CIDER

Magmatic connections:

The interplay of magmatic systems with their crustal containers

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Supplemental Reading: Dufek, Huber and Karlstrom, Chpt. 2. Magma chamber dynamics and thermodynamics, *Modeling Volcanic Processes: The Physics and Mathematics of Volcanism*

Questions:

- **What is the flux of mass and enthalpy of melt into the crust, and what does this imply for long term continental growth?**

  Thermal aspects and melting efficiency

-**By what means and rates is melt separated from its residue?**

  Efficiency of melt-crystal dynamics

-**What is the nature of the volcanic-plutonic connection?**

  A link between complimentary outcrops?

-**How much do chamber and eruptive processes mix material?**

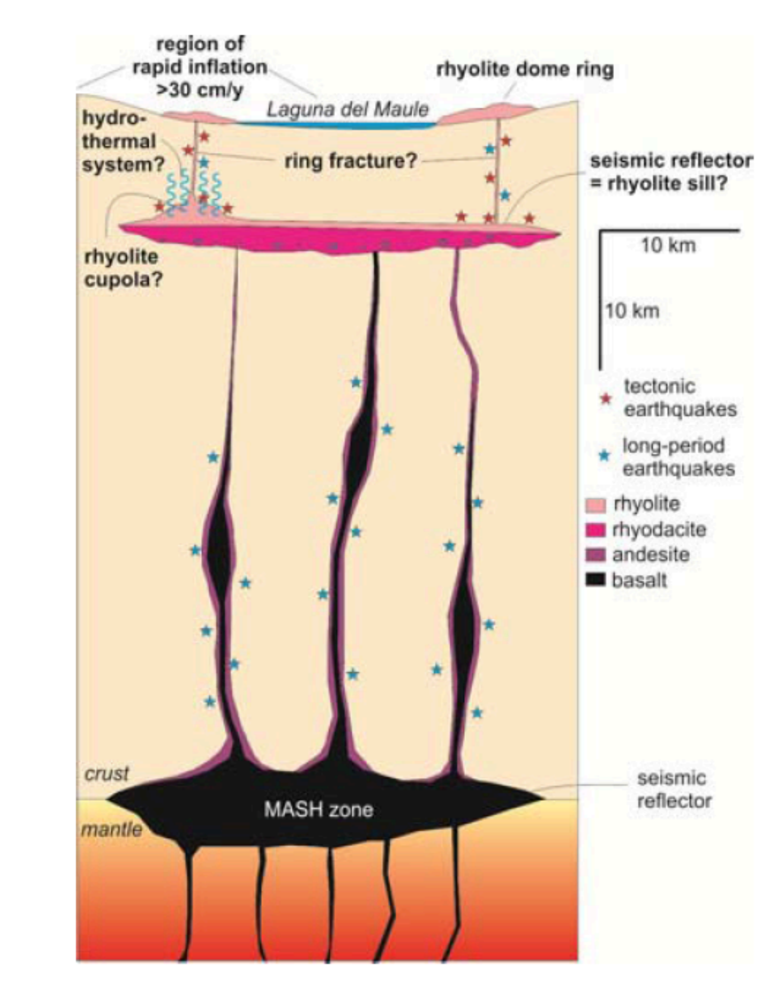
  Interpretation of outcrops and deposits

Outline:

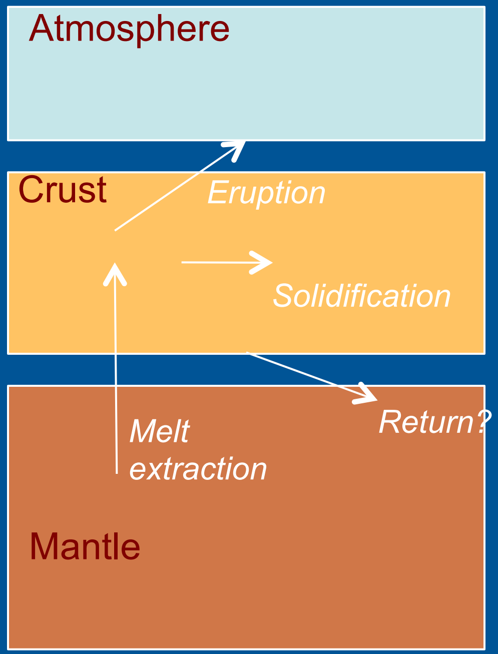
1. Magma chambers in context
2. Thermal system
3. Intro/thermodynamics
4. Surrounding stresses
5. Multiphase Dynamics/Melt Evolution
6. Link to eruptive dynamics
7. Some persistent open questions


Silas Canyon, Wy (Dufek, 2012)

The advection of mass and heat from mantle melts ultimately drives crustal magmatism



Singer et al. 2013



Magma Chambers/intrusions influence:

1. Compositional diversity of the crust

2. Physical properties of the crust

3. Staging region for accumulation of magmas prior to eruption

4. Concentrating incompatible elements and relationships to ore deposits

Modern approaches have focused on examining two parallel sources of data

Plutons

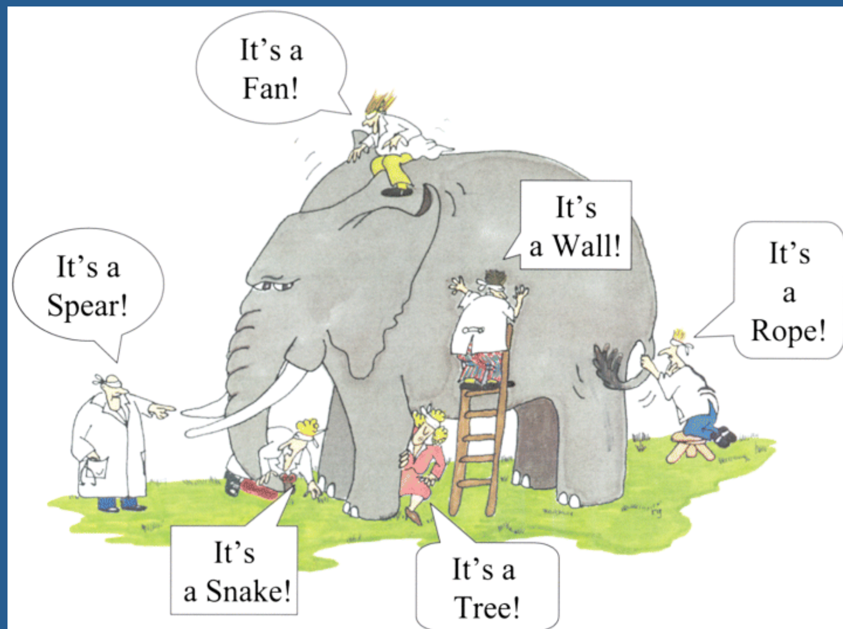
Volcanic deposits

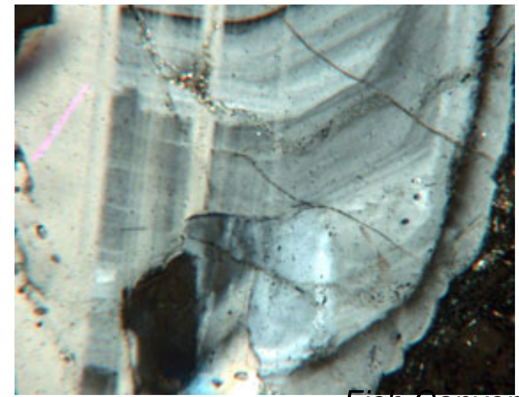
Preserves an integrated record

Closer to instantaneous snap-shot

Connections?

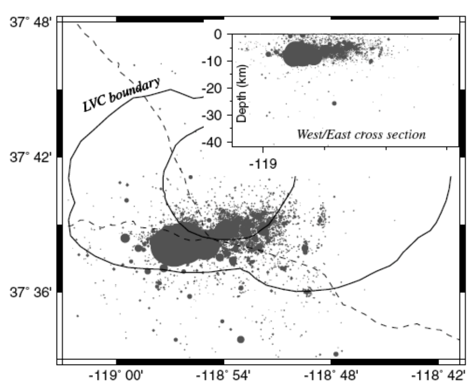
Magmatic processes are recorded across a range of length and timescales



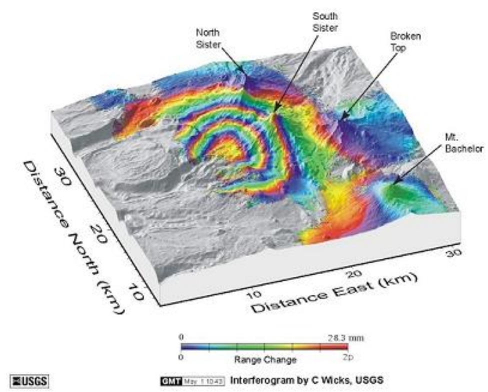


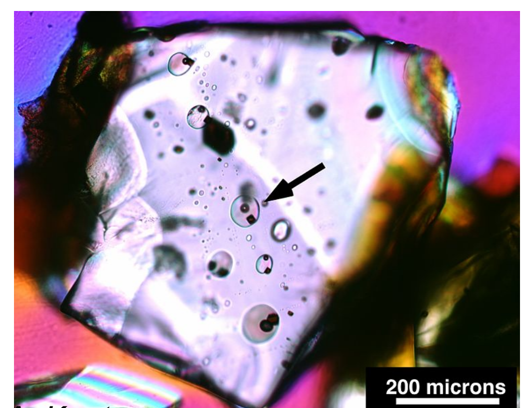
Fish Canyon

Jones



Newman et al., 2001

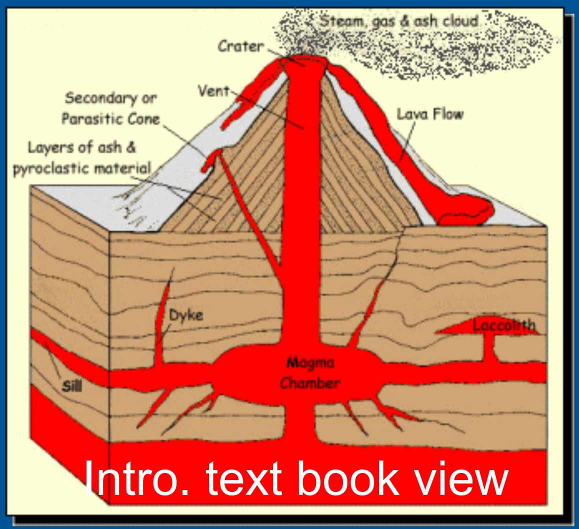




A. Kent

What is a magma chamber?

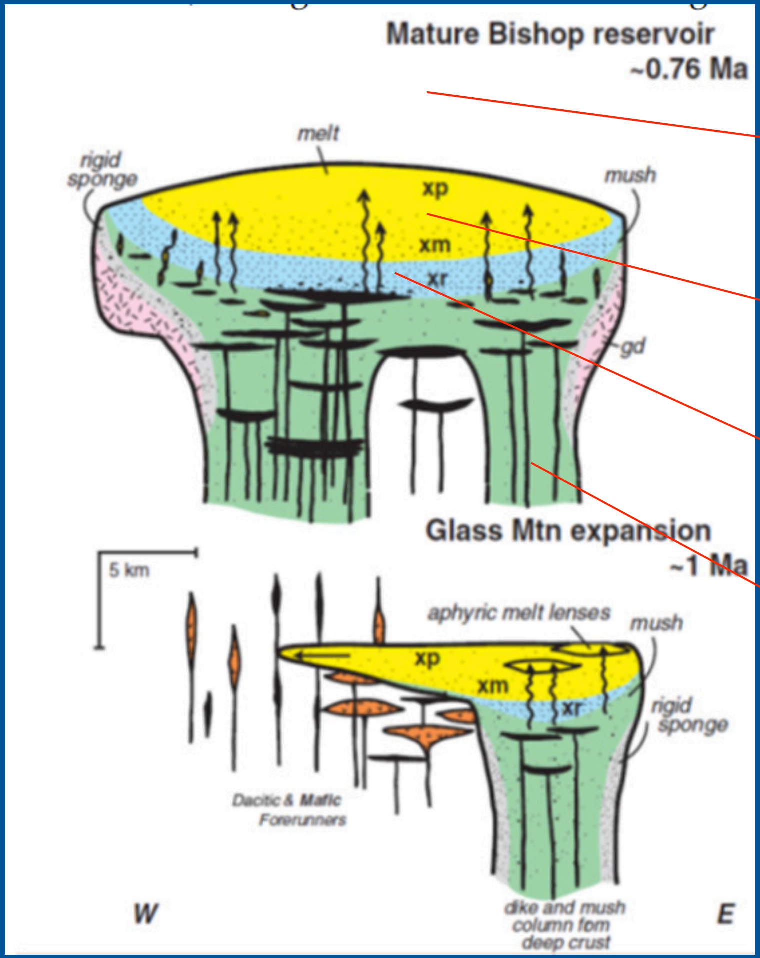
Definitions/usage has varied over time.



Here I'll used the more relaxed definition of a *spatially connected body of magma in a suprasolidus state.*

1. *Magma chambers exist at least transiently --- as evidenced from large eruptions.*
2. *Thermally, chambers are transient features.*
3. *Likely locations of major element evolution in composition due to:*
    1. *Fractional crystallization*
    2. *Melting*
    3. *Mixing of melts*
    4. *Assimilation of crystals*

Transient and crystal-rich mush dominated model



Heating of surrounding crust, hydrothermal systems.

Buoyant rise of interstitial melt.

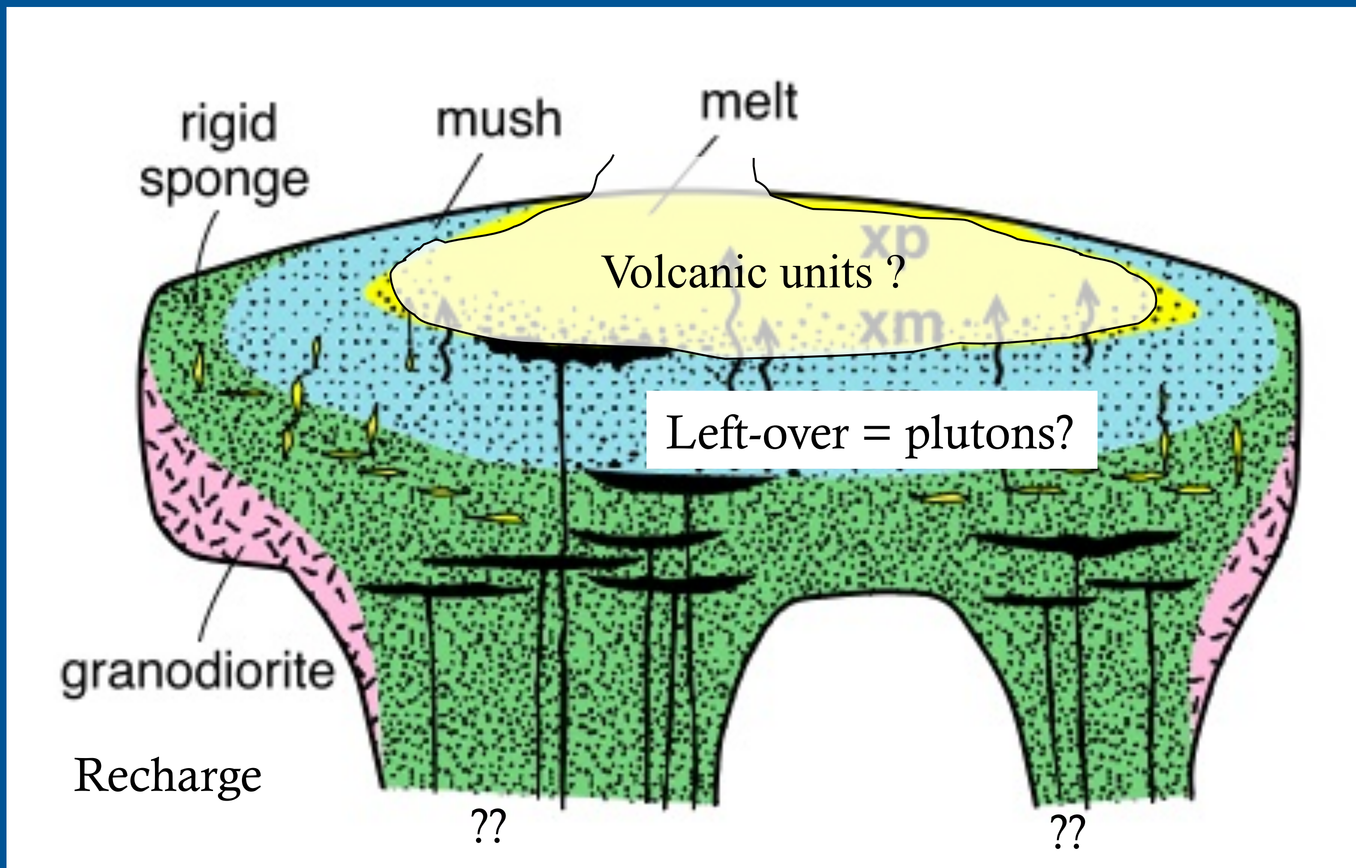
Cooling and crystallizing of residual melt.

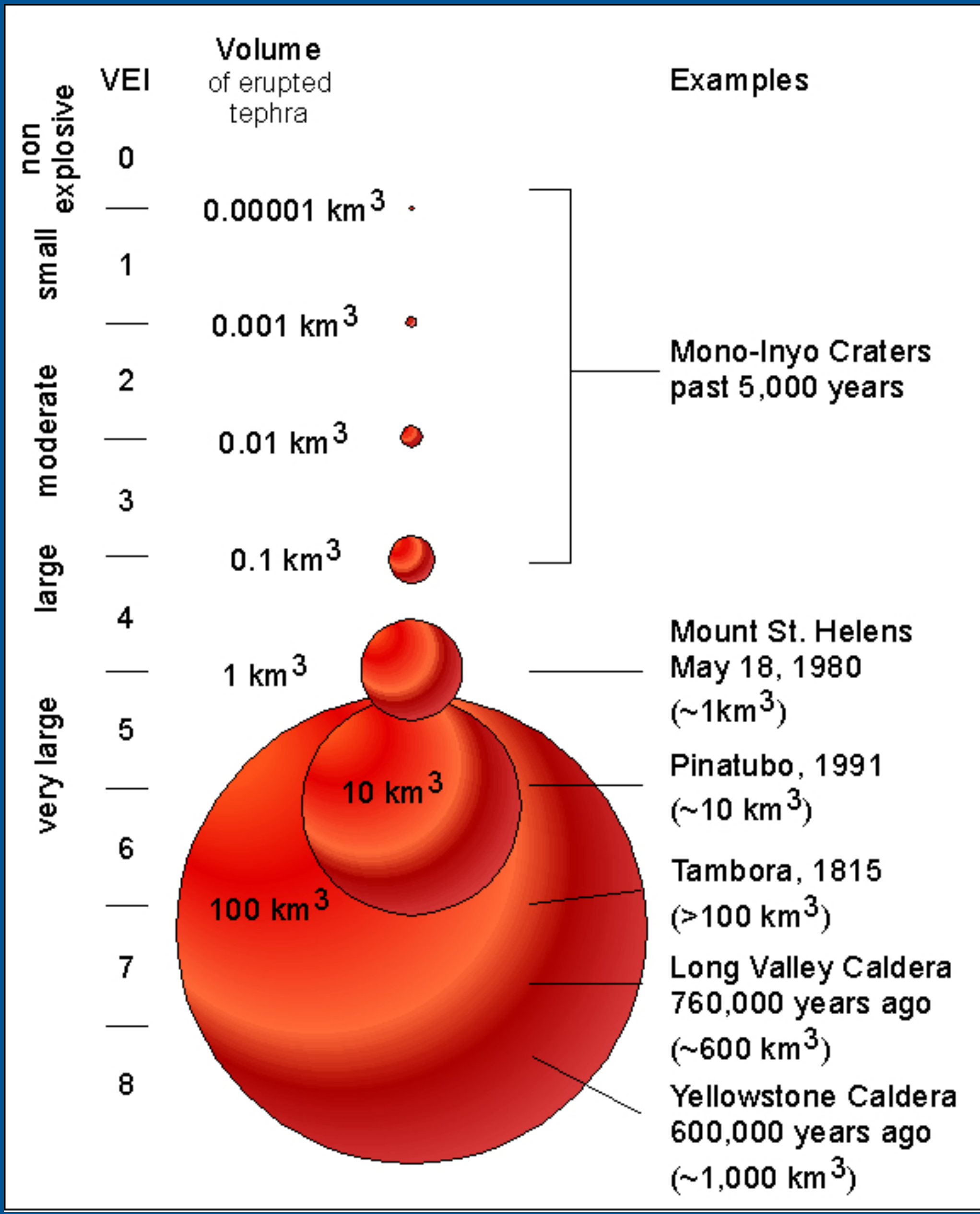
Intrusion of magmas as source of mass and enthalpy

Incremental emplacement

Hildreth, 2004

Hypothesis: A link between volcanic-plutonic realms?



