Granular mass flows and Coulomb’s friction in shear cell experiments: Implications for geophysical flows

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Granular mass flows of rock fragments are studied in the lab by means of a high-speed video camera at 2000 frames per second. These granular flows are generated using beds of pumice fragments positioned on a rough rotating disk, whose angular velocity is controlled by a motor. The experimental apparatus allows an understanding of the arrangement of the particles in granular mass flows with relatively small and relatively large values of the Savage number (the Savage number represents the ratio between grain collision stresses and gravitational grain contact stresses). In particular, these flows develop a basal layer of agitated and colliding particles underneath a relatively rigid upper layer. Our experimental results suggest the validity, on average, of the Coulomb’s relationship between shear and normal forces at the base of granular mass flows irrespective of their Savage number value. In Coulomb’s equation the shear stresses do not depend on the shear rate. We expect the Coulomb friction law to be valid also in moving pyroclastic flows. Our experiments suggest that the collisions and subsequent comminution of pumice fragments in moving pyroclastic flows could provide ash for the overriding ash clouds. In our experiments the amount of ash generated by particle-particle and particle-boundary interactions increases as the value of the Savage number increases. In nature, part of this ash may also simply move toward the base of the flows because of kinetic sieving.

INDEX TERMS: 1824 Hydrology: Geomorphology; 3210 Mathematical Geophysics: Modeling; 5104 Physical Properties of Rocks: Fracture and flow; 8414 Volcanology: Eruption mechanisms; KEYWORDS: rock fragments, collisions, friction, granular mass flows, rock avalanches, debris flows, pyroclastic flows


1. Introduction

[2] We present high-speed video camera studies of non-fluidized, dry pumice granular flows. The purpose of our lab experiments is to better understand natural geophysical flows. Geophysical flows with pumice fragments that travel on ground surfaces occur as pyroclastic flows [Cas and Wright, 1988]. The results of our analyses, however, are also applicable to granular mass flows with rock fragments different from pumice, in particular, pyroclastic flows generated by dome collapse (i.e., block-and-ash flows) but also rock avalanches and probably debris flows that develop a rigid raft [Iverson, 1997]. Field studies of moving geophysical flows are difficult. For this reason, lab experiments can be useful to establish basic principles quantitatively and as guidance for improving theoretical models and computer simulations. Even the most sophisticated quantitative models rely, in reality, on tacit and unproven assumptions such as that of the validity of the concept of coefficient of restitution in rock-rock collisions (the coefficient of restitution applies to collisions where the energy losses are only due to plastic deformations and vibrations [Cagnoli and Manga, 2003]) and that of the validity of the concept of viscosity in moving pyroclastic flows. Our lab experiments are meant to test these underlying assumptions using real granular material.

[3] Pyroclastic flows are one of the most deadly and destructive of all volcanic phenomena. They consist of mixtures of ash, blocks, and gas capable of traveling long distances at great speed. The understanding of their flow mechanisms is thus of great importance in the assessment of volcanic hazards. The observation that pyroclastic flows come to rest on substantial slopes, transport very large blocks, present a quasi-rigid upper portion (raft) and steep flow fronts have led researchers to believe that they flow as Bingham fluids with a high yield strength [Cas and Wright, 1988]. However, in Bingham fluids, shear stresses are function of the shear rate. The purpose of our experiments is to test whether in granular mass flows the shear stresses depend on the shear rate.

[4] In our experiments we use irregularly shaped pumice fragments to produce more realistic flows than those obtained with glass beads (glass beads have a value of the coefficient of restitution that is too large to be able to simulate properly natural geophysical flows). We compare
flows with relatively small and relatively large values of the Savage number [Savage and Hutter, 1989]. Although in natural geophysical flows the role played by interstitial fluids (such as water with fines in debris flows and gases in some pyroclastic flows) can be important (for example, partly supporting clasts and reducing friction), in this paper we focus on the properties of the solid granular component. For a more general discussion on mobility of geophysical flows we refer the reader to Legros [2002].

[5] Here we show that the ratio between the average shear and normal forces acting on the lower surface of a relatively rigid upper layer can be considered equal to the coefficient of internal friction (as in Coulomb’s equation) regardless of the Savage number value of the flows. The Coulomb friction equation, whose validity in moving granular mass flows has been suggested, for example, by Hung and Morgenstern [1984a, 1984b], is used in computer models of debris flows and pyroclastic flows, and it is useful also because it can easily accommodate the effect of an interstitial fluid overpressure [Iverson and Vallance, 2001; Iverson and Denlinger, 2001; Denlinger and Iverson, 2001; Iverson et al., 2004; Denlinger and Iverson, 2004].

[6] In this paper, before a description of the main features of the granular flows (section 4), we delineate the characteristics of the experimental apparatus (section 2), and we explain how the coefficients of friction are measured (section 3). Two sections will then follow, where the vertical profiles of velocity (section 6) and the forces acting on the upper layers (section 7) are discussed. Our flows are considered in two different reference systems: a lab one and one on a slope (section 5). In section 8 we describe some important features of particle collisions in the basal layers underlying the relatively rigid upper layers. In section 9 we compare our flows with those moving on slopes. When the Savage number value of the flows increases (section 10), more energy is dissipated, as suggested by the increase in the amount of ash produced at larger flow speeds in the disk (i.e., slope) reference frame (section 11).

