, the slopes of \( \sum_i \mu^q_i(L) \log[\mu^q_i(q, L)] \) and \( \sum_i \mu^q_i(L) \log[\mu_i(L)] \) versus \( \log L \) are estimated to obtain \( f(\alpha) \) and \( \alpha \), respectively. Care must be taken to only consider the range of \( \log L \) over which these graphs do actually represent linear trends. If they do not represent linear trends, then the record is not truly multifractal, but could still result in a singularity spectrum with multifractal character. More details are provided in the caption to Figure 11; meanwhile, let it suffice to say that in our subsequent analysis, we were careful to avoid the potential pitfalls associated with multifractal analysis ([43] and references therein).

### 3.2.2 Results

The singularity spectrum (Figure 11) indicates that banding structure of BGM and MI obsidian are a multifractal, indicating that banding may have formed through a multiplicative processes. To illustrate that this is indeed feasible, we show \( f(\alpha) \) for a multifractal record produced by a simple mathematical model for multiplicative processes (Figure 11). This record is based on the two-scale Cantor set (MC) [47]. To construct this record, a unit interval is randomly subdivided into three segments: two of given length \( l_2 \), and one of length \( l_1 \) (Figure 12). The two former intervals each receive a proportion, \( p_2 \), of the total measure, and the latter interval receives a proportion given by \( p_1 \). This process
is recursively repeated for each segment from the previous iteration (Figure 12). In an idealized sense, this is equivalent to a fragmentation process with a given scaling for fragment size ($l_1$ and $2l_2$) and an embedded enrichment or depletion ($p_1$ and $p_2$) of some measure (e.g., volatile content, crystal or vesicle content, chemical or isotopic component). This idealized multiplicative process with $l_1 = p_1 = 1/5$ and $l_1 + 2l_2 = p_1 + 2p_2 = 1$ results in the banding shown in Figure 8c and provides an excellent match to the BGM singularity spectrum (Figure 11). The important point here is that microlite growth or development of nonuniform vesicularity may be integral to the formation of banding. Banding in BGM and MI obsidian samples, indeed, seems to preserve a record of such a multiplicative process.

3.3 Null hypotheses

To assess the limitations of our results, we perform several ‘null-hypothesis’ tests, summarized in Table 1.

3.3.1 Black and white banding

Figure 8d shows the banding structure generated by the same multiplicative process as shown in Figure 12, but with banding defined by only two alternating values of the measure $\phi$ (black or white). In contrast to the BGM and MI samples, as well as the MC model, this binary banding is not multifractal, and its singularity spectrum consists of a single point, shown in Figure 13. As already discussed by Turcotte [50], this result indicates the importance of a binomial or log-normal measure to produce a multifractal record, and this example illustrates that the formation of binomial measures is well explained by a multiplicative enrichment or fractionation process (e.g., [51–53]).

3.3.2 Randomized BGM

Figure 8e shows the measure of the BGM sample (Figure 8a), sorted from darkest to lightest normalized color index, $\phi$. For our second test, we randomly redistributed this measure. The resultant record (Figure 8f) is not multifractal (Figure 13). Not surprisingly, we can conclude that a binomial measure alone does not suffice for a multifractal record, implying either that a random formational process resulting in a log-normal measure, or that a random “mixing” process of a preexisting binomial measure, is not a viable alternative for the formation of banding in BGM or MI samples.
3.3.3 Chaotic mixing via Baker’s map

In our third test, we examine the possibility that a preexisting measure (e.g., microlite content) of identical probability distribution as the BGM sample (Figure 8e) could be redistributed through a chaotic mixing process to result in banding with similar characteristics as the BGM sample. This hypothesis is motivated by the possibility that microlite content or vesicularity may vary, for example, radially or vertically before the actual mixing process, as suggested by Castro and Mercer [28].

