Cooperative Institute for Dynamic Earth Research

Introduction to Seismology - Body Waves

Seismology Lecture #1

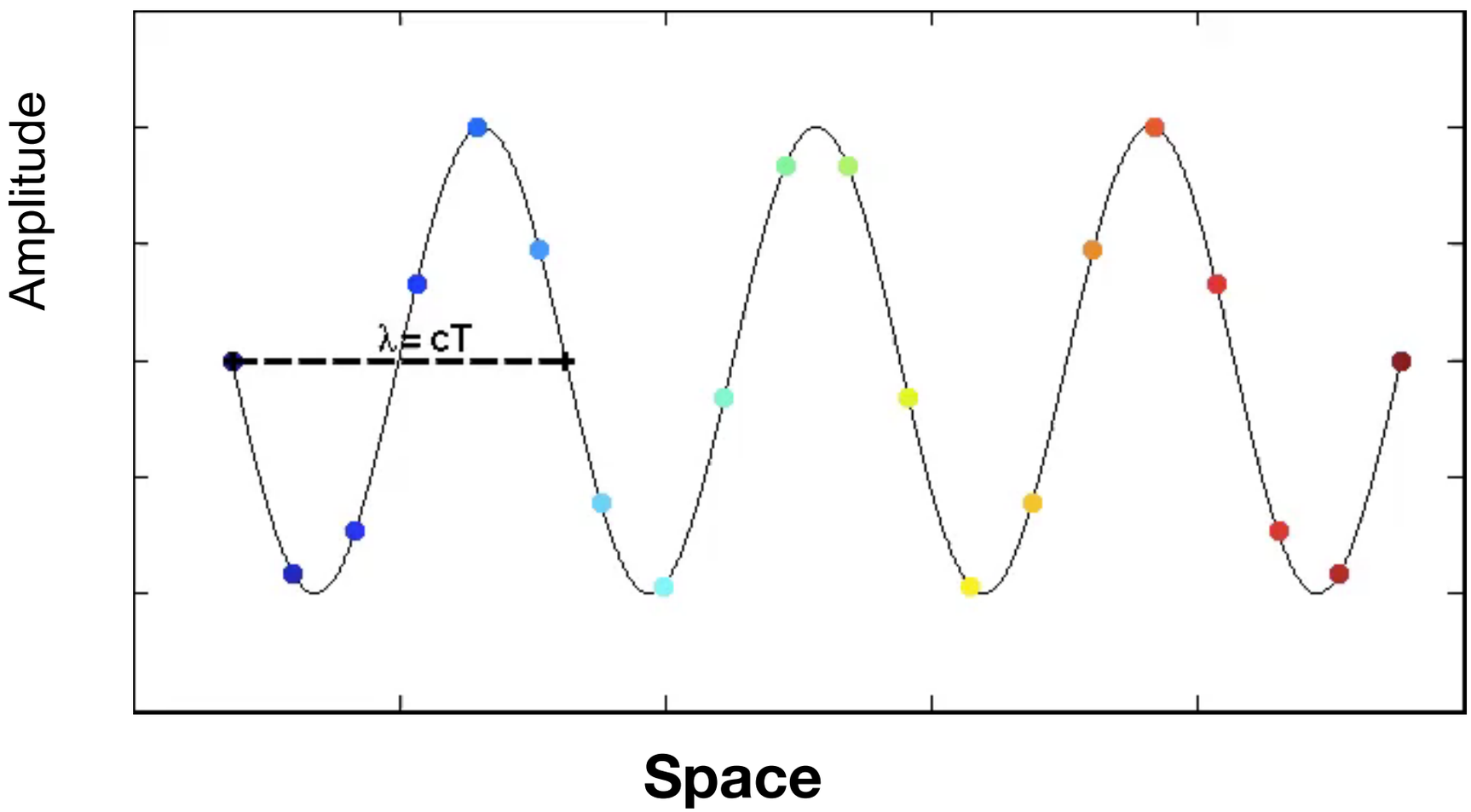
Sanne Cottaar
University of Cambridge

Types of seismology

- ***Theoretical Seismology -*** Study of the physics of wave propagation
- ***Observational Seismology -***
  - ***Source seismology*** - study of earthquakes and other seismic sources
  - ***Structural seismology*** - study of the structures waves propagate through
    - Active source seismology - exploration
    - Local and regional imaging
    - **Global seismology**

Waves

single frequency wave



Period = Frequency $^{-1}$
$T = 1/f$

Wavelength = Wavenumber^{-1}
$\lambda = 1/k$

Speed (c):
$c = \lambda f = f/k = \lambda/T$

The Wave Equation…

The Wave Equation

- Newton's 2nd law: $\boldsymbol{ma} = \boldsymbol{F}$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \sigma_{ij} + f_i$$

density

acceleration

stress tensor

body forces:
e.g. earthquakes,
gravity

volume integration is dropped out of each term of this equation

The Wave Equation

- Newton's 2nd law: $\boldsymbol{ma} = \boldsymbol{F}$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \sigma_{ij} + f_i$$

- $\boldsymbol{i}$ is 1,2, or 3, representing three equations in 3D
- $\boldsymbol{j}$ represents a summation

$$\partial_j \sigma_{ij} = \partial_1 \sigma_{i1} + \partial_2 \sigma_{i2} + \partial_3 \sigma_{i3}$$

The Wave Equation

- Newton's 2nd law: $\boldsymbol{ma} = \boldsymbol{F}$

$$\rho(\boldsymbol{x}, t)\frac{\partial^2 u_i(\boldsymbol{x}, t)}{\partial t^2} = \partial_j \sigma_{ij}(\boldsymbol{x}, t) + f_i(\boldsymbol{x}, t)$$

- Everything is a function of **space and time**

The Wave Equation

- Newton's 2nd law: $\boldsymbol{ma} = \boldsymbol{F}$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \sigma_{ij} + f_i$$

$$\rho(\boldsymbol{x},t)\frac{\partial^2 u_1(\boldsymbol{x},t)}{\partial t^2} = \partial_1\sigma_{11}(\boldsymbol{x},t) + \partial_2\sigma_{12}(\boldsymbol{x},t) + \partial_3\sigma_{13}(\boldsymbol{x},t) + f_1(\boldsymbol{x},t)$$

$$\rho(\boldsymbol{x},t)\frac{\partial^2 u_2(\boldsymbol{x},t)}{\partial t^2} = \partial_1\sigma_{21}(\boldsymbol{x},t) + \partial_2\sigma_{22}(\boldsymbol{x},t) + \partial_3\sigma_{23}(\boldsymbol{x},t) + f_2(\boldsymbol{x},t)$$

$$\rho(\boldsymbol{x},t)\frac{\partial^2 u_3(\boldsymbol{x},t)}{\partial t^2} = \partial_3\sigma_{31}(\boldsymbol{x},t) + \partial_2\sigma_{32}(\boldsymbol{x},t) + \partial_3\sigma_{33}(\boldsymbol{x},t) + f_3(\boldsymbol{x},t)$$