2. Apparatus and Experimental Method

[7] The granular mass flows described in this paper are generated using beds of pumice particles located on a rotating steel disk, whose angular velocity is controlled by a motor (Figure 1). Viewed from above, the disk rotates in a counterclockwise direction. In the apparatus the particles are positioned between two coaxial vertical glass cylinders (Figure 1). The inner diameter of the larger cylinder (i.e., the container) is 12.6 cm, and the outer diameter of the smaller cylinder is 6.4 cm. This results in a 3.1 cm gap between the two cylinders. The container is supported by a heavy aluminum and steel stand to damp vibrations. The basal disk presents eight radial and equally spaced 1-mm-high steps, whose erosive effect is attenuated by covering the surface of the disk with duct tape. After each run the abraded tape is replaced so that an identical disk surface is used in all experiments.

[8] The pumice fragments come from the pyroclastic deposits of Medicine Lake Volcano in northern California, United States. They are angular, with an average density of 550 ± 39 kg/m³ and a bubble size mode smaller than 1 mm. These irregularly shaped fragments are sieved, and only those retained between the sieve openings of 8.0 and 9.5 mm are selected. Thus the average sieve diameter of our particles is 8.75 mm. We use 100 g of particles in each experimental run, corresponding, on average, to 327 pumice fragments.

[9] The experiments are carried out at three different angular disk speeds. Each experiment at each angular disk speed is repeated three times to assess its reproducibility. Each experimental run lasts 1 min, and the masses of the bed before and after the experiment are compared. The internal features of the flows (that are visible through the container glass surface) are studied using a high-speed video camera at 2000 frames per second. These high-speed movies allow the measurement of the velocity components of the pumice particles within the flows at a distance from the longitudinal axis of the cylindrical container approximately equal to its inner radius $R$ (Figure 1). The linear velocities are obtained simply by tracking the positions of the center of mass of the particles in the high-speed images. Likewise, the angular velocities can be easily estimated by counting the rotations of the particles around their center of mass. These measurements provide average velocities (i.e., ratios of displacements to time intervals).

3. Friction Coefficients

[10] Ancillary experiments are carried out to measure the angle of internal friction of pumice particles. This angle of internal friction is estimated as the angle from the horizontal formed by the lines of demarcation between stationary and moving fragments in a narrow rectangular glass container (31 cm high, 31 cm long, and 1.5 cm wide) with a small
opening in the center of the base from where the particles can fall freely [Zenz and Othmer, 1960]. An average angle of 55.8° is obtained from eight measurements, which results in a mean friction coefficient $\mu_P$ and a related standard error of 1.47 ± 0.08 (this coefficient is equal to the tangent of the angle). This compares well with an angle of 60° for crushed rock fragments reported by Holtz and Kovacs [1981]. This coefficient of internal friction could change in natural geophysical flows if this parameter is, for example, a function of the normal stresses that are larger than in small-scale experiments.

It is important here to explain the difference between angle of internal friction and angle of repose. The angle of internal friction and the angle of repose differ in the nature of the interface they represent, and therefore we expect them to have different values [Zenz and Othmer, 1960]. The angle of repose represents static equilibrium between unconfined solids and surrounding fluid medium. The angle of internal friction represents internal dynamic equilibrium between grains moving in contact along an interface. In general, the angle of internal friction is larger than the angle of repose [Zenz and Othmer, 1960].

We measure also the angle of sliding of pumice particles on the surface of the glass container. The inclination from the horizontal of the container glass surface at which a pumice particle starts sliding ranges between 21° and 36°, which results in static friction coefficients between 0.38 and 0.73 respectively (from 20 measurements). However, because the force necessary to start sliding is usually greater than the force required to maintain sliding [Rabinowicz, 1995], it is common to assume that the dynamic coefficient of friction is, in general, smaller than the maximum value of the static coefficient of friction [Benenson et al., 2002]. In this paper we are, of course, interested in dynamic coefficients of friction because our granular material is moving.

4. Main Flow Features

The high-speed movies show that in the experiments at the lowest angular disk speed the flows consist of the superposition of horizontal and parallel rows of particles (Figure 2a). Conversely, in the experiments at the two higher angular disk speeds a relatively rigid upper layer of particles is located above a basal layer of colliding particles (Figure 2b). The angle of internal friction in angular granular material is so high that the deformation is mostly concentrated where it is unavoidable, i.e., in the basal layer that is (in a vertical cross section) a narrow band along the contact with the disk. In the quasi-rigid upper layer the particles touch each other (Figure 2b) and form what is known in the literature as raft or plug [Iverson, 1997]. In reality, a basal layer and an upper layer can be identified also in the flows obtained at the lowest angular disk speed (Figure 2a). Here, however, the vertical mean free paths of the particles of the basal layer are significantly smaller than those at intermediate and larger angular disk speeds. Thus in our flows (as expected in natural geophysical flows) the particles of a quasi-rigid upper layer are not dispersed, and they cannot be modeled in the manner of gas molecules (i.e., with the kinetic theory of gases).
The high-speed movies reveal that on average, at the three angular disk speeds, approximately three quarters of the particles are in contact with the container glass wall. Therefore, in this paper, because the flows are relatively thin in the radial direction, when we speak about the linear velocities of the particles, we refer to those measured at a distance from the longitudinal axis of the cylindrical container that is approximately equal to its inner radius $R$ (i.e., the speed of the particles visible through the container glass wall). The vertical profiles of velocity in the flows generated at the different angular disk speeds do not change significantly along the perimeter of the cylindrical glass container. For this reason, these flows can be considered virtually one-dimensional with respect to the vertical. However, these flows are the result of forces acting in all three dimensions. The purpose of this paper is to study these forces to estimate the friction force at the base of natural granular mass flows.