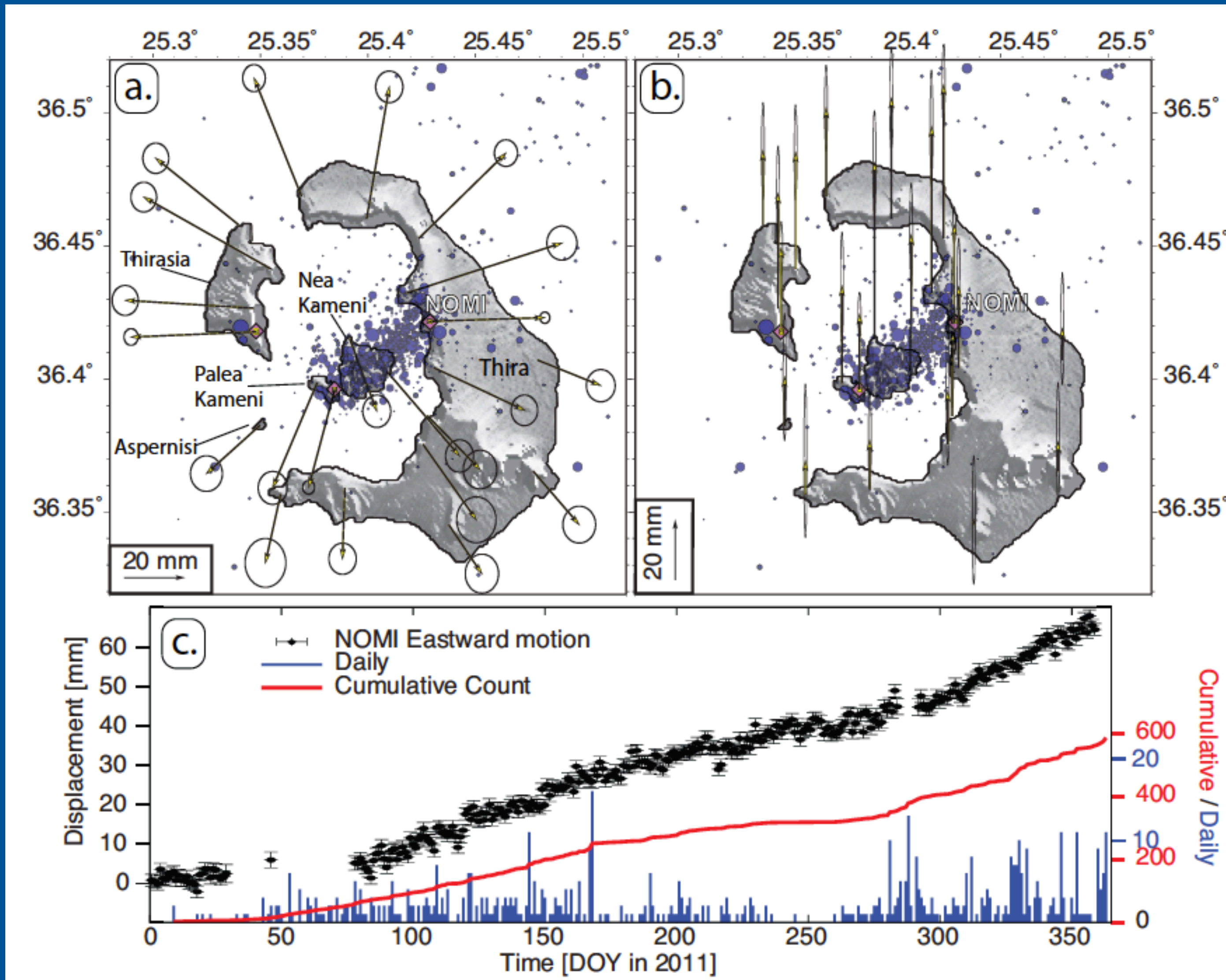

non explosive
small
moderate
large
very large
VEI
0
1
2
3
4
5
6
7
8
Volume
of erupted tephra
0.00001 km3
0.001 km3
0.01 km3
0.1 km3
1 km3
10 km3
100 km3
Examples
Mono-Inyo Craters past 5,000 years
Mount St. Helens May 18, 1980 (~1km3)
Pinatubo, 1991 (~10 km3)
Tambora, 1815 (>100 km3)
Long Valley Caldera 760,000 years ago (~600 km3)
Yellowstone Caldera 600,000 years ago (~1,000 km3)


Peach Springs Tuff, 2012

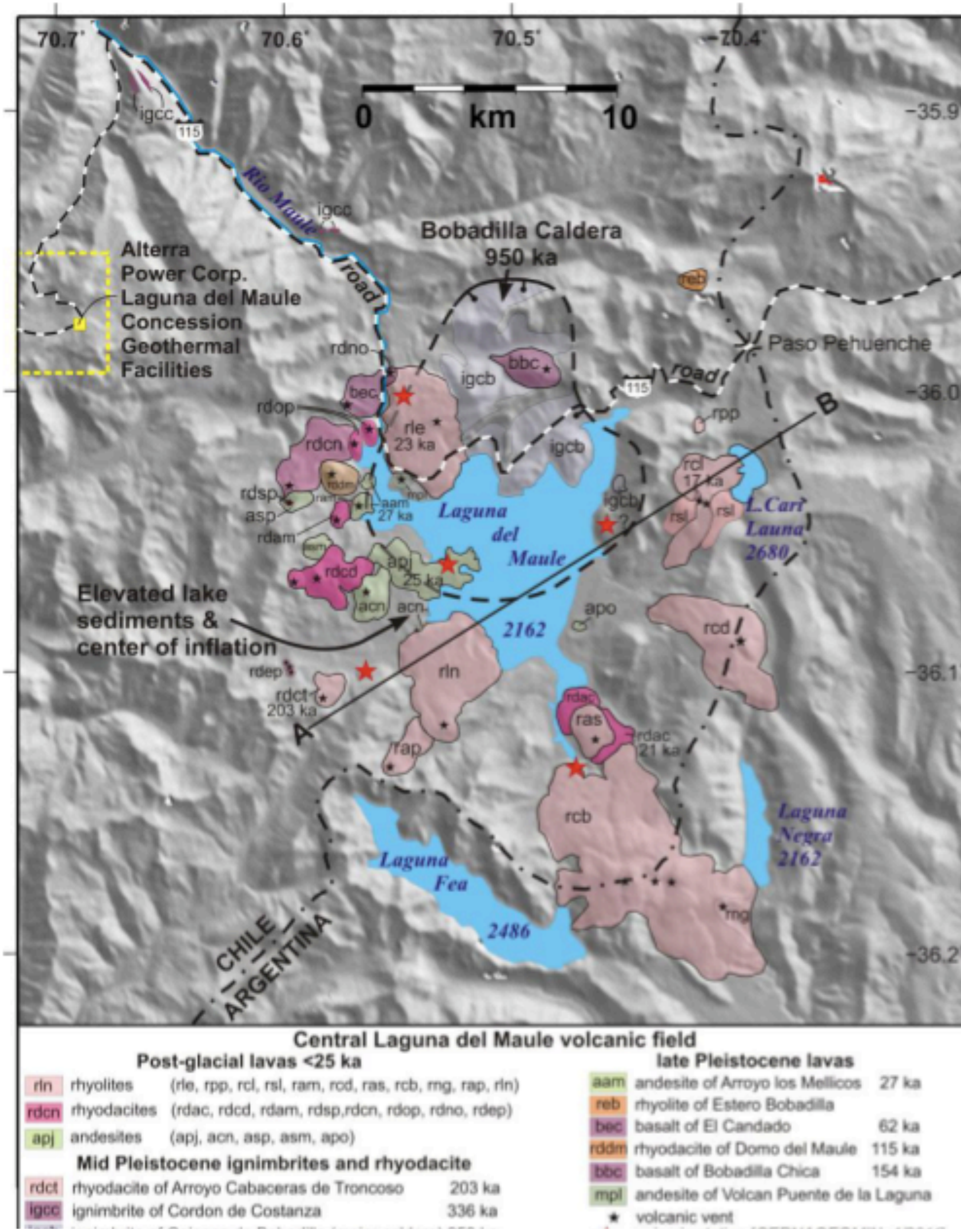

Lake Toba

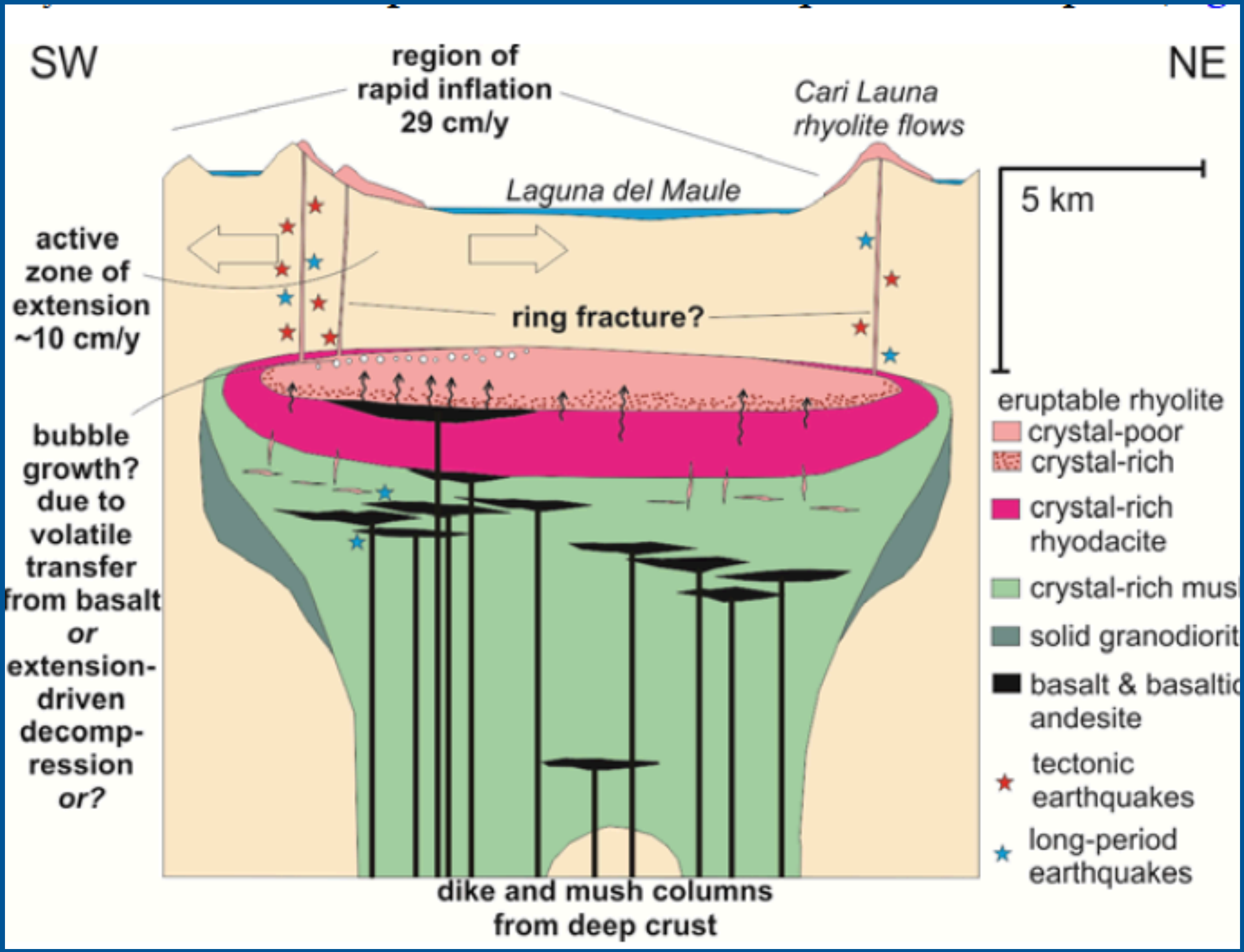
USGS

Active deformation

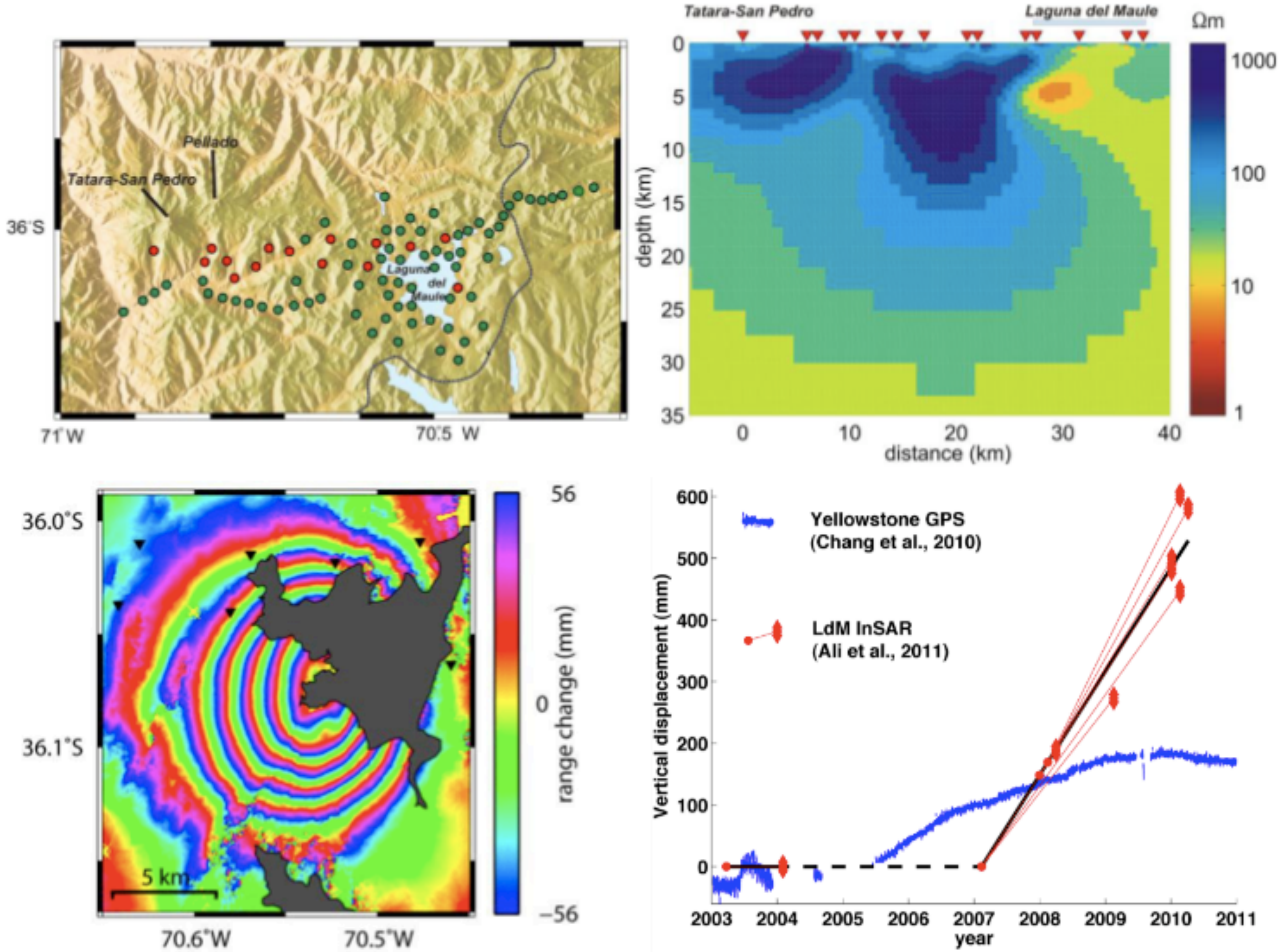


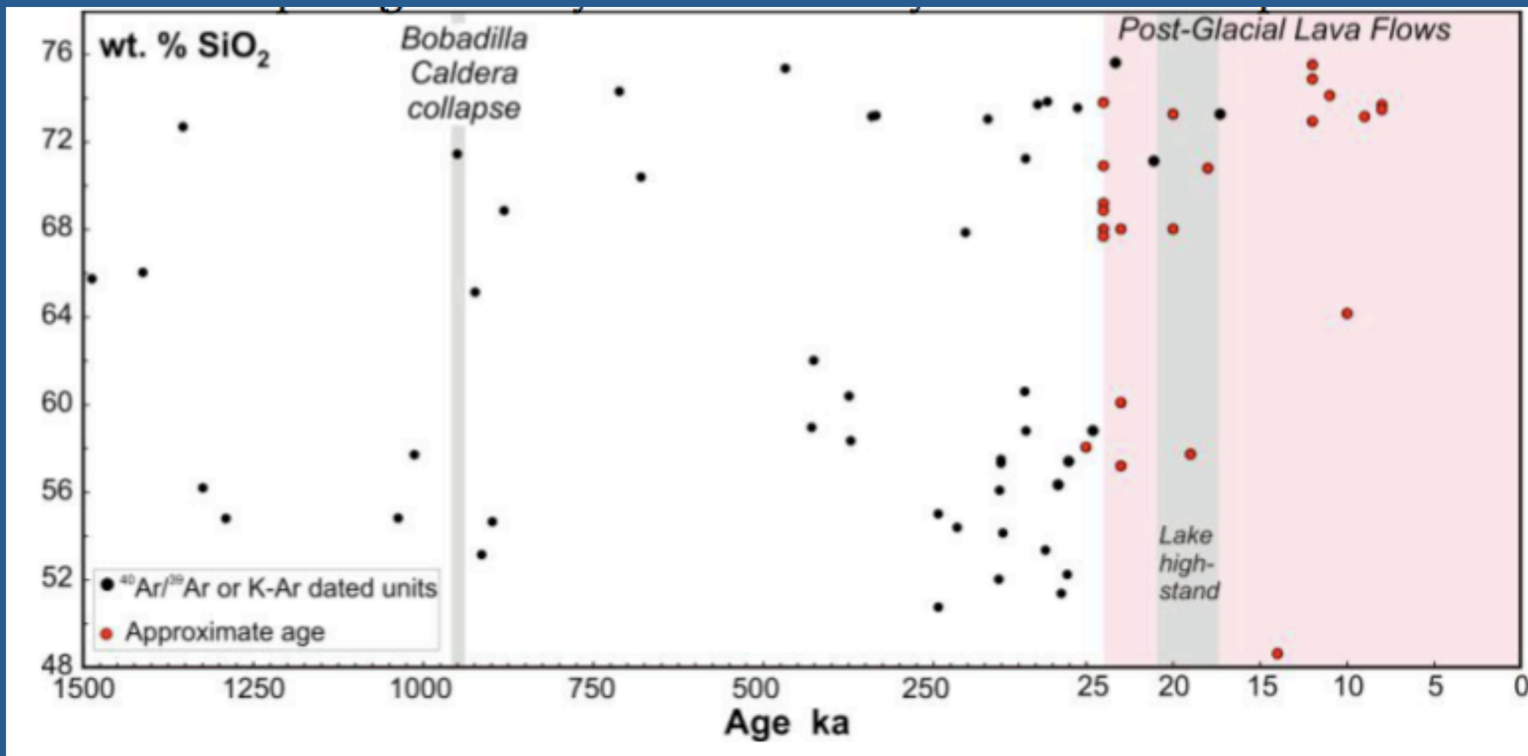
Newman et al., 2012


70.7°
70.6°
70.5°
-70.4°
-35.9°
-36.0°
-36.1°
-36.2°
0 km 10
igcc
115
Rio Maule
Bobadilla Caldera
950 ka
Alterra
Power Corp.
Laguna del Maule
Concession
Geothermal
Facilities
road
Paso Pehuenche
road
reb
rdno
bbc
igcb
bec
rdop
rle
23 ka
rpp
rdcn
rcl
17 ka
rdsp
asp
aam
27 ka
rsl
rdam
Laguna
del
Maule
L.Cari
Launa
2680
rdcd
apj
25 ka
Elevated lake
sediments &
center of inflation
acn
apo
2162
rcd
rdep
rln
rdct
203 ka
rdac
ras
rdac
21 ka
A
B
rap
rcb
Laguna
Negra
2162
Laguna
Fea
2486
rng
CHILE
ARGENTINA
Central Laguna del Maule volcanic field
Post-glacial lavas <25 ka
rln rhyolites (rle, rpp, rcl, rsl, ram, rcd, ras, rcb, rng, rap, rln)
rdcn rhyodacites (rdac, rdcd, rdam, rdsp,rdcn, rdop, rdno, rdep)
apj andesites (apj, acn, asp, asm, apo)
Mid Pleistocene ignimbrites and rhyodacite
rdct rhyodacite of Arroyo Cabaceras de Troncoso 203 ka
igcc ignimbrite of Cordon de Costanza 336 ka
late Pleistocene lavas
aam andesite of Arroyo los Mellicos 27 ka
reb rhyolite of Estero Bobadilla
bec basalt of El Candado 62 ka
rddm rhyodacite of Domo del Maule 115 ka
bbc basalt of Bobadilla Chica 154 ka
mpl andesite of Volcan Puente de la Laguna
volcanic vent



Laguna del Maule System Model


Pellado
Tatara-San Pedro
Laguna del Maule
36°S
71°W
70.5°W
Tatara-San Pedro
Laguna del Maule
Ωm
1000
100
10
1
depth (km)
0
5
10
15
20
25
30
35
distance (km)
0
10
20
30
40
36.0°S
36.1°S
70.6°W
70.5°W
5 km
56
0
−56
range change (mm)
Yellowstone GPS
(Chang et al., 2010)
LdM InSAR
(Ali et al., 2011)
Vertical displacement (mm)
600
500
400
300
200
100
0
2003
2004
2005
2006
2007
2008
2009
2010
2011
year



Singer et al. 2013

Some Thermal Considerations

$$\frac{\partial H_T}{\partial t} + \frac{\partial}{\partial \mathbf{x}_i}\left(\mathbf{u}_i H_T\right) = \frac{\partial}{\partial \mathbf{x}_i} k \left(\frac{\partial}{\partial \mathbf{x}_i} T \right)$$

$$H_T = \overbrace{\rho \int_{\mathrm{Tref}}^{\mathrm{T}} c_p dT}^{\text{Sensible Heat}} + \overbrace{\rho \ fL}^{\text{Latent Heat}}$$

$$\rho^* \left[\frac{\partial \mathrm{T}^*}{\partial t^*} + \mathbf{u}_i^* \frac{\partial \mathrm{T}^*}{\partial \mathrm{x}_i^*} \right] = \left[\frac{1}{\mathrm{Pe}} \right] \frac{\partial^2 \mathrm{T}^*}{\partial \mathrm{x}_i^{2*}} - \left[\frac{1}{\mathrm{Ste}} \right] R^*$$

$$R^* = \rho * \frac{\partial f}{\partial t *}$$

Dimensionless rate of production of melt

$$\mathrm{Pe} = \left[\frac{c_p \rho_0 \delta}{k} \right] \mathbf{u}_0$$

Peclet: Advective/diffusive heat transport

$$\mathrm{Ste} = \frac{c_p T_0}{L}$$

Stefan: Sensible/latent heat contribution

Numerous modeling approaches applied to the magmatic problem from 1-D conduction to 3D multiphase dynamic simulations

Some Thermal Considerations

$$\frac{\partial H_T}{\partial t} + \frac{\partial}{\partial \mathbf{x}_i}\left(\mathbf{u}_i H_T\right) = \frac{\partial}{\partial \mathbf{x}_i} k \left(\frac{\partial}{\partial \mathbf{x}_i} T \right)$$

$$H_T = \overbrace{\rho \int_{\mathrm{Tref}}^{\mathrm{T}} c_p \, dT}^{\text{Sensible Heat}} + \overbrace{\rho \; fL}^{\text{Latent Heat}}$$

Related to temperature change

Related to phase change

Thermal calculations can be quite sensitive to the variation in the physical parameters

$$\rho^* \left[\frac{\partial \mathrm{T}^*}{\partial \mathrm{t}^*} + \mathbf{u}_i^* \frac{\partial \mathrm{T}^*}{\partial \mathrm{x}_i^*} \right] = \left[\frac{1}{\mathrm{Pe}} \right] \frac{\partial^2 \mathrm{T}^*}{\partial \mathrm{x}_i^{2*}} - \left[\frac{1}{\mathrm{Ste}} \right] R^*$$

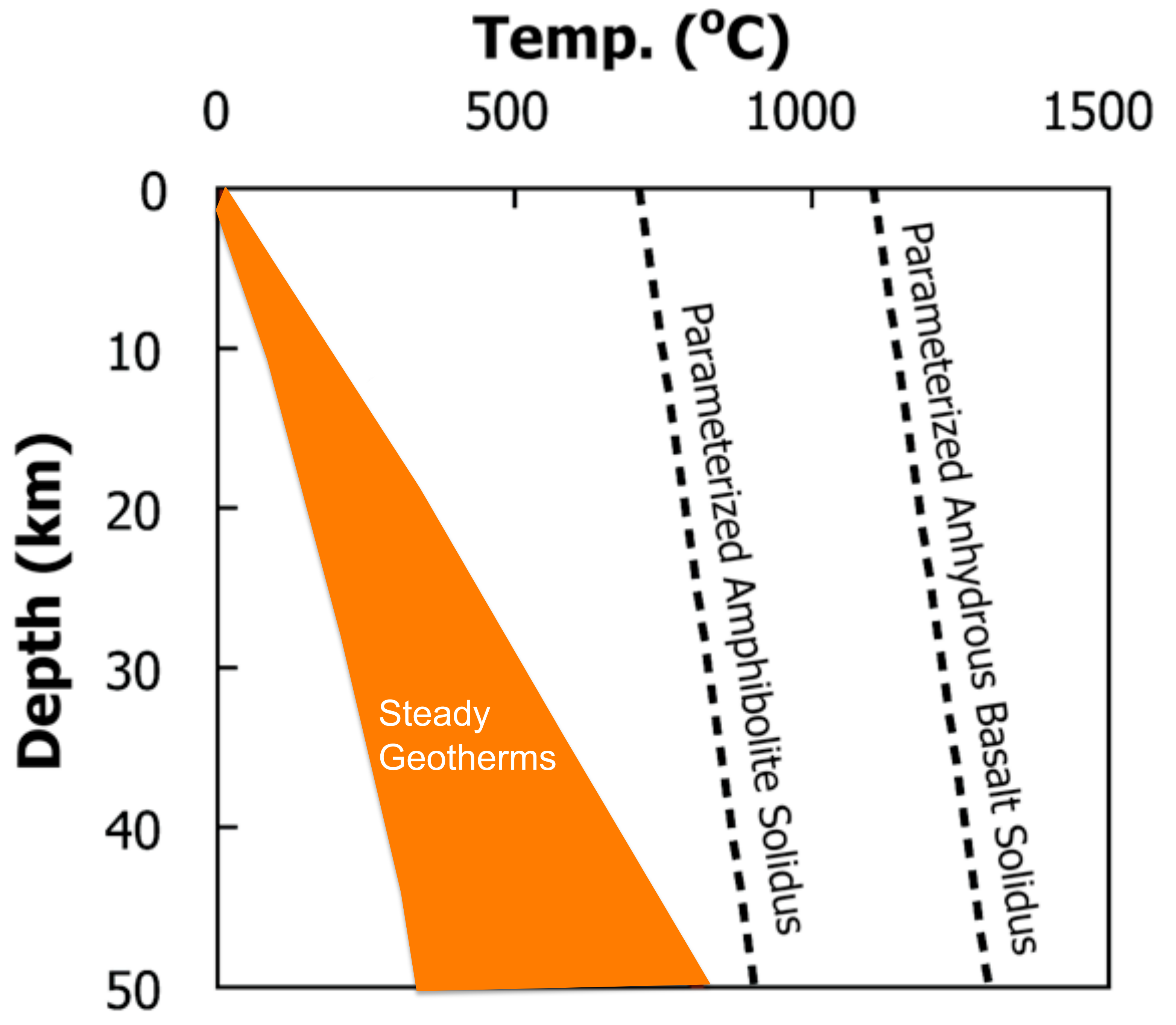
$$\mathrm{Pe} = \left[\frac{c_p \rho_0 \delta}{k} \right] \mathbf{u}_0$$