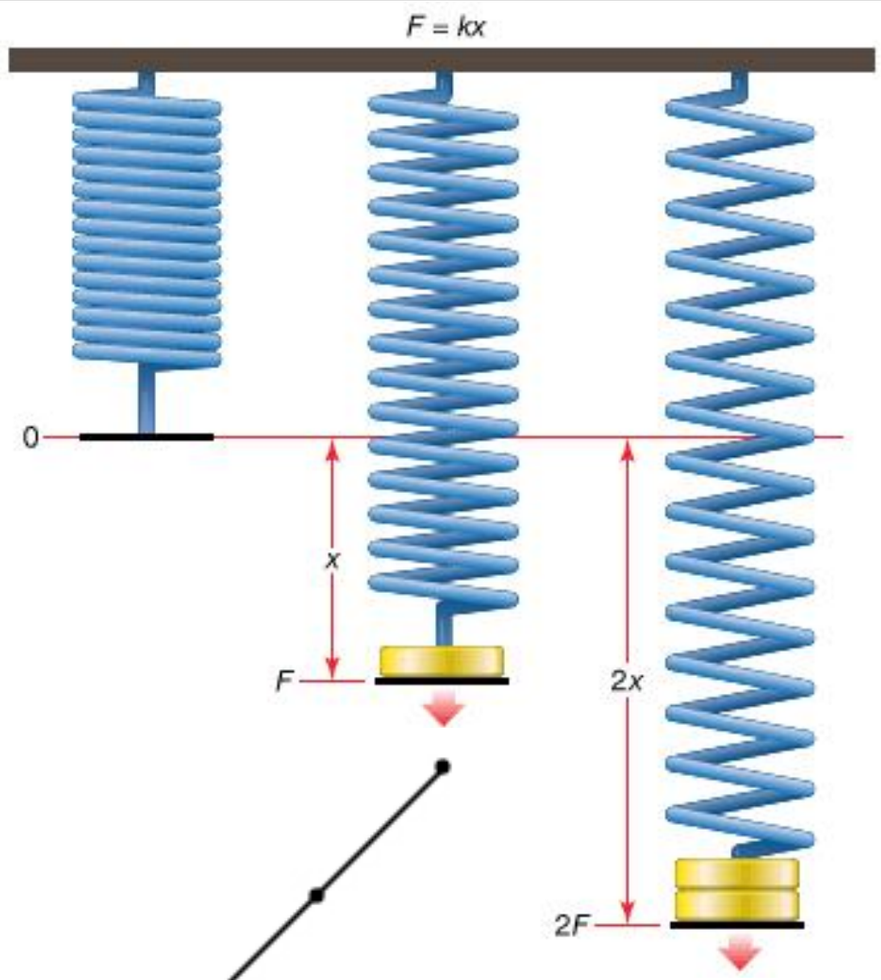
The Wave Equation

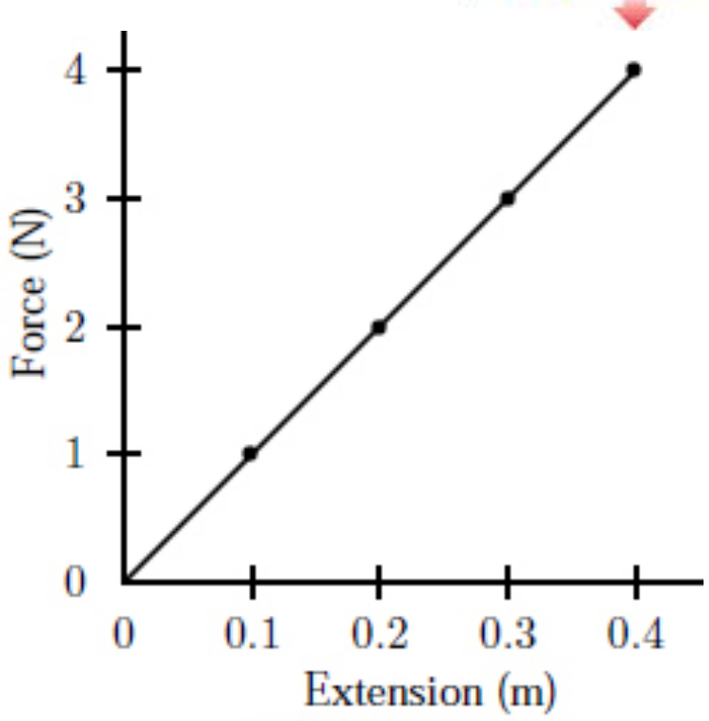
- Newton's 2nd law: $\boldsymbol{ma} = \boldsymbol{F}$

$$\rho(\boldsymbol{x},t)\frac{\partial^2 u_i(\boldsymbol{x},t)}{\partial t^2} = \partial_j \sigma_{ij}(\boldsymbol{x},t) + f_i(\boldsymbol{x},t)$$

- What forces are created when a medium is deformed?
- This is described by so-called ***constitutive relationships***

