Seismic methods: Refraction I

Pre-Critical incidence

Reflection and refraction

Snell’s Law:

\[
\frac{\sin i_P}{V_{P1}} = \frac{\sin R_P}{V_{P1}} = \frac{\sin r_P}{V_{P2}} = \rho
\]

where \( \rho \) is the ray parameter and is constant along each ray.

Reflection and transmission coefficients for a specific impedance contrast
Critical incidence

When $r_p = 90^\circ$ $i_p = i_C$ the critical angle

$$\sin i_C = \frac{V_{p1}}{V_{p2}}$$

The critically refracted energy travels along the velocity interface at $V_2$ continually refracting energy back into the upper medium at an angle $i_C$.

$\Rightarrow$ a head wave

Post-Critical incidence

The angle of incidence $> i_C$

$\Rightarrow$ No transmission, just reflection
Horizontal interface

Traveltime equations

Direct wave:

\[ T = \frac{x}{V_1} \]

Head wave:

\[ T = T_{SB} + T_{ED} + T_{AD} \]

\[ T = \frac{2h_1}{V_1 \cos i_c} + x - 2h_1 \tan i \frac{V_1}{V_2} \]

\[ T = \frac{x}{V_2} \pm \frac{2h_1 \sqrt{V_2^2 - V_1^2}}{V_2 V_1} \]

\[ T = ax + b \]

slope: \( \frac{1}{V_2} \)

intercept: gives \( h_1 \)

Crossover distance, \( x_{co} \)

Where the direct and head wave cross. Their travel times are equal:

\[ \frac{x_{co}}{V_1} = \frac{x_{co}}{V_2} + 2h_1 \frac{\sqrt{V_2^2 - V_1^2}}{V_2 V_1} \]

\[ x_{co} = 2h_1 \frac{\sqrt{V_2^2 + V_1^2}}{\sqrt{V_2^2 - V_1^2}} \]

Another approach to obtaining layer thickness
Horizontal interface

Reflections

The critical reflection is the closest head wave arrival.

At shorter offsets there are low amplitude reflections (used in reflection seismology).

At greater offsets there are wide-angle reflections.

Three-layer model

Traveltime

\[ T_{SG} = \frac{SA}{V_1} + \frac{AB}{V_2} + \frac{BC}{V_3} + \frac{CD}{V_2} + \frac{DG}{V_1} \]

\[ T_{SG} = \frac{2z_1}{V_1 \cos \theta_1} + \frac{2z_2}{V_2 \cos \theta_2} + \frac{x - 2z_1 \tan \theta_1 - 2z_2 \tan \theta_2}{V_3} \]

With some manipulation

\[ T_{SG} = \frac{x}{V_3} + \frac{2z_1 \sqrt{V_3^2 - V_1^2}}{V_1 V_3} \]

\[ + \frac{2z_2 \sqrt{V_3^2 - V_2^2}}{V_2 V_3} \]

1. Determine \( V_1, V_2, V_3 \) from slopes
2. Determine \( z_1 \) from 1st intercept
3. Determine \( z_2 \) from 2nd intercept
Multiple-layered models

For multiple layered models we can apply the same process to determine layer thickness and velocity sequentially from the top layer to the bottom.

Head wave from top of layer 2:

\[ T = \frac{x}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_2V_1} \]

Head wave from top of layer 3:

\[ T = \frac{x}{V_3} + \frac{2h_1\sqrt{V_3^2 - V_2^2}}{V_3V_2} + \frac{2h_2\sqrt{V_3^2 - V_2^2}}{V_2V_1} \]

Head wave from top of layer \( n \):

\[ T = \sum_{j=2}^{n-1} \left( \frac{2h_j\sqrt{V_n^2 - V_j^2}}{V_jV_{j+1}} \right) + \frac{x}{V_n} \]

Horizontal vs. vertical velocity contrasts

A three-horizontal layer model can produce the same traveltime curve as a single horizontal layer over a vertical velocity contact.

Head wave continues into 2b.
Horizontal vs. vertical velocity contrasts

**Use a long-offset shot**

- Leave the geophones fixed and move shot to greater offset

In horizontal layers case the shape of the traveltime curve is unchanged, just shifted in space.

In vertical velocity contrast case the crossover distance remains fixed but is time shifted.

