On the Generation of Core Dynamo Action

CIDER Meeting, KITP 7/7/16

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Beyond record players?…

Treat core dynamics as a rotating magneto-convection problem.

Thoretical insight

Reduction in problem complexity (?)

Cheng et al. GJI 15
Movie: Daphné Lemasquerier
The Geodynamo

- Generates a magnetic fields and then continually regenerate that field
- Magnetohydrodynamic (MHD) process
- **Converts kinetic energy of flowing conductor into magnetic energy**
Successful MHD Modeling

- Navier-Stokes Equation
  \[
  \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\Omega \times \mathbf{u} = -\frac{1}{\rho_o} \nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o} \mathbf{J} \times \mathbf{B}
  \]

- Induction Equation
  \[
  \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}
  \]

- Energy Equation
  \[
  \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + S
  \]

- Current Density
  Ampere’s Law: \[
  \mathbf{J} = \frac{1}{\mu_o} \nabla \times \mathbf{B}
  \]

- Continuity
  \[
  \nabla \cdot \mathbf{u} = 0 \quad \nabla \cdot \mathbf{B} = 0
  \]
Successful MHD Modeling

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  \]
Lorentz Force on a Particle $i$

$$m_i \frac{d\vec{v}_i}{dt} = q_i \vec{v}_i \times \vec{B}$$

- Moving charged particle pushed off to its right

$$R_{gr} = \frac{m_i v_i}{q_i B}$$

$B$ out of the board
Lorentz Force in a Continuum

\[ m_i \frac{d\vec{v}_i}{dt} = q_i \vec{v}_i \times \vec{B} \]

- Using the current density \( J \), where \( n \) is the charge density

\[ \vec{J} = nq < \vec{v}_i > = \frac{I}{A} \]

- Gives:

\[ \rho \frac{d\vec{u}}{dt} \delta V = nq\vec{v}_d \delta V \times \vec{B} \]
Lorentz Force in a Continuum

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- *NB*: \( u \) is now the material velocity!
Lorentz Force in a Continuum

- An example maybe?
- MHD Pump
  - “Opposite” of a dynamo: $J$ and $B$ drive $u$

\[ 0 = \vec{J} \times \vec{B} + \mu \nabla^2 \vec{u} \]
MHD Pump

Top View

Cu walls

Cross-Section

$J_x B$

Leads

0.05 M CuSO₄ solution

Austin Chadwick,
Lorraine Esturas,
Alex Kerelsky

B out of the board
MHD Pump
MHD Pump

\[ J = \frac{I}{A} \]
\[
J = \frac{I}{A} = \frac{I}{2\pi r H} \propto \frac{1}{r}
\]

- Thin layer: \( H << r_{\text{inner}} \)

\[
0 = \vec{J} \times \vec{B} + \mu \nabla^2 \vec{u}
\]

\[
0 = \frac{IB}{2\pi r H} + \mu \frac{d^2 u_\phi}{dz^2}
\]

\[
u_\phi(z = 0) = 0
\]

\[
u_\phi(H) = \left( \frac{IB H}{4\pi \mu} \right) \frac{1}{r}
\]
MHD Pump

\[ u_\phi(H) = \left( \frac{IBH}{4\pi\mu} \right) \frac{1}{r} \]

\[ r_i \simeq r_o/2 \rightarrow u_i \simeq 2u_o \]
\[ C_i \simeq C_o/2 \rightarrow t_i \simeq t_o/4 \]
MHD Pump

- “Opposite” of a dynamo
- $J \times B$ drives flow, $u$, in an electrically conducting fluid

- Now to dynamos where $u$ generates $(J, B)$...
Dynamo Essentials

• Navier-Stokes Equation

\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\Omega \times \mathbf{u} = -\frac{1}{\rho_o} \nabla p + \alpha T g + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o} \mathbf{J} \times \mathbf{B} \]

• Induction Equation

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \]

• Current Density

Ampere's Law: \[ \mathbf{J} = \frac{1}{\mu_o} \nabla \times \mathbf{B} \]

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The Induction Equation

\[ \frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times \vec{u} \times \vec{B} \]

- Compare to the (scalar) heat equation

\[ \frac{dT}{dt} = \kappa \nabla^2 T + \epsilon \]
The Induction Equation

\[ \frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} \]
\[ \rightarrow \tau \sim \frac{L^2}{\eta} \]