The presence of a basal layer of colliding particles is due to the rough surface of the disk, and it does not depend on the curvature of the container glass wall. Flows with a basal layer of colliding particles occur also when moving along straight lines [Iverson, 1997]. This important feature is likely to exist also in natural geophysical flows [Iverson, 1997]. Thus although our simulations have a direct counterpart in flows within channel bends, our one-dimensional flows can be thought (as far as our analysis is concerned) as moving along straight lines (Figure 2c).

5. Reference Frames

The description of the velocity profiles (as visible through the container glass wall) is done in two rectangular Cartesian frames of reference. The first one is the lab reference frame (attached to the surface of the glass container), where the horizontal linear (i.e., tangential) speeds of the particles decrease upward and the disk has the highest angular speed (Figure 3). The second one is the disk reference frame (attached to the perimeter of the disk), where the horizontal linear (i.e., tangential) speeds of the particles are obtained subtracting the speed of the disk from the horizontal speeds of the particles in the other reference frame. In the disk reference frame the horizontal speeds increase upward (the speed of the disk is zero), and they have a direction opposite to those in the lab reference frame (Figure 3). The lab reference system is where the measurements are carried out, whereas the velocities in the disk reference frame are important because they correspond to those in a ground reference system fixed with respect to a slope upon which the natural granular material whose properties we intend to study would be moving (Figure 3). The disk reference frame is necessary to transpose the velocity profiles onto a slope. We are, of course, not interested in rotating reference systems (here, as far as we are concerned, the rotations of the disk reference system can be ignored because the features we consider in the disk reference frame do not depend on its rotations). In this work the properties of the granular material are studied in the lab reference frame that is inertial. These properties are valid in our apparatus, and they are valid in natural granular mass flows on slopes.

6. Velocity Profiles

We have selected a continuous sequence of frames in each high-speed movie where the flows are not disrupted by unusual, strong collisions. In our analysis we split the flows obtained at the three different angular disk speeds into two portions: a basal layer (that corre-
[18] In the high-speed movies of the experiments at the smallest angular disk speed (Figure 2a) the flows are divided into 1-cm-thick horizontal stripes. The average horizontal velocity of the center of mass of 25 consecutive particles traveling within each stripe is assigned to the stripe midpoint. Also the upper layers of the flows at intermediate and largest angular disk speeds (Figure 2b) are divided into 1-cm-thick horizontal stripes and again, in each experiment, the average horizontal velocity of the center of mass of 25 consecutive particles traveling within each stripe is assigned to the stripe midpoint. Concerning the basal layers at intermediate and largest angular disk speeds (Figure 2b), we have studied the velocity components of 25 colliding particles in each experiment before and after the collisions with the upper layer. The components of the linear velocities of their centers of mass are \( V_{HA}, V_{NB}, V_{HB} \), and \( V_{NA} \), whereas \( \omega_A \) and \( \omega_B \) are their angular velocities around their centers of mass (Figure 4). Here the subscript \( H \) stands for horizontal, \( N \) stands for normal, \( B \) stands for before collision with the upper layer, and \( A \) stands for after collision with the upper layer (Figure 4).

[19] Figure 5a presents a typical vertical profile of the horizontal linear (i.e., tangential) velocity in flows at the lowest angular disk speed, whereas Figure 5b presents an example of a vertical profile of the horizontal linear (i.e., tangential) velocity in flows at the largest angular disk speed. Both profiles consist of velocities in the disk reference frame at a distance from the longitudinal axis of the cylindrical container that is approximately equal to its inner radius \( R \). In Figure 5b, below the upper layer, the average horizontal velocity components \( V_{HB} \) of the basal layer particles immediately before collision with the upper layer is also shown. In Figure 5 the speed values are fitted, in a least squares sense, with an exponential equation. The error bars represent the errors of the means. In the upper portions of the upper layers the error bars overlap, suggesting indistinguishable speed values. Table 1 presents the average horizontal linear speed in the lab reference frame of the particles of the upper layers in the experiments at the three different angular disk speeds. The average horizontal speeds of the particles of the upper layers at the three angular disk speeds are, in the lab reference frame, indistinguishable (Table 1). In the lab reference frame, on average, also the vertical profiles of the horizontal velocities of the particles in the upper layers (and the upper layer thickness) do not differ significantly at the three different angular disk speeds. The discussion on the forces in section 7 refers to the same continuous sequence of frames selected for the velocity measurements.

7. Forces Acting on the Upper Layers

[20] The distinction between basal and upper layers allows a direct comparison between flows generated at different angular disk speeds. In these flows we are interested in the forces acting on the upper layers. The fact that the average horizontal velocity of the particles of the upper layers in the lab reference frame is the same at the three different angular disk speeds (Table 1) suggests that the shear force acting on the lower surface of the upper layers does not depend on the shear rate (here the shear rate is computed as explained in section 10). This is consistent
with Coulomb’s equation for cohesionless granular material, where the magnitude of the shear force is the product of a coefficient of internal friction and the magnitude of the force normal to the relevant surface [Coulomb, 1776]. In this section we will elaborate on the validity of Coulomb’s equation in our granular mass flows and on alternative models.

