1. A few words about EarthScope and USArray
2. Surface-wave studies of the crust and mantle
3. Tomography using noise and Aki’s method
4. Remarkable images of US crust (and basins)!
Unlocking the Secrets of the North American Continent

An EarthScope Science Plan for 2010–2020
Transportable Array Installation Plan
as of October 2009

Year
- 2004
- 2005
- 2006
- 2007
- 2008
- 2009
- 2010
- 2011
- 2012
- 2013
- RefNet
- Existing

Station removal follows in 24 months

Beginning 2014
The speed of surface waves (Love and Rayleigh) depend on the shallow structure of the Earth
Sensitivity of surface wave velocities to elastic structure at depth

200 seconds

300 km

20 seconds

30 km
Rayleigh wave sensitivity to Vs
Requirements for high-resolution (~50 km) surface-wave tomography:

1. short paths to resolve small structures
2. short periods (5 < T < 25 sec) to resolve shallow (crustal) structure
3. evenly distributed sources (earthquakes) to create a tomographic image

These are not met by traditional earthquake-based techniques
Proposition:
The cross correlation of background seismic noise recorded at two stations provides information about the propagation speed (the phase velocity) of surface waves between the two stations.

Explored by many, e.g., Aki, Campillo, Cox, Lobkis, Ritzwoller, Sabra, Shapiro, Snieder and many others, also in other fields
Gaussian white noise, 1 Hz sampling
Gaussian white noise spectrum
Two stations, P and Q, separated by L km

What is the cross correlation of noise signals recorded at P and Q?

\[ R_{PQ}(\tau) = \frac{1}{T} \int_{0}^{T} s_P(t)s_Q(t + \tau) \, dt \]
Auto-correlation function of noise

The exponent in the integrand of equation ?? has stationary points at \( \theta = 0 \) and \( \theta = \pi \). With two contributions to the stationary-phase approximation we obtain

\[
\mathcal{R}_{PQ}(t) = A_0 \theta \exp(\iota \omega t) \theta^2 \kappa L \frac{1}{2} \exp \left( \frac{-\kappa L}{4} \right) + \exp \left( \frac{\iota \kappa L}{4} \right),
\]

(14)

which can also be written

\[
\mathcal{R}_{PQ}(t) = A_0 \theta^2 \kappa L \frac{1}{2} \exp \left( \iota \left( \omega t + \frac{\kappa L}{4} \right) \right) + \exp \left( \iota \left( \omega t + \frac{\kappa L}{4} \right) \right),
\]

(15)

highlighting the \( \pi/4 \) phase advance with respect to the phase delay resulting from plane-wave propagation between \( P \) and \( Q \). It should be noted, however, that for a single frequency \( \omega \) the integral also simplifies to

\[
\mathcal{R}_{PQ}(t) = A_0 \theta^2 \kappa L \frac{1}{2} \cos \left( \frac{\kappa L}{4} \right),
\]

(16)

which is a simple cosinusoid with the amplitude modulated by the distance between the two stations. That is, the separation of the cross-correlation function into two contributions observed in equation ?? is only apparent; since \( A_0 \theta \) normally is unknown, the argument of the cosine cannot be recovered from \( \mathcal{R}_{PQ}(t) \).

\[
\mathcal{R}_{PQ}(\tau) = \mathcal{R}_{PP}(t + L_c \cos \theta),
\]

(17)

\[
\mathcal{R}_{PP}(t) = \mathcal{R} \mathcal{P}(t)
\]

(18)

\[
\Re \{ s(\omega) \} = c_0 J_0 \left( \frac{\kappa L_c}{4} \right)
\]

(19)

\[
\Im \{ s(\omega) \} = c_1 J_1 \left( \frac{\kappa L_c}{4} \right)
\]

(20)

\[
s(\omega) = a_0 \exp(\iota \kappa t)
\]

(21)

1.3 The basic equations for the spectrum

Following Cox (1973), we write that the normalized cross-spectral density function \( \mathcal{Q}(\omega, \kappa'; \theta) \) is

\[
\mathcal{Q}(\omega, \kappa'; \theta) = \frac{1}{2} \pi \mathcal{G}(\kappa', \omega) \exp \left\{ \iota \left( \frac{\kappa L_c}{4} \right) \cos \left( \frac{\kappa L_c}{4} \right) \right\}
\]

