Rheology cont.: Influence of melt

TEM images of three-grain edges

Solgel olivine

San Carlos olivine
Melt distribution in polycrystalline olivine

$P = 1 \text{ GPa, } 1350^\circ \text{C, 432 hours}$
Serial sectioning + high res. imaging:
Thin (~100 nm) layers on two-grain boundaries
Figure 5. View from the top of the reconstructed 3D melt distribution. The thin layers on two-grain boundaries seen in the 2-D images (Figure 1) persist in 3-D, confirming that they are sheet-like rather than a sectioning artifact. Some small grains are completely surrounded by melt.

Figure 6. Vertical 2-D sections through the assembled 3-D image of the pore space, parallel to Y-Z plane. The same features observed in 2-D horizontal surfaces are present here, triple-junctions, large melt pockets and thin inclusions.

3-D view of the melt distribution
3.6% melt, 30 \( \mu \text{m} \) grain size

Garapić, Faul and Brisson, G\(^3\), 2013
Influence of melt on rheology

Small amount (1%) of melt enhances strain rate by ~ 1.5 orders of magnitude

Connected melt produces short circuit diffusion paths

Faul and Jackson, 2007
Theoretically predicted viscosity reduction due to melt

(b) $10^0$

normalized shear viscosity, $\frac{\eta^P}{\eta_{cc}}$

$R = 7.5\mu m$

(TH2)

$\exp(-25\phi)$

$\lambda = 25$

$A = 2$

$A = 2.3$

$\theta_d = 20^\circ$

$\theta_d = 30^\circ$

Takei & Holtzman, 2009
Deformation at a range of time scales

\[ \varepsilon < 10^{-4} \]

\[ \varepsilon >> 1 \]

\[ t (\text{sec}) \]

Yuen & Peltier, 1982
Microcreep experiments (time domain)

Temperature, °C

Torsional mode distortion, rad/Nm

Time, s

Stress

Time

$\varepsilon < 10^{-4}$

1200

1100

1000

2000 s
Attenuation/dissipation (frequency domain)
amplitude decreases with each cycle

\[ Q^{-1} = \frac{\delta E}{E} \cdot \frac{1}{2\pi} \]

Data - seismograms

Mw ~9.2 2004 26 December, Sumatra
Experiments: Measurement of shear modulus (G) and attenuation (1/Q)

Experiments at
• temperatures to 1300°C
• periods 1 - 1000s
• 200 MPa confining pressure

Measure shear modulus G and dissipation/attenuation

Attenuation (1/Q): energy loss per cycle

Research School of Earth Sciences, Australian National University
Elastic behavior (Spring): Hooke’s Law

\[ \sigma = E \varepsilon \]

\[ \theta = \tan^{-1} E \]
Viscous behavior (leaky dashpot)

\[ \sigma = \eta \dot{\varepsilon} \]

increasing strain rate results in increasing stress, with viscosity constant of proportionality

For a step function application of stress:
Maxwell body: viscoelastic

relevant elastic modulus for deformation of solid Earth is shear modulus $G$
The relevant elastic modulus for deformation of solid Earth is shear modulus $G$.

For constant strain:

with $\dot{\varepsilon} = 0$: $\sigma = \sigma_0 \exp(- (G/\eta) t)$, exponential decay of stress with Maxwell relaxation time $\tau_M = \eta/G$. 

\[
\varepsilon' = \frac{\sigma}{\eta} + \frac{\dot{\sigma}}{G}
\]
Maxwell relaxation time: how quickly does stress decay in the Earth?
Microcreep experiments

\[ \varepsilon < 10^{-4} \]
Anelastic behavior (transient creep)

\[ \sigma = \eta \dot{\varepsilon} + G \varepsilon \]

time-dependent, unique equilibrium, recoverable
Viscoelastic behavior: Burgers Model

\[ \varepsilon(t) = \varepsilon_e + \varepsilon_t(t) + \dot{\varepsilon}t \]
Timescales of deformation in the Earth

- **Elastic**
- **Anelastic**
- **Viscous**
- **Steady-state creep**
- **Transient creep**