Hooke's Law





- For a spring, this is described by linear 1D Hooke's law

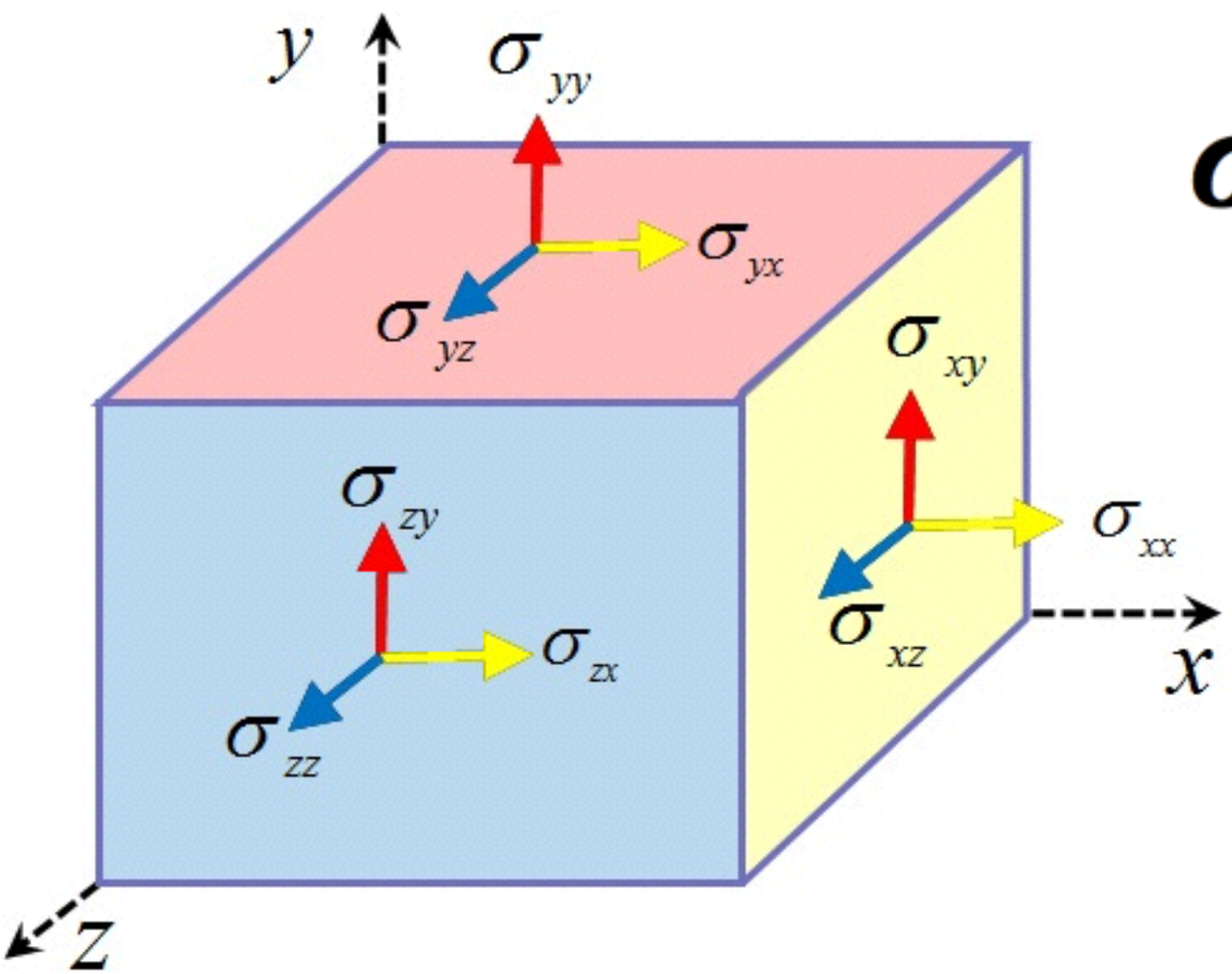
$$F = k\,x$$

force

spring constant

deformation

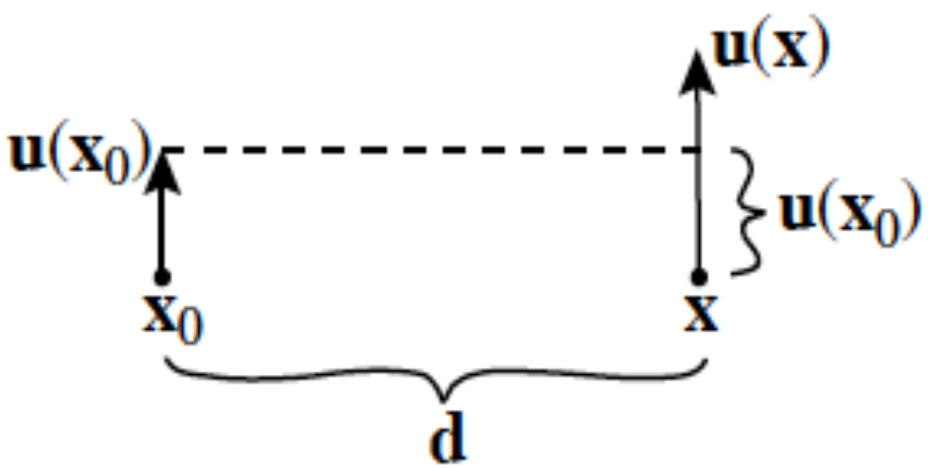
Stress tensor - 3D

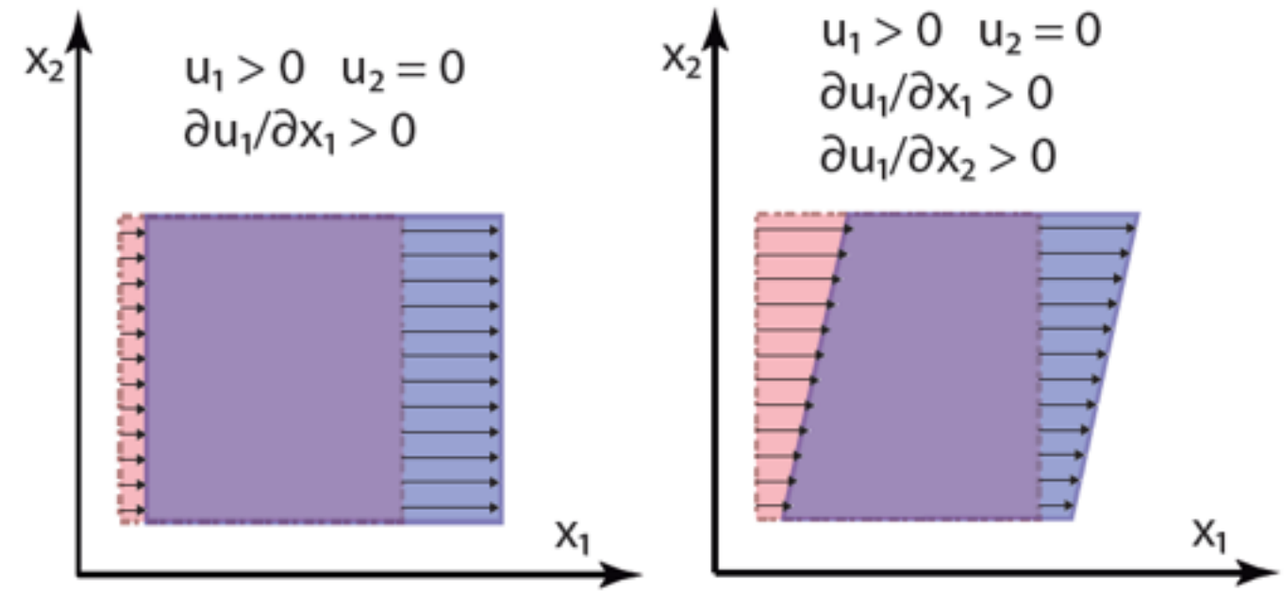


$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

- 3 Normal stresses
- 6 Shear stresses

Strain tensor - 3D deformation tensor





$$u_i(\boldsymbol{x}+\delta\boldsymbol{x}) = u_i(\boldsymbol{x}) + \frac{\partial u_i(\boldsymbol{x})}{\partial x_j}\delta x_j$$

- To first order: $\delta u_i(\boldsymbol{x}) = \dfrac{\partial u_i(\boldsymbol{x})}{\partial x_j}\delta x_j$

component of deformation tensor [dimensionless]

Strain tensor - 3D

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) =$$

$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right) & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

Strain tensor - symmetric part
(anti-symmetric component represents rotation, which has no internal deformation)

Strain tensor - 3D

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) =$$

$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right) & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

Volume change
or *dilatation*

$$\theta = \varepsilon_{ii} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \nabla \cdot \boldsymbol{u}$$

Hooke's Law for a continuous medium

- Material is assumed *linearly elastic*, which is valid over small strains and short times scales:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

[Pa] [Pa] [dimensionless]

C_{ijkl} : 81 (=3^4) elastic constants relating the 9 stress component to the 9 strain components

- Applying symmetries $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$

'only' 21 components are retained.

Assuming isotropy - only 2 elastic constants

- Simplifying Hooke's Law to an isotropic continuum:

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}$$

Kronecker Delta
=1 if i=j

Lamé constants

Assuming isotropy - only 2 elastic constants

- Simplifying Hooke's Law to an isotropic continuum:

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}$$

Kronecker Delta Lamé constants

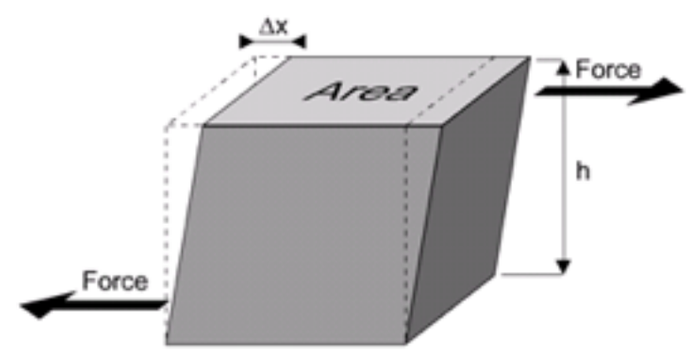
$$\boldsymbol{\sigma} = \begin{bmatrix} \lambda\theta + 2\mu\varepsilon_{11} & 2\mu\varepsilon_{12} & 2\mu\varepsilon_{13} \\ 2\mu\varepsilon_{12} & \lambda\theta + 2\mu\varepsilon_{22} & 2\mu\varepsilon_{23} \\ 2\mu\varepsilon_{13} & 2\mu\varepsilon_{23} & \lambda\theta + 2\mu\varepsilon_{33} \end{bmatrix}$$

Assuming isotropy - only 2 elastic constants

- Simplifying Hooke's Law to an isotropic continuum:

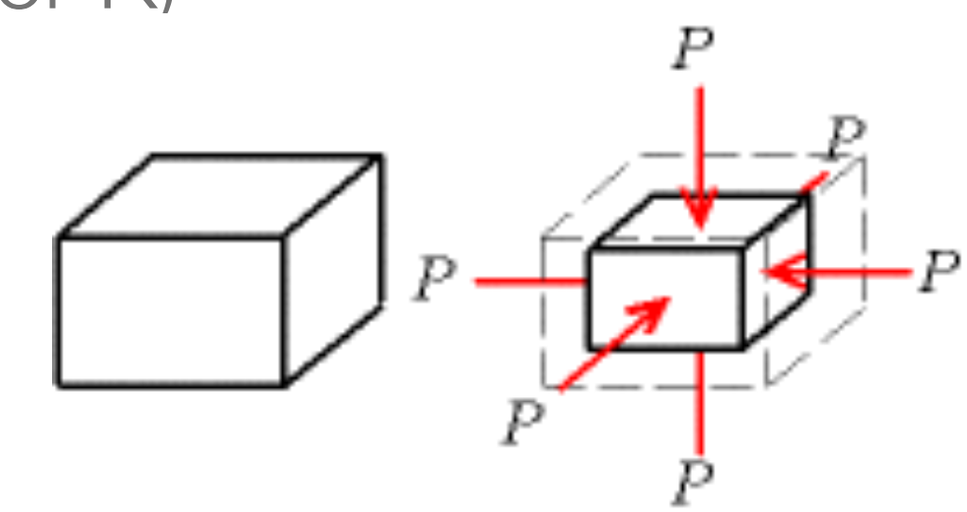
$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}$$

- μ: shear modulus (also G)