(23)

where \( \mathcal{G}(\kappa', \omega) \) is a function that describes the power of the azimuthal density of plane wave incoming from azimuth \( \kappa' \) at the frequency \( \omega \). \( \mathcal{G}(\kappa', \omega) \) is a real and nonnegative function that can be expressed using a cylindrical harmonic expansion,

\[
\mathcal{G}(\kappa', \omega) = \sum_{m=0}^{\infty} a_m(\omega) \cos(\kappa' \theta) + b_m(\omega) \sin(\kappa' \theta)
\]

(24)
distance: $L=200$ km
speed: $c=3$ km/s

one noise source
station azimuth: $\theta=0$ deg.

Plane wave of noise incident on two stations, P and Q
The exponent in the integral of equation 10 has stationary points at $t = 0$ and $t = \pm \frac{L \cos \theta}{c}$.

With two contributions to the stationary-phase approximation we obtain

$$R_{PQ}(t) = A_0 \exp(i \frac{L}{c} \cos \theta) + \exp(i \frac{L}{c} \cos \theta),$$

highlighting the phase advance with respect to the phase delay resulting from plane-wave propagation between $P$ and $Q$.

It should be noted, however, that for a single frequency $\omega$ the integral also simplifies to

$$R_{PQ}(t) = A_0 \exp(i \frac{L}{c} \cos \theta),$$

which is a simple cosine with the amplitude modulated by the distance between the two stations.

That is, the separation of the cross-correlation function into two contributions observed in equation 15 is only apparent; since $A_0$ normally is unknown, the argument of the cosine cannot be recovered from $R_{PQ}(t)$.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$
Plane wave of noise incident on two stations, P and Q

one noise source station azimuth 180 deg.
Cross-correlation function, P and Q

one noise source
station azimuth: $\theta = 180$ deg.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$

deep}

Cross-correlation function, P and Q

one noise source
station azimuth: $\theta = 180$ deg.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$

Cross-correlation function, P and Q

one noise source
station azimuth: $\theta = 180$ deg.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$

Cross-correlation function, P and Q

one noise source
station azimuth: $\theta = 180$ deg.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$

Cross-correlation function, P and Q

one noise source
station azimuth: $\theta = 180$ deg.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$

Cross-correlation function, P and Q

one noise source
station azimuth: $\theta = 180$ deg.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$
one noise source
station azimuth 90 deg.
The exponent in the integral of equation 10 has stationary points at \( t = 0 \) and \( t = \pm \varepsilon \).

With two contributions to the stationary-phase approximation we obtain

\[
R_{PQ}(t) = A_0 \exp\left(i\varepsilon t - k L \frac{1}{2} t^2\right) + \exp\left(i k L + i\varepsilon t \right),
\]

which can also be written

\[
R_{PQ}(t) = A_0 \exp\left(2k L \frac{1}{2} t^2\right) \cos\left(k L + \varepsilon t\right) + \exp\left(i k L + i\varepsilon t \right),
\]

highlighting the \( \varepsilon / 4 \) phase advance with respect to the phase delay resulting from plane-wave propagation between P and Q.

It should be noted, however, that for a single frequency \( \varepsilon \), the integral also simplifies to

\[
R_{PQ}(t) = A_0 \exp\left(2k L \frac{1}{2} t^2\right) \cos\left(k L \varepsilon^2\right),
\]

which is a simple cosinusoid with the amplitude modulated by the distance between the two stations.

That is, the separation of the cross-correlation function into two contributions observed in equation 15 is only apparent; since \( A_0 \) normally is unknown, the argument of the cosine cannot be recovered from \( R_{PQ}(t) \).
one noise source
station azimuth 45 deg.
The exponent in the integrand of equation 10 has stationary points at $t = 0$ and $\pm \xi$. With two contributions to the stationary-phase approximation we obtain

$$R_{PQ}(t) = A_0 \exp(i \xi t) 2 \pi \xi L \exp(-i \frac{\xi}{4} L) + \exp(i \xi t + \frac{\pi}{4} L + i \xi \xi L),$$

(14)

which can also be written

$$R_{PQ}(t) = A_0 2 \pi \xi L \exp(i \xi t + \frac{\pi}{4} L + i \xi \xi L) + \exp(i \xi t + \frac{\pi}{4} L + i \xi \xi L),$$

(15)

highlighting the $\xi / 4$ phase advance with respect to the phase delay resulting from plane-wave propagation between $P$ and $Q$.