- Compare to the (scalar) heat equation

\[ \frac{dT}{dt} = \kappa \nabla^2 T \]
\[ \rightarrow \tau \sim \frac{L^2}{\kappa} \]
The Induction Equation

\[ \frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} \rightarrow \tau \sim \frac{L^2}{\eta} \]

- Magnetic diffusivity \( \sim 1 \text{ m}^2/\text{s} \) in core fluids
- Magnetic diffusion time \( \sim 15 \text{ kyr} \) or so
- Geologically rapid decay
The Induction Equation

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The Induction Equation

\[ \frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times \vec{u} \times \vec{B} \]

- For a self-sustaining dynamo, the inductive source term must be able to exceed the diffusion term.
The Induction Equation

\[ \frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E} + \nabla \times \vec{j} \]

For a self-sustaining dynamo, the inductive source term must be able to exceed the diffusion term.

Magnetic Reynolds number:

\[ Rm = \frac{UL}{\eta} \geq 40 \quad \text{for dynamo action} \]
The Induction Equation

\[
\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times \vec{u} \times \vec{B}
\]

- For a self-sustaining dynamo, the inductive source term must be able to exceed the diffusion term.
- Magnetic Reynolds number:

\[
Rm = \frac{UL}{\eta}
\]

- NB: magnetic field lines are carried along with the fluid at large \( Rm \).
The Omega-Effect

- Shearing out of magnetic field orthogonal to its direction (axisymmetric)
- Inductive stretching term:

\[ \nabla \wedge (u \hat{\phi}) \wedge (B_0 \hat{s}) = B_0 \hat{s} \cdot \nabla u \hat{\phi} = B_0 \frac{\partial u}{\partial s} \hat{\phi} \]
The Omega-Effect

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The Omega-Effect

Omega generates axisymmetric toroidal field from poloidal field

But nothing regenerates initial poloidal field…

citation?
The Alpha-Effect

- **Helical** twisting of magnetic field lines (Non-axisymmetric!!!)
- Can generate $J$ parallel to toroidal $B$
  - Ensemble parallel $J$ generates toroidal $B$
  - Now Omega-effect can re-generate toroidal field... **DYNAMO!**

\[
\frac{1}{\mu_0} \nabla \times \vec{B} = \vec{J}
\]
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\frac{1}{\mu_0} \nabla \times \vec{B} = \vec{J}
\]

Roberts 2008
Alpha-Omega Dynamo

Self-sustaining process!

citation?
Models


\[ B_r \text{ CMB} \]

\[ L_{\text{max}} \sim 13 \]

Soderlund et al., *EPSL* 2012

\[ B_r \text{ CMB} \]

Tangent

Cylinder
The Earth's magnetic field is generated in the fluid core of the Earth (radius $R_E$) by the dynamo mechanism. This mechanism has sustained the field from decay over most of the lifetime of the Earth, and convective motions associated with the flow of the fluid core are responsible for the 'secular variation', or slow changes in the magnetic field over timescales of decades to centuries which are seen on the Earth's surface.

The sensitivity of the data to the model is generally a quadratic smoothness criterion implemented in the solution; generally a quadratic smoothness criterion is implemented. The results from certain dynamo simulations show that it is possible to recover the two positive intensities of the core field with a known kernel. This kernel, the equivalent of the Green's function in the linearized inverse problem, is unique and therefore requires some prior information in its selection of data. Properties of this method are well-known; it is frequently used in virtually any inverse problem, the problem of reconstructing the magnetic fields from numerical dynamo models confirm this.

Table 1

<table>
<thead>
<tr>
<th>Model</th>
<th>No. of data</th>
<th>Misfit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Br CMB</td>
<td>3,684</td>
<td>2.25 nT</td>
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Jackson, Nature 2003

Soderlund et al. EPSL 2012

$Z^2$
The Earth's magnetic field is generated in the fluid core of the Earth (radius 3,485 km) by a 'self-exciting dynamo' mechanism. This mechanism has sustained the field from decay over most of the lifetime of the Earth, and convective motions associated with the dynamo mechanism are responsible for the 'secular variation', or slow changes in the magnetic field over timescales of decades to centuries which are seen on the Earth's surface.