- **Strain**

- **Time**

- **Recoverable**
- **Irrecoverable**

- **Seismic waves**
- **Post-glacial rebound**
- **Mantle convection**

(Earthquakes)
Data fitting: Burgers model

**Time domain:** strain as a function of time (creep function)

\[ J(t) = J_M + t/\tau_M + J_V (1 - \exp(-t/\tau_V)) \]

\[ J_M = 1/G_M; \tau_{M,V} \text{ relaxation times} \]

**Frequency domain:**

\[ J_1(\omega) = J_M + J_V / (1 + \omega^2 \tau_V^2), \]

\[ J^*(\omega) = J_1(\omega) + i J_2(\omega) \]

\[ J_2(\omega) = J_V \omega \tau_V / (1 + \omega^2 \tau_V^2) + 1/\omega \tau_M \]

\[ G(\omega) = [J_1^2(\omega) + J_2^2(\omega)]^{-1/2} \]

\[ Q(\omega) = J_1(\omega)/J_2(\omega) \]
Burgers model

Maxwell

Voigt

\( G_M \)

\( \eta_M \)

\( G_V \)

\( \eta_V \)

elastic

anelastic

viscous

Dissipation, \( Q^{-1} \)

Log_{10}(frequency)

- short time scales
- long time scales

strain

time

elastic

anelastic

viscous

Dissipation, \( Q^{-1} \)

Log_{10}(frequency)

- short time scales
- long time scales
Burgers model replicates experimental observations

\[ \varepsilon < 10^{-4} \]
**Alternative models for transient creep**

**Burgers model**

\[
J(t) = J_M \left(1 + \frac{t}{\tau_M}\right) + J_V \left(1 - \exp\left(-\frac{t}{\tau_V}\right)\right)
\]

**Andrade model**

\[
J(t) = J_M \left(1 + \frac{t}{\tau_M}\right) + \beta t^n.
\]

Transient term of Andrade model contributes indefinitely, unsuitable for extrapolation.
Forced torsional oscillation (frequency domain): Temperature, grain size and frequency dependence of dry, melt-free polycrystalline olivine

![Graphs showing shear modulus vs. oscillation period for different temperatures](image)

Jackson and Faul, 2010
Here \( J_U \) is the unrelaxed (high frequency) compliance of the Maxwell element, equated with the anharmonic shear modulus of olivine at a reference temperature \[ \text{Jackson and Faul, 2010} \]. The compliance of the Voigt element has been replaced with a relaxation strength \( P,B \) for the peak (P) and background (B), respectively. The relaxation strength relates to \( J_V \) as

\[
J_U = \frac{J_R + J_M}{J_M},
\]

with \( J_R = J_M + J_V \), and characterizes the increase in compliance (drop in modulus) due to an anelastic process (see Figure ??).

Temperature (\( T \)), pressure (\( P \)) and grain size (\( d \)) dependence are incorporated in the relaxation times for the peak (\( \tau_P \)), the cut-off times (\( \tau_{L,H} \)) and the Maxwell time (\( \tau_M \):

\[
\tau_i(T,P,d) = \tau_{io} d^m \exp \left( \frac{-E}{PV \cdot R \cdot T} \right)
\]

where \( \tau_{io} \) are reference values, \( m \) is a grain size exponent, \( E \) the activation energy, \( V \) activation volume and \( R \) the gas constant. The grain size exponent for the Maxwell time, \( m_v \), is fixed at a value of 3, corresponding to diffusion creep limited by grain boundary diffusion, the exponent for anelastic processes \( m_a \) is determined from the data.