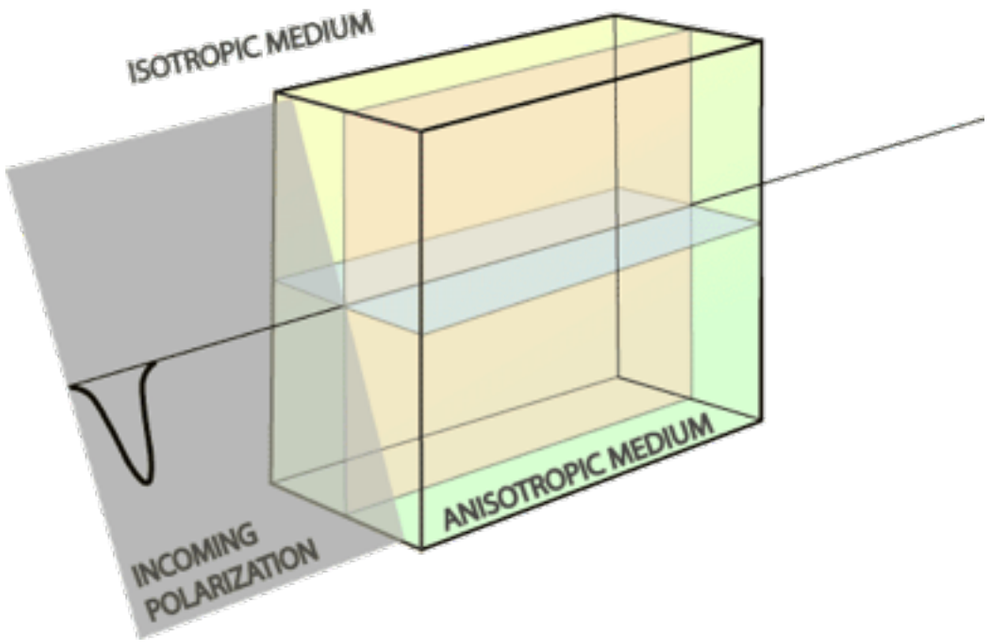
- λ relates to the bulk modulus (κ or K)

$$\kappa = \lambda + \frac{2}{3}\mu = -V\frac{dP}{dV}$$



Spoiler alert!

- The Earth is not purely elastic, there is an anelastic component
- The Earth is not purely isotropic, there is also has an *anisotropic* component and we need more than two *elastic moduli* to describe this



Maureen Long's lecture on anisotropy and attenuation: Seismology #2

The Wave Equation

- Using Hooke's law for an isotropic medium $\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}$
- And the definition of the strain tensor
- And ignoring body force (assuming far-field from an earthquake)
- And using $\nabla^2\boldsymbol{x} = \nabla(\nabla\cdot\boldsymbol{x}) - \nabla\times(\nabla\times\boldsymbol{x})$

We get the ***Seismic Wave Equation*** for an isotropic medium:

$$\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2} = \nabla\lambda(\nabla\cdot\boldsymbol{u}) + \nabla\mu\cdot[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda + 2\mu)\nabla\nabla\cdot\mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u}$$

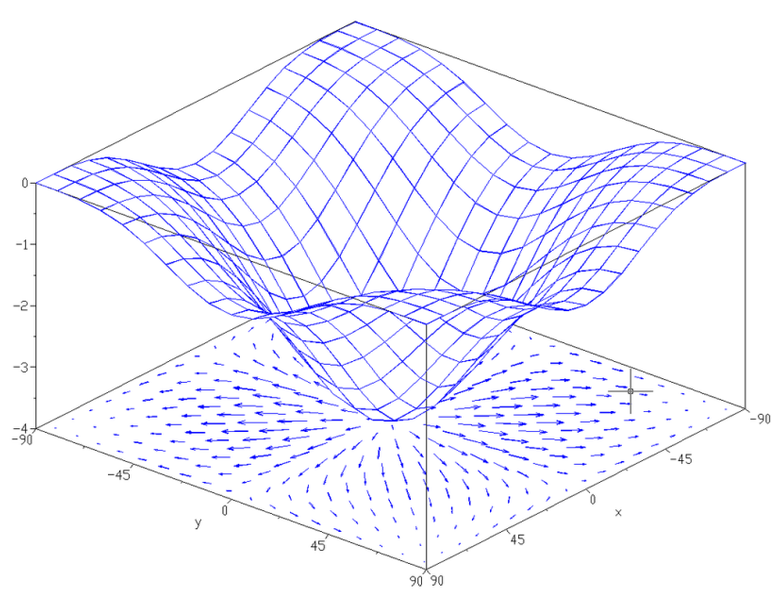
The Wave Equation

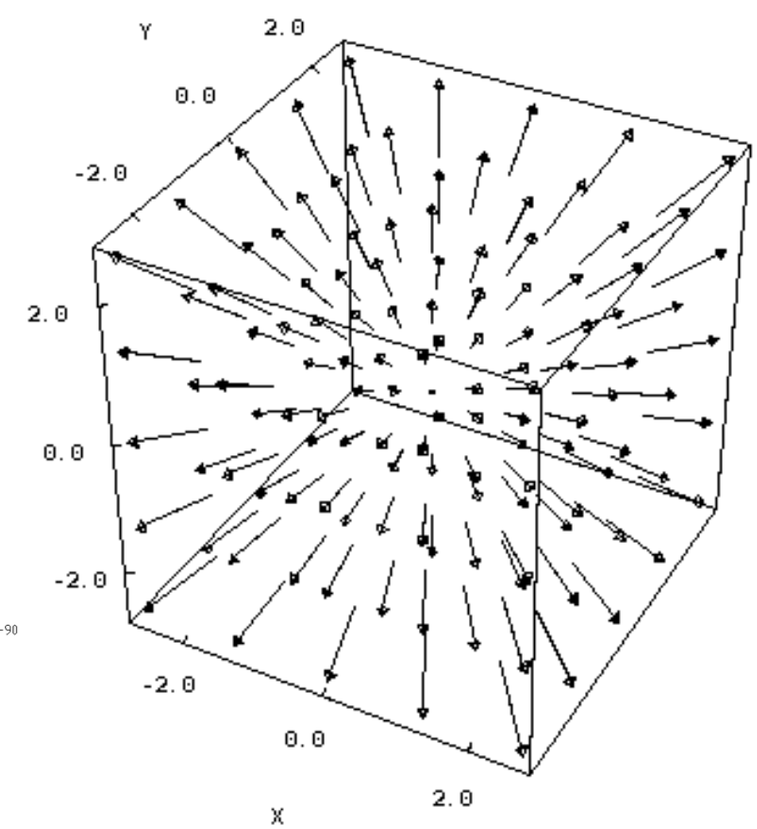
$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \nabla\lambda(\nabla \cdot \boldsymbol{u}) + \nabla\mu \cdot [\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda + 2\mu)\nabla\nabla \cdot \mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u}$$

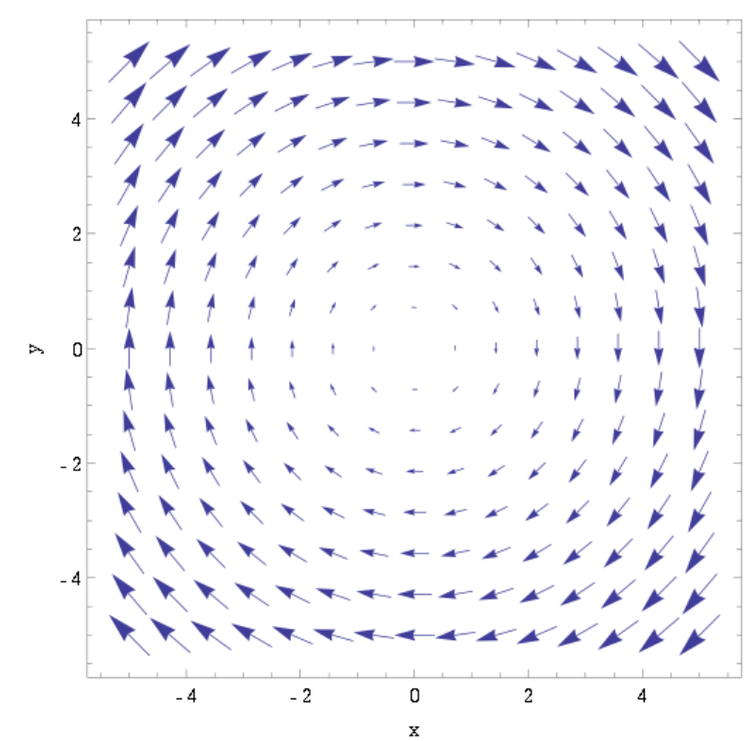
Gradient: Vector describing the direction and magnitude of change in a quantity