Mapping vertical contacts

**Small offsets**

A vertical step causes an offset on the traveltime curve

- The relation of velocity to the slope remains unchanged
- The offset can be calculated from the time offset, \( \Delta T \)

\[
z_i = \frac{\Delta TV_i V_i}{\sqrt{V_2^2 - V_1^2}}
\]

- Diffractions link the two head wave curves
- Depth, \( z_1 \), is calculated from the intercept in the usual way
Mapping vertical contacts

For infinite/large vertical offsets there is no secondary head wave

**Three segments**
- Direct wave
- Head wave
- Diffracted wave
  - Will have the velocity close to the direct wave

**Reverse the line**
- Shooting to the same string of geophones from the other end
- Two traveltime segments: direct and head wave
  - Head wave generated from energy entering the high velocity layer at the vertical interface

Dipping layers

- Dipping layers still produce head waves but the traveltimes are affected by the dip
  - **Shooting up-dip:** the velocity appears greater
  - **Shooting down-dip:** the velocity is reduced
Reversing lines  …shooting to a line of geophones from both ends

For horizontal layers the traveltime curves are symmetrical

For dipping layers layer velocities appear different for each end – the dip and true velocity can be determined from the up-dip and down-dip velocities

Figure 2-10: Contrast of a travel-time graph with more paths to geophones at equal distance from an energy source for a forward and a reverse survey.

Dipping layer traveltime

Down-dip

\[
T_d = \frac{SC}{V_1} + \frac{CD}{V_2} + \frac{DS}{V_1}
\]

\[
T_d = \frac{h + h_d}{V_1\cos i} + \frac{x - (h_c + h_d\tan i)}{V_2}
\]

With trigonometric transformations, an exercise for the class:

Down-dip traveltime

\[
T_d = \frac{x\sin(i + \phi)}{V_1} + \frac{2h_d\cos i}{V_1}
\]

Down-dip apparent velocity

\[
V_d = \frac{V_i}{\sin(i + \phi)}
\]

Up-dip traveltime

\[
T_u = \frac{x\sin(i - \phi)}{V_1} + \frac{2h_c\cos i}{V_1}
\]

Up-dip apparent velocity

\[
V_u = \frac{V_i}{\sin(i - \phi)}
\]

…where is \(V_2\) dependence?

Applied Geophysics – Refraction I
Dipping layer traveltime

Given
\[ V_d = \frac{V_i}{\sin(i_c + \phi)} \quad V_u = \frac{V_i}{\sin(i_c - \phi)} \]

We can solve for:
\[ \phi = \frac{1}{2} \left[ \sin^{-1}\left( \frac{V_i}{V_d} \right) - \sin^{-1}\left( \frac{V_i}{V_u} \right) \right] \]
\[ i_c = \frac{1}{2} \left[ \sin^{-1}\left( \frac{V_i}{V_d} \right) + \sin^{-1}\left( \frac{V_i}{V_u} \right) \right] \]

\( V_i \) then obtained from: \( \sin i_c = \frac{V_i}{V_j} \)

Finally, the intercept times can be used to determine the perpendicular distance to the reflector:
\[ T_{id} = \frac{2h_i \cos i_i}{V_1} \quad T_{iu} = \frac{2h_i \cos i_u}{V_1} \]

Dipping layer

**Example**

**Direct arrivals**

Velocities from slopes: 1780 m/s and 2250 m/s ⇒ average: 2015 m/s

**Head waves**

Up-dip velocity, \( V_i = 3200 \) m/s
Down-dip velocity, \( V_d = 2870 \) m/s

Using
\[ \phi = \frac{1}{2} \left[ \sin^{-1}\left( \frac{V_i}{V_d} \right) - \sin^{-1}\left( \frac{V_i}{V_u} \right) \right] \]
\[ i_c = \frac{1}{2} \left[ \sin^{-1}\left( \frac{V_i}{V_d} \right) + \sin^{-1}\left( \frac{V_i}{V_u} \right) \right] \]

we obtain:
\[ \phi = 2.8^\circ \]
\[ i_c = 42^\circ \]
Dipping layer

Example

Now obtain $V_2$ from

$$\sin i_c = \frac{V_1}{V_2}$$

$V_2 = 3000 \text{ m/s}$

To determine the perpendicular depths, $h_u$ and $h_d$ use

$$T_{id} = \frac{2h_d \cos i}{V_1} \quad T_{iu} = \frac{2h_u \cos i}{V_1}$$

$h_u = 155 \text{ m}$ and $h_d = 95 \text{ m}$