It should be noted, however, that for a single frequency $\omega$ the integral also simplifies to

$$R_{PQ}(t) = A_0 2 \pi \xi L \cos(\xi L + \xi \xi L),$$

(16)

which is a simple sinusoid with the amplitude modulated by the distance between the two stations. That is, the separation of the cross-correlation function into two contributions observed in equation 15 is only apparent; since $A_0$ normally is unknown, the argument of the cosine cannot be recovered from $R_{PQ}(t)$.

$$R_{PQ}(t) = R_{PP}(t + \frac{L}{c} \cos \theta),$$

(17)
two noise sources
station azimuth 0 deg.
Cross-correlation function, P and Q
four noise sources
station azimuth 0 deg.
Cross-correlation function, P and Q
10 noise sources
azimuth 0 deg.
Cross-correlation function, P and Q
180 noise sources

stochastic surface waves
Cross-correlation function, P and Q

10 hours of noise data
Cross-correlation function, P and Q

200 hours of noise data
sine function
\[ y = \sin(\theta) \]

pdf of sine function
\[ x = p(y) \]
Suppose a point of stationary phase exists such that

\[ f = \frac{1}{\sqrt{(L/c)^2 - t^2}}, \]

\[ |t| < L/c. \]

The stationary-phase approximation can be used to evaluate integrals of the form

\[ \left[ \int_{-\infty}^{\infty} \exp(i x) \, dx \right] \sim \left[ \int_{-\infty}^{\infty} \exp(i x) \, dx \right] \text{ if } \pm x > \text{large}, \]

\[ \sim 0 \text{ if } |x| < \text{large}. \]
What about the Fourier transform?

\[
\frac{1}{\sqrt{(L/c)^2 - t^2}} \quad \rightarrow \quad J_0\left(\frac{\omega L}{c}\right)
\]
The spectrum of the cross-correlation function is shown in the graph. The real and imaginary parts are highlighted separately. The graph displays the function 

$\text{Re} \{ s(\omega) \} = c_0 J_0 \left( \frac{\omega L}{c} \right)$

and 

$\text{Im} \{ s(\omega) \} = c_1 J_1 \left( \frac{\omega L}{c} \right)$

for various values of frequency (Hz).
\[
\bar{\rho}(r, \omega_0) = J_0 \left( \frac{\omega_0}{c(\omega_0)} r \right)
\]

“This formula clearly indicates that if one measures \(\bar{\rho}(r, \omega_0)\) for a certain \(r\) and for various \(\omega_0\)'s, he can obtain the function \(c(\omega_0)\), i.e., the dispersion curve of the wave for the corresponding range of frequency \(\omega_0\).”

Aki, 1957
Matching zero crossings for dispersion

\[ c(\omega_n) = \frac{\omega_n r}{z_n} \]

D07A-B04A
282 km
Recipe for tomographic success:

1. Correlate continuous recorded signals at all pairs of USArray stations in 4-h windows (note - this is a big calculation)
2. Stack all correlation functions for each pair
3. Determine zero crossings of stacked cross-correlation spectra
4. Determine phase velocities using Aki’s formula
5. Invert phase-velocity observations to determine phase-velocity maps
What are we looking at?

Elastic structure of the crust

Including strong signals of slow sediments
Correlation of low velocities with thick sediments (sediment data from Mooney and Kaban, 2010)

Love waves, 5 sec period

Phase velocity (km/s)

Sediment thickness (km)
1. The Transportable Array of USArray allows spatially uniform mapping of surface-wave dispersion across the US using noise tomography

2. Aki’s spectral approach works well for automation

3. Extremely slow Love and Rayleigh velocities along the Gulf coast (and in other areas) are not matched by current models of the crust

4. Very low VS is needed (high VP/VS ratio) to explain the signals from the basins