Advective/diffusive heat transport

$$\mathrm{Ste} = \frac{c_p T_0}{L}$$

Sensible/latent heat contribution

Numerous modeling approaches applied to the magmatic problem from 1-D conduction to 3D multiphase dynamic simulations


Temp. (°C)
0
500
1000
1500
Depth (km)
0
10
20
30
40
50
Steady
Geotherms
Parameterized Amphibolite Solidus
Parameterized Anhydrous Basalt Solidus

Summary of some thermal model results

Study	Model type[1]	Intr. style[2]	Total intr.[3] (km)	T_{init} (°C)	Rock type[4]	T_{melt}–T_{solid} (°C)	*E* (%)
Younker and Vogel, 1976	1-D, cond., no bottom heat loss	single intrusion	2.0	500	basalt, biotite-granite	L:1200, S:1100 L:1100, S:800	32
Wells, 1980	1-D, cond., over-accretion	multiple intrusion	40.0	200	tonalite	L:1050, S:800	8
Huppert and Sparks, 1988	1-D, param. convection, no bottom heat loss	single intrusion	0.5	500	basalt, grano-diorite	L:1200, S:1091 L:1000, S:850	44
Bergantz, 1989	1-D, cond., no bottom heat loss	single intrusion	16.6	700	basalt, pelite	L:1250, S: 980 L:1200, S: 725	38
Bittner and Schmeling, 1995	2-D, convection	single intrusion	5.0	756	basalt, granite	L:1100, S: 950 L:1050, S:760	NA
Barboza and Bergantz, 1996	2-D, convection	fixed *T* bottom boundary	NA	600	pelite	L:1200, S:750	NA
Raia and Spera, 1997	2-D, convection	fixed *T* bottom boundary	NA	1195	($CaAl_2Si_2O_8$-$CaMgSi_2O_6$)	L:1547, S:1277	NA
Pedersen *et al.,* 1988	1-D, cond., over-accretion	multiple intrusion	10.0	650	basalt, grano-diorite	L:1250, S:1100 L:1000, S:710	5
Petford and Gallagher, 2001	1-D, cond., over-accretion	multiple intrusion	1.0	650	basalt. amphibolite	L:1250, S:1050 L:1075, S:1010	4
Annen and Sparks, 2002	1-D, cond, over-accretion	multiple intrusion	8.0	variable, based on depth (600)	basalt, amphibolite	L:1300, S:620 L:1075, S:1010	8
Dufek and Bergantz, 2005a	2-D, cond. and convection, stochastic	multiple intrusion	variable (5.0)	variable, based on depth (640)	basalt, amphibolite	pressure dependent L:1240, S:640 L: 1100, S:850	0.5-10.4 (7.1)

Efficiency of Generating Crustal Melt

An enthalpy balance can give the maximum amount of melting:

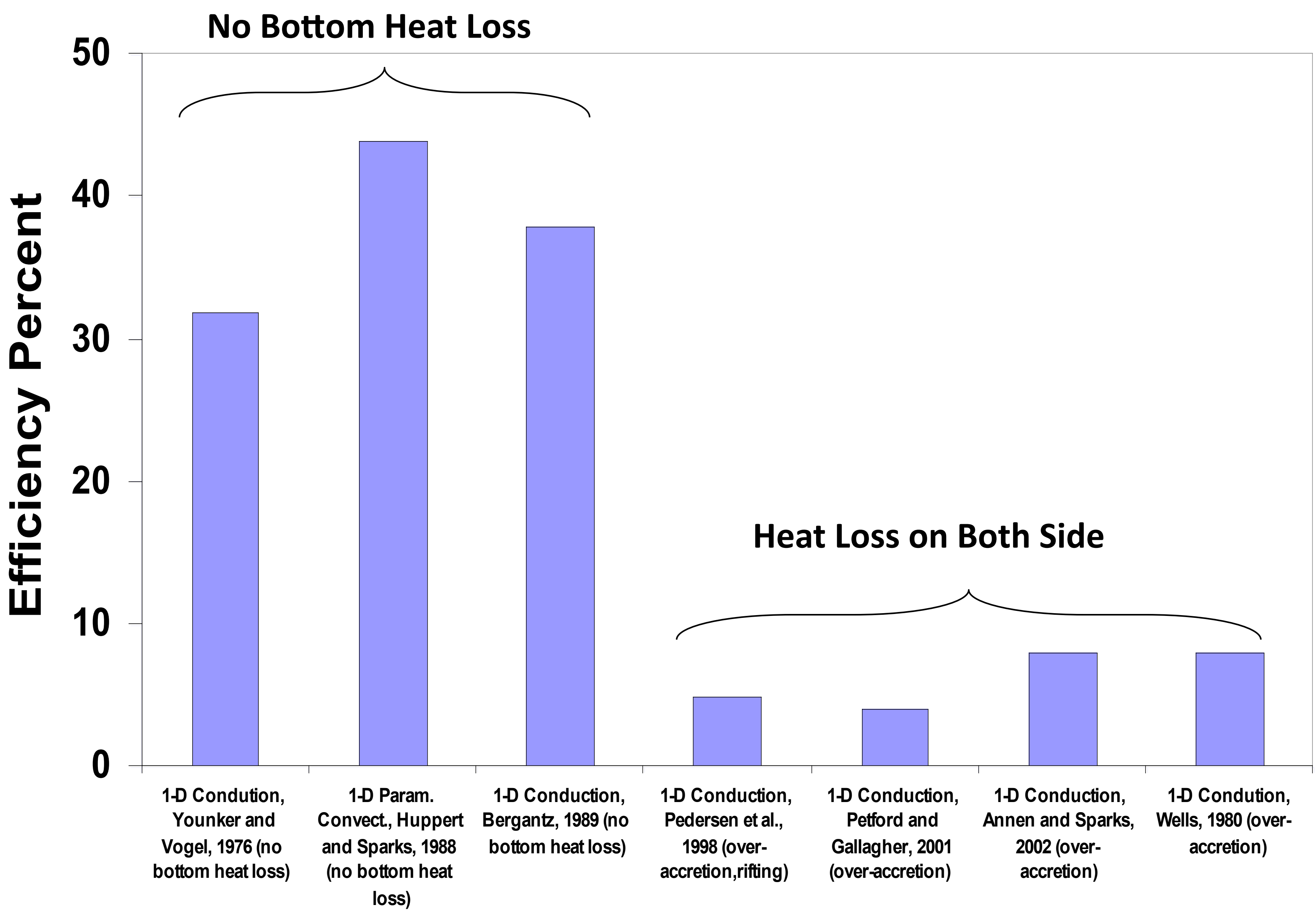
$$V_{crust}^{eff} = \frac{\rho^b V^b \left[c_p^b (T_L^b - T) + L^b \right]}{\rho^c c_p^c (T_s^c - T_R) + \rho^c \left[c_p^c (T - T_s^c) + L^c f \right]}$$

This assumes all energy from an intrusion (b) is extracted and applied to only that area of the crust (c) that melts.

$$E^{\%} = 100 \times V_{crust}^{\mathrm{mod.}} / V_{crust}^{eff.}$$

Models typically show less melting than this end-member.

Summary of 1-D Conduction/Melting Simulations

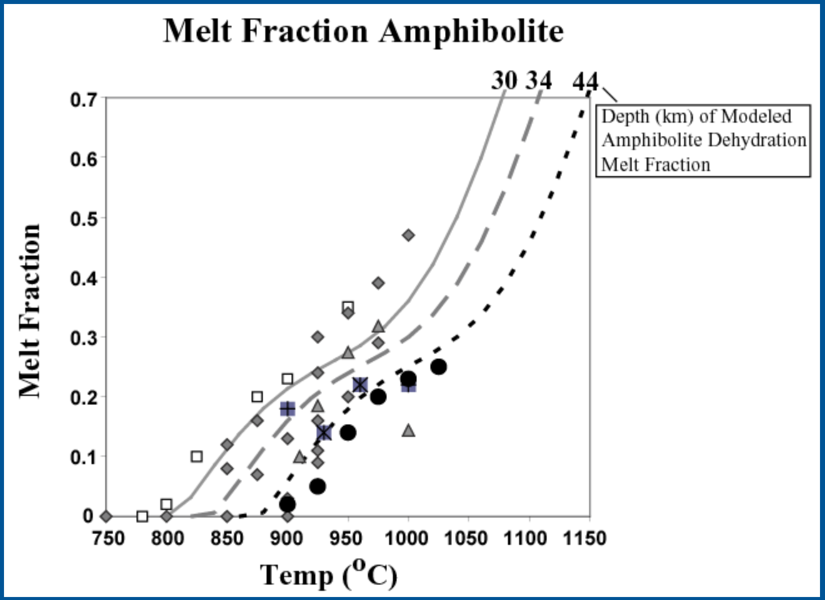


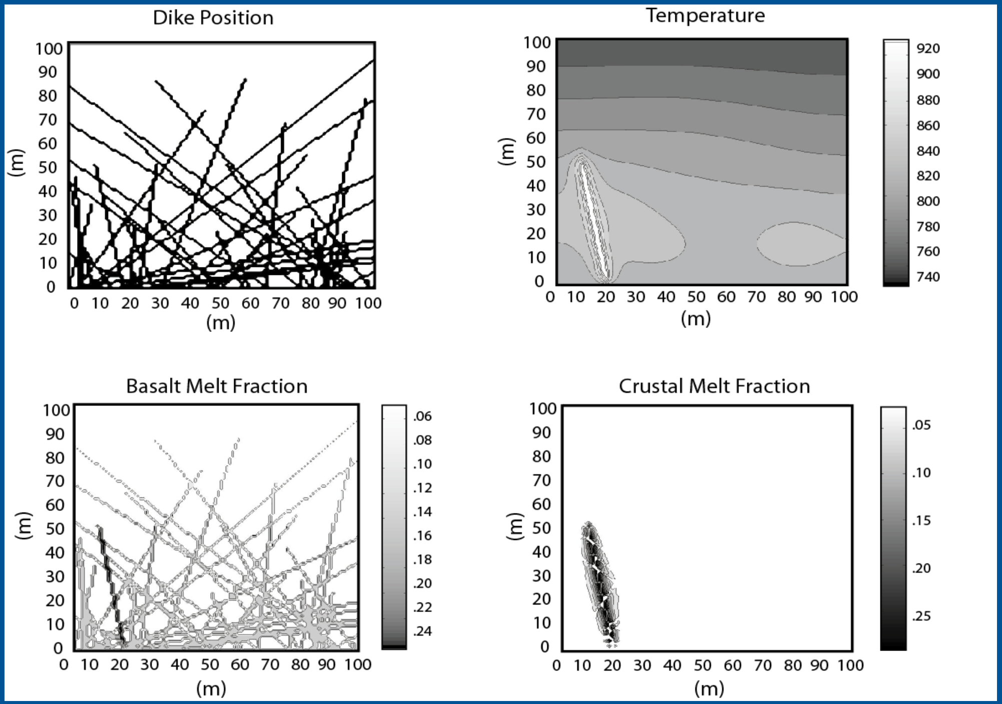
Stochastic simulations of magma intrusion into the lower crust:

- Survey of basalt flux and crustal thickness variations.

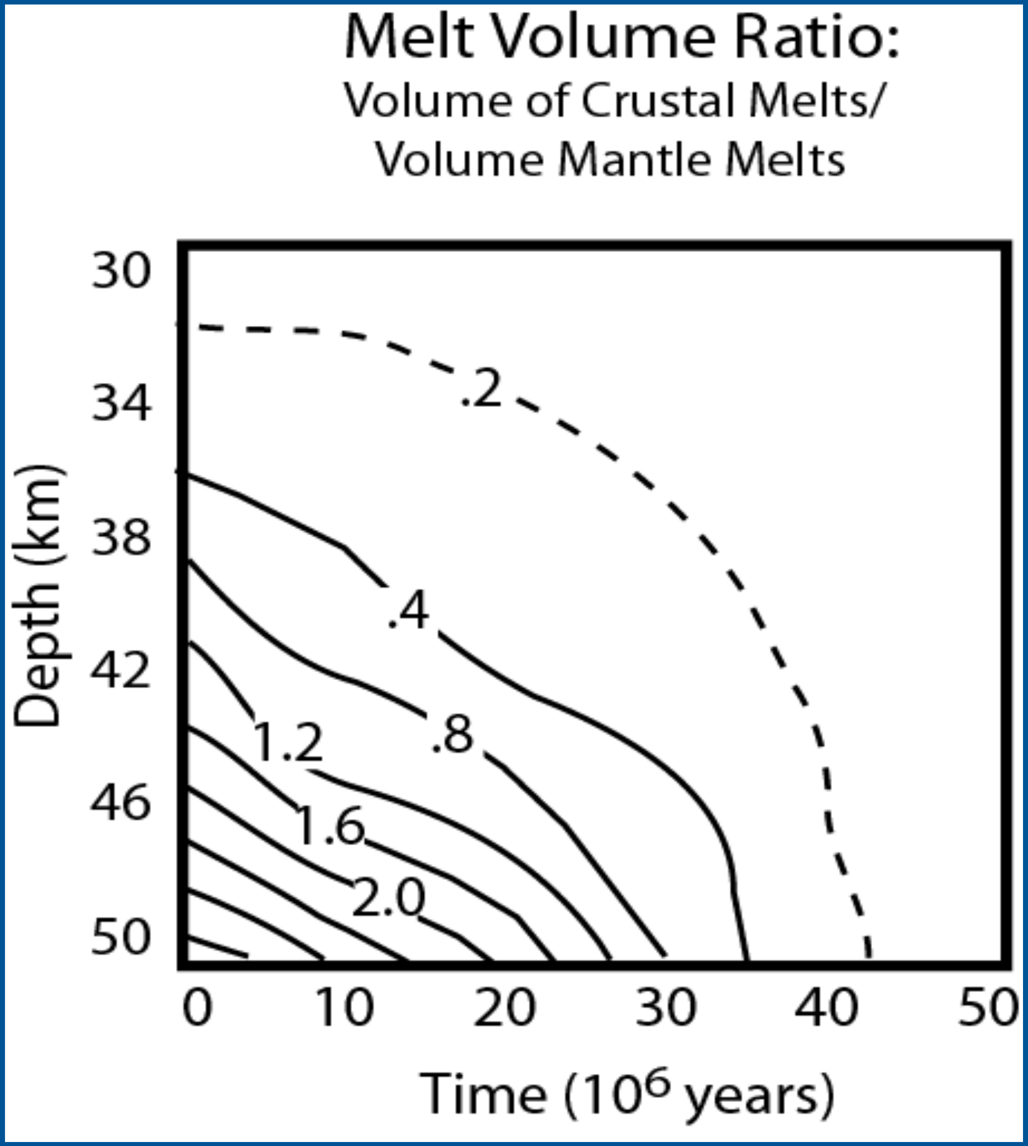
- Compilation of numerous realizations to examine probability of production.

- Long term melt productivity and dynamic response.





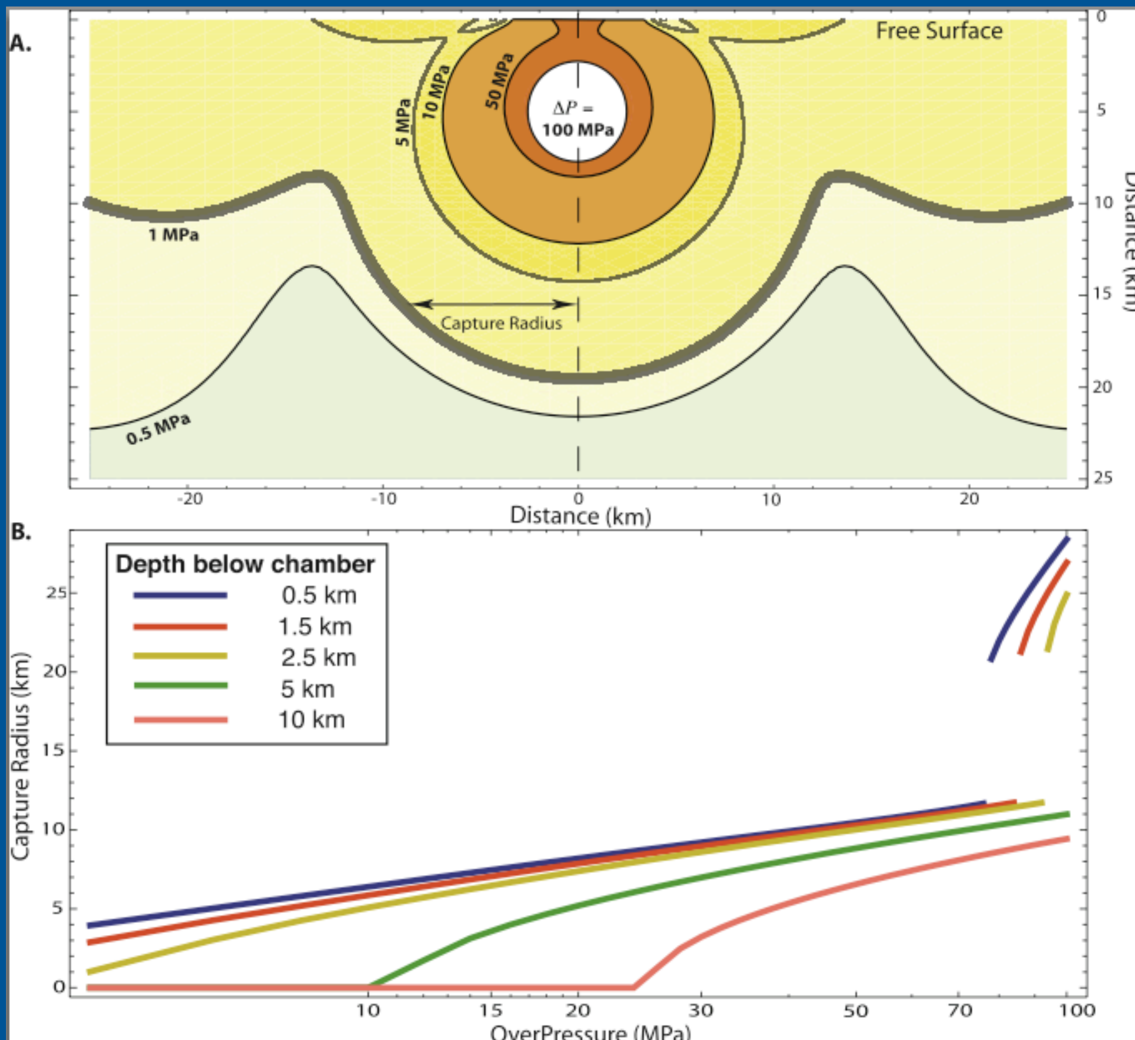
Dufek and Bergantz,J. Pet. 2005



Thin crust (on average) leads to less overall melt, although thin crust is more prone to variability.

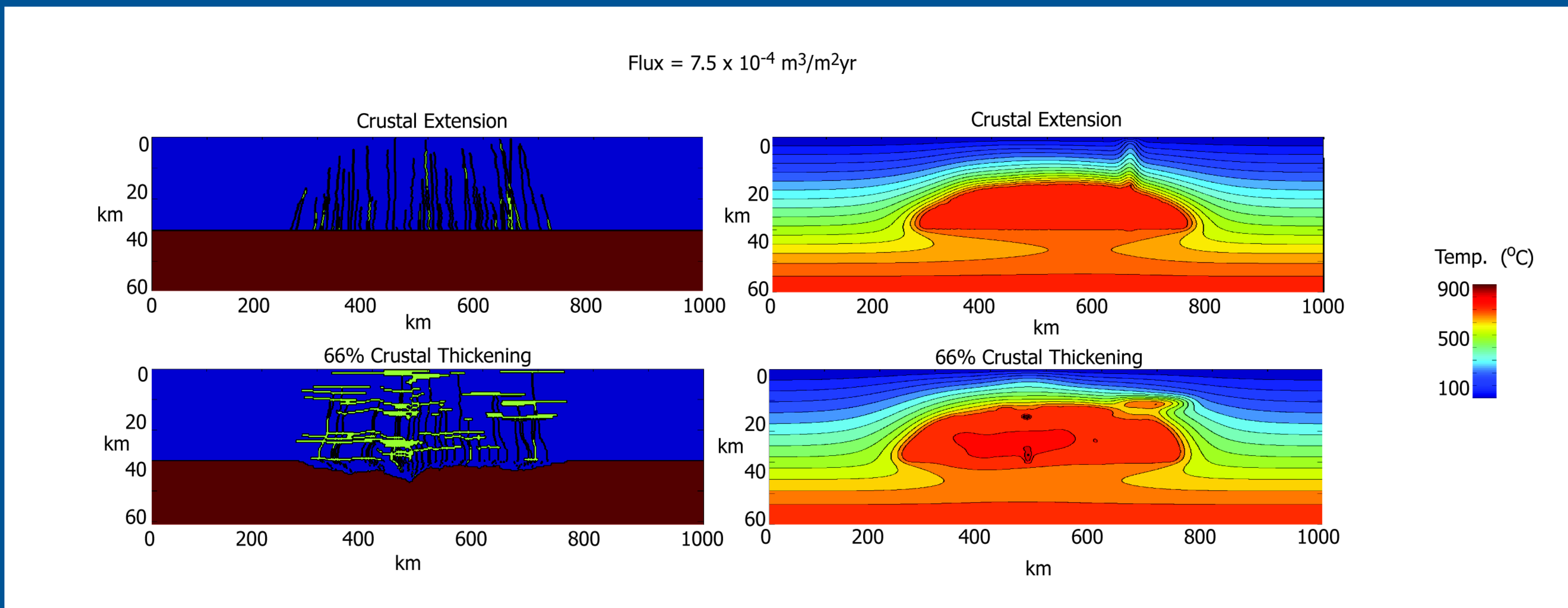
Note also that this is overall melt, and what we can sample at surface can be considerably limited.

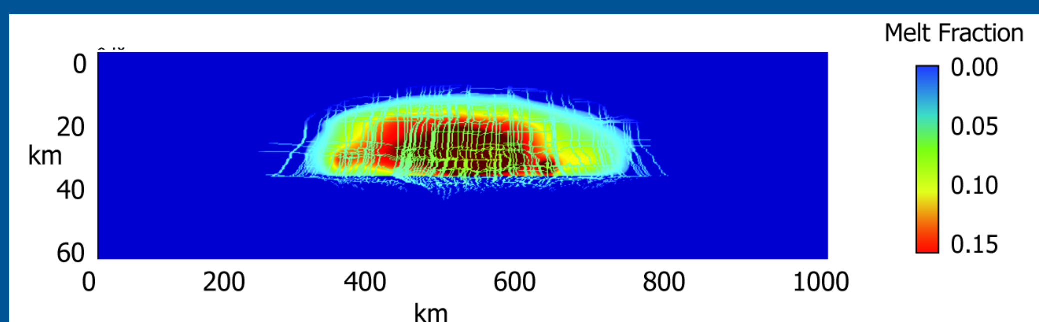
Structural heterogeneity and stress feedback can also lead to concentration of magma.