[21] Because the angular speed of the upper layers is, on average, constant in time, the net torque acting on the upper layers must be zero in the horizontal plane. Here we consider that the upper layers are relatively thin in the radial direction because at the three different angular disk speeds, approximately three quarters of the particles are in contact with the container glass surface so that the position vectors of the forces forming the torques are not significantly different. Thus in the lab reference frame the rightward (when seen through the container glass surface) average horizontal force exerted by the basal layer (i.e., Coulomb friction) can be considered equal in magnitude but opposite in direction to the average horizontal component \( \overline{C_H} \) of the friction force exerted on the oscillating upper layer by the container glass surface (at the three angular disk speeds the particles of the flows do not touch the inner glass cylinder):

\[
\overline{C_H} = \mu_r m_1 g^*,
\]

where \( \mu_r \) is the coefficient of internal friction, \( m_1 \) is the mass of the upper layer, and \( g^* \) is an effective acceleration of gravity. Considering an ideal upper layer that does not deform in the vertical direction, the average total friction force \( G \) exerted on the oscillating upper layer by the container glass surface is the product of the dynamic coefficient of friction \( \mu_G \) (between particles and container glass wall) and the centripetal force that is normal to the lateral surface of the granular flow:

\[
G = \mu_G \left( m_1 \frac{v_{2m1}^2}{R} \right),
\]

where \( R \) is the inner radius of the glass container and \( v_{2m1} \) is the average horizontal linear speed of the particles of the upper layers at a distance \( R \) from the longitudinal axis of the cylindrical container. In the lab reference frame the average horizontal force exerted by the faster particles of the basal layer on the slower upper layer pushes the upper layer rightward (i.e., counterclockwise when viewed from above) at a speed lower than that of the disk. Conversely, in the disk reference frame the faster leftward moving upper layer is resisted by the average horizontal force exerted by the slower particles of the basal layer. Thus in the disk reference frame, because the horizontal speed of the upper layer is constant in time, the average horizontal force exerted by the basal layer on the upper layer is equal in magnitude but opposite in direction to a virtual force that can be thought to be pulling the upper layer leftward. On an inclined plane (section 9) this virtual force would correspond to the slope-parallel component of the force of gravity.

[22] The effective acceleration of gravity \( g^* \) arises from the fact that the friction force exerted by the container glass wall affects the weight of the upper layers during their vertical oscillations. Thus we add to the magnitude of the

![Figure 5](image_url)

**Figure 5.** Examples of vertical profiles of the horizontal (i.e., tangential) components of velocity (measured at a distance from the longitudinal axis of the cylindrical container approximately equal to its inner radius) (a) in flows obtained at the lowest angular disk speed and (b) in flows generated at the largest angular disk speed. Both profiles are in the disk reference frame that is attached to the perimeter of the disk (negative values indicate that the velocity vectors point leftward). In Figure 5b the average horizontal velocity \( V_{HIB} \) of the particles of basal layer immediately before collision with the upper layer is also shown. The error bars are the errors of the means. The data points are fitted with an exponential equation in a least squares sense.

### Table 1. Horizontal Linear Speeds in Lab Reference Frame

<table>
<thead>
<tr>
<th>Disk Speed</th>
<th>Intermediate Disk Speed</th>
<th>Higher Disk Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower</td>
<td>2.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Upper layers</td>
<td>0.97 ± 0.01</td>
<td>0.98 ± 0.01</td>
</tr>
</tbody>
</table>

*These linear speeds (m s\(^{-1}\)) are measured at a distance from the longitudinal axis of the cylindrical container equal to its inner radius. Each upper layer speed is the mean of the velocities of 375 particles in the three experiments at the same disk speed.
acceleration of gravity $g$ (9.8 m s$^{-2}$) a term due to friction against the glass wall:

$$g^* = g - \frac{G_H}{m_1},$$

(3)

where $G_H$ is the magnitude of the average vertical component of the friction force exerted by the container glass surface on the oscillating upper layer at the time of interaction with the basal layer. The minus sign is due to the fact that (as revealed by the high-speed movies) the interaction between upper layer and basal layer is considered here to occur mainly when the upper layer moves downward so that $g^*$ is smaller than $g$. When the oscillating upper layer is moving upward, its less numerous interactions (due to the increased distance from the disk) with the particles of the basal layer are probably less effective (but not necessarily zero) because, in this case, the upper layer can expand, and its particles can also temporarily lose contact. A similar phenomenon may happen also in natural geophysical flows.

[23] In equation (1), considering $G_H$ a component of or equal to $G$, the average horizontal speed $v_{x\text{avg}}$ of the upper layers in the lab reference frame does not depend on the shear rate. Moreover, in our apparatus, $v_{x\text{avg}}$ does not depend on the mass $m_1$ of the upper layer (i.e., any possible different redistribution of particles between upper and basal layer at the different angular disk speeds) because $m_1$ appears on both sides of equation (1), and it can be deleted. There is a relatively large uncertainty in the estimate of $G_H$ because the average of the slip angles discussed in section 3 provides only an upper bound for the dynamic coefficient of friction. However, equation (1) predicts an average horizontal speed of the upper layers in the lab reference system at a distance $R$ from the longitudinal axis of the cylindrical container that is equal to the measured values shown in Table 1 (i.e., 0.98 m s$^{-1}$) if, for example, because of the uncertainty in the value of $G_H$, we suppose that the vertical and horizontal components of the friction force due to the container glass wall are approximately equal to the right-hand side of equation (2), with $G_H$ set equal to 0.38 (the minimum value in the range of static coefficients of friction obtained measuring the slip angle). In this calculation our estimate of the coefficient of internal friction of the particles, $G_H = 1.47$, is also used because the upper layers are in physical contact with the basal layers, and they do not touch the disk. Here $G_H$ and $G_V$ are the time and space averages of the horizontal and vertical components, respectively, of the resultant friction force exerted by the glass surface on the different (but connected) portions of the oscillating annular upper layer that can move upward and downward at different locations and in different moments. In any case, although the actual magnitude of $G^*$ and $G_H$ are masked by uncertainties in the value of $G_H$, there is no evidence that they are, on average, significantly different at the three different disk speeds.