A simple mathematical model that has been used to represent the characteristics of chaotic mixing processes is the generalized Baker’s map (e.g., [16,17,44]). We performed calculations using several different parameterizations of the generalized Baker’s map. Although details varied between different versions, the overall results were the same. We therefore only show results for one version of this map. This version has a ratio of segmentation lengths identical to that of a multiplicative Cantor process. The Baker’s map spatially redistributes a given measure on a unit square and is equivalent to the process of stretching and folding in chaotic fluid mixing, illustrated in Figure 14. Mathematically, the process is given by the mapping

\[
\begin{align*}
x_{n+1} &= \begin{cases} 
l_2^* x_n, & \text{if } y_n < l_2^* \\
l_2^* + l_1^* x_n, & \text{if } l_2^* < y_n \leq l_2^* + l_1^* \\
l_2^* + l_1^* + l_1^* x_n, & \text{if } l_2^* + l_1^* < y_n
\end{cases} \\
y_{n+1} &= \begin{cases} 
l_2^* y_n/l_2^* & \text{if } y_n < l_2^* \\
l_2^* + l_1^* y_n/l_2^*/l_1^* & \text{if } l_2^* < y_n \leq l_2^* + l_1^* \\
l_2^* + l_1^* + l_1^* y_n/l_2^*/l_1^* & \text{if } l_2^* + l_1^* < y_n
\end{cases}
\end{align*}
\]

(7)

Here \( l_1^* = l_1 + \theta \), \( l_2^* = (1 - l_1^*)/2 \), and \( \theta \) is a number chosen randomly at each iteration, with \( \theta \leq l_1/2 \leq \theta \). The Baker’s map is analogous to closed-system chaotic mixing, in which the original measure is conserved throughout (i.e., for all iterations \( \sum_{i=1}^{N} \phi_i = 1 \)). With respect to BGM, the Baker’s map implies that microlite growth and banding formation are decoupled. The singularity spectrum (Figure 11) obtained from this map is multifractal and is very similar to the BGM sample (Figure 13). However, the power spectrum allows a clear distinction. Whereas power spectra of BGM, MI (Figure 10), and the MC model are characterized by \( S \sim k^{-1} \), the power spectrum for the Baker’s map with initial measure derived from the BGM sample has \( S \sim k^{-\beta} \), with \( \beta \approx 1.5 - 2 \). This result indicates, consistent with results from mixing studies (e.g., [4,16,17]), that a magma with preexisting nonuniform microlite content, or vesicularity, can be mixed, resulting in banding with multifractal properties. However, the power spectrum is not likely to have the \( S \sim k^{-1} \) scaling observed in BGM and MI samples, and the mixing process itself is not multiplicative, so...
the generation of variable microlite content or vesicularity still requires some prior process that yields a binomial distribution.

4 Brittle deformation of magma

In this section, we provide a brief review of the physics of brittle magma deformation. If strain rates during viscous deformation of silicic melt exceed the viscous relaxation rate, then the melt will no longer deform in a viscous manner but, rather, in a brittle manner. This transition is referred to as the glass transition (e.g., [54] and references therein). In general, it is thought that magma fragmentation occurs when strain rates, for example, resulting from flow deformation or bubble expansion [23,55] exceed the glass transition of the melt. Recent work indicates that shear strain rates during the ascent of silicic magmas exceed the glass transition [14,15,56], even for effusive eruptions that produce lava flows and domes. The glass transition for silicate melts is \( \dot{\gamma}_c \sim 0.01 \frac{G}{\mu} \), where \( G \) is the elastic modulus of order 10 GPa (e.g., [54]), \( \mu \) is viscosity, and \( \dot{\gamma} \) is the local shear strain rate. Joseph [57] suggests that a liquid undergoing simple shear, such as magma ascending in a conduit or dike, can cavitate if the maximum tensile stress, \( 2\mu\dot{\gamma} \), exceeds ambient pressure and the liquid’s tensile strength (i.e., the glass transition). At the glass transition, \( \gamma = \gamma_c \), implying a tensile stress of the order of 100 MPa. As a consequence, shear-induced fragmentation should be accompanied by cavity formation at depths of up to several kilometers. This is important because it indicates that a shear-induced fragmentation event can create a short-lived network of fractures at significantly lower pressures than the surrounding magma. Moreover, these fractures should be highly permeable to gas flow, and the adjacent magma would be subjected to significant undercooling. The latter, in turn, may result in highly variable crystallization kinetics [18–20,58] over relatively short distances, thereby providing a possible mechanism for the generation of textural heterogeneity in direct association with the deformation/fragmentation process.