Divergence: Scalar describing volume change

Curl: Vector describing infinitesimal rotation (think paddle wheel)







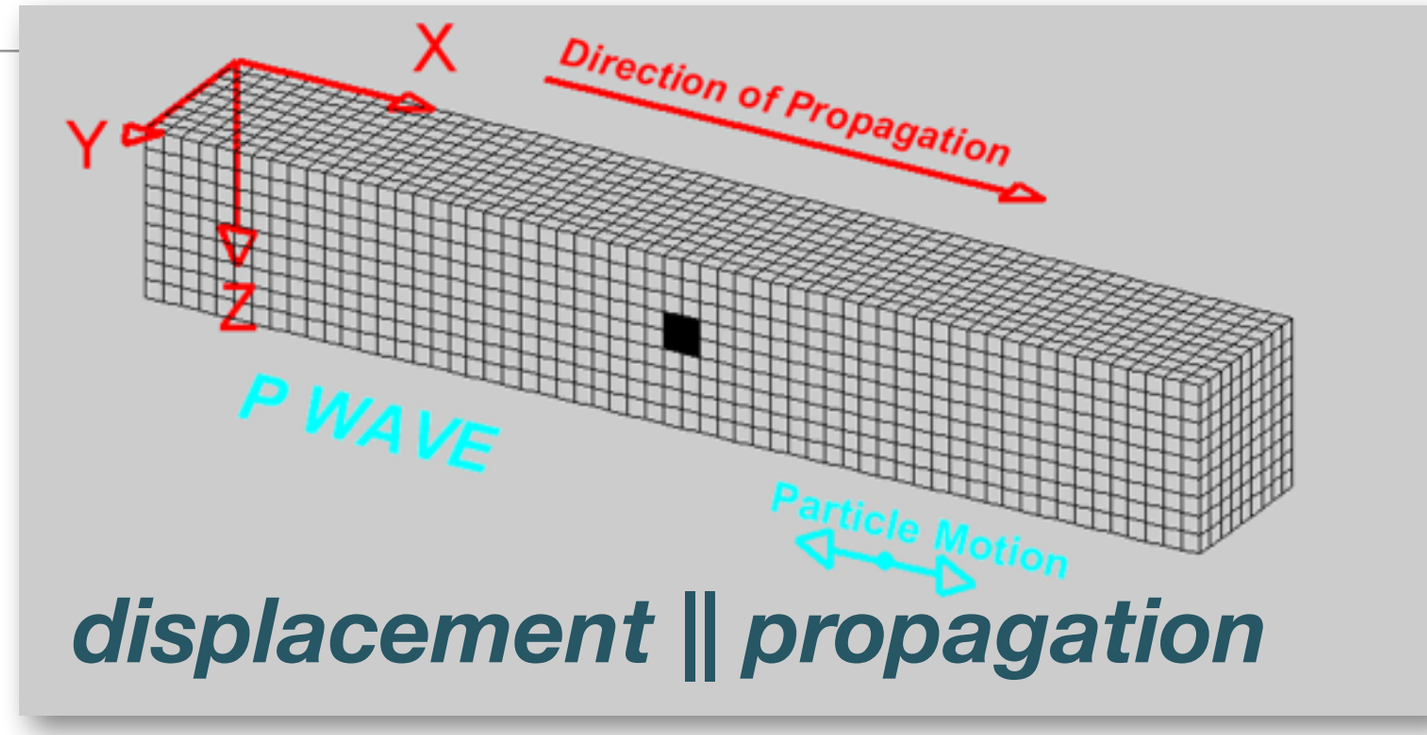
Spoiler Alert!

We can solve the wave equation

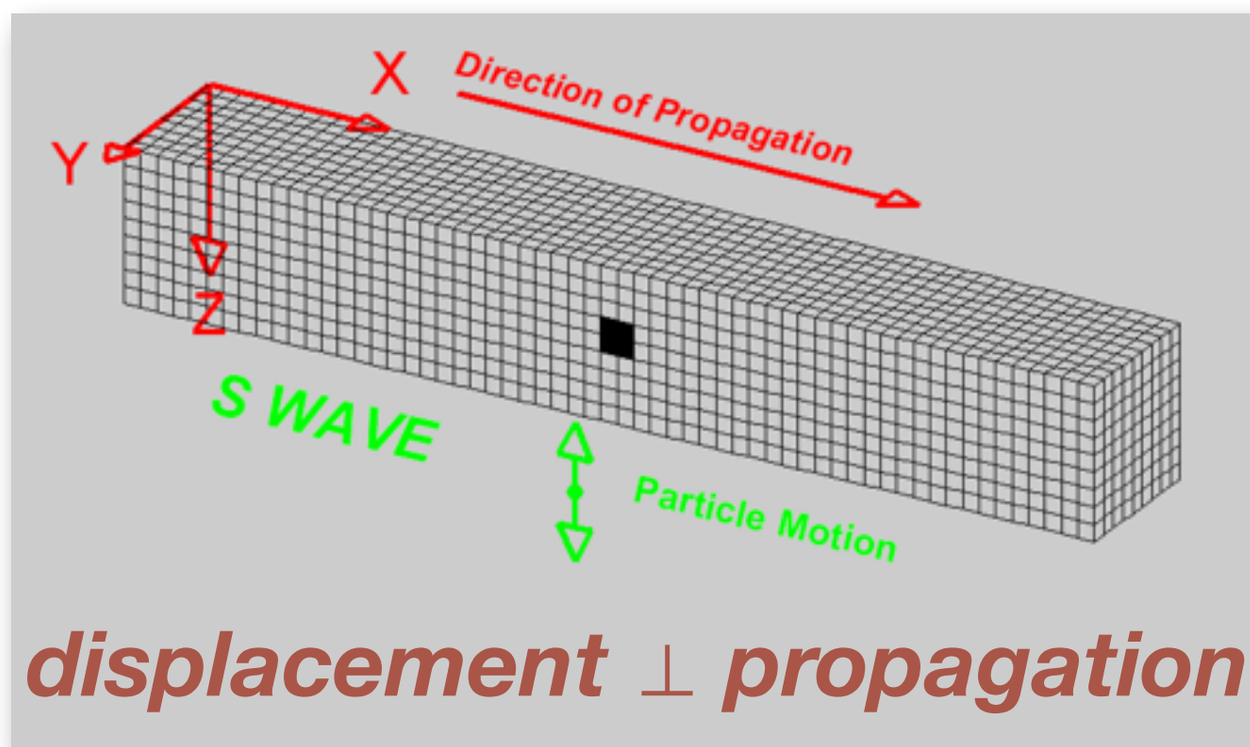
Ebru Bozdag's lecture on solving the wave equation
Seismology #4

Wave types: Body waves

P wave
Compressional or longitudinal waves



S wave
Shear or transverse waves



Wave types: Body waves

- Assuming a homogeneous medium $\nabla\lambda = \nabla\mu = 0$

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \nabla\lambda(\nabla \cdot \boldsymbol{u}) + \nabla\mu \cdot [\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda + 2\mu)\nabla\nabla \cdot \mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u}$$

Wave types: Body waves

- Assuming a homogeneous medium

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = (\lambda + 2\mu)\nabla\nabla \cdot \mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u}$$

- Represent by two Helmholtz potentials

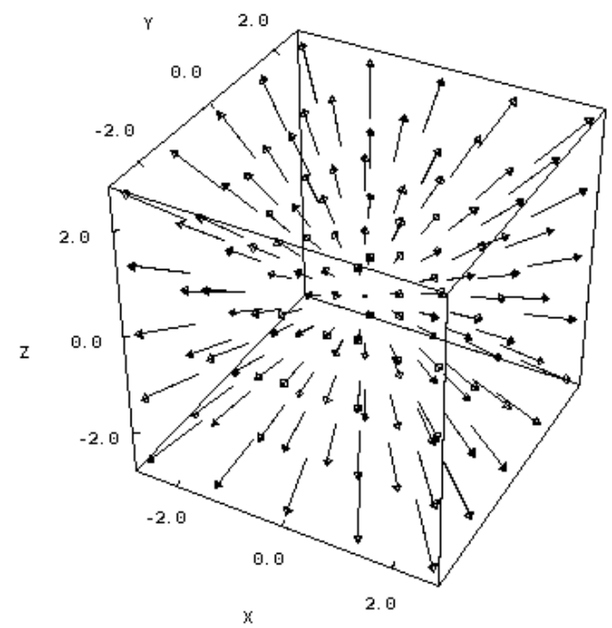
$$\boldsymbol{u} = \nabla\varphi + \nabla \times \boldsymbol{\psi}$$