[24] We can imagine that during the initial acceleration immediately after the motor is turned on the force represented by the left-hand side of equation (1) increases, whereas that represented by the right-hand side decreases until a speed value is reached at which the opposite forces become equal in magnitude. Consequently, the average angular velocity of the upper layer can be considered fixed. However, the important point is that this behavior is incompatible, for example, with Newtonian and Bingham fluids [Fox and McDonald, 1985] and with Bagnoldian grain flows [Bagnold, 1954], where the shear forces increase when the shear rate increases, and they would not depend on the effective acceleration of gravity $g^*$.

The expressions for the shear forces in Newtonian, Bingham, and Bagnoldian flows on the right-hand side of equation (1) would predict an increase of the velocity of the upper layers (considering, for example, $G_H$ equal to $G$) as the angular disk speed and the shear rate increase. For these other models, changes in the values of $m_1$ and other parameters (such as the particle concentration in Bagnold's equation) at the three different disk speeds could also affect the upper layer velocity. In any case, tests in our apparatus with a Newtonian fluid (i.e., a Bingham fluid with zero yield strength) confirm that its angular velocity increases when that of the disk increases. With granular material, this does not happen. Therefore our experiments suggest the validity, on average, of the Coulomb friction law [Coulomb, 1776] at the base of moving granular mass flows, where the shear forces do not depend on the shear rate. Bagnoldian grain flows also seem unsuitable to describe natural pyroclastic flows because they are obtained by suppressing the force of gravity on dispersed particles in solid-liquid systems [Bagnold, 1954], whereas our experiments demonstrate that gravity matters.

[25] Thus the same equation used to predict the forces acting on the lower surface of the upper layer in shear-dominated flows, i.e., the Coulomb's equation with zero cohesion on the right-hand side of equation (1), can also be used in flows with a basal layer of colliding particles. This is also true in the disk (i.e., slope) reference frame irrespective of the shear rate (the speed of the upper layers does not change in the lab reference frame, but it changes in the disk, i.e., slope, reference frame). Consequently, considering an average friction force at the base of the upper layers that is indistinguishable at different flow speeds, as implied by Coulomb's equation for same weight upper layers, the formation of a basal layer of agitated and colliding particles cannot be considered a way to lubricate the flows, as suggested, for example, by Campbell [1989].

8. Basal Layers of Colliding Particles

[26] At intermediate and largest angular disk speeds the magnitude of the velocity component $V_{NA}$ (Figure 4) is, on average, not larger than that of a falling body. In particular, the high-speed movies show that the particles of the basal layer hit the lower surface of the upper layer and then are entrained into the upper layer or fall because of the force of gravity. Although it is not appropriate to use the concept of coefficient of restitution with fragile rock fragments that shatter upon impacts [Cagnoli and Manga, 2003], with glass beads we would say that this is due to the fact that the coefficient of restitution ($\left| V_{NAR} V_{NBR} \right|$ [Goldsmith,
2001], where the subscript $R$ refers to velocities relative to those of the upper layer) between one bead and a cluster of several other beads is much smaller than that between two glass beads. For the same reason a group of glass beads held together by a net and hitting a horizontal surface will rebound at a much lower height than a single glass bead. This is the result of the larger energy dissipation due to the numerous collisions between all the beads in the cluster. The collisions of the basal layer particles constitute an important mechanism to dissipate the energy difference between flows and disk. Thus a particularly small average value of the velocity ratio $|V_{NBR}/V_{NAR}|$ may enable us to bypass its complex dependency on the several impact variables such as impact angle, shape of the fragments, impact velocity, etc., discussed by Cagnoli and Manga [2003]. This complex dependency would prevent the suggestion of a single, constant value of the velocity ratio (analogous to the coefficient of restitution) that is representative of the rock fragments collisions.

[27] If we consider the collision with the disk in a relatively thin basal layer, the above ratio is reversed $(|V_{NBR}/V_{NAR}|)$ (Figure 4), and its value is, on average, relatively large. This value is large because upon collision with the disk, part of the horizontal motion is diverted upward [Iverson, 1997]. This happens because the particles hit asperities and irregularities (such as the indentations on the disk surface) whose surface inclinations differ from that of the disk (or ground in the slope reference frame). This phenomenon is very important because as $V_{NA}$ is relatively small, the particle would not have had enough energy after collision with the disk or ground to rebound upward. Particles collisions with the ground at the base of natural pyroclastic flows could be responsible for impact marks similar to those described by Pittari and Cas [2004].