Because of the displacement and rotational component associated with simple shear, fragments from different regions in a conduit or a flow can easily become juxtaposed. Once strain rates have relaxed to a value smaller than the viscous relaxation rate, the fragmented magma can reanneal and once again deform in a viscous manner. If the juxtaposed fragments are texturally (or compositionally) heterogeneous, either initially or as a consequence of the deformation process itself, viscous stretching of reannealed fragments will result in banding. The degree of textural heterogeneity recorded in bands is often small. For example, clinopyroxene microlite content, which defines banding in BGM and similar obsidian samples, varies by less than a few volume percent [59]. It appears reasonable that this degree of heterogeneity can exist or
form over relatively small distances within the conduit. Fragmentation can be repeated during continued ascent and is, except for reannealing and viscous deformation, essentially equivalent to the comminution of rocks in fault or shear zones banding [60].

5 Conclusion

The essential feature of a multiplicative process, within the context of banding formation in silicic volcanic rocks, is the repeated reworking of magma and concurrent development of textural heterogeneity. In the case of the analyzed banded obsidian from BGM, textural heterogeneity is predominantly a result of variable microlite content, whereas in the spatter-fed obsidian sample from MI it is caused by varying vesicularity. A multiplicative process capable of banding formation requires a deformational process that repeatedly results in a spatial redistribution of preexisting banding structure and subsequent viscous deformation into thin, flow-aligned bands. This creates increasingly finer scales of banding or in other words, a cascade to shorter and shorter wavelengths until some cutoff value, given by bubble or crystal size, is reached [44]. In the BGM obsidian sample, for example, the boundaries between individual bands are often very sharp, even at the smallest length scales. In contrast to typical mixing processes (e.g., [4]), this indicates that microlite dispersion or molecular diffusion were relatively insignificant during banding formation. We therefore conclude that the observed binomial distribution of textural heterogeneity is best explained if there is a change in crystallinity or vesicularity that is a small random multiple of the amount already present and that this change occurs concurrently with, or is integral to, the mechanical deformation process.

Although banding formation in the samples under consideration probably involved some deformation during surface emplacement, considerations of crystal nucleation and growth kinetics [18–20,58] are perhaps indicative of a process occurring, at least in part, within the conduit during magma ascent. There is observational evidence for a genetic relationship between brittle deformation and banding (Figure 4), and furthermore, brittle deformation of magma in the conduit seems plausible on the basis of theoretical considerations (Section 4). One may therefore speculate that breakup of magma and concurrent enhanced degassing of individual fragments may result in spatially variable crystallinity or vesicularity. Nonetheless, it should be noted that it is possible to produce multifractal banding through viscous mixing of a magma with an “inherited” textural heterogeneity. However, such a scenario would imply some prior formational process to result in the required binomial distribution of heterogeneity. Probably the most plausible scenario for a viscous mixing origin of banding would be the development of textural heterogeneity concurrent with the mixing process. One aspect of possible importance in this
context might be viscous dissipation [61]. However, pervasive banding formation over the observed spatial scales requires repeated stretching and folding, in which nearby parcels of magma can diverge strongly from each other. Such large degree of divergent stretching is not easily achieved in a highly viscous magma in the volcanic conduit, where flow is presumably dominated by simple shear. Because of these considerations, and based on the powerspectral and multifractal analysis presented here, we conclude that banding in the BGM and MI obsidian samples analyzed here is best explained as the record of a multiplicative process, represented by repeated cycles of magma breaking, reannealing, and viscous deformation. On the basis of theoretical considerations, it is feasible that such a process can occur during ascent in the volcanic conduit, as well as during surface emplacement.