Scalar potential
has no curl
$\nabla \times \nabla\varphi = 0$
-> Compressional waves

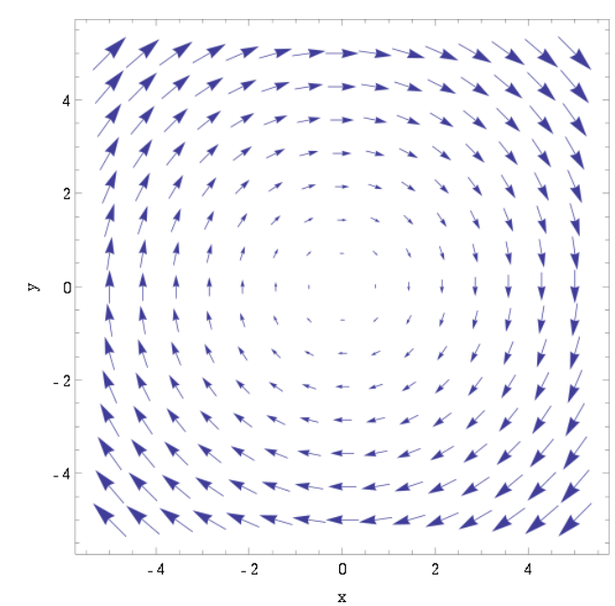
Vector potential
has no divergence
$\nabla \cdot (\nabla \times \boldsymbol{\psi}) = 0$
-> Shear waves

Wave types: Body waves

- $\boldsymbol{u} = \nabla\varphi + \nabla\times\boldsymbol{\psi}$ and again using $\nabla^2\boldsymbol{x} = \nabla(\nabla\cdot\boldsymbol{x}) - \nabla\times(\nabla\times\boldsymbol{x})$
  - gives: $\nabla^2\varphi = \nabla\cdot\boldsymbol{u}$



  - and : $\nabla^2\boldsymbol{\psi} = -\nabla\times(\nabla\times\boldsymbol{\psi}) = -\nabla\times\boldsymbol{u}$



Wave types: Body waves

- fill in $\boldsymbol{u} = \nabla\varphi + \nabla \times \boldsymbol{\psi}$ and using $\nabla^2 \boldsymbol{x} = \nabla(\nabla \cdot \boldsymbol{x}) - \nabla\times(\nabla\times\boldsymbol{x})$

$$\nabla\left[(\lambda + 2\mu)\nabla^2\varphi - \rho\frac{\partial^2\varphi}{\partial t^2}\right]$$
$$= -\nabla \times \left[\mu\nabla^2\boldsymbol{\psi} - \rho\frac{\partial^2\boldsymbol{\psi}}{\partial t^2}\right]$$

- One solution: both sides of the equation equal zero.

Wave types: Body waves

$$\nabla\left[(\lambda+2\mu)\nabla^2\varphi-\rho\frac{\partial^2\varphi}{\partial t^2}\right]$$
$$=-\nabla\times\left[\mu\nabla^2\boldsymbol{\psi}-\rho\frac{\partial^2\boldsymbol{\psi}}{\partial t^2}\right]$$

$$\nabla^2\varphi(\boldsymbol{x},t)=\frac{1}{\alpha^2}\frac{\partial^2\varphi(\boldsymbol{x},t)}{\partial t^2}$$

scalar wave equation

$$\alpha=\sqrt{\frac{\lambda+2\mu}{\rho}}$$

$$\nabla^2\boldsymbol{\psi}(\boldsymbol{x},t)=\frac{1}{\beta^2}\frac{\partial^2\boldsymbol{\psi}(\boldsymbol{x},t)}{\partial t^2}$$

vector wave equation

$$\beta=\sqrt{\frac{\mu}{\rho}}$$

Plane waves

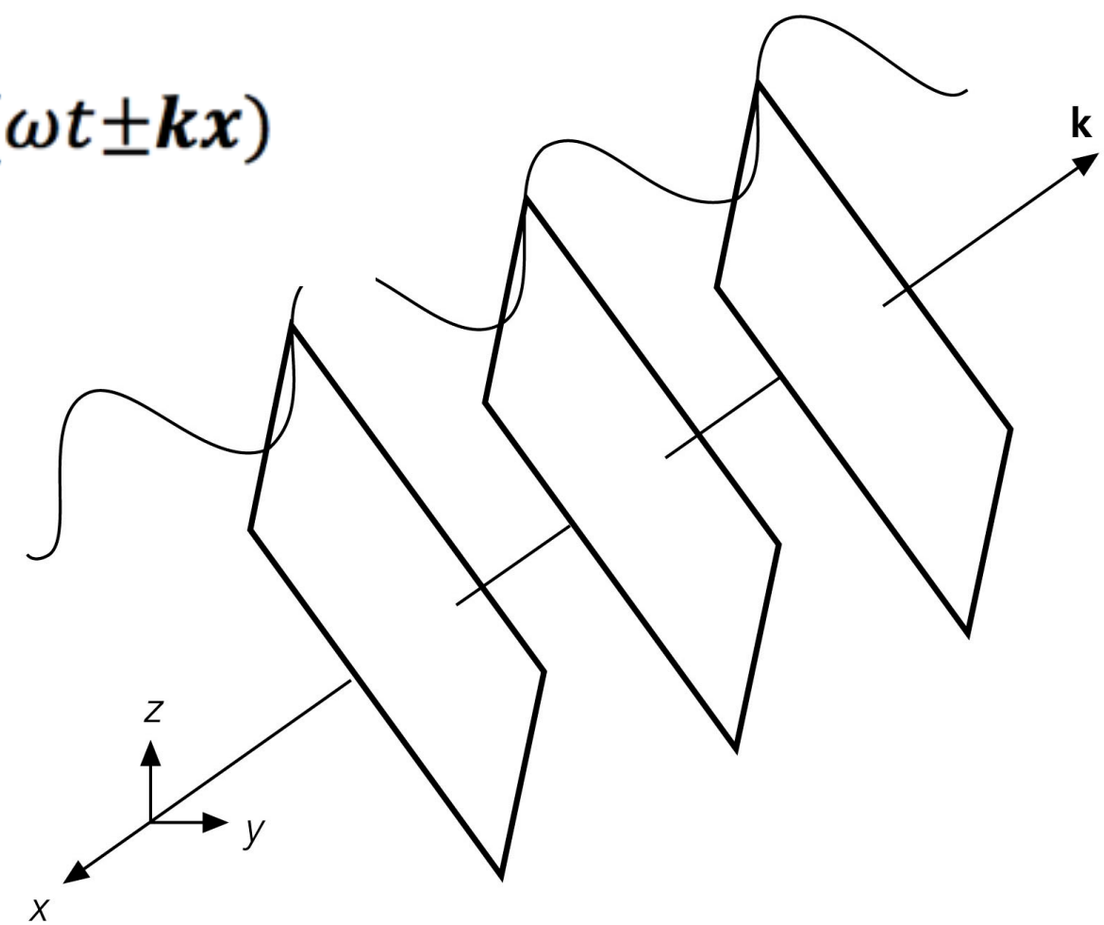
$$\nabla^2 \varphi(\boldsymbol{x}, t) = \frac{1}{\alpha^2} \frac{\partial^2 \varphi(\boldsymbol{x}, t)}{\partial t^2}$$

- A solution to this equation takes the form:

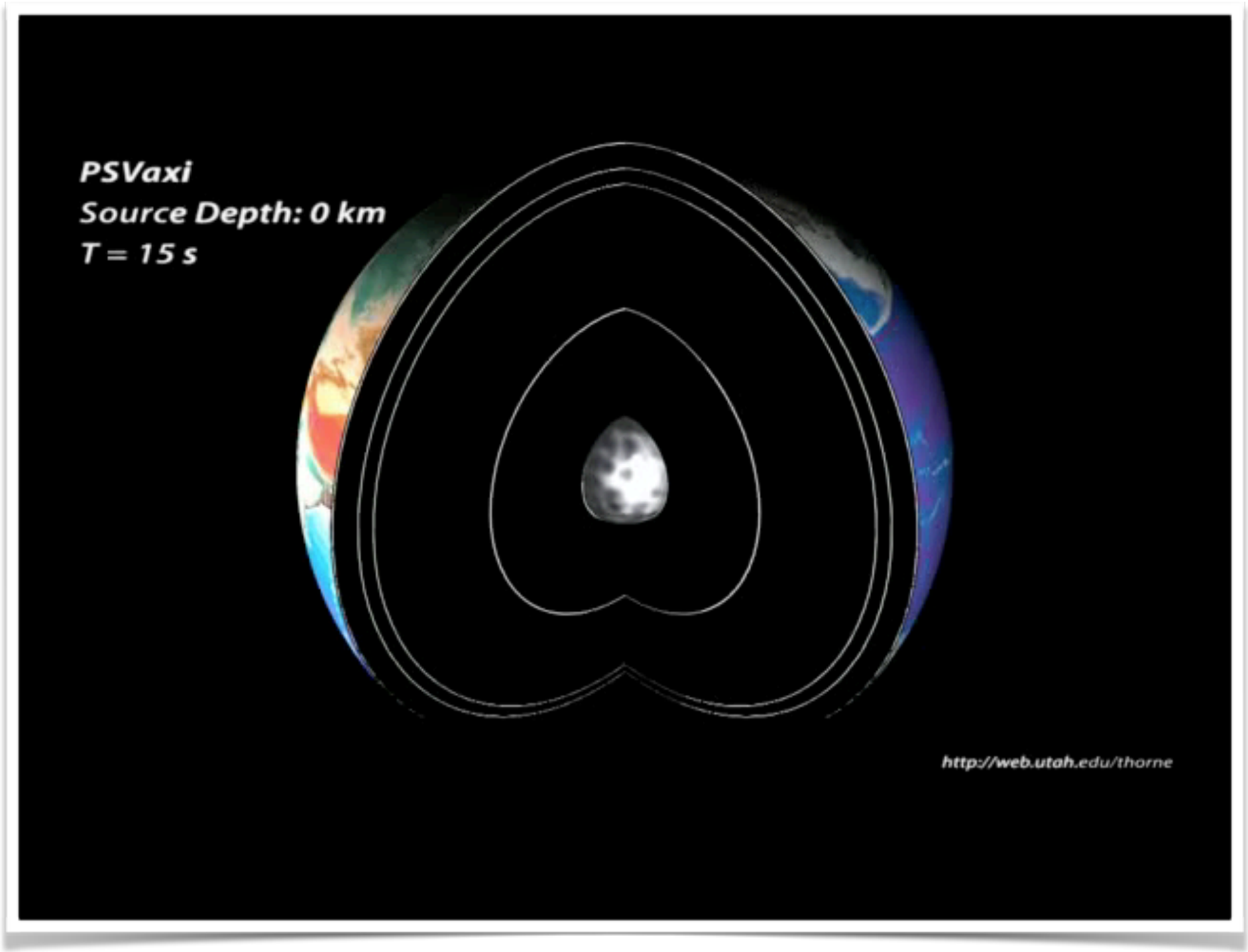
$$\varphi(\boldsymbol{x}, t) = f(\boldsymbol{x} \pm vt) = Ae^{i(\omega t \pm \boldsymbol{k}\boldsymbol{x})}$$

- With the velocity:

$$v = \alpha = \frac{\omega}{|\boldsymbol{k}|}$$



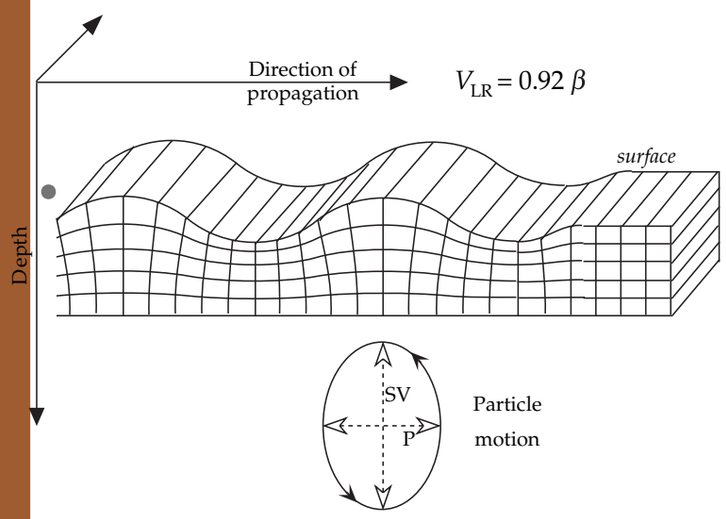
Wave types: Body waves



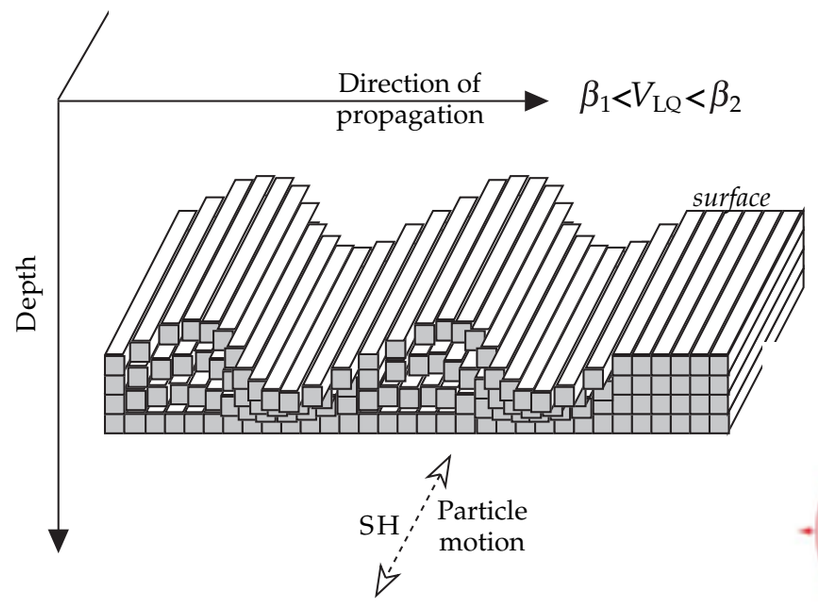
Spoiler alert!