Smooth images of the core field can be reliably constructed back to the 16th century using data collected on land and sea. The Oersted data have lower errors and are treated using the anisotropic error models the Magsat data are assigned errors of 10 nT, commensurate with previous estimates put the level of violation at a few per cent per century.

The forward problem of reconstructing the core magnetic field from surface observation is formally non-convex, and is common to most inverse problems. There is no zero contribution to the sensitivity matrix. The unsigned flux of the two models differs by less than 0.5%.

The point-spread function in astronomical imaging, is very wide and localized flux lines or 'ropes' in which strong localizations heavily towards zero. A method based on regularization methods will produce poor images, because they bias the answer to this problem, the Maximum Entropy method, has had considerable success in astronomy and medical physics fields and will be obeyed to a high degree, and indeed numerical simulations.

The sensitivity of the data to the model is a good one. Thus, Alfven's celebrated frozen-flux theorem will be obeyed to a high degree, and indeed numerical simulations. The Earth's magnetic field is generated in the fluid core of the Earth by a 'self-exciting dynamo' mechanism. This mechanism has sustained the field from decay over most of the lifetime of the Earth, and convective motions associated with the dynamo mechanism are responsible for the 'secular variation', or slow changes in the magnetic field over timescales of decades to centuries which are seen on the Earth's surface.

Models

Christensen, Enc. Solid Earth Geophys. 2011

Jackson, Nature 2003

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<td>z-vorticity</td>
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$B_r$ CMB

Magnetic flux ($\mu$T)

- 900
- 800
- 700
- 600
- 500
- 400
- 300
- 200
- 100
- 0
- 100
- 200
- 300
- 400
- 500
- 600
- 700
- 800
- 900

$L_{max} \sim 13$

$\Omega$
Dynamo model concept and equations

The magnetic induction equation, obtained from Maxwell’s equations, is written in non-dimensional form. A possible scheme for dynamo models, which is defined as a field line structure resulting from the interaction of fluid and magnetic fields, is

\[
\frac{\partial B}{\partial t} + \nabla \times (u \times B) = \alpha \nabla^2 B + \kappa \nabla^2 B + \epsilon B
\]

where \(B\) is the magnetic field, \(u\) is the fluid velocity, \(\alpha\) is the magnetic Prandtl number, \(\kappa\) is the magnetic diffusivity, and \(\epsilon\) is the magnetic diffusivity.

The advection-diffusion equation for the temperature field is

\[
\frac{\partial T}{\partial t} + \nabla \cdot (u T) = \nabla \cdot (k \nabla T)
\]

where \(T\) is the temperature, \(u\) is the fluid velocity, and \(k\) is the thermal conductivity.

The Navier–Stokes equation for the momentum equation is

\[
\rho \left( \frac{\partial u}{\partial t} + u \cdot \nabla u \right) = -\nabla P + \mu \nabla^2 u + \rho g - \rho \beta T \mathbf{g}
\]

where \(\rho\) is the fluid density, \(P\) is the pressure, \(\mu\) is the viscosity, \(\beta\) is the thermal expansion coefficient, \(T\) is the temperature, and \(g\) is the gravitational acceleration.

The equation of state for an ideal gas is

\[
P = \rho R T
\]

where \(R\) is the specific gas constant.

Inside these vortices, the uneven distribution of buoyancy along the flow direction creates a thermal wind secondary circulation (Olson 1995) and enhances the instability of the fluid flow.

In contrast to earlier kinematic dynamo models, where the flow patterns of a certain complexity are driven by the rotation of the Earth, dynamo models based on fluid flow and magnetic field interaction provide a more self-consistent treatment of the geodynamo. The coupling of fluid flow and magnetic field generation in the Earth's core is addressed.

The Proudman–Taylor constraint, which limits the possible flow structures, can be satisfied when the fluid flow is aligned with the magnetic field. This alignment is consistent with the observations of the geomagnetic field.

The magnetic field can be identified by the distribution of buoyancy forces and the concentration of light components. The buoyancy forces are due to temperature differences and are represented by the temperature gradient.