6 Acknowledgments

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7 Figure Captions

Figure 1: Schematic illustration of our working hypothesis for banding formation: brittle deformation of magma (see Section 4 for a detailed discussion). (a) Brittle deformation during simple shear. The single fragment shown here undergoes a change in texture (e.g., microlite content or vesicularity), indicated by the darker color. Note the presence of fractures and the sigmoidal geometry, which is similar to tension gashes. Tension gashes often form in en echelon arrays. They may be precursors to shear localization (e.g., [62] and references therein), and they can be rotated by deformation either during or after formation. (b) The magma has reannealed and is deforming viscously by simple shear into a band. (c) Repeated brittle deformation and multiplicative change in microlite content or vesicularity indicated by the change in color. Note that the band that formed during the previous cycle is broken into several pieces and becomes “reworked”. Repetition of this process results in a broad range of textural heterogeneity and in an increasingly finer banding structure.
Figure 2: Map of Big Glass Mountain, Medicine Lake volcano and vicinity.

Figure 3: Photograph of banded obsidian sample from Big Glass Mountain, Medicine Lake Volcano, California.

Figure 4: Photograph of obsidian from Big Glass Mountain (cut at mutually perpendicular directions). This sample shows a continuous gradation from angular, reannealed fragments (right third), to deformed fragments (middle), to well-developed banding (left third), indicating that brittle deformation, reannealing, and subsequent deformation can result in banding.

Figure 5: Photomicrograph of obsidian from Big Glass Mountain. Banding is generally defined by varying mircolite content in a glassy matrix. Some dark bands also contain oxides.

Figure 6: Polished sample of Mayor Island banded obsidian (collected by K. V. Cashman, University of Oregon).

Figure 7: Photomicrograph of obsidian from Mayor Island. Banding is defined by varying vesicularity.

Figure 8: Banding for (a) Big Glass Mountain (BGM) (200 mm length), (b) Mayor Island (55 mm length), (c) Multiplicative cascade (MC) based on the two-scale cantor set, (d) MC with alternating black and white bands only, (e) BGM pixels sorted by color index, (f) BGM with random redistribution of pixels, (g) Baker’s map of the sorted BGM from panel (e) after four iterations, (h) Baker’s map of sorted BGM from panel (e) after six iterations.

Figure 9: Cumulative frequency distribution of the measure (normalized grayscale color index) for Big Glass Mountain and Mayor Island obsidian samples.

Figure 10: Power spectra for Big Glass Mountain and Mayor Island obtained from averaging the spectra from 200 rasters of the digital photomicrograph of each obsidian sample. As discussed by Namenson et al. [44], spectral averaging is a robust and recommended procedure in the search for power-law spectral characteristics. Spectral estimation is made using the multitaper method (e.g., [46] and references therein), where data windows are given by discrete prolate spheroidal sequences [63] that effectively minimize spectral leakage. We
tested different taper sequences to minimize leakage effects or other artifacts and find that an estimator with NW = 4, the time bandwidth for the Slepian sequences, was well suited for our analysis.