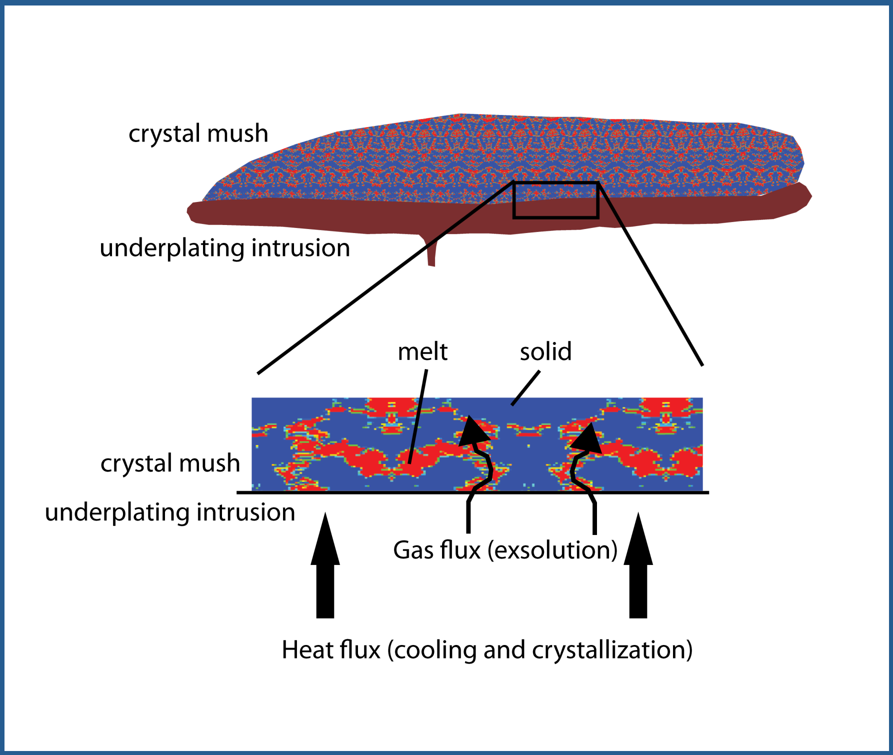
Karlstrom, Dufek and Manga 2009

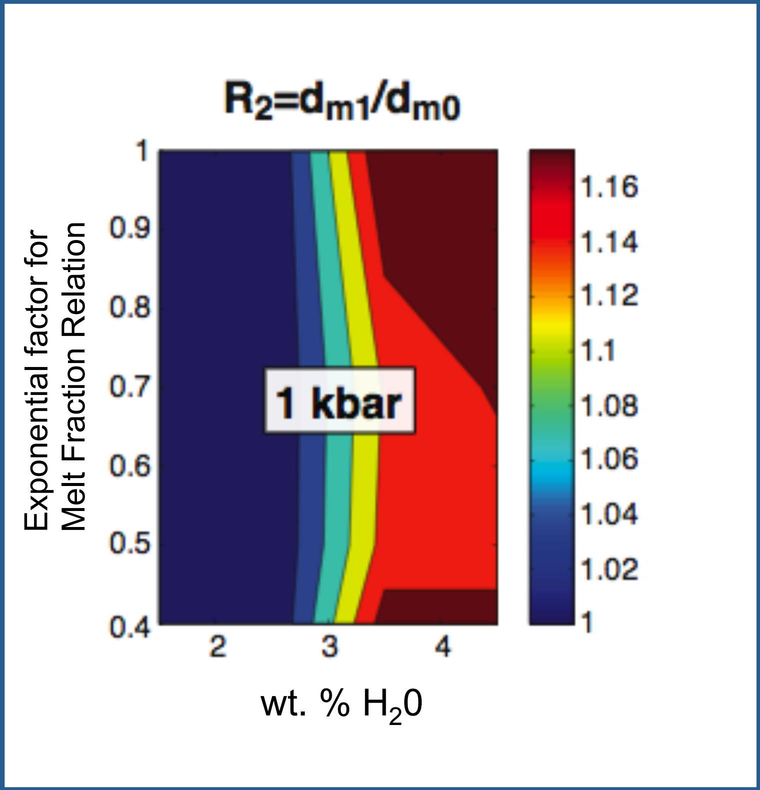
How does the method of accommodation influence crustal evolution?





What about volatiles?

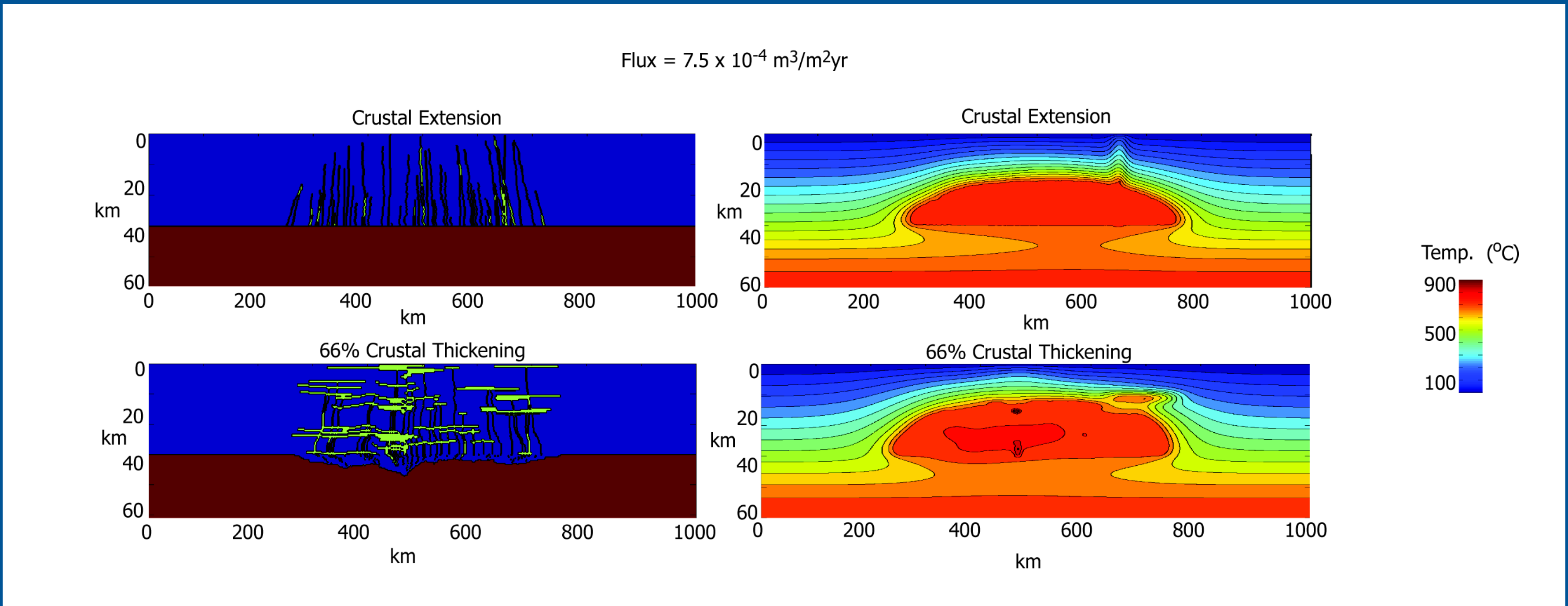




Huber et al., 2009

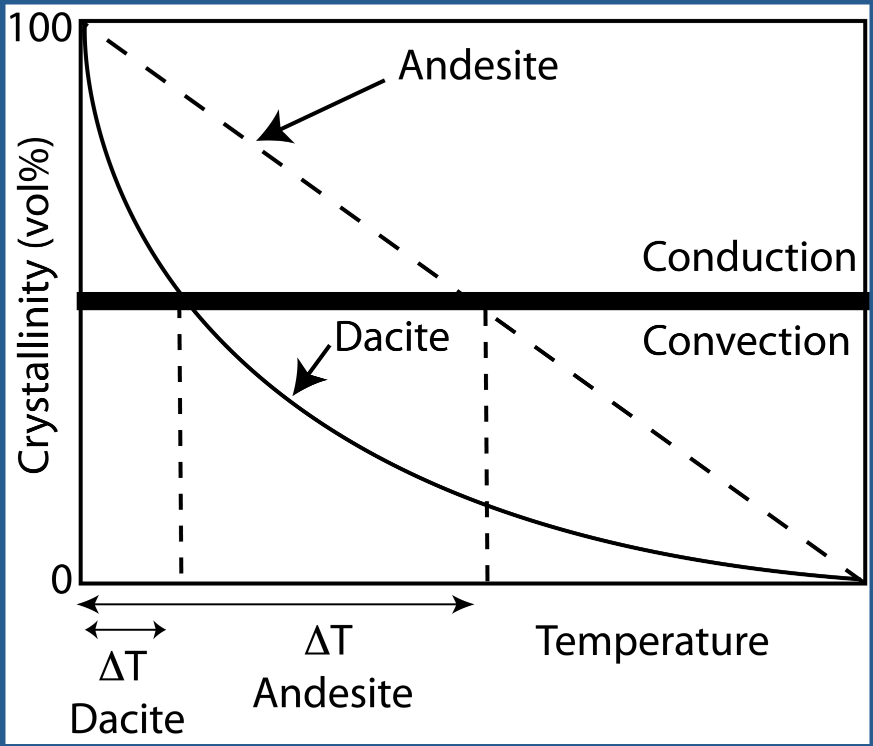
Volatiles can increase melting by a factor of ~10% beyond the dry case.

Even with various considerations (i.e. variable flux, thickened crust, focused magmas, volatile rich, etc) melting is, averaged over the entire crustal column, a relatively inefficient process - a good rule of thumb is ~10% efficient *(Dufek and Bergantz, 2005; Karlstrom, Dufek and Manga, 2009)*



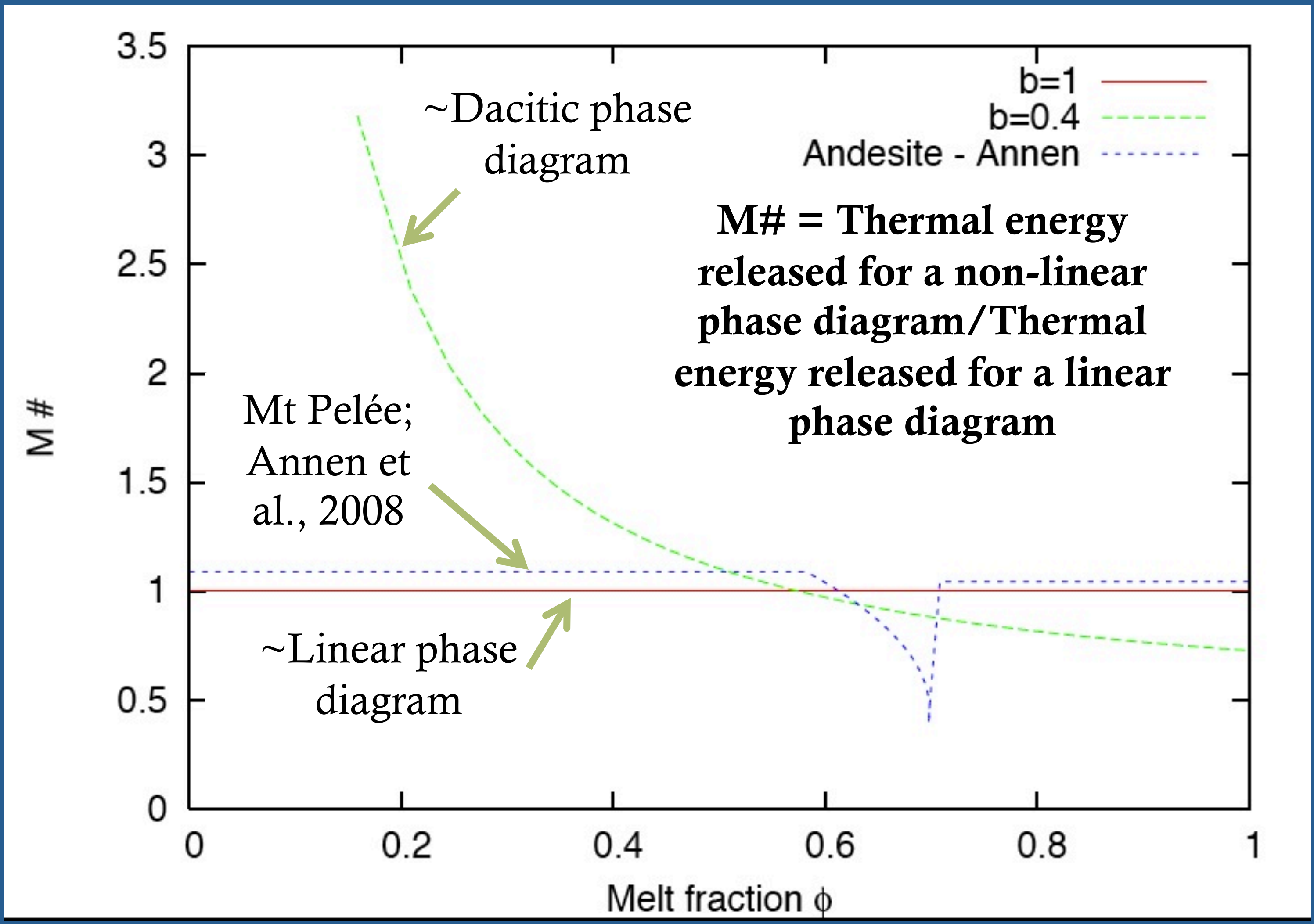
Magma reservoirs spend most of the active time at high crystallinities

- Reduced thermal gradients at higher crystallinity
- Latent heat effect
  - Dacitic upper crust: **high latent/sensible heat ratio near eutectic**

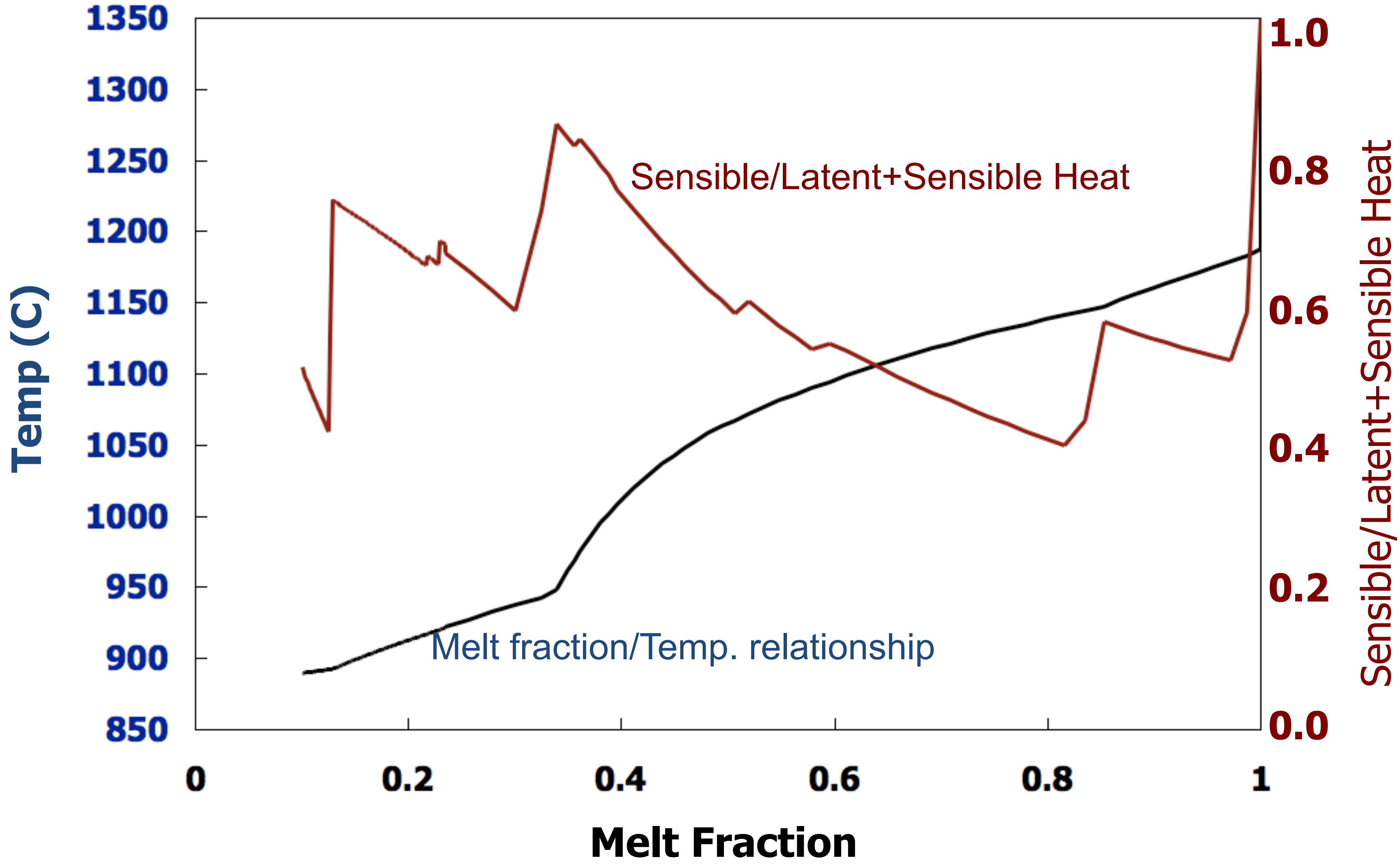


Simplified crystallinity relationship

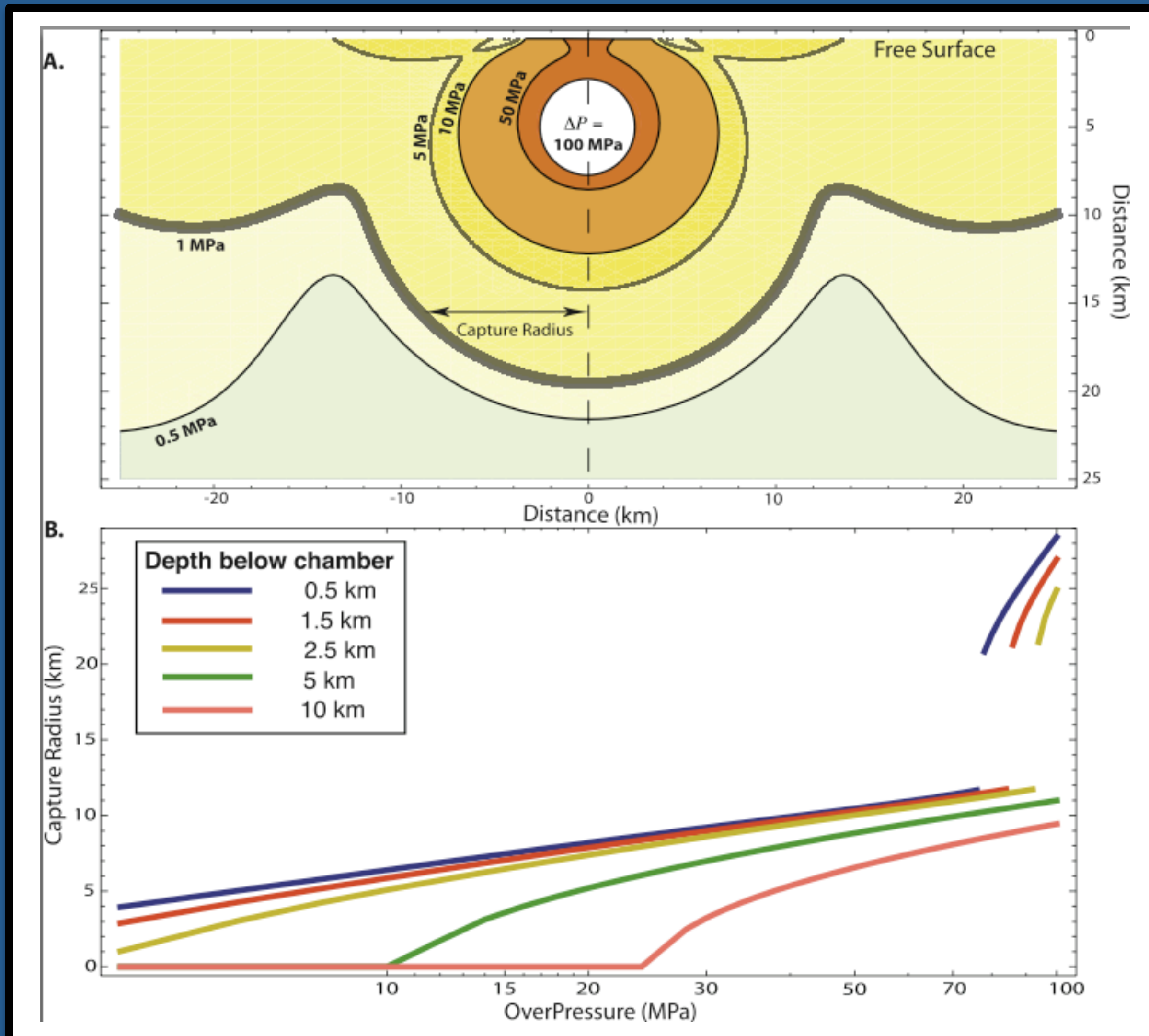
Latent heat effect in silicic magmas



Partitioning of latent and sensible heating can have complex relationships with non-trivial results for the cooling history.

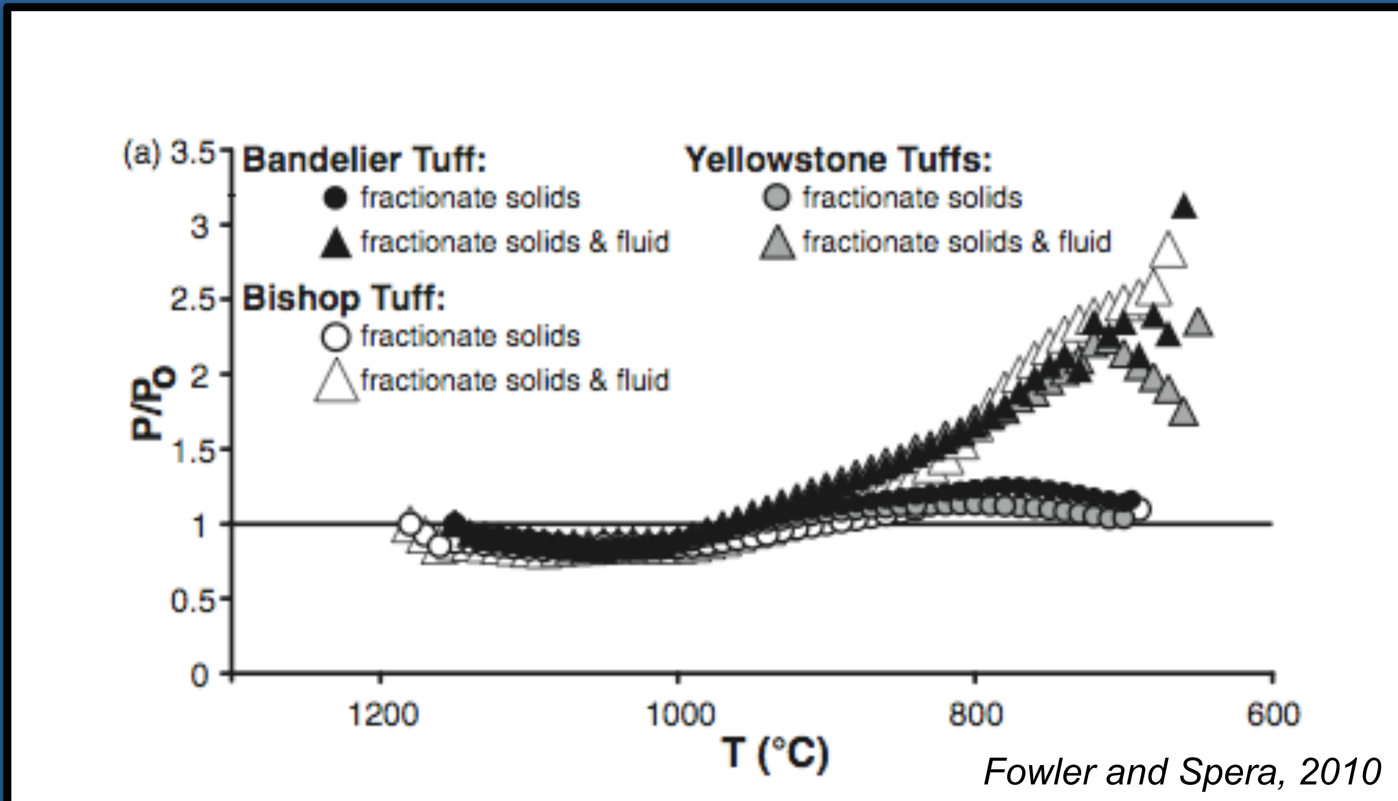


Magma overpressure can be generated by melting and intrusion, which can influence crustal stress fields.