Visualizations of magnetic cyclones and anticyclones, as displayed in Figs 4 and 5, suggest an axial vorticity distribution biased towards flow cyclones. Very large-scale magnetic cyclones (times 4.3617, 4.3811), which are found in geomagnetic field models, stretch field lines to form a magnetic cyclone. Fig. 6 relates DMFI (Dynamic Magnetic Field Imaging) to the Proudman–Taylor constraint. The sparse character of the magnetic field lines correlates well with the flow structures in our models. The radial magnetic field as seen from the Earth's surface is represented in z-direction, due to the Earth's surface field from 945–956 GJI.

The inner and outer boundaries of the model are colour-coded with the radial magnetic field (a red patch denotes outwards oriented field). In addition, the magnetic field lines correlate well with the flow structures in our models. The visual confirmation of previously published dynamo mechanisms is based on visualizations of magnetic cyclones, as displayed in Figs 4 and 5. Fig. 6 relates DMFI (Dynamic Magnetic Field Imaging) to the Proudman–Taylor constraint. The sparse character of the magnetic field lines correlates well with the flow structures in our models. The radial magnetic field as seen from the Earth's surface is represented in z-direction, due to the Earth's surface field from 945–956 GJI.

A strong axial flow cyclone (red isosurface in Fig. 4) moves close to the equator, moving jointly with fairly stable field lines clustered close to the equator, moving jointly with fairly stable field lines. Fig. 6 relates DMFI (Dynamic Magnetic Field Imaging) to the Proudman–Taylor constraint. The sparse character of the magnetic field lines correlates well with the flow structures in our models. The radial magnetic field as seen from the Earth's surface is represented in z-direction, due to the Earth's surface field from 945–956 GJI.

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“Present” paradigm: helical columns extrapolate to planetary cores; continually regenerate field
Summary

- Dynamos require:
  - Energy source
  - Organized flow
  - Conducting fluid
  - Large length scales

- Two stage regeneration process
  - Non-axisymmetric flow in at least one stage

- Paradigm: Large-scale helical columns

$$Rm = \frac{UL}{\eta}$$
Sheyko’s Turbulent Dynamo

\( E = 3 \times 10^{-7}, \ P_m = 0.05 \); \( R_m = 3 \times 10^3, \ R_e \sim 5 \times 10^4, \ R_o \sim 0.02 \)

Andrey Sheyko (2014)
Sheyko’s Turbulent Dynamo

Andrey Sheyko (2014)

\(Br\) Slice

\(E = 3e^{-7}, Pm=0.05\); \([Rm = 3e3, Re \sim 5e4, Ro \sim 0.02]\)
Sheyko’s Turbulent Dynamo

\( Br \) CMB

\((E = 3e-7, Pm=0.05); [Rm = 3e3, Re \sim 5e4, Ro \sim 0.02]\)

Andrey Sheyko (2014)
Sheyko’s Turbulent Dynamo

$L_{\text{max}} = 12$

\( E = 3 \times 10^{-7}, Pm=0.05 \); \([Rm]=3 \times 10^3\)

Br CMB

\( L_{\text{max}} \sim 13 \)

(\textit{Jackson, Nature} 2003)

\[ \rho_{\text{c}} \left( \frac{\partial^2 \Phi}{\partial x^2} \right) = -\nabla \cdot q \left( \frac{\partial^2 \Phi}{\partial x^2} \right) \]
Sheyko’s Turbulent Dynamo

- Extreme model: magnetic flux patches no longer directly related to columns (scale separation)
Length Scale Questions

- Linear HD onset scale: $\ell \sim E^{1/3} H$
- In a turbulent HD core, $\ell \sim Ro^{1/2} H$
Length Scale Questions

- Linear HD onset scale: \( \ell \sim E^{1/3} H \)
- In a turbulent HD core, \( \ell \sim Ro^{1/2} H \)
MHD Experiments

- Rotating magneto-convection in liquid metal
- Strong magnetic field can balance Coriolis
- Different behaviors predicted
Top View:
Axial Velocity
Lab MHD Experiments

- In **liquid metals**, fast **oscillatory** columns and slow wall modes
- **Wall modes** related to low latitude waves in observations
  - Missing in present dynamo models
Summary II

- “2000’s” model: large, helical columns associated with B-flux patches
- State-of-the-art: helical flow, but flux patches no longer directly related to columns
- Many open questions
  - e.g., Dynamo action driven by rotating turbulence in liquid metals (Featherstone CIG/ALCF)
    - Paradigm shift? Or waiting for Godot?
Thanks...