### Physical Processes of Attenuation

Energy dissipation in melt-free polycrystalline rocks can potentially occur due to point defects, line defects (dislocations) or planar defects (grain boundaries). Calculated relaxation strengths for point defect processes are based on the distribution of relaxation times.

\[
\begin{align*}
J_1(\omega) &= J_U \left\{ 1 + \frac{\alpha \Delta B}{\tau_H^\alpha - \tau_L^\alpha} \int_{\tau_L}^{\tau_H} \frac{\tau^{\alpha-1}}{1 + \omega^2 \tau^2} d\tau \\
&\quad + \frac{1}{\sigma \sqrt{2\pi}} \Delta P \int_0^{\infty} \frac{1}{\tau} \exp \left( \frac{-|\ln(\tau/\tau_P)/\sigma|^2}{2} \right) d\tau \right\}
\end{align*}
\]

\[
\begin{align*}
J_2(\omega) &= J_U \left\{ \frac{\omega \alpha \Delta B}{\tau_H^\alpha - \tau_L^\alpha} \int_{\tau_L}^{\tau_H} \frac{\tau^\alpha}{1 + \omega^2 \tau^2} d\tau \\
&\quad + \frac{\omega}{\sigma \sqrt{2\pi}} \Delta P \int_0^{\infty} \exp \left( \frac{-|\ln(\tau/\tau_P)/\sigma|^2}{2} \right) d\tau + \frac{1}{\omega \tau_M} \right\}
\end{align*}
\]
Physical Model?
- needs to account for:

• grain size dependence
• temperature dependent
• different frequency regimes:
  broad region w. mild frequency dependence, i.e. absorption band; plateau/broad peak; transition to viscous behavior
Grain size-dependent dissipation mechanism: Grain boundary sliding

parameters:
- \(d\) grain size,
- \(\delta\) grain boundary width
- \(\eta_{gb}\) grain boundary viscosity
- \(D_{gb}\) grain boundary diffusivity

results in three distinct processes:
1. Elastically accommodated sliding

viscous sliding of grain boundaries leads to elastic stress concentrations at grain corners

time scale: $\tau_E = \eta_{gb} d/G \delta$

recoverable strain, anelastic process, dissipation peak

After Raj and Ashby, 1971; Raj, 1975
2. Diffusionally assisted sliding

- stress concentrations cause diffusion away from corners
- transient phase is characterised by diffusion over increasing length scales

distribution of relaxation times, transient, recoverable
3. Diffusionally accommodated sliding (steady state)

- time scale: $\tau_D \sim T d^3/G \delta D_{gb}$
- gb normal stresses are highest in center between grain corners (steady state diffusion creep)

1. end of elastically accommodated sliding

2. diffusionally assisted sliding

3. steady state creep

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Raj 1975, Gribb and Cooper, 1998
Frequency domain model
(Morris and Jackson, 2009, Lee et al., 2011)

grain size dependence changes from
~ linear (transient) to cubic (steady state)
Extended Burgers model fit to forced oscillation data for olivine

![Graph showing dissipation (1/Q) vs normalized frequency (f/f_M) for different particle sizes: 3 μm, 30 μm, and 150 μm.](image)
Application to the Upper Mantle
Comparison with velocity models for the Pacific

Nishimura & Forsyth, 1989
Gaherty et al., 1996
Additional mechanism for velocity reduction and attenuation: melt

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Grain-scale melt network, Garapić, Faul and Brisson, G³ 2013
Partially molten rocks: melt ‘squirt’ as dissipation mechanism

Melt squirt:
pressure driven melt flow

After Mavko and Nur, 1975

![Diagram of melt squirt](image)

observed aspect ratios

Aspect ratio

observed aspect ratios

After Schmeling (1985)

viscosity, Pas

unrelaxed

relaxed

basaltic melt

After Schmeling (1985)

dissipation maximum at a particular frequency
Melt present: dissipation peak due to melt ‘squirt’

Jackson et al., 2004, Faul et al., 2004
Shear modulus reduction due to melt

Jackson et al., 2004
At longer periods: normal modes - whole Earth deformation

Sumatra, 26 décembre 2004

Park et al., 2005
Lekic et al., 2009: absorption band from surface waves, normal modes

attenuation decreases again at periods > 1000 s
At even longer periods: tidal dissipation

Benjamin et al., 2006: attenuation becomes frequency dependent and increases at > 10000s
Frequency domain model
(Morris and Jackson, 2009, Lee et al., 2011)