**Figure 11:** Singularity spectra obtained by the Chhabra-Jensen method [49]. \( f(\alpha) \) for Big Glass Mountain (BGM) and Mayor Island (MI) are broad, indicating a multifractal record. The spectrum of the multiplicative cascade provides an excellent match to BGM. \( f(q) \) and \( \alpha(q) \) are determined from the slope of the graphs \( \sum_i \mu_i^q (L) \log [\mu_i^q (q, L)] \) and \( \sum_i \mu_i^q (L) \log [\mu_i (L)] \) versus \( \log L \). The error of estimation is evaluated from the variance in the estimation of the slope. To evaluate the slope, we use only the linear segment of the aforementioned graphs. Therefore, \( f(q) \) and \( \alpha(q) \) were estimated for \( \max(\delta_{BGM}, \delta_{MI}) \leq L \leq \min(X_{BGM}, X_{MI})/10 \), where \( \delta_{BGM} \) represents the width of the smallest resolved band of the BGM sample, \( \delta_{MI} \) is the average diameter of the largest bubbles of the MI sample, \( X_{BGM} \) is the length of the BGM sample, and \( X_{MI} \) is the length of the MI sample. These ranges correspond to linear segments of the aforementioned graphs. They are also identical for both samples, so there is no bias in the analysis. Furthermore, it should be noted that we compared our results for the entire BGM sample (200 mm in length) with analyses of segments of the BGM sample with lengths identical to that of the MI sample (55 mm). In all cases, the BGM singularity spectrum essentially has the multifractal characteristics shown in this figure. Finally, we compared results from the Chhabra-Jensen method with the Legendre-transform method [47] and found the results shown here to be robust.

**Figure 12:** Multifractal record generated by a multiplicative cascade (after four iterations) based on the two-scale Cantor set [47]. The segment of unit length and uniform measure (denoted by gray-scale color) shown at step 0 is cut into three segments, one of length \( l_1 \) and two of length \( l_2 \). The measure of the first segment is multiplied by a factor of \( p_1 \), and that of the two other segments is multiplied by a factor of \( p_2 \), where \( p_1 > p_2 \). Each of the resulting three segments shown at step 1 is, in turn, cut into three segments, one of length \( l_1 l_{seg} \) and two of length \( l_2 l_{seg} \). Here \( l_{seg} \) denotes the length of the given segment shown at step 1. The resultant segment lengths are either \( l_1, l_2, l_1^2 \), or \( l_2^2 \). Concurrently, the measure of each segment from step 1 is again multiplied by either \( p_1 \) if its length is \( l_1 l_{seg} \), or by \( p_2 \) otherwise. The resultant segments and measures, denoted by different shades of gray (\( p_1, p_2, l_1^2 \), or \( l_2^2 \)), are shown at step 2. The process is then repeated for steps 3 and 4. It should be noted that the spatial sequence into which each segment at any given step is subdivided (i.e., whether \( l_1 l_{seg} \) is at the first, second, or third position) is chosen randomly.
**Figure 13:** Singularity spectra for the different null-hypothesis tests. Big Glass Mountain with randomized pixels (Figure 8f) and the multiplicative cascade with only black-and-white bands (Figure 8d) are monofractal and plot as single points. The spectrum for the generalized Baker’s map after six iterations (Figure 8g) is multifractal and similar to that of the Big Glass Mountain sample (Figure 8a), shown as the dashed line.

**Figure 14:** Schematic representation of the modified Baker’s map (e.g., [16,17]), where the initial measure is based on the sorted Big Glass Mountain of Figure 8e. This map represents a chaotic redistribution of a preexisting measure.

Fig. 1. (b&w)
Fig. 3. (b&w)
Fig. 4. (b&w)
Fig. 5. (b&w)
Fig. 6. (b&w)
Fig. 7. (b&w)
Fig. 9. (b&w)
Fig. 10. (b&w)
Fig. 11. (b&w)
Fig. 12. (b&w)
Fig. 13. (b&w)
Fig. 14. (b&w)
Table 1: Comparison of obsidian samples Cantor map with null hypothesis tests

<table>
<thead>
<tr>
<th>Record</th>
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<th>Implications</th>
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<tr>
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<td>Y</td>
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<tr>
<td>Mayor Island (MI)</td>
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<td>Y</td>
<td>Y</td>
<td>Concurrent formation of variable vesicularity and banding</td>
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<tr>
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<td>Y</td>
<td>Concurrent development of heterogeneous measure and banding</td>
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<tr>
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<td>N</td>
<td>Decoupled microlite growth and banding formation</td>
</tr>
</tbody>
</table>

\(^a\) Multifractal.
\(^b\) \(S\) is spectral power and \(k\) is wavenumber.
\(^c\) Multiplicative process.

References