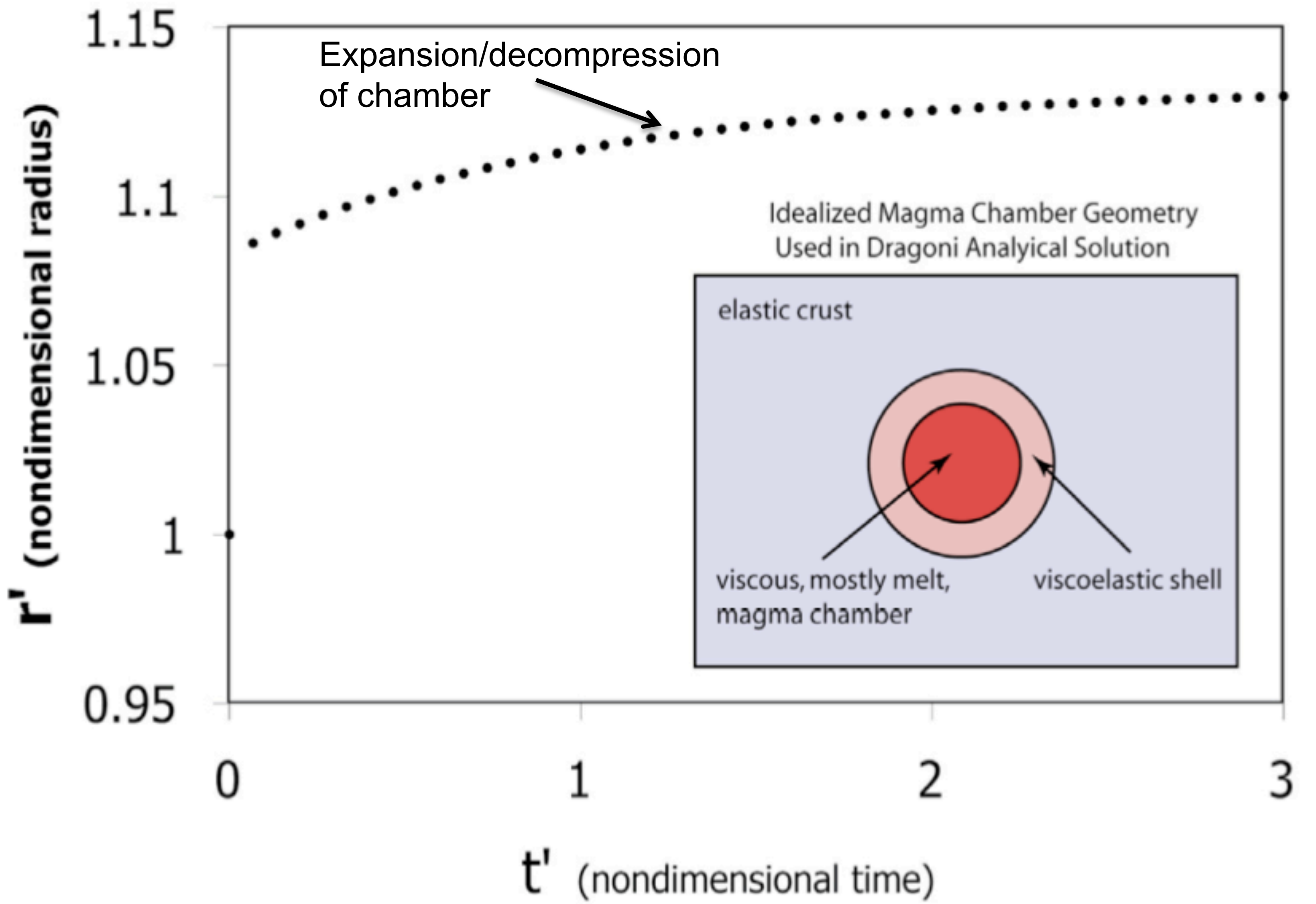
Karlstrom, Dufek and Manga 2009

Overpressure can also influence phase equilibria. Below are examples of isochoric (constant volume) calculations performed by Fowler and Spera (2010).

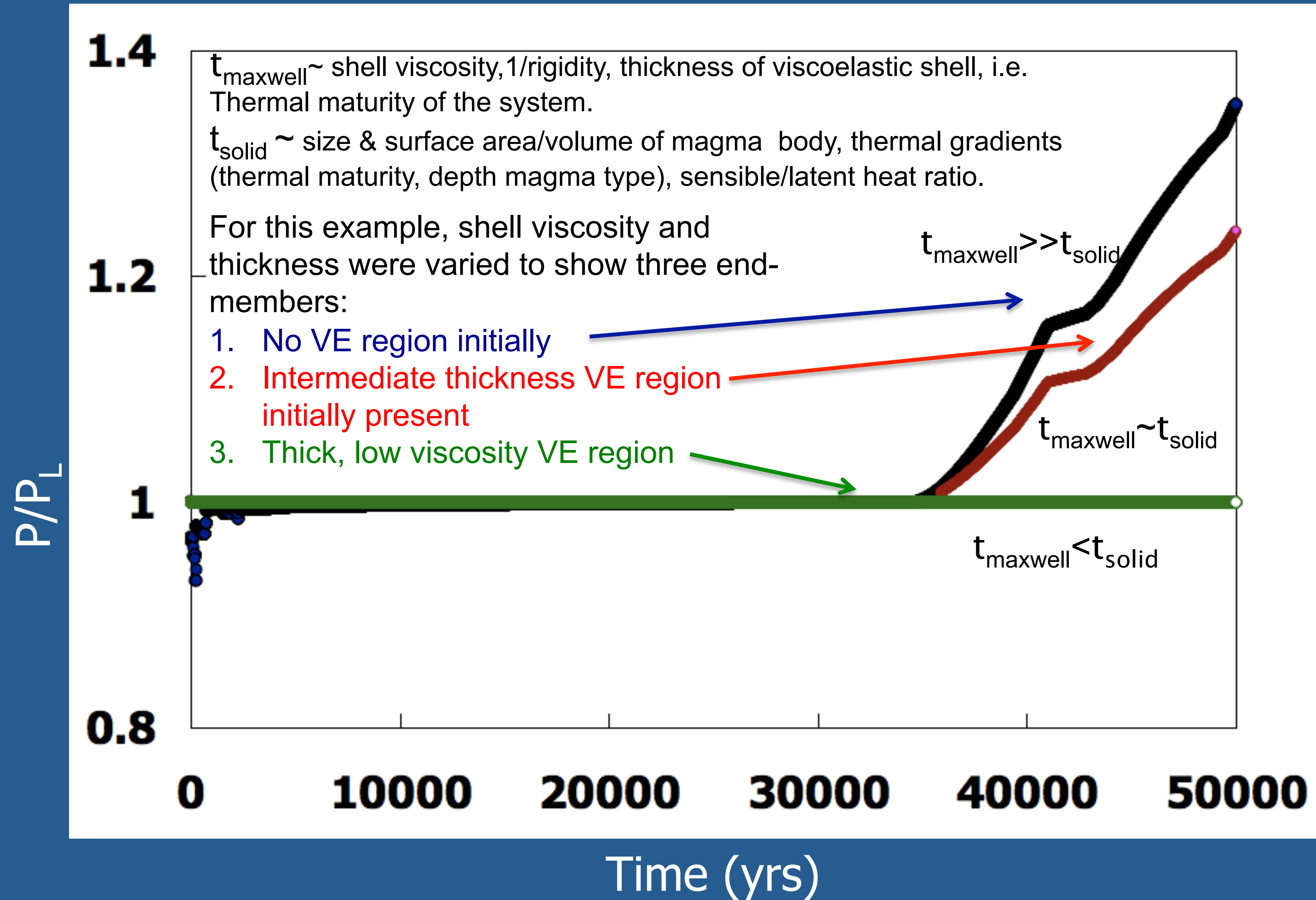


We can think of these calculations as end-members assuming a perfectly rigid crustal container, and identical P-T conditions throughout the chamber.

Crustal container is not, in general, completely rigid and can have elastic and viscoelastic response.



As an example, consider the pressure evolution of a 1 km diameter dacitic magma chamber with 5 wt% water.

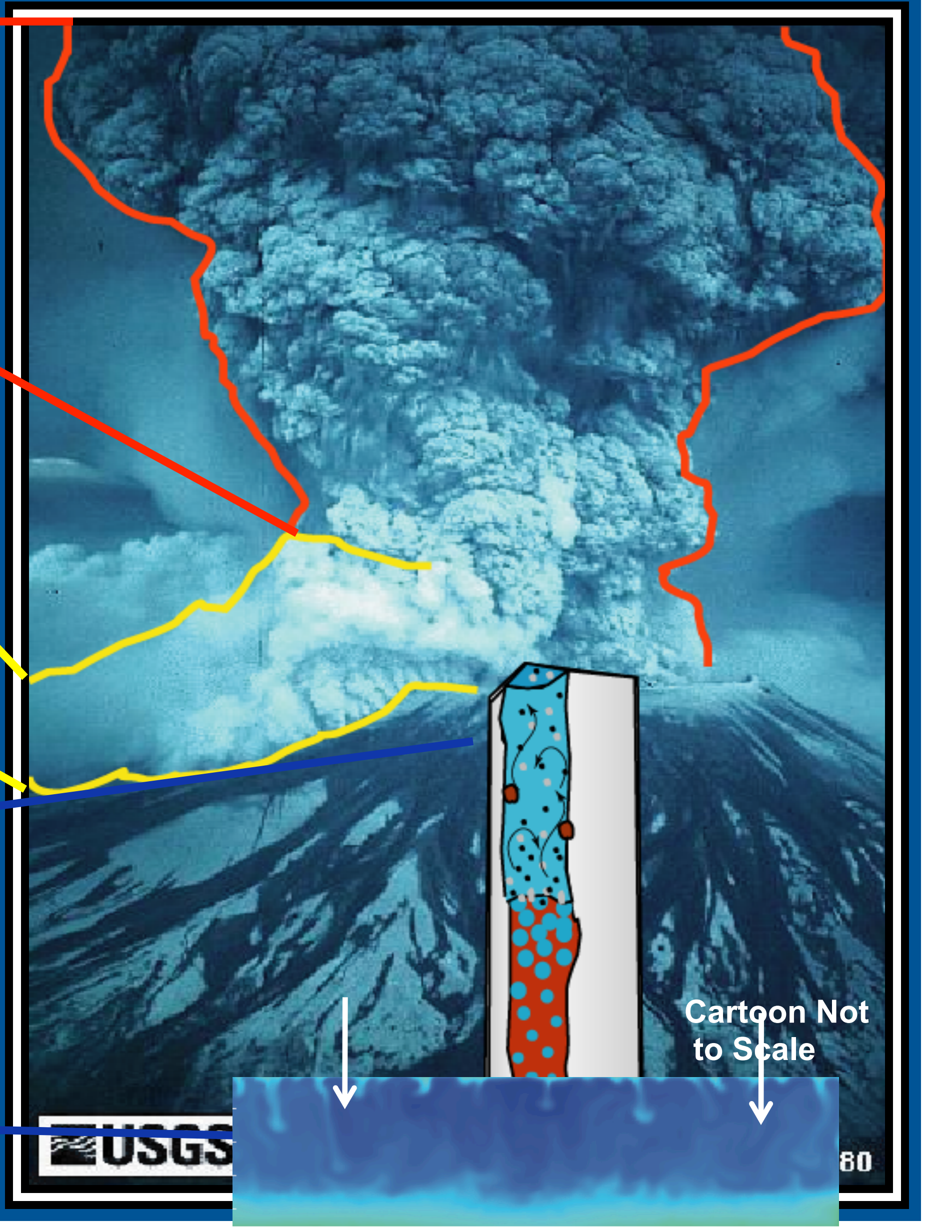


Multiphase Flow in Explosive Volcanic Eruptions

Plinian Column:
- Buoyant plume
- Particle+Gas Flow

Pyroclastic Flow:
- Particulate gravity current
- Particle+Gas Flow

Magma Chambers:
- Bubbles and crystal discrete phases

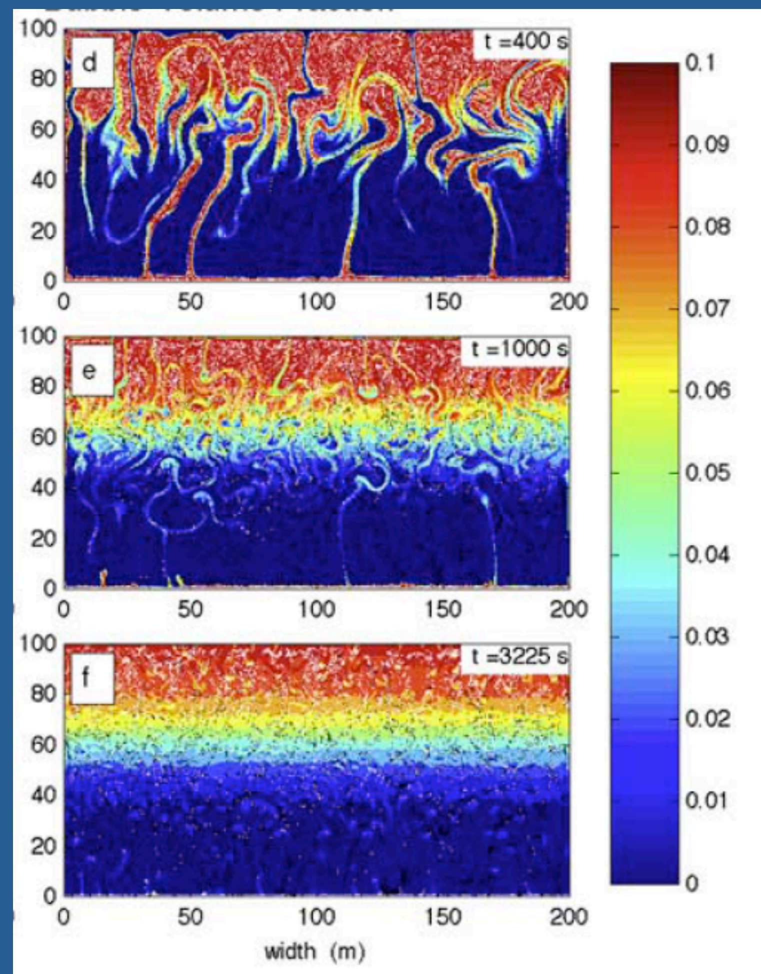


Dynamics

Density Drivers:

1. Composition
2. Thermal Buoyancy
3. Crystal Phases
4. Bubble Phase

Bubble plumes driving dynamics



BBO and Stokes Number

Lagrangian equation of motion

$$\rho_d \frac{d\mathbf{V}_i}{dt} = \underbrace{(\rho_d - \rho_m)\mathbf{g}_\mathbf{j}\delta_{ij}}_{\text{buoyancy}} + \underbrace{\rho_m \frac{D\mathbf{u}_\mathbf{i}}{Dt}\bigg|_{Y(t)}}_{\text{Shear and Pressure terms}} \underbrace{- \frac{1}{2}\rho_m \frac{d}{dt}\left[\mathbf{V}_\mathbf{i}(t) - \mathbf{u}_\mathbf{i}[Y(t),t] - \frac{1}{6}r^2\left[\frac{\partial^2}{\partial \mathbf{x}_\mathbf{j}^{\;2}}\mathbf{u}_\mathbf{i}\right]\bigg|_{Y(t)}\right]}_{\text{Virtual Mass Term}}$$

$$\underbrace{-6\pi r\mu\left[\mathbf{V}_\mathbf{i}(t) - \mathbf{u}_\mathbf{i}[Y(t),t] - \frac{1}{6}r^2\left[\frac{\partial^2}{\partial \mathbf{x}_\mathbf{j}^{\;2}}\mathbf{u}_\mathbf{i}\right]\bigg|_{Y(t)}\right]}_{\text{Stokes Drag Term}} \underbrace{- 6\pi f^2\mu\int_0^t d\tau \left(\frac{d/d\tau\left[V_i(\tau) - \mathbf{u}_\mathbf{i}[Y(\tau),\tau] - \frac{1}{6}r^2\left[\frac{\partial^2}{\partial \mathbf{x}_\mathbf{j}^{\;2}}\mathbf{u}_\mathbf{i}\right]\bigg|_{Y(\tau)}\right]}{[\pi\nu(t-\tau)]^{\frac{1}{2}}}\right)}_{\text{Basset history term}}$$

BBO and Stokes Number

Lagrangian equation of motion

$$\rho_d \frac{d\mathbf{V}_i}{dt} = \underbrace{(\rho_d - \rho_m)\mathbf{g}_j\delta_{ij}}_{\text{buoyancy}} + \underbrace{\rho_m \frac{D\mathbf{u}_i}{Dt}\bigg|_{Y(t)}}_{\text{Shear and Pressure terms}} \underbrace{-\frac{1}{2}\rho_m \frac{d}{dt}\left[\mathbf{V}_i(t) - \mathbf{u}_i[Y(t),t] - \frac{1}{6}r^2\left[\frac{\partial^2}{\partial \mathbf{x}_j^2}\mathbf{u}_i\right]\bigg|_{Y(t)}\right]}_{\text{Virtual Mass Term}}$$

$$\underbrace{-6\pi r\mu\left[\mathbf{V}_i(t) - \mathbf{u}_i[Y(t),t] - \frac{1}{6}r^2\left[\frac{\partial^2}{\partial \mathbf{x}_j^2}\mathbf{u}_i\right]\bigg|_{Y(t)}\right]}_{\text{Stokes Drag Term}} \underbrace{-6\pi f^2\mu\int_0^t d\tau \left(\frac{d/d\tau\left[V_i(\tau) - \mathbf{u}_i[Y(\tau),\tau] - \frac{1}{6}r^2\left[\frac{\partial^2}{\partial \mathbf{x}_j^2}\mathbf{u}_i\right]\bigg|_{Y(\tau)}\right]}{[\pi\nu(t-\tau)]^{\frac{1}{2}}}\right)}_{\text{Basset history term}}$$

For most magmatic conditions the added mass, Bassett history, and pressure gradient terms can be neglected leaving (in dimensionless form):

$$\frac{d\mathbf{V}_i^*}{dt^*} = -\frac{1}{\mathbf{St}}\left[\mathbf{V}_i^* - \mathbf{u}_i^*\right] + \frac{1}{\mathbf{Fr_d}^2}\delta_{ij}$$

Densimetric Froude #

$$\mathbf{Fr_d} = \frac{u_0}{\sqrt{\frac{\rho_d - \rho_m}{\rho_d} g u_0 \tau_m}}$$

$$\mathbf{St} = \frac{\tau_m}{\tau_d}$$

Stokes #: Magmatic timescale/ disperse phase timescale

Disperse Phase Timescale

$$\tau_d = \frac{\rho_d r^2}{6\pi\mu}$$

Direct Numerical Simulation (DNS) of Multiphase Wake Flow, Re~100.

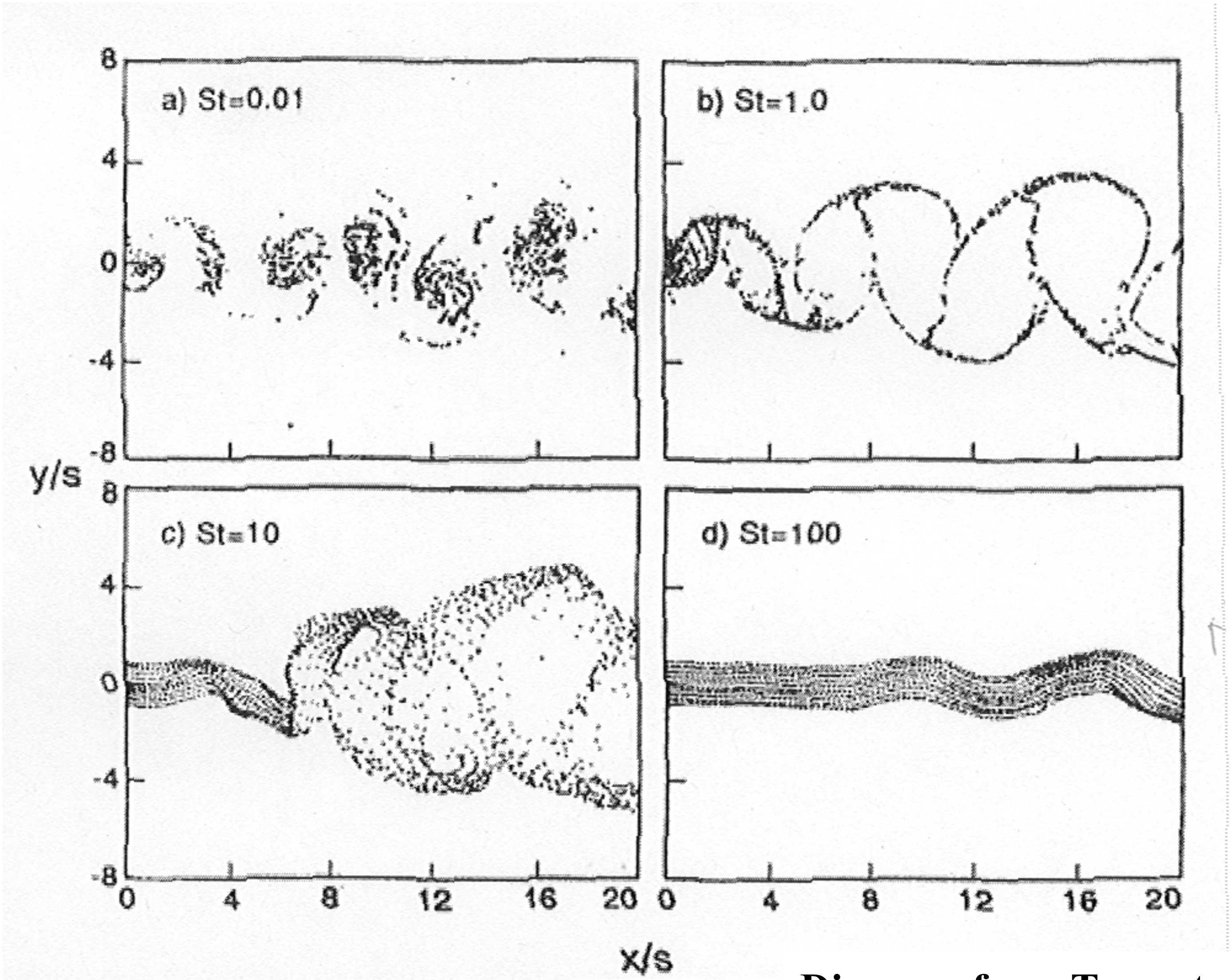
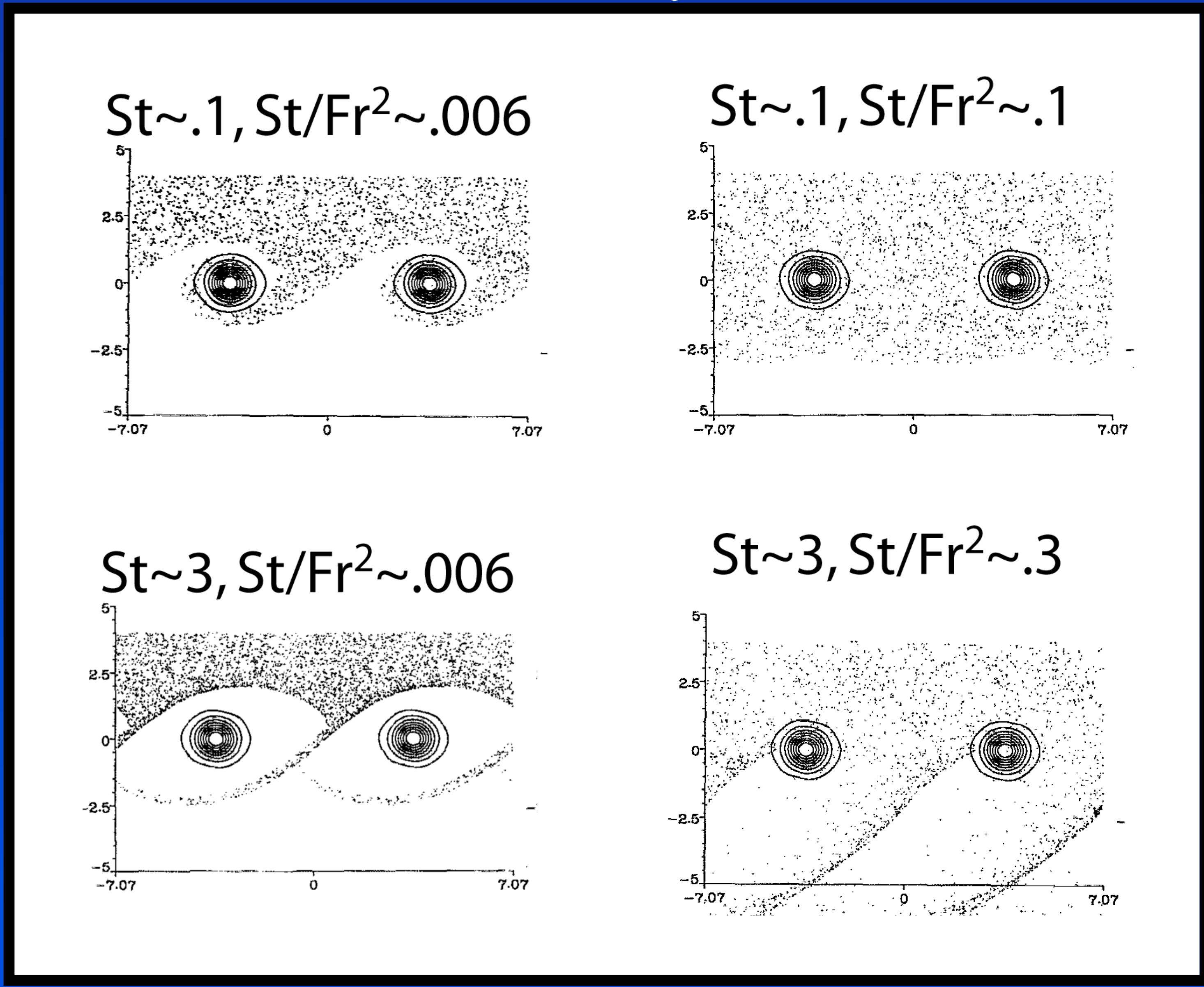


Diagram from Tang et al. 1992

Stokes Numbers and Stability Factor

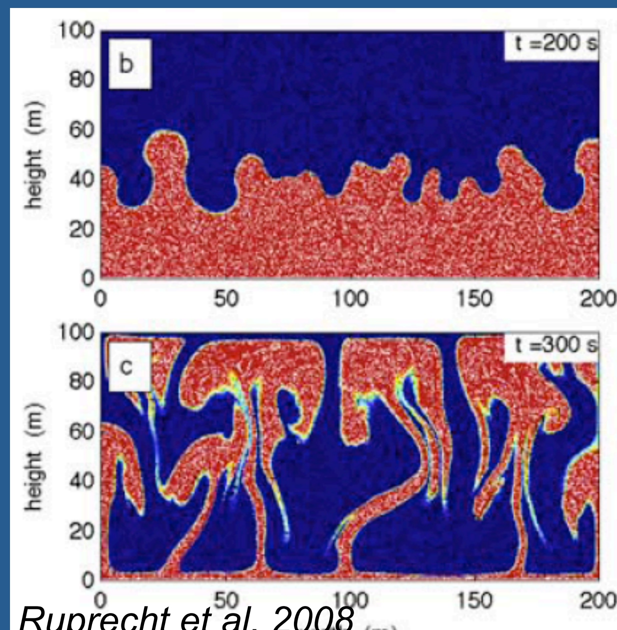


From Raju and Meiburg, 1994

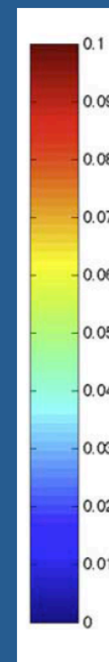
- Crystal and bubble phases have largest density difference.