Grain size dependence changes from linear (transient) to cubic (steady state)

\[ Q^{-1} \sim (\omega \tau_D)^{-1} \]
\[ Q^{-1} \sim d^{-m} \]

\[ Q^{-1} \sim (\omega \tau_D)^{-\alpha} \]
\[ Q^{-1} \sim d^{-m\alpha} \]

\[ Q^{-1} \sim (\omega \tau_A)^{-1} \]
\[ Q^{-1} \sim d^{-1} \]
Two key parameters used in deciphering both lunar evolution and the lunar samples are the average thickness of the crust and how the crustal thickness varies from place to place. As an example, the average crustal thickness is believed to be a direct by-product of the Moon's initial differentiation; it therefore depends upon several factors, including the depth of the magma ocean and the efficiency with which crystallizing plagioclase was able to float (e.g. Solomatov 2000). Data obtained from seismometers placed on the lunar surface at the Apollo 12, 14, 15, and 16 stations offer the most direct means of assessing this quantity (see Figure 1 for the locations of the Apollo and Luna sampling stations and the outline of the Apollo seismic network). Analyses during the Apollo era suggested initially that the crust beneath the “Apollo zone” was about 60 km thick. However, recent independent analyses by two different research groups now suggest that the crustal thickness is much less in this area, probably somewhere between about 30 and 38 km (Khan and Mosegaard 2002; Lognonné et al. 2003). While one might think that this dramatic revision would have large consequences on estimates of the bulk composition of the Moon, it is now realized that the globally averaged thickness of the crust is somewhat greater than that measured beneath the Apollo stations (the globally averaged thickness is probably between 40 and 45 km; see Chenet et al. 2006 and Hikida and Wieczorek 2007). Nevertheless, as a complicating factor, several measurements show that there are also large lateral and vertical variations in the composition of the crust, which were not fully appreciated until after the Clementine and Lunar Prospector missions (Jolliff et al. 2000). In particular, orbital gamma-ray data and in situ heat-flow measurements indicate that heat-producing and incompatible elements are concentrated within a single geologic province that encompasses Mare Imbrium and Oceanus Procellarum (i.e. the Procellarum KREEP Terrane; see Wieczorek and Phillip 2000), and remote sensing data show that the surrounding highlands crust becomes increasingly mafic with depth. One of the problems with the Apollo seismic data is that the network spans only a small portion of the Moon's central nearside hemisphere. Fortunately, the thickness of the crust can be estimated outside of the Apollo zone by using a combination of the Moon's surface relief, gravitational field, and reasonable assumptions about the density of the crust and mantle. Figure 2 shows the topography of the Moon (upper left) obtained by the Clementine mission (Smith et al. 1997; USGS 2002), as well as the best estimate of the radial gravitational acceleration at the surface (upper right), derived primarily from the Lunar Prospector mission (Konopliv et al. 2001). Since the gravitational field is obtained by measuring small Doppler shifts in radio signals emitted by orbiting spacecraft, the resolution over the farside hemisphere of the Moon, where spacecraft are not visible from Earth, is considerably poorer than for the nearside hemisphere. Immediately visible in the topographic image are the depressions associated with numerous giant impact basins—the giant basin on the farside hemisphere is the South Pole–Aitken basin, which is currently the largest recognizable impact structure in the solar system. Furthermore, some of the impact basins are seen to possess large central positive gravitational anomalies, which are colloquially referred to as “mascons,” short for mass concentrations. While the origin of mascons is not completely resolved, it is likely that they result from a combination of dense mare basaltic lava flows (which are perhaps several kilometers thick in places) and structural uplift of relatively dense mantle materials (for a review, see Wieczorek et al. 2006).
Model from seismic data

Weber et al., 2011
temperature increases - velocity has to decrease for fixed composition
Attenuation: no melt required
geodetic and dissipation data and model

inferred temperature

Core heat flux

T at base of mantle (~1400 km depth)
Mars

Geophysical lander in 2016 (InSight mission), including a seismometer
Existing data provides constraints on density, composition etc. allows forward modelling of temperature, seismic properties.
Tidal deformation (Phobos) places constraints on Q and k2

As for Moon, tidal deformation indicates overall moderate temperatures in the interior, substantially below solidus.