[28] At intermediate and largest angular disk speeds the angular velocity of the particles of the basal layer around their centers of mass is always counterclockwise in both the lab and the disk reference frames and both before $(\omega_{BL})$ and after $(\omega_{A})$ the collisions with the upper layer. In the lab reference frame the particles of the basal layer have a higher rightward horizontal speed before collision and thus after hitting the slower upper layer their angular velocity is still counterclockwise (Figure 4). In the disk reference system the particles of the basal layer have a slower leftward horizontal speed before collision and thus after hitting the faster upper layer their angular velocity is again counterclockwise (Figure 4).

9. Flows on Inclined Planes

[29] Although our experiments are meant to study the properties of loose rock fragments that (in a channel) turn out to behave en masse as a quasi-rigid body, we can attempt a more direct comparison between our flows and those on an inclined plane. Thus because the two average horizontal (i.e., disk-parallel) forces acting on the upper layers are equal in magnitude and opposite in direction, our flows in the disk reference frame can be thought on a slope with uniform inclination equal to the angle of internal friction (where, also, the coefficient of basal friction with the ground is equal to the coefficient of internal friction). In this case, using a reference frame attached to the ground (Figure 3), we can write

$$W \sin \alpha = \mu_s W \cos \alpha,$$

where $W$ is the weight of the upper layer and $\alpha$ is the slope angle. Here it is straightforward to show that $\sin \alpha/\cos \alpha = \mu_s$, where $\alpha$ is both the uniform slope inclination and the angle of internal friction. Relatively large values of the angle of internal friction of granular material explain deposits of block-and-ash flows on slopes with inclination exceeding $30^\circ$ that have been described on Colima Volcano, Mexico [Saucedo et al., 2004].

[30] On a ground surface with slope inclination equal to the angle of internal friction the initial velocity can be set, in the case of pyroclastic flows, by the volcanic explosion. For example, different pressure drops in a volcanic conduit generate flows with different initial velocities [Cagnoli et al., 2002]. The speed is also affected by the slope angle because in a schematic model a flow accelerates when the slope angle is larger than $\alpha$, whereas it decelerates when it is smaller. Pyroclastic flows with a visible change in velocity in correspondence to a break in slope have, for example, been observed at Ngauruhoe Volcano, New Zealand [Nairn and Self, 1978].

[31] Our experiments show that a force (in our case the centripetal force) that is perpendicular to the direction of motion sustains a portion of the weight of the oscillating upper layer and slows its vertical movements thanks to the friction that it generates on a lateral surface (equation (3)). In nature this probably does not happen only in channel bends. Particle collisions with the lateral surfaces of a straight channel (such as the levees of the flow) result in forces perpendicular to the direction of motion. It may also be possible that part of the weight of a granular mass flow traveling within a straight channel is redirected toward the lateral surfaces. This happens in stationary granular materials within containers where the vertical stress $\sigma_v$ is redirected horizontally according to $\sigma_h = K\sigma_v$, where $\sigma_h$ is the horizontal stress and $K$ is the coefficient of redirection [Janssen, 1895; Fan and Zhu, 1998]. Levees with height above the adjacent interiors between 1 and 2 m have, for example, been described in the pumice flow deposits of the Mount St. Helens, United States, 1980 eruption [Rowley et al., 1981].

[32] Neglecting their radial variation in thickness due to their rotations around the axis of the cylindrical container (because our flows are relatively thin in the radial direction), these flows can be thought, as far as our analysis is concerned, as a relatively thin longitudinal section of a flow with infinite length (traveling within an inclined straight channel with slope angle $\alpha$) that is wrapped against the interior surface of our glass container (Figure 2c). In this case the friction force of the container wall can be thought to correspond, qualitatively, to the force that in a natural flow is exerted by other longitudinal sections on the central one that we are studying (Figure 2c). We can expect that the presence of rough lateral surfaces in the channel would produce lateral velocity gradients in the upper layers as well as lateral layers of colliding particles similar to our basal layers. In short, our apparatus is successful in producing a
the three experiments at the same angular disk speed. The error bars are the errors of the means. The data points are fitted with an exponential equation in a least squares sense.

virtually rigid body from an otherwise loose mixture of granular material as, for example, we expect to occur in natural geophysical flows traveling within channels. The applicability of a sliding friction law (such as Coulomb’s equation) to a rigid body is thus not surprising.

10. Savage Number

[33] We characterize our flows using the Savage number [Savage and Hutter, 1989] recast in the following form [Iverson and Denlinger, 2001]:

\[
N_s = \frac{\rho_s \gamma^2 \delta^2}{(\rho_s - \rho_f) gh},
\]

where \(\rho_s\) and \(\rho_f\) are the mass density of solid grains and interstitial fluids (air in our case), respectively, \(\gamma\) is the shear rate, \(\delta\) is the diameter of the particles, and \(h\) is the depth below the surface of the flow. This dimensionless scaling parameter is considered the ratio between grain collision stresses and gravitational grain contact stresses [Iverson and Denlinger, 2001]. Here we use the Savage number as a parameter that quantifies the transformations experienced by a granular mass flow when its velocity increases, as revealed by our high-speed movies (Figure 2).