-Low St --- crystals travel with melt.

-However mixing can still be efficient.



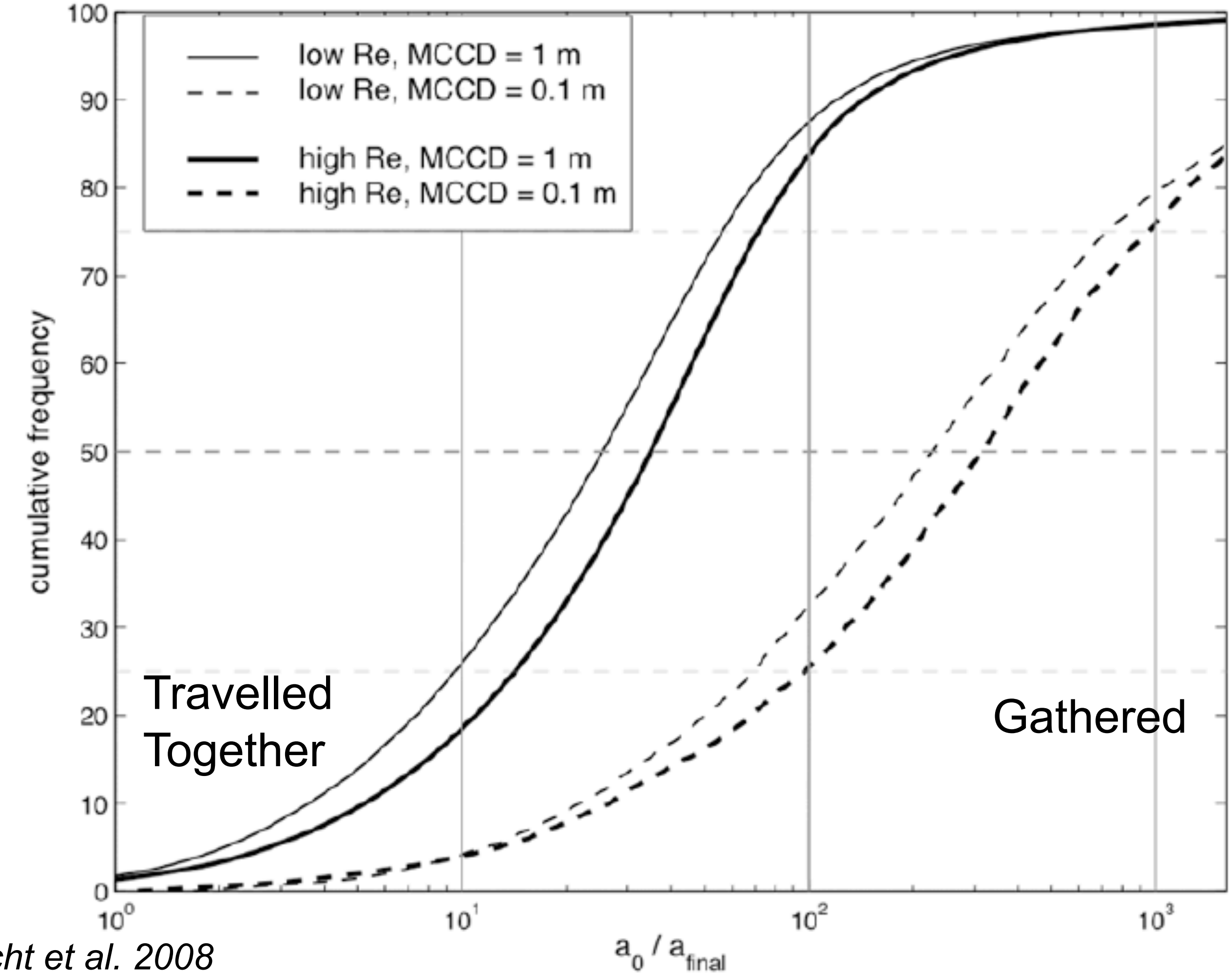
Ruprecht et al. 2008



Bubble Volume Fraction

In this overturn simulation a significant proportion (>20%) of the crystal cargo may have originated from a distance of 100 m or more from each other. Therefore, a single mixing event is capable of juxtaposing crystals that originated from spatially distinct regions of the chamber, and with completely different histories (compositional zoning, textures, etc.).

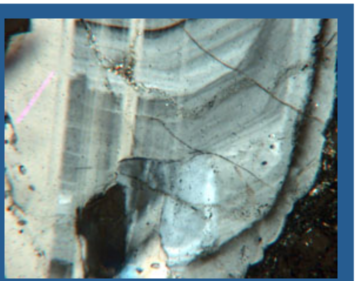
How much do hand samples represent crystals that have travelled together?

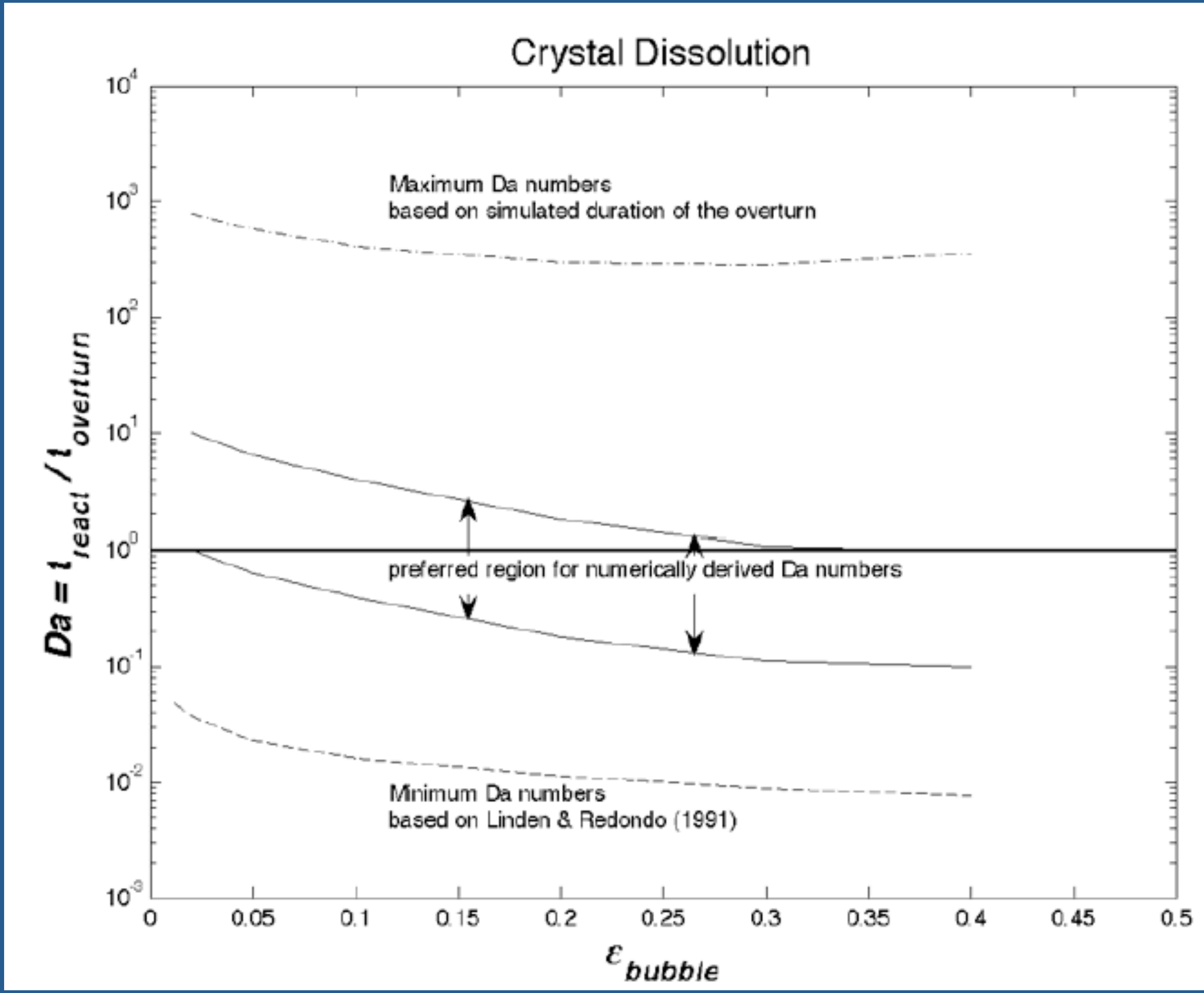


Ruprecht et al. 2008

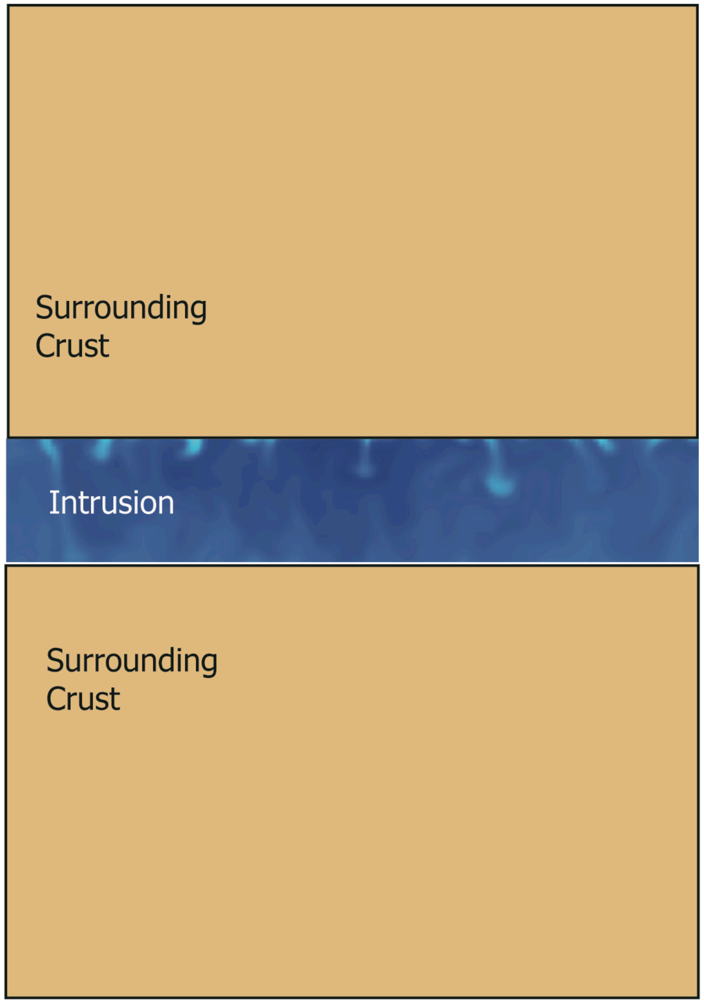
Relative distance between crystals

If crystals are gathered do they react quickly enough to preserve the signal?





Coupled Thermal and Mechanical Multiphase Model



- Considers both the thermal and dynamics aspect of this multiphase system.
- Included phase change.
- Modeled evolving physical properties (density, viscosity, etc using major oxides from MELTS).
- Included a drag formulation to consider a wide range of crystal fraction from dilute suspensions to compaction flows.

Multiphase Equations for Magma Chamber

Volume fraction of all phases equals 1

$$\sum_k \phi_k = 1$$

Conservation of Mass

$$\frac{\partial}{\partial t}\left(\phi_k \rho_k\right) + \frac{\partial}{\partial \mathbf{x_i}}\left(\phi_k \rho_k \mathbf{u_{k,i}}\right) = R_k$$

Conservation of Momentum

$$\frac{\partial(\phi_k \rho_k \mathbf{u_{k,i}})}{\partial t} + \frac{\partial(\phi_k \rho_k \mathbf{u_{k,i}} \mathbf{u_{k,j}})}{\partial \mathbf{x_i}} =$$

$$-\phi_k \frac{\partial P}{\partial \mathbf{x_i}} \delta_{ij} + \frac{\partial}{\partial \mathbf{x_i}}\left[\tau_{\mathbf{ij}}\right] + \mathbf{D}_i + \rho_k \phi_k \boldsymbol{g}_2 \delta_{\mathbf{i2}} + R_k \mathbf{u_{k,i}}$$

Conservation of Thermal Energy

$$\phi_k \rho_k c_k \left[\frac{\partial \mathrm{T_k}}{\partial \mathrm{t}} + \mathbf{u_i} \frac{\partial \mathrm{T_k}}{\partial \mathbf{x_i}}\right] = \delta_{\mathrm{km}} \frac{\partial \mathrm{q_k}}{\partial \mathbf{x_i}} + \pi k_m \mathrm{d}\ \mathbf{Nu}\left(\mathrm{T}_m - \mathrm{T_c}\right) + \phi_k R_k L$$

Conservation of Chemical Species

$$\frac{\partial}{\partial t}\left(\phi_k \rho_k C_{SiO_2}\right) + \frac{\partial}{\partial \mathbf{x_i}}\left(\phi_k \rho_k \mathbf{u_{k,i}} C_{SiO_2}\right) = \beta_{(f)}$$

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Crystals and magma have distinct sets of conservation equations (denoted by *k* in these equations)

Conservation of Thermal Energy

$$\phi_k \rho_k c_k \left[\frac{\partial \mathrm{T_k}}{\partial \mathrm{t}} + \mathbf{u_i} \frac{\partial \mathrm{T_k}}{\partial \mathbf{x_i}}\right] = \delta_{\mathrm{km}} \frac{\partial \mathrm{q_k}}{\partial \mathbf{x_i}} + \pi k_m \mathrm{d}\ \mathbf{Nu}\left(\mathrm{T}_m - \mathrm{T_c}\right) + \phi_k R_k L$$

Conservation of Chemical Species

$$\frac{\partial}{\partial t}\left(\phi_k \rho_k C_{SiO_2}\right) + \frac{\partial}{\partial \mathbf{x_i}}\left(\phi_k \rho_k \mathbf{u_{k,i}} C_{SiO_2}\right) = \beta_{(f)}$$

Multiphase Equations for Magma Chamber

Volume fraction of all phases equals 1

$$\sum_k \phi_k = 1$$

Conservation of Mass

$$\frac{\partial}{\partial t}\left(\phi_k \rho_k\right) + \frac{\partial}{\partial \mathbf{x_i}}\left(\phi_k \rho_k \mathbf{u_{k,i}}\right) = \boxed{R_k}$$

Conservation of Momentum

$$\frac{\partial(\phi_k \rho_k \mathbf{u_{k,i}})}{\partial t} + \frac{\partial(\phi_k \rho_k \mathbf{u_{k,i}} \mathbf{u_{k,j}})}{\partial \mathbf{x_i}} =$$

$$-\phi_k \frac{\partial P}{\partial \mathbf{x_i}} \delta_{ij} + \frac{\partial}{\partial \mathbf{x_i}}\left[\tau_{\mathbf{ij}}\right] + \mathbf{D}_i + \rho_k \phi_k g_2 \delta_{\mathbf{i2}} + \boxed{R_k \mathbf{u_{k,i}}}$$

Conservation of Thermal Energy

$$\phi_k \rho_k c_k \left[\frac{\partial \mathrm{T_k}}{\partial \mathrm{t}} + \mathbf{u_i} \frac{\partial \mathrm{T_k}}{\partial \mathbf{x_i}}\right] = \delta_{\mathrm{km}} \frac{\partial \mathrm{q_k}}{\partial \mathbf{x_i}} + \pi k_m \mathrm{d}\ \mathbf{Nu}\left(\mathrm{T}_m - \mathrm{T_c}\right) + \boxed{\phi_k R_k L}$$

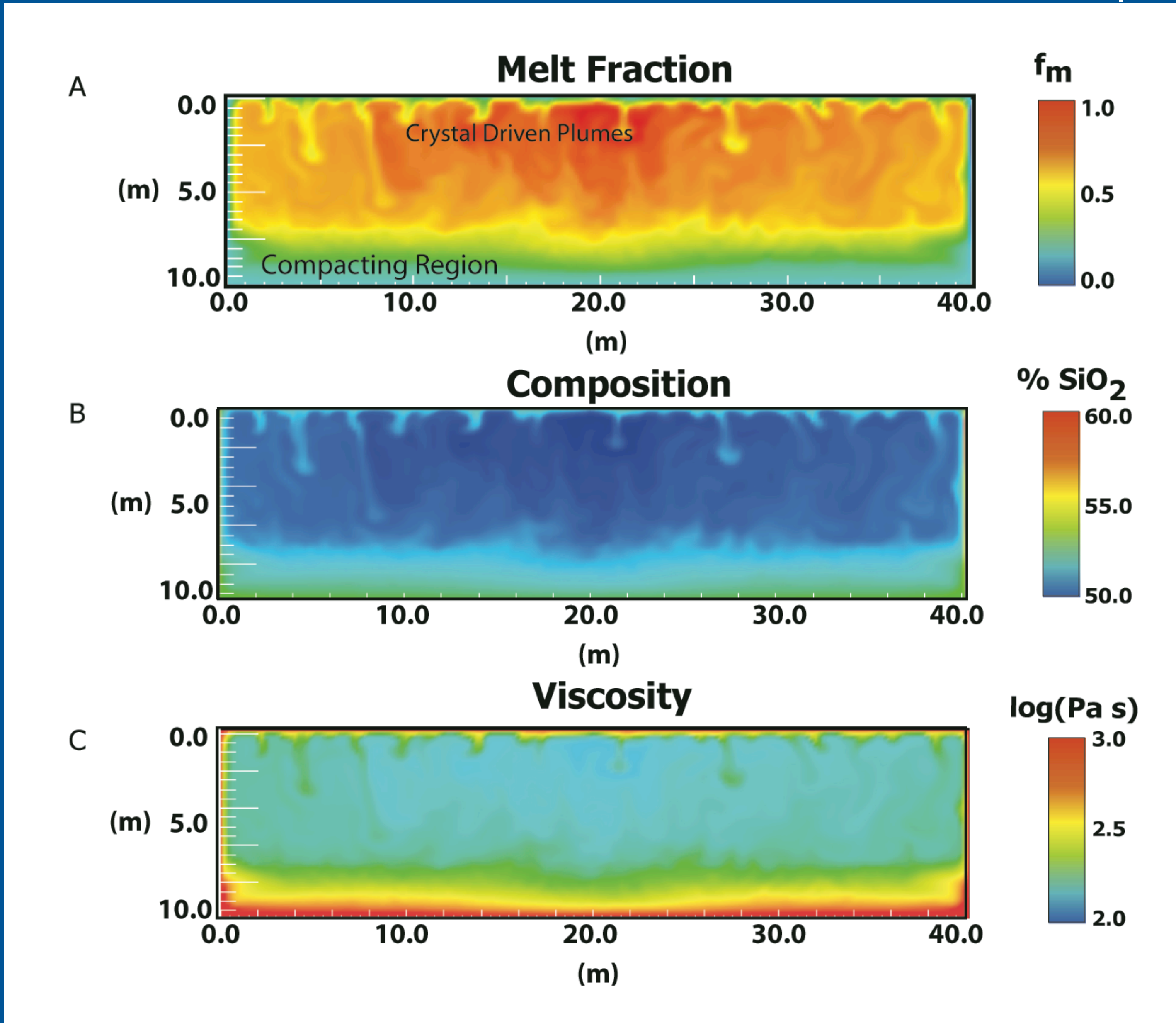
Conservation of Chemical Species

$$\frac{\partial}{\partial t}\left(\phi_k \rho_k C_{SiO_2}\right) + \frac{\partial}{\partial \mathbf{x_i}}\left(\phi_k \rho_k \mathbf{u_{k,i}} C_{SiO_2}\right) = \boxed{\beta_{(f)}}$$

Crystallization

An Example Simulation:

Basaltic intrusion, modeled intrusion depth: 24 km

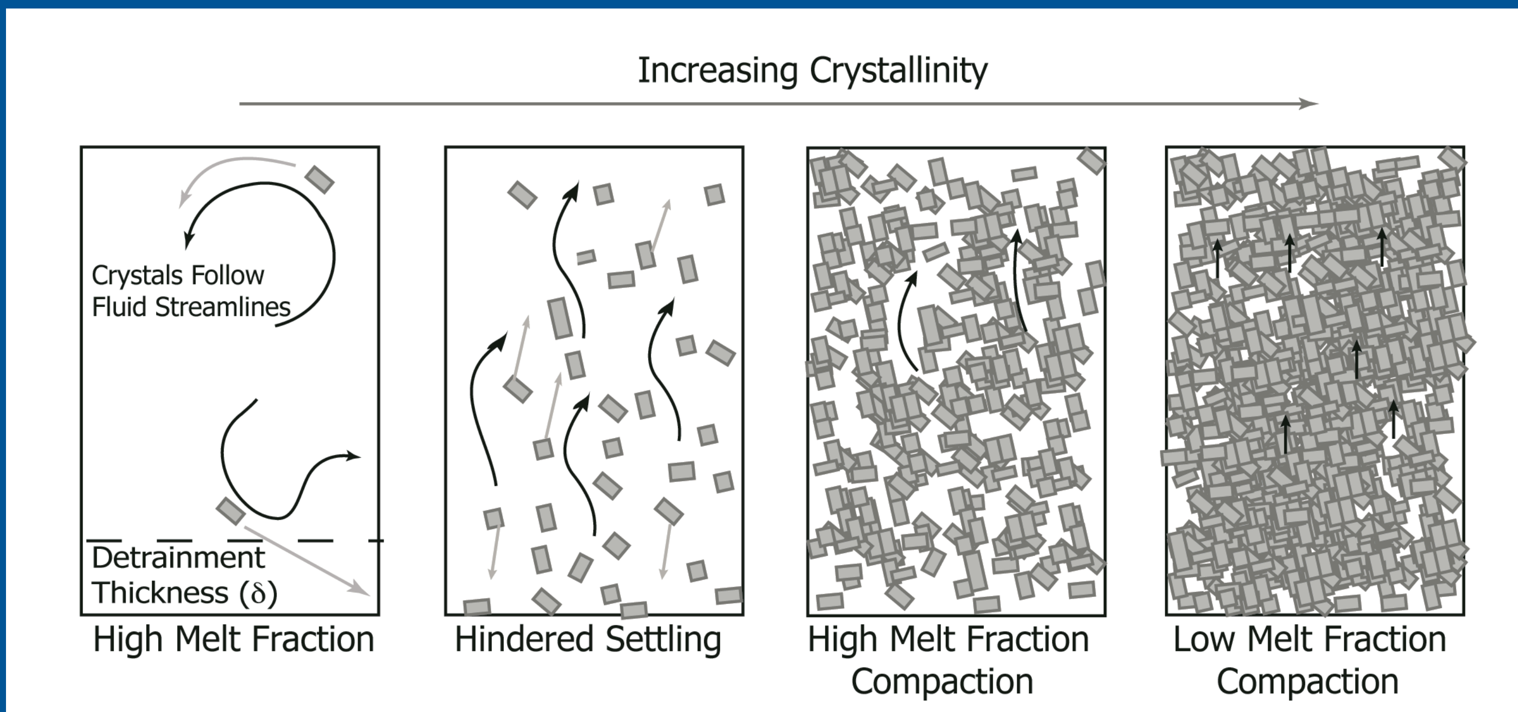


Dufek and Bachmann, 2010

Melt extraction probability is modulated by two factors:

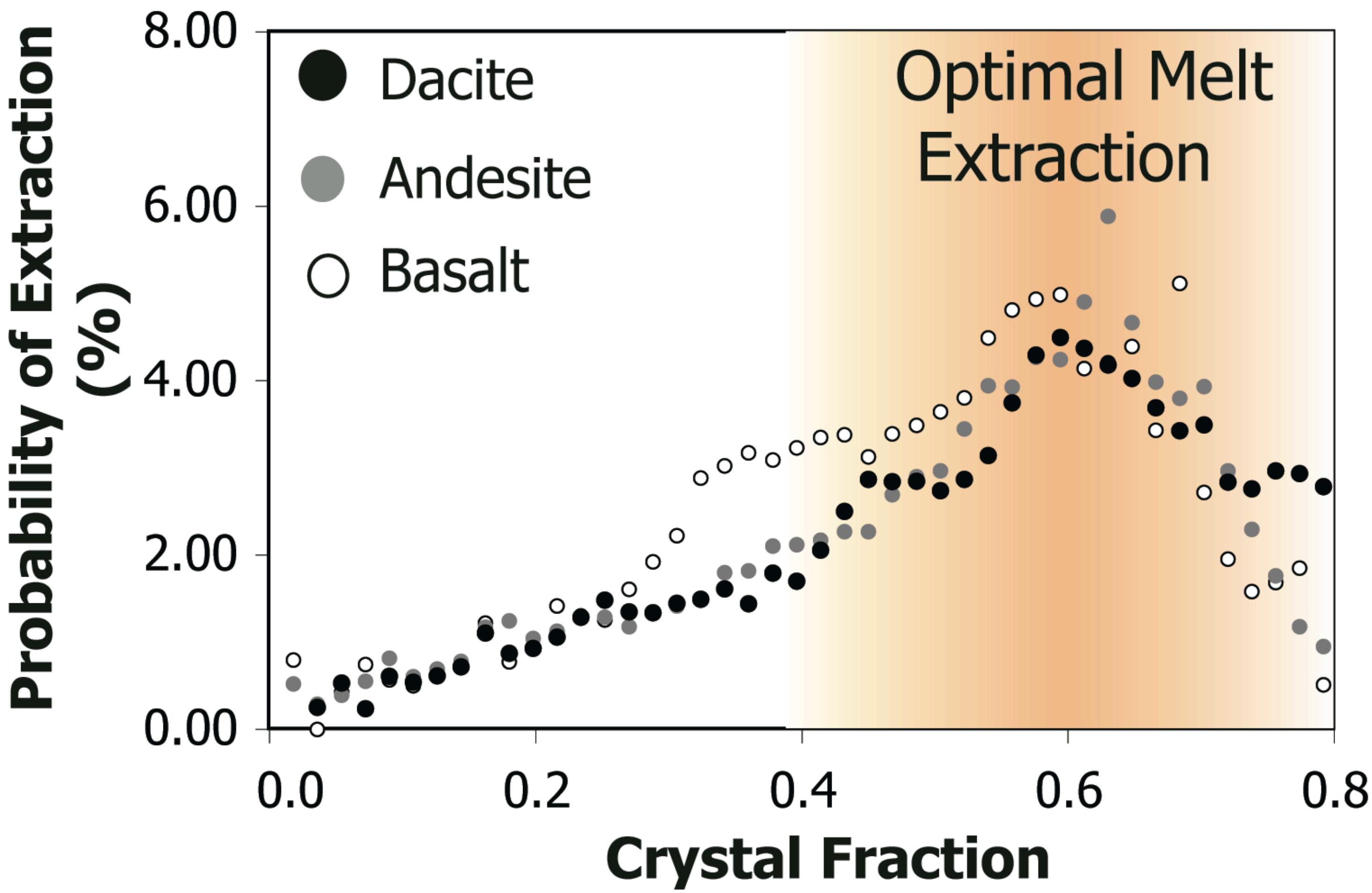
1. The length of time a given magmatic composition exists (thermal problem).

2. Separation velocity between crystal and melt phases.

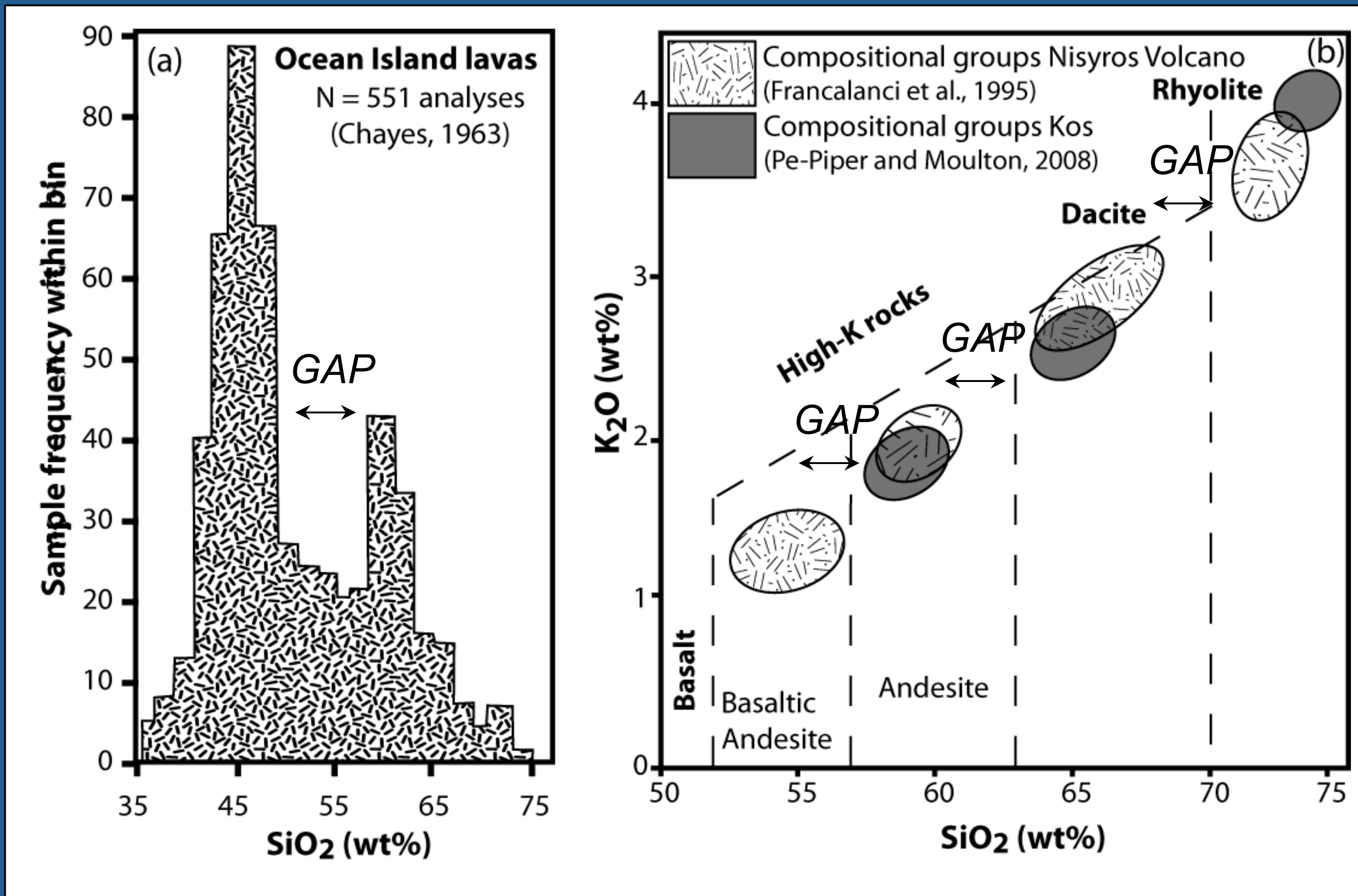


Melt Extraction probability

Reference Depth: 20 km

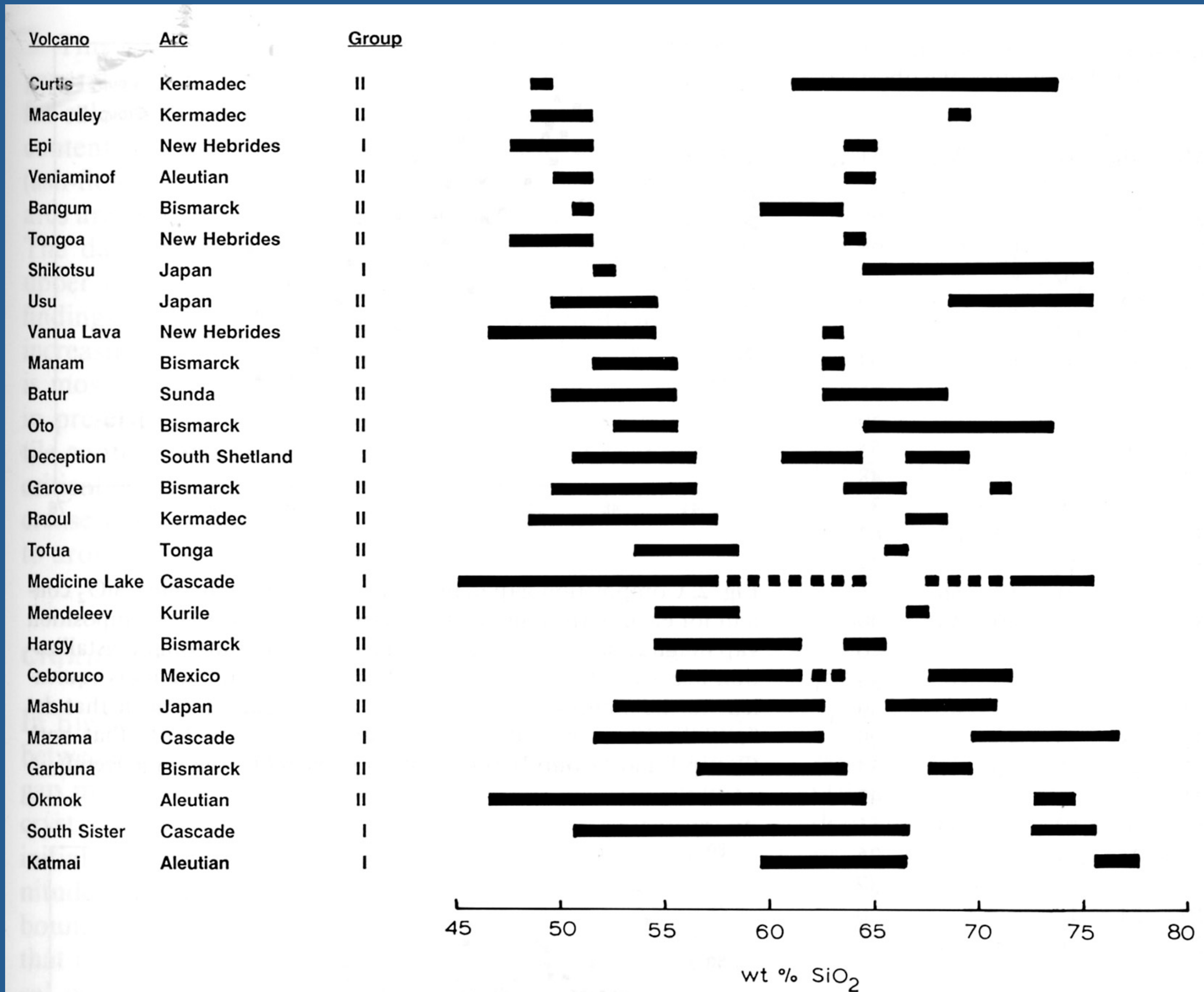


Compositional Gaps, or Daly Gaps - A paucity in the occurrence of intermediate erupted compositions.



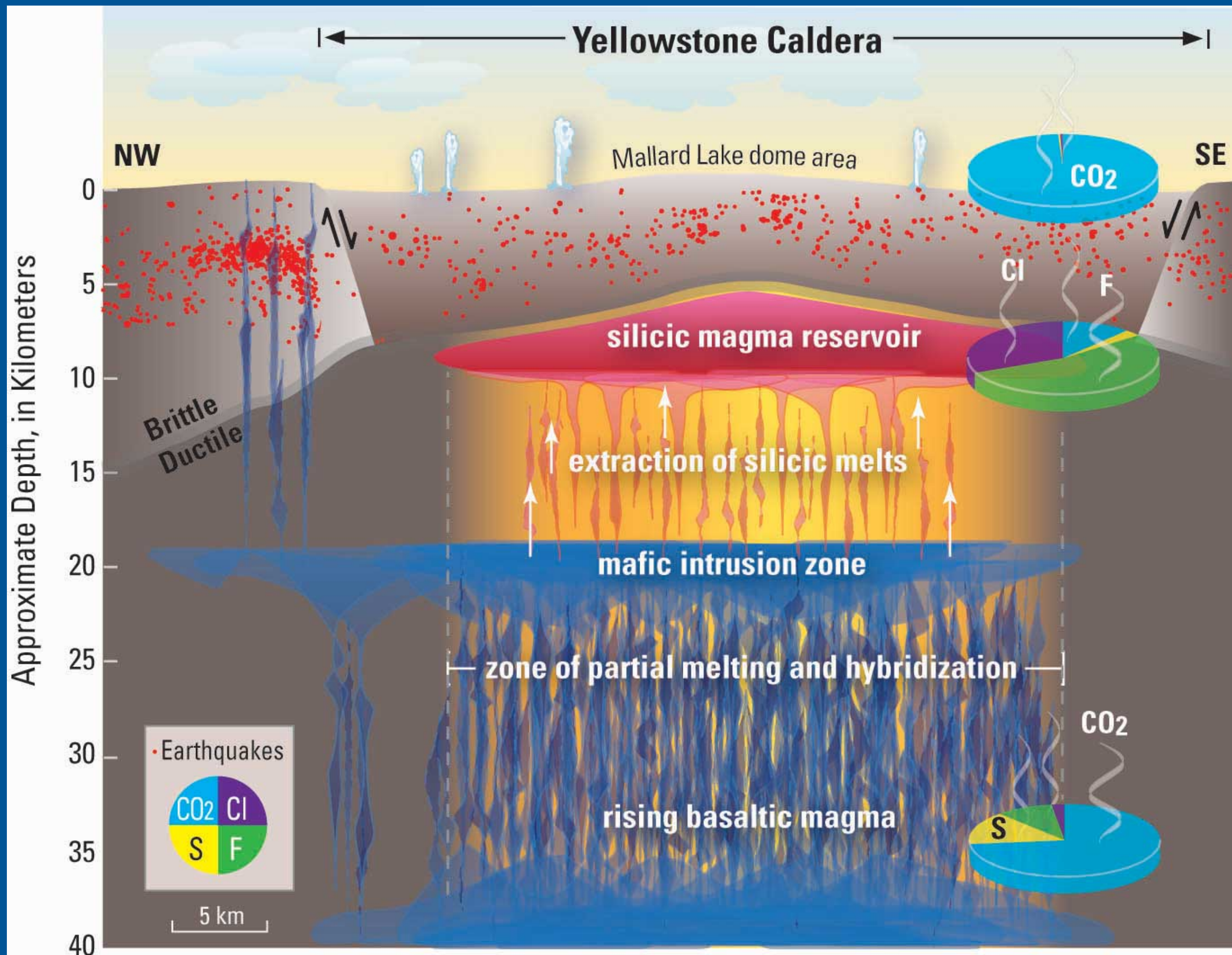
A gap in this framework does not necessarily imply complete absence of certain compositions (such assertions can be hard to make rigorously) but the relative dearth of compositions.

Observations of Gaps are abundant - Here a compilation from Brophy in SiO_2.



