Appendix G

Sample Calculations of Magnitudes, Moment, and Energy of an Earthquake

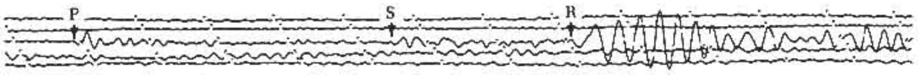
The following calculations are for an Alaskan earthquake recorded at Oroville, California. The energy factor (equation 4) gives us an idea of the scale of energy release that is possible for earthquakes of different magnitude. For instance, 30 earthquakes of magnitude 6 are needed to release the equivalent amount of energy in the Earth's crust that is released by just one magnitude 7 earthquake; and 900 earthquakes of magnitude 5 are needed to produce the same energy. It follows, therefore, that even if small earthquakes occur in swarms in a particular area, they do very little to reduce the reservoir of strain energy needed for a major earthquake. But tectonic energy is drained away into heat and seismic waves in a truly gigantic way by a major earthquake like that of 1906 along the San Andreas fault with a magnitude M_W of 7.9. This earthquake released about 10^{17} joules of strain energy within 60 seconds! (Only a fraction went into ground shaking.)

It is well known that, as the threshold of earthquake size being considered in a seismic region is lowered, the number of earthquakes above that magnitude rapidly increases (see Appendix A, page 316). The rate of occurrence n of shocks above a given magnitude is again logarithmic and is measured by a parameter b (see equation 6). Suppose a is constant. Then the smaller b is, the more numerous are the earthquakes in a given time span. When b is determined for a seismically active region, the total seismic energy released over a period can be calculated by using the energy factor.

Magnitude is also sometimes roughly estimated from the length of surface fault rupture L (in kilometers—see equation 7).

These calculations all follow from substitutions of measurements made directly on a seismogram into empirical formulas.

Let A be the ground amplitude (in microns) and T the period of a wave measured at a distance Δ (degrees) from the source.



Typical P, S, and Rayleigh waves on seismogram. One minute between gaps.

Measured values from above seismogram (reduced to ground motion)

$$\text{P wave, } A = 1.4 \text{ microns } (10^{-6} \text{ meters}),\ T = 12 \text{ seconds}$$
$$\text{Rayleigh wave, } A = 4.3 \text{ microns},\ T = 20 \text{ seconds}$$
$$\Delta = 28^\circ$$

Body-wave magnitude m_b $(25^\circ < \Delta < 90^\circ)$

$$\begin{aligned} m_b &= \log A - \log T + 0.01\Delta + 5.9 \\ &= 0.15 - 1.08 + 0.28 + 5.9 \\ &\approx 5.3 \end{aligned} \tag{1}$$

Surface-wave magnitude M_S $(25^\circ < \Delta < 90^\circ)$

$$\begin{aligned} M_S &= \log A + 1.66 \log \Delta + 2.0 \\ &= 0.63 + 2.40 + 2.0 \\ &\approx 5.0 \end{aligned} \tag{2}$$

Moment M_O (Newton meters)

$$\log M_O = 16.1 + 1.5\, M_S \tag{3}$$
$$\text{Therefore } M_O \approx 4 \times 10^{23} \text{ Newton meters}$$

(*Note:* Equation 3 is not applicable to the largest earthquakes.)
Seismic energy E

$$\begin{aligned} \log E &= 11.8 + 1.5\, M_S \\ &= 19.3 \\ E &= 2.0 \times 10^{19} \text{ ergs} \end{aligned} \tag{4}$$

Relation between moment magnitude M_W and seismic moment M_O in Newton-meters (see Chapter 7)

$$M_W = \tfrac{2}{3} \log M_O - 10.7 \approx 4.9 \tag{5}$$

Relation between n and M_S

$$\log n = a - bM_S \tag{6}$$

Relation between M_S and fault rupture length in kilometers L (worldwide data)

$$M_S = 6.10 + 0.70 \log L \tag{7}$$