[34] We compute the Savage number using an effective average acceleration of gravity estimated as in section 7. At the lowest angular disk speed, \(N_s\) is computed at the base of the upper layer, using to estimate the shear rate, the difference in horizontal speed between uppermost and lowermost rows of particles in the flows (the lowermost particles are those in the basal layers). At the intermediate and largest angular disk speeds, \(N_s\) is computed again at the base of the upper layer, using to estimate the shear rate, the difference in horizontal speed between the uppermost row of particles of the upper layer and the average horizontal components of velocity \((V_{HUP})\) of the particles of the basal layer immediately before impact with the upper layer. Therefore at all disk speeds the vertical distance between particles used to compute the shear rate is approximately equal to the upper layer thickness that is virtually the same at the three disk speeds. Moreover, because \(\rho_f\) is much smaller than \(\rho_s\), here the Savage number can be considered independent from \(\rho_f\).

[35] The average values of this dimensionless number for the three runs at each angular disk speed are shown in Figure 6, where the mean \(N_s\) values are 0.03, 0.26, and 0.41 at the smallest, intermediate, and largest angular disk speeds, respectively, and the error bars are the errors of the means obtained from Gauss’s law of error propagation [Benenson et al., 2002, p. 1102]. Our apparatus provides an excellent opportunity to interpret the Savage number values. For example, the threshold \(N_s = 0.1\) suggested by Savage and Hutter [1989] separates, in our case, flows with an average basal layer thickness equal to the average particle diameter and flows with thicker basal layers. Our range of values includes some of those estimated for other natural and artificial granular mass flows [Iverson and Vallance, 2001], however, care must be taken in comparing \(N_s\) values obtained by different authors because these values may refer to different depths within the flows (our \(N_s\) values are estimated at the same depth in all flows, i.e., at the base of the upper layers, where there is the largest difference among the flows at the three disk speeds). Also, field estimates of the Savage number in natural geophysical flows are not straightforward because of the difficulties in assessing the real vertical profile of velocity within the flows as well as the effective acceleration of gravity. Flow features in pyroclastic flows may also change in time and space [Cagnoli, 1998]. For example, debris flows are considered unsteady and nonuniform [Iverson, 2003], however, our experiments focus on some key properties of granular material, and they do not intend to reproduce in its entirety the complexity of natural granular mass flows.

11. Energy Dissipation

[36] The angular speed of the disk is the time-constant speed that the upper layer would have had if all the energy that the apparatus could provide was converted into rotational motion of the upper layer. Thus a dimensionless parameter representing the energy that is not available in the rotational motion and that is, in part, dissipated by the collisions in the basal layer and against the friction force exerted by the container glass wall can be defined as

\[
\Pi = \frac{1}{2} I_{up} \omega_{up}^2 - \frac{1}{2} I_{up} \omega_{up}^2 = \frac{\omega_{disk}^2 - \omega_{up}^2}{\omega_{up}^2},
\]

where \(\omega_{disk}\) and \(\omega_{up}\) are the angular speeds of disk and upper layer, respectively, and \(I_{up}\) is the rotational inertia of the upper layer around the longitudinal axis of the cylindrical container. Therefore \(\Pi\) represents the difference between the rotational kinetic energy that the upper layer would have...
had with the angular velocity of the disk and the rotational energy that it actually has. This difference has been divided by the actual rotational kinetic energy of the flows (that is the same at the three angular disk speeds) so that II provides relative values of the energy difference, and it does not depend on the mass, $m_1$, of the upper layers.

[37] Figure 6 presents the plot of the average II versus $Ns$ values, where the error bars are the errors of the means obtained from Gauss’s law of error propagation [Benenson et al., 2002, p. 1102]. This figure shows an increase of the energy that is not effective in the rotational motion as $Ns$ increases (the data points are fitted with an exponential equation in a least squares sense). It is clear that as the angular velocity of the upper layer in the lab reference system is the same at the three different angular disk speeds, the parameter II is smaller at smaller Savage number values. The II increase can, in part, be explained by an increase in the energy dissipated, as suggested by the larger amount of ash at increasingly larger flow speeds in the disk reference frame (Figure 7). Part of the energy is also diverted into the vertical direction because of, for example, the oscillations of the upper layers that result in an oscillating vertical expansion of the flows.

[38] After the 1 min experimental runs at the smallest, intermediate, and largest angular disk speeds the 100 g beds lost approximately 3, 8, and 12 g, respectively (Figure 7). The differences between the mass losses at the same angular disk speed are always <1 g (Figure 7). This indicates a good reproducibility of the experiments. These relatively large mass losses in experiments that lasted only 1 min suggest that relatively large pumice fragments in moving pyroclastic flows can be totally eroded by collisions and turned completely into ash. The ash that is billowing within the container above the flows during the experiments can be thought to correspond, on a qualitative ground, to the overriding ash clouds that accompany natural pyroclastic flows, whereas the ash that is deposited underneath the disk, in nature, would cover the ground, and we wonder whether the segregation by percolation of fine particles (such as that caused by shaking and vibrations in particle mixtures) is responsible for the formation of the layer 2a of the standard ignimbrite depositional sequence [Sparks, 1976]. Interestingly, particle grading in the relatively rigid plug region can also occur simply because of density contrasts and with the absence of fluidizing agents [Mitani et al., 2004].

[39] The mass losses incurred by our beds are due to both particle-particle and particle-boundary interactions (where the boundaries are both disk and container glass surfaces), but this is probably true also in natural geophysical flows. Because of the linear relationship between quantity of produced ash and speed of the flows in the disk reference frame (Figure 7), we can expect that, also in nature, faster pyroclastic flows (the other features constant) produce more ash than slower ones in the same interval of time. Thin deposits of fine ash due to overriding ash clouds can be found (preserved between different flow units) also with nonpumiceous block-and-ash flow deposits [Cagnoli et al., 1994]. Nonpumiceous block-and-ash flow deposits can also contain the layer 2a [Cagnoli et al., 1994].