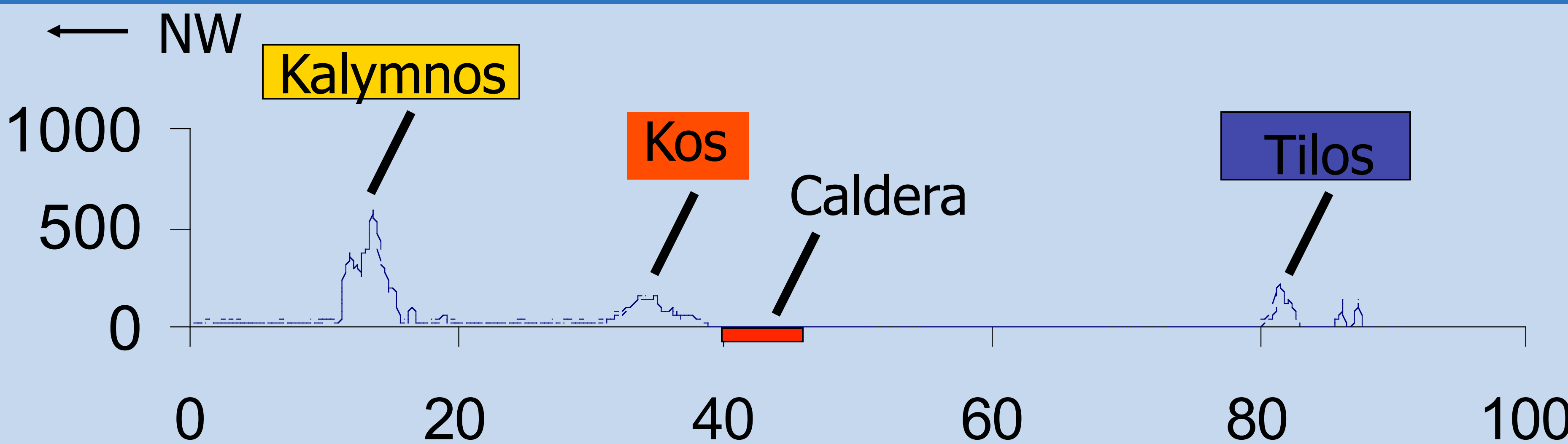
Brophy, 1991

Link to eruptions and syn-eruptive mixing

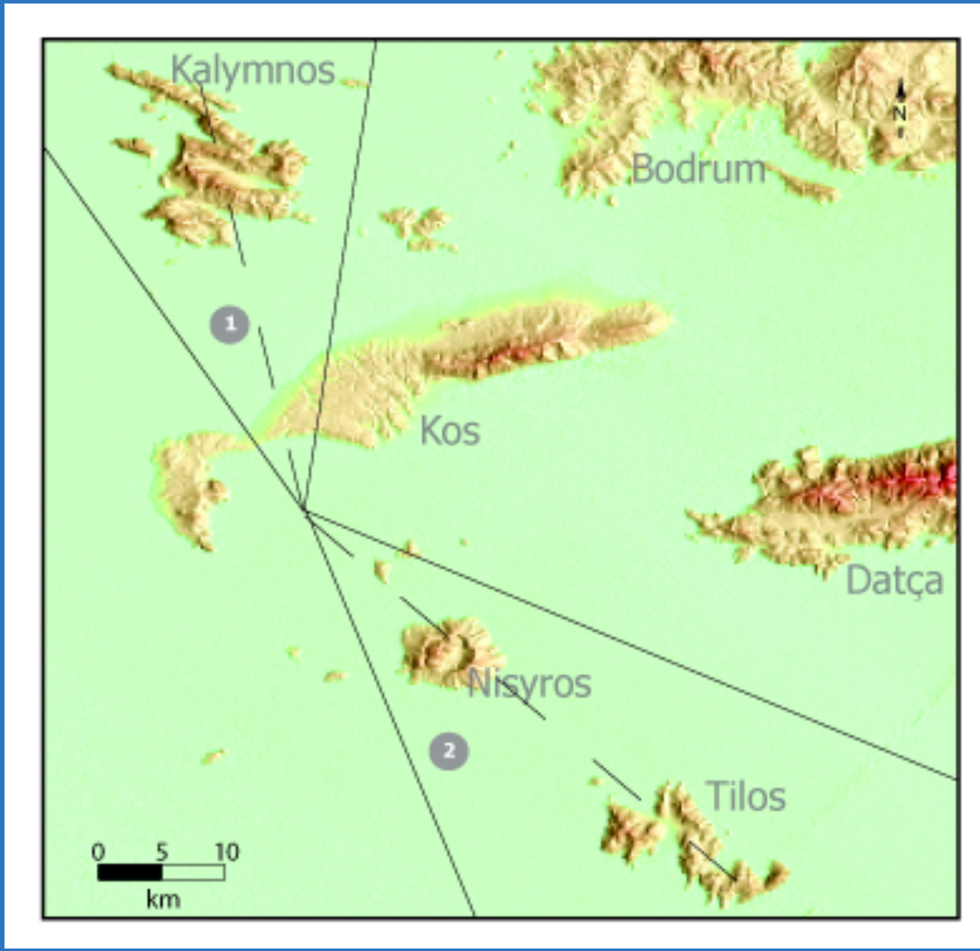


Lowenstern and Hurwitz, 2008

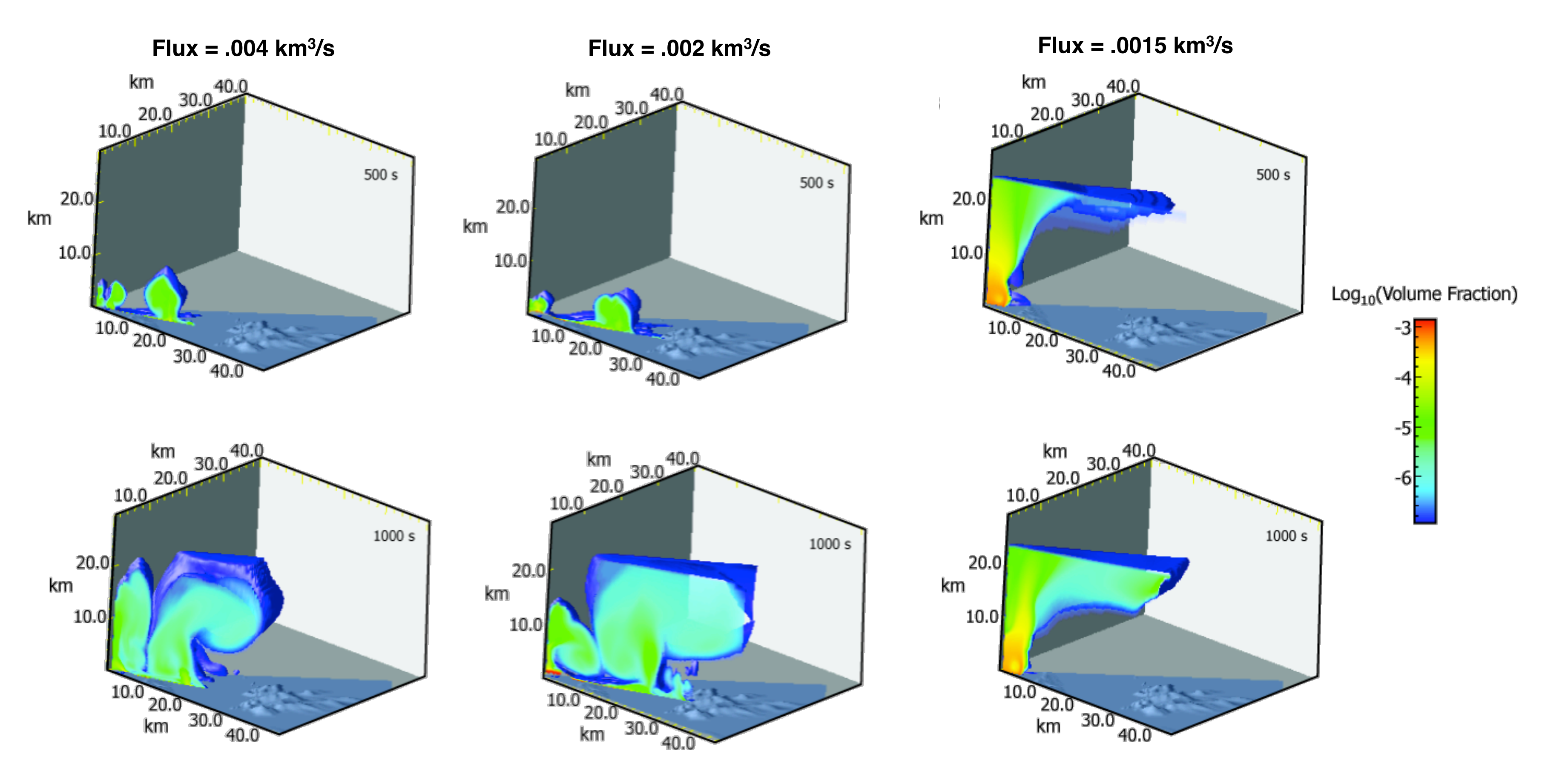

~1 m

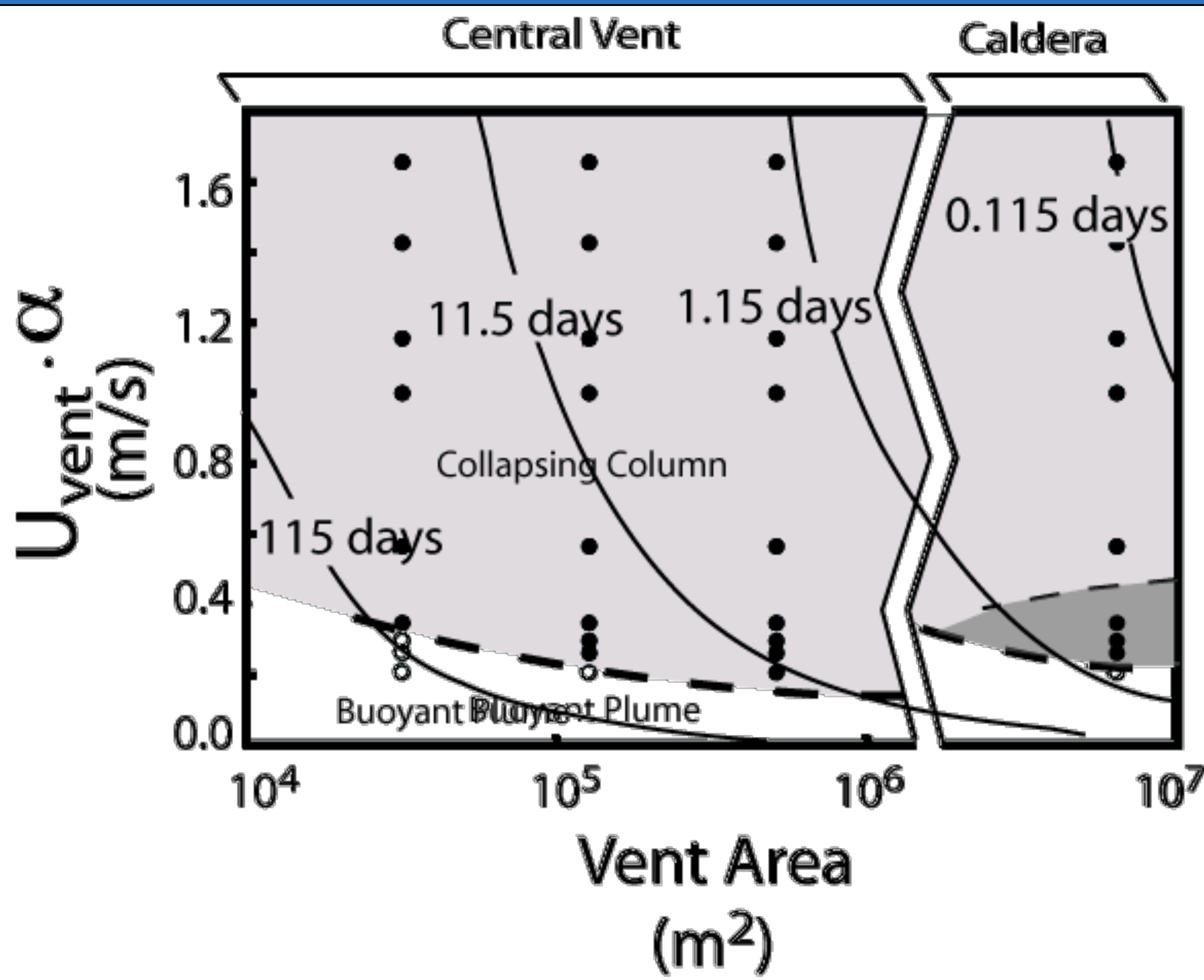

NW
Kalymnos
Kos
Caldera
Tilos
1000
500
0
0
20
40
60
80
100

Kos Plateau Tuff - Using EEL to constrain eruptive conditions

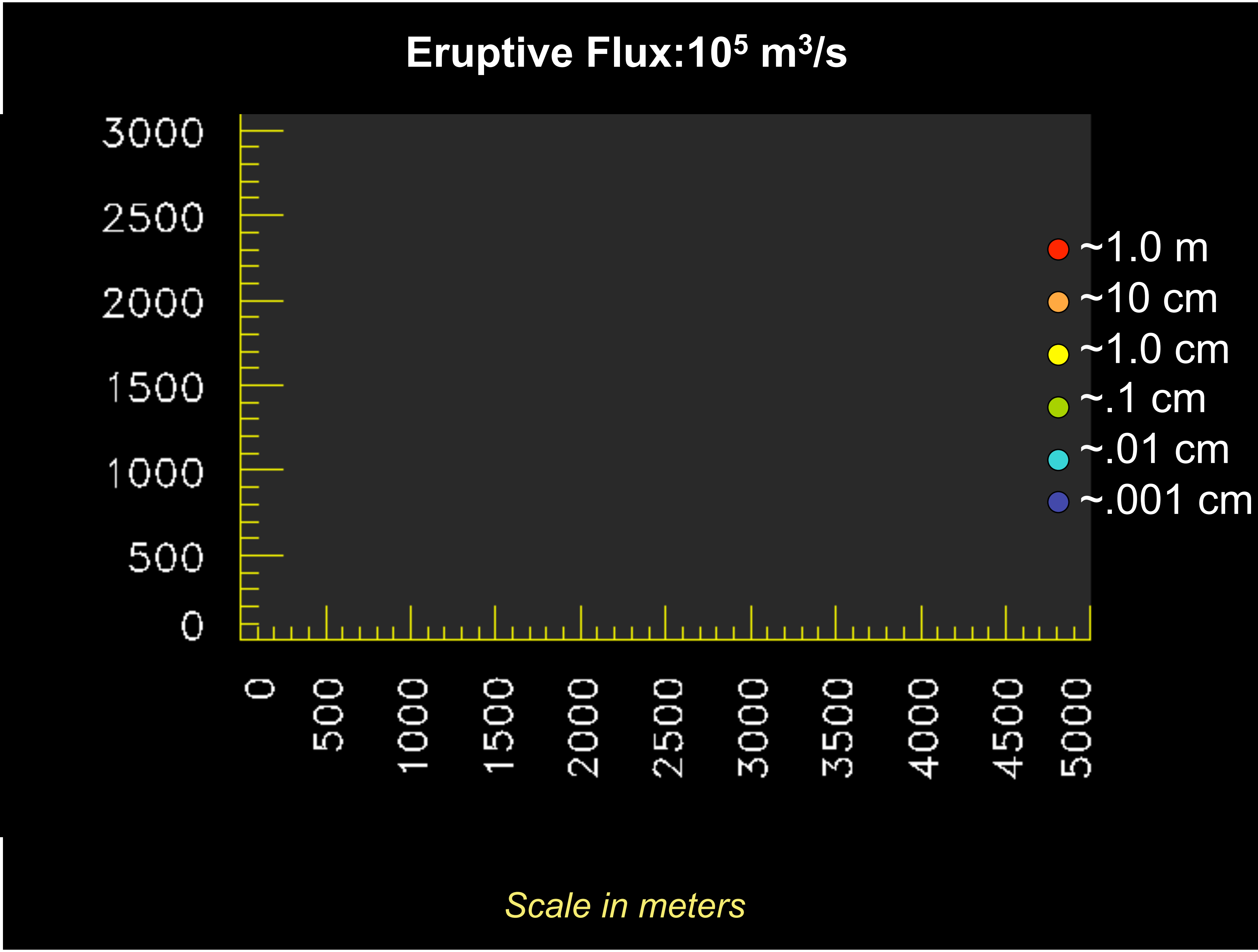


A survey of 3D dynamics



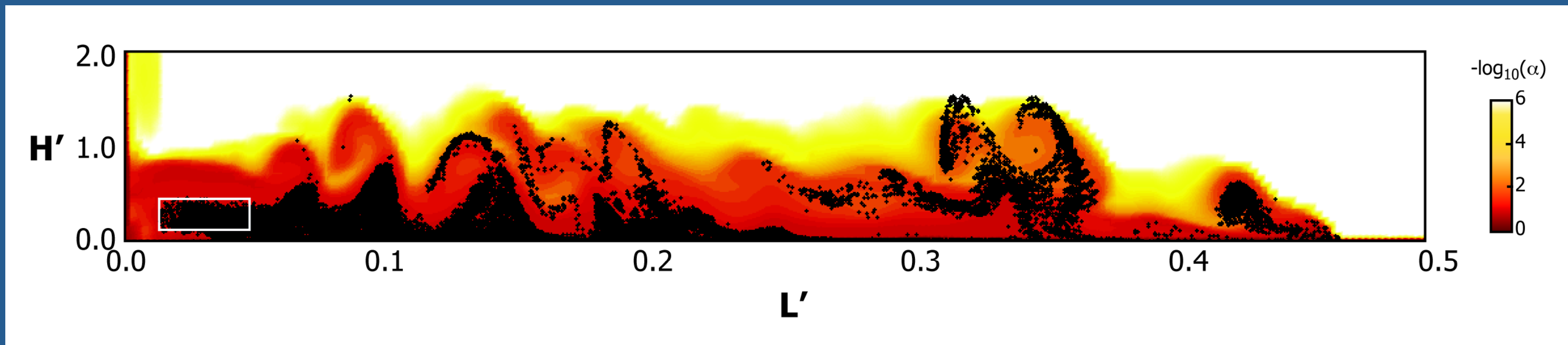


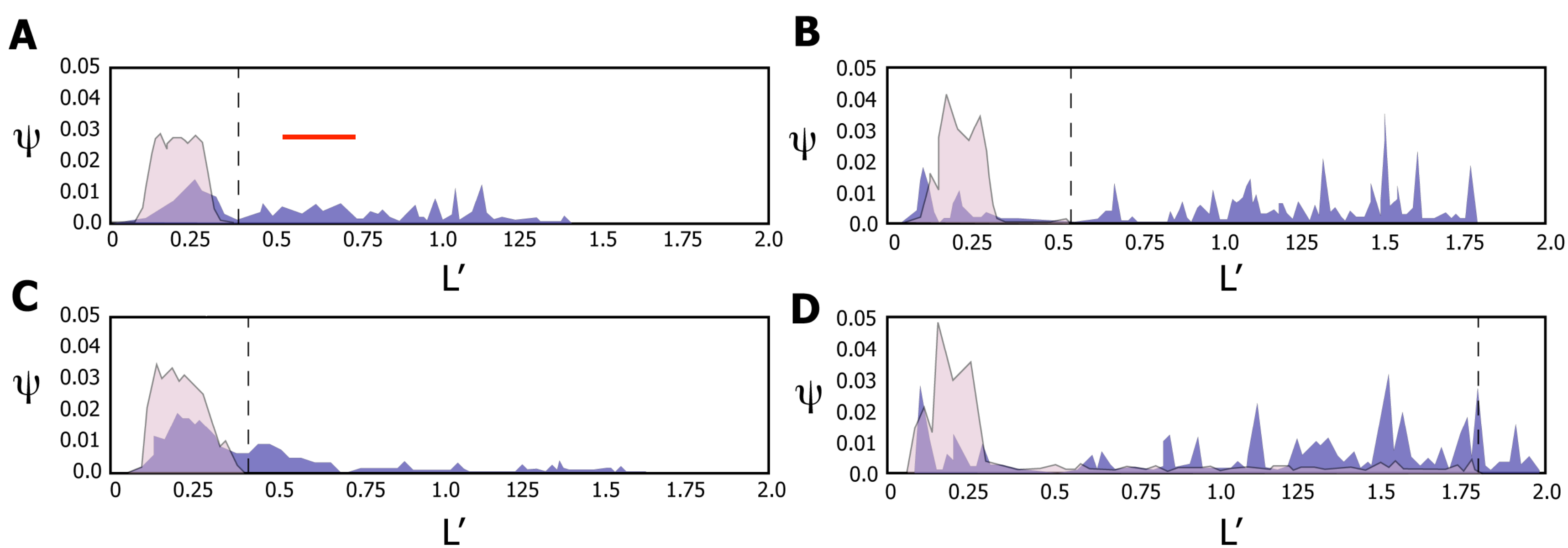
Dufek and Bergantz, G3, Submitted



Eruptive Flux:10^5 m^3/s
3000
2500
2000
1500
1000
500
0
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
~1.0 m
~10 cm
~1.0 cm
~.1 cm
~.01 cm
~.001 cm
Scale in meters

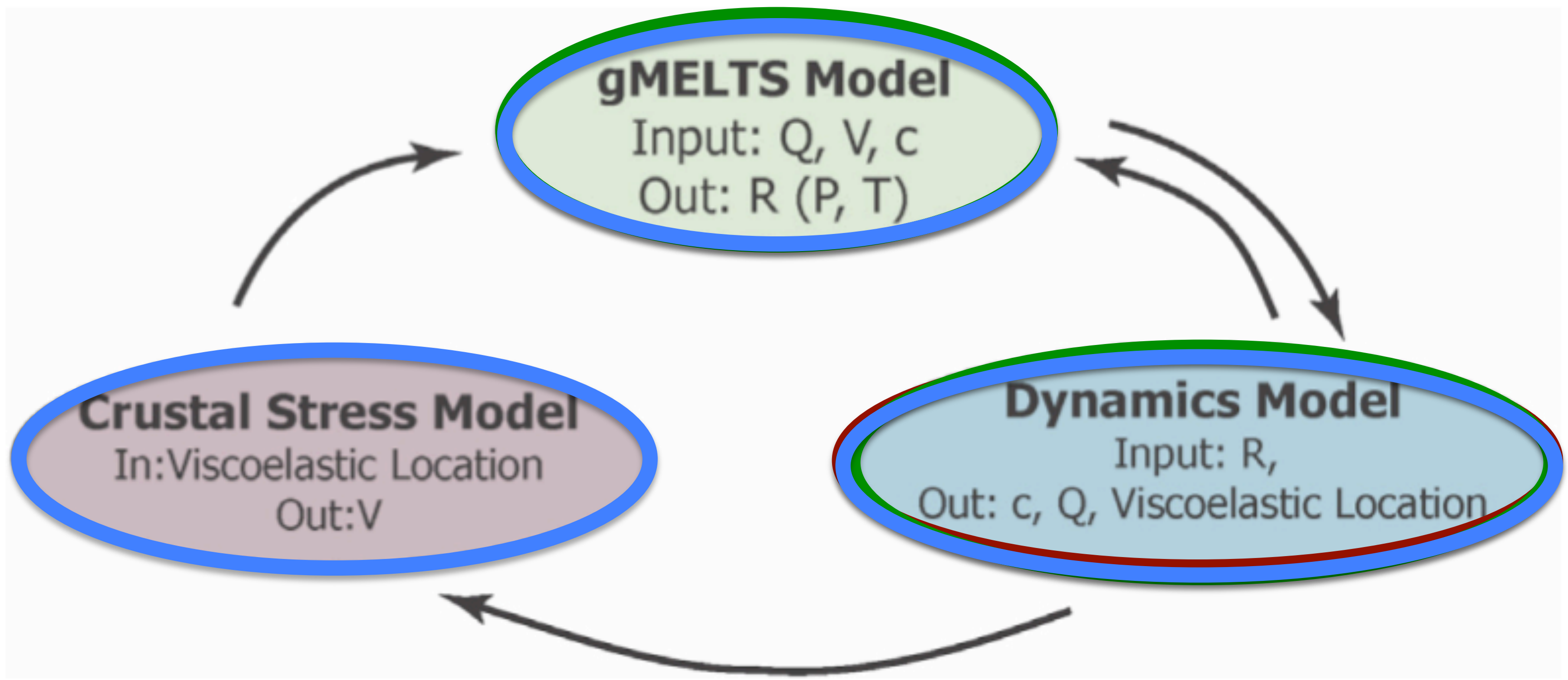
Lagrangian Analysis





Well-mixed ash (St<1) particles in pyroclastic density currents - welded ignimbrites likely eruptively well mixed.

Coupled multiphase dynamics, stress field and thermodynamics models provide the context to integrate disparate observations.



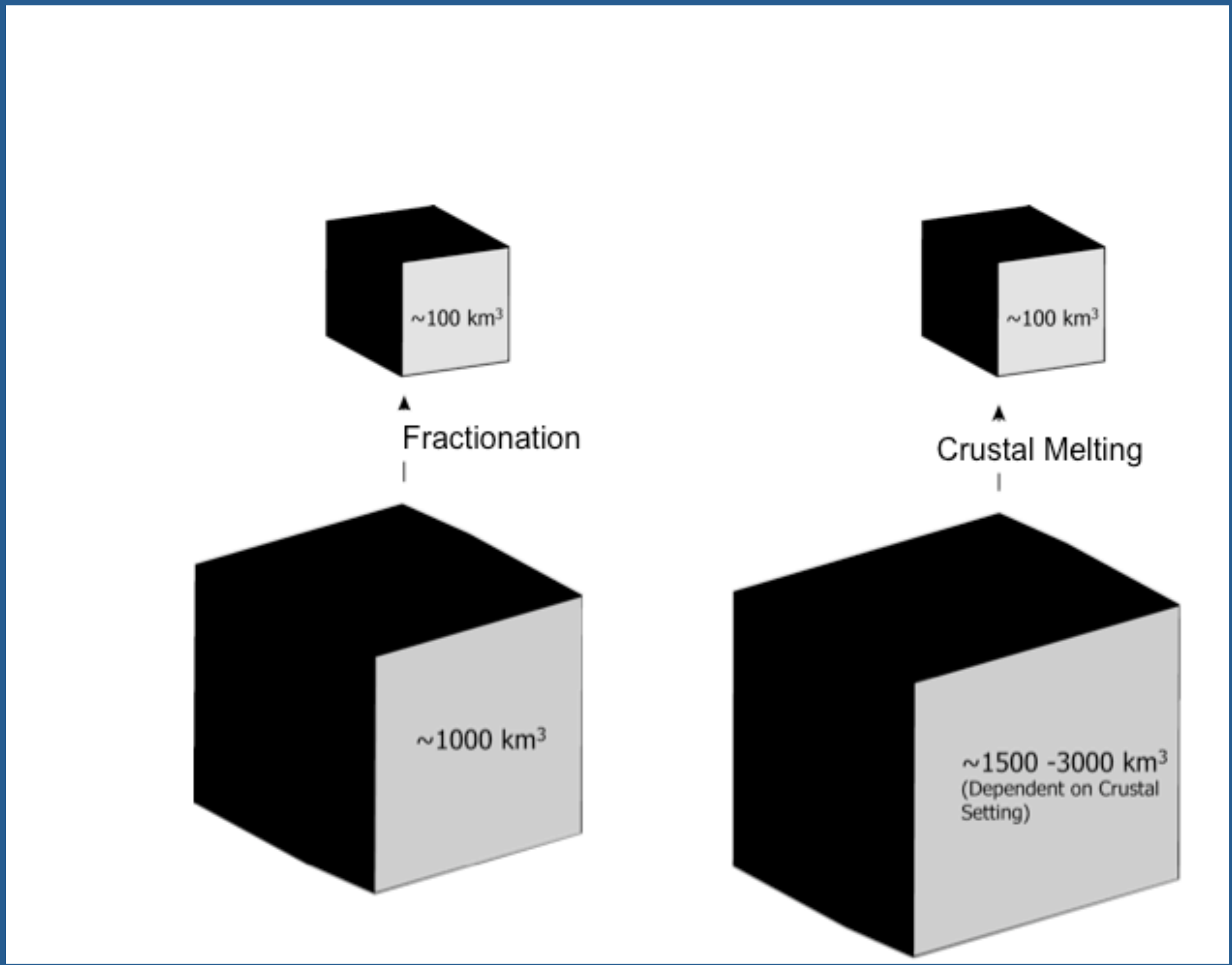
Multiphase dynamics in magma chambers and its role in melt extraction and compositional (Daly) gaps.

Determination of detailed phase equilibria, melt residence times and accurate calculations of sensible to latent heat ratios.

Magma chamber pressure evolution, modification of crustal stress fields, and influence on phase equilibria.

Persistent questions….

Both Fractionation and Melting Create an Apparent Crustal Mass Balance Issue



Continental Crust Paradox

(Kay and Kay, 1988; Rudnick, 1995; C.T.A. Lee et al. 2006)

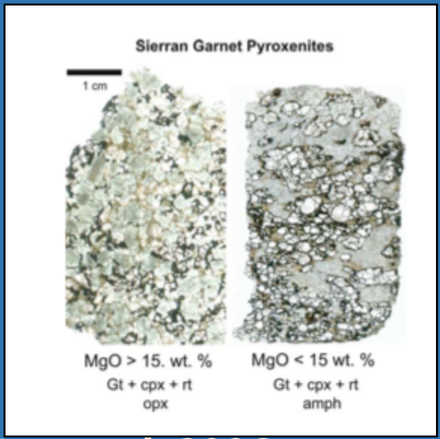
- **Crust is more silicic than primitive mantle melt input.**

Crustal Compositions
Compilation from *Rudnick and Gao, 2003*

	Lower Crust	Middle Crust	Upper Crust	Bulk Crust
SiO_2	53.4	63.5	66.6	60.6
Al_20_3	16.9	15	15.4	15.9
FeO	8.57	6.02	5.03	6.7
MgO	7.24	3.59	2.48	4.7
CaO	9.59	5.25	3.59	6.4
Na_2O	2.65	3.39	3.27	3.1
K_2O	.61	2.3	2.8	1.8

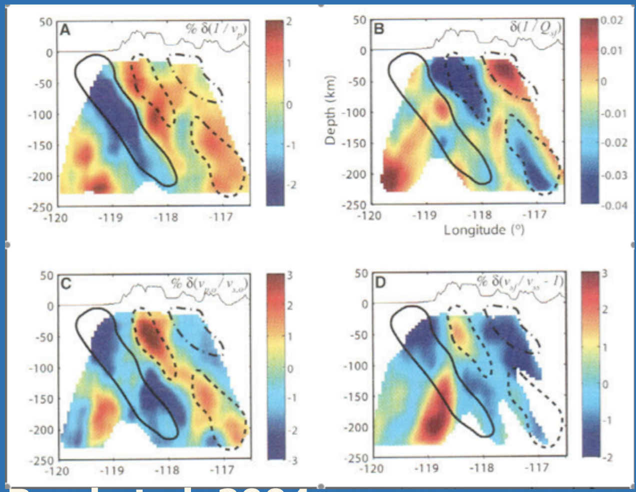
A Potential Resolution - Mass return back to the mantle (R-T instabilities, delamination, erosion…)

Xenoliths



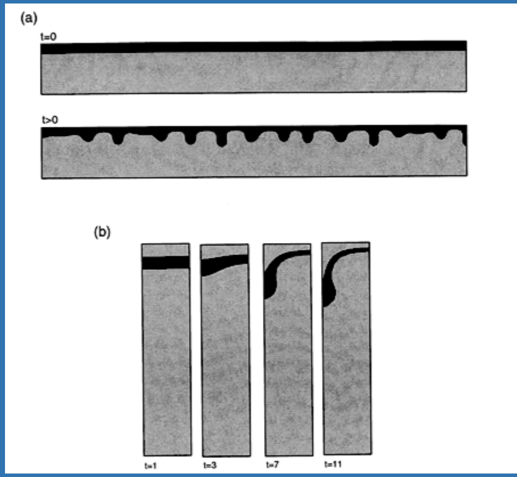
Lee et al, 2006

Tomography



Boyd et al, 2004

Dynamic Models



Jull and Kelemen, 2001

How might foundering be related to an actively growing crust, being forced by mass and enthalpy input?

Magmatic Rates in the Crust