12. Conclusions

[40] Our experiments suggest the validity, on average, of the Coulomb friction law in moving granular mass flows (for example, in pyroclastic flows) irrespective of their Savage number value. This applies to the base and the sides of relatively rigid upper layers that form between confining lateral surfaces (such as the so-called levees) in channels. As such, the applicability of a sliding friction law to a virtually rigid body should not be a surprise. In particular, our experimental results suggest that granular mass flows do not behave as Bingham fluids (and they do not behave as Bagnoldian grain flows). Therefore pyroclastic flow features such as steep flow fronts, relatively rigid upper layers, their transport of large blocks, and their deposits left on substantial slopes can be explained by relatively large values of the coefficient of internal friction in Coulomb’s equation, whose validity is not restricted to a quasi-static regime. Thus, for example, the presence of a rigid plug in moving geophysical flows does not require a yield strength different from zero. Here we should point out that in Coulomb’s law the shear stresses do not depend on the shear rate, and our experimental results are a manifestation of this fact.

[41] In granular mass flows the formation of a basal layer of colliding particles underneath a relatively rigid upper layer is a necessary and important mechanism to dissipate the energy difference between flows and ground. The basal layer forms naturally in mass flows of angular granular material because the deformation is concentrated where it is unavoidable, i.e., in a narrow band in contact with the boundary surfaces. In particular, the validity of the Coulomb friction law irrespective of the Savage number value implies that friction at the base of same weight upper layers does not change with the formation of a basal layer of colliding and agitated particles. Of course, in this case, the concept of no-slip condition on boundary surfaces is meaningless.
Our experiments also suggest that in natural pyroclastic flows, relatively large pumice fragments can be totally eroded by collisions and turned, at least in part, into the fines that form the overriding ash clouds. Another part of the relatively fine grains produced by particle-particle and particle-boundary interactions may simply move (because of kinetic sieving) toward the base of the flows and forms the lower fine-grained portion of pyroclastic flow deposits (such as the layer 2a of the standard depositional sequence).

Notation

- $g$ acceleration of gravity [L T$^{-2}$].
- $g^*$ effective acceleration of gravity [L T$^{-2}$].
- $G$ magnitude of the average total friction force exerted on an oscillating upper layer by the container glass surface [M L T$^{-2}$].
- $G_H$ magnitude of the average horizontal component of $G$ [M L T$^{-2}$].
- $G_V$ magnitude of the average vertical component of $G$ at the time of interaction with the basal layer particles [M L T$^{-2}$].
- $h$ depth below the surface of flow [L].
- $I_{pp}$ rotational inertia of upper layer [M L$^2$].
- $K$ coefficient of redirection.
- $m_1$ mass of upper layer [M].
- $N_s$ Savage number.
- $R$ inner radius of glass container [L].
- $V_{HA}$ average horizontal velocity component of the particles of basal layer after collision with upper layer [L T$^{-1}$].
- $V_{HB}$ average horizontal velocity component of the particles of basal layer before collision with upper layer [L T$^{-1}$].
- $V_{NA}$ average normal velocity component of the particles of basal layer after collision with upper layer [L T$^{-1}$].
- $V_{NAR}$ relative (with respect to the upper layer) average normal velocity component of the particles of basal layer after collision with upper layer [L T$^{-1}$].
- $V_{NB}$ average normal velocity component of the particles of basal layer before collision with upper layer [L T$^{-1}$].
- $V_{NBR}$ relative (with respect to the upper layer) average normal velocity component of the particles of basal layer before collision with upper layer [L T$^{-1}$].
- $V_{x_{max}}$ average horizontal linear speed of the upper layer particles at a distance from the longitudinal axis of the cylindrical container approximately equal to its inner radius $R$ [L T$^{-1}$].
- $W$ weight of upper layer in flows moving on slope [M L T$^{-2}$].
- $x$ unit vector pointing rightward in the rectangular reference frames.
- $y$ unit vector pointing upward in the rectangular reference frames.
- $z$ unit vector pointing toward the reader in the rectangular reference frames.
- $\alpha$ slope angle.
- $\gamma$ shear rate [T$^{-1}$].
- $d$ diameter of the particles [L].

$\mu_g$ dynamic coefficient of friction between pumice particles and container glass surface.

$\mu_p$ coefficient of internal friction of pumice particles.

$\Pi$ dimensionless energy parameter.

$\rho_f$ air density [M L$^{-3}$].

$\rho_s$ particle density [M L$^{-3}$].

$\sigma_k$ horizontal stress [M L$^{-1}$ T$^{-2}$].

$\sigma_v$ vertical stress [M L$^{-1}$ T$^{-2}$].

$\omega_A$ average angular velocity (around their center of mass) of the particles of basal layer before collision with upper layer [T$^{-1}$].

$\omega_B$ average angular velocity (around their center of mass) of the particles of basal layer after collision with upper layer [T$^{-1}$].

$\omega_{disk}$ angular speed of the disk around longitudinal axis of cylindrical container [T$^{-1}$].

$\omega_{up}$ angular speed of upper layer around longitudinal axis of cylindrical container [T$^{-1}$].

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