- There are also **surface waves**

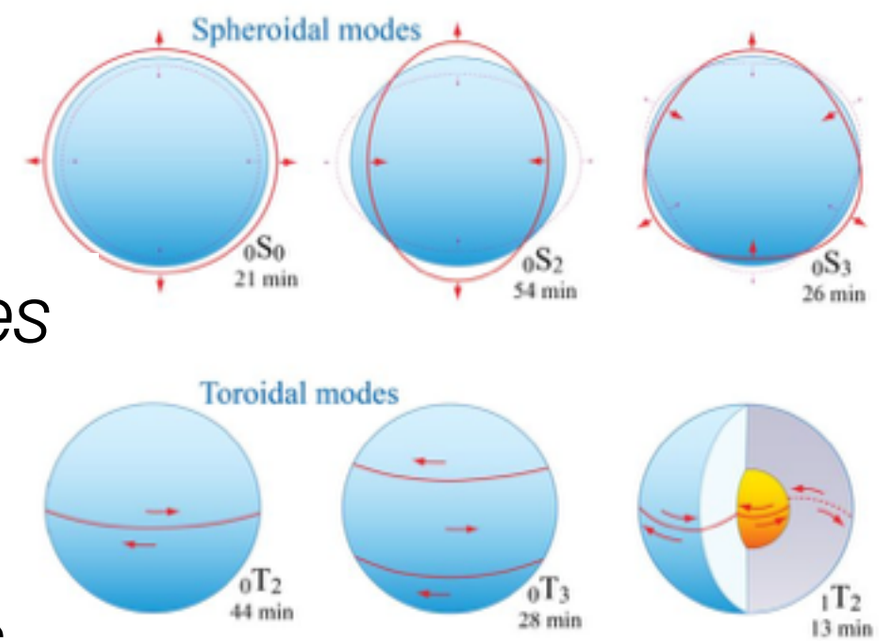
Rayleigh wave



Love wave



and **normal modes**

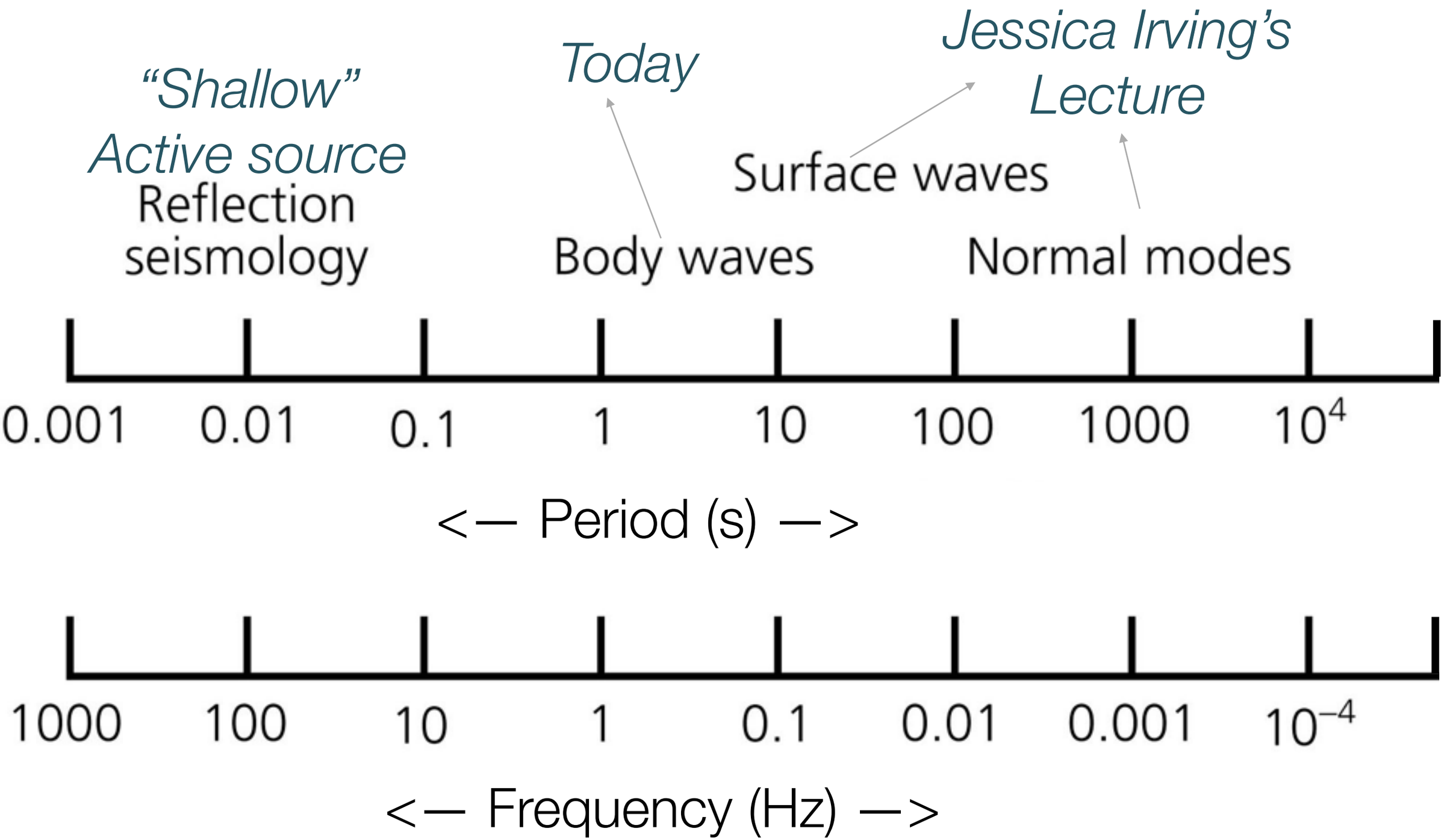


Spheroidal modes

Toroidal modes

Jessica Irving's lecture on normal modes and surface waves Seismology #5

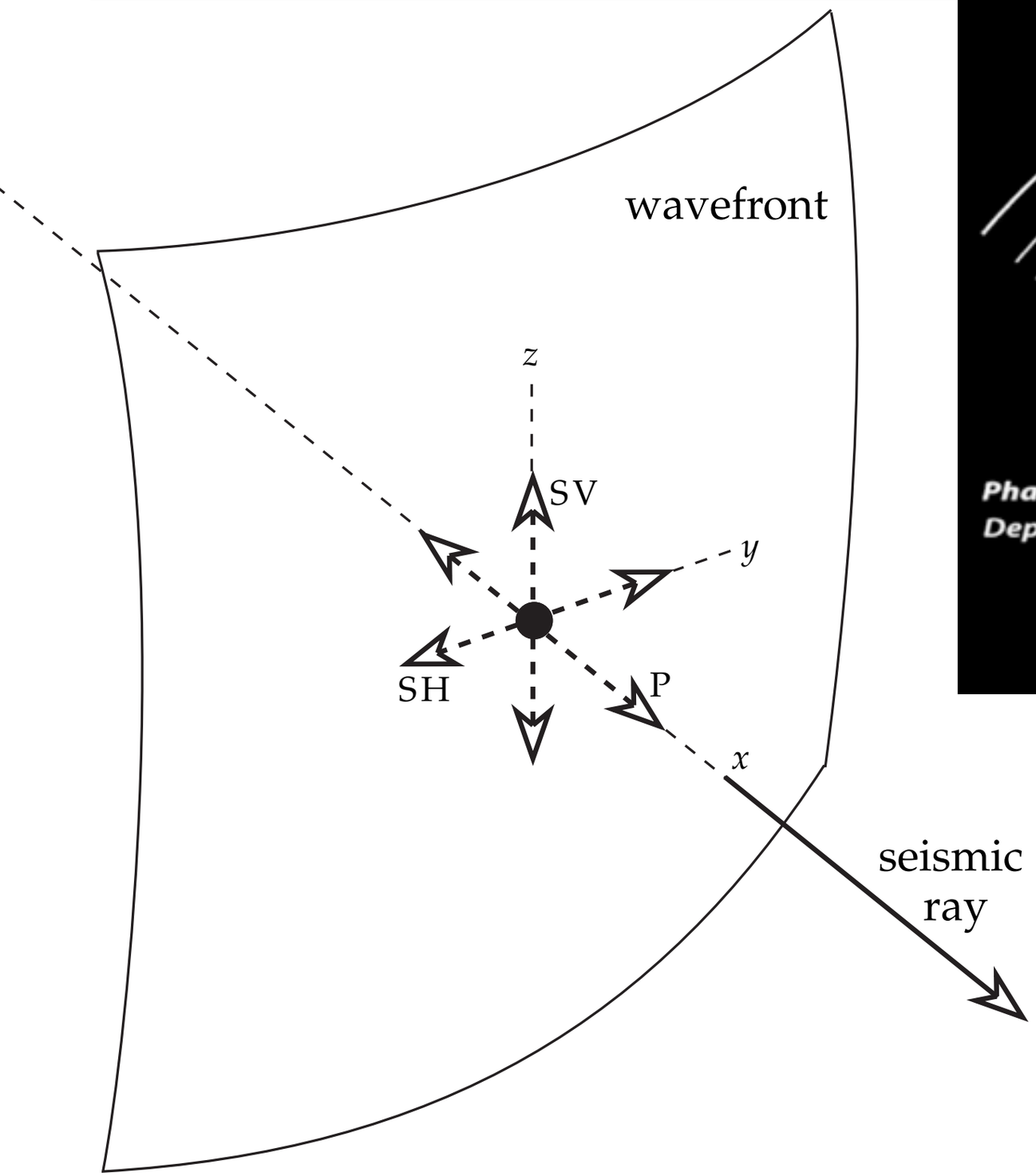
Wave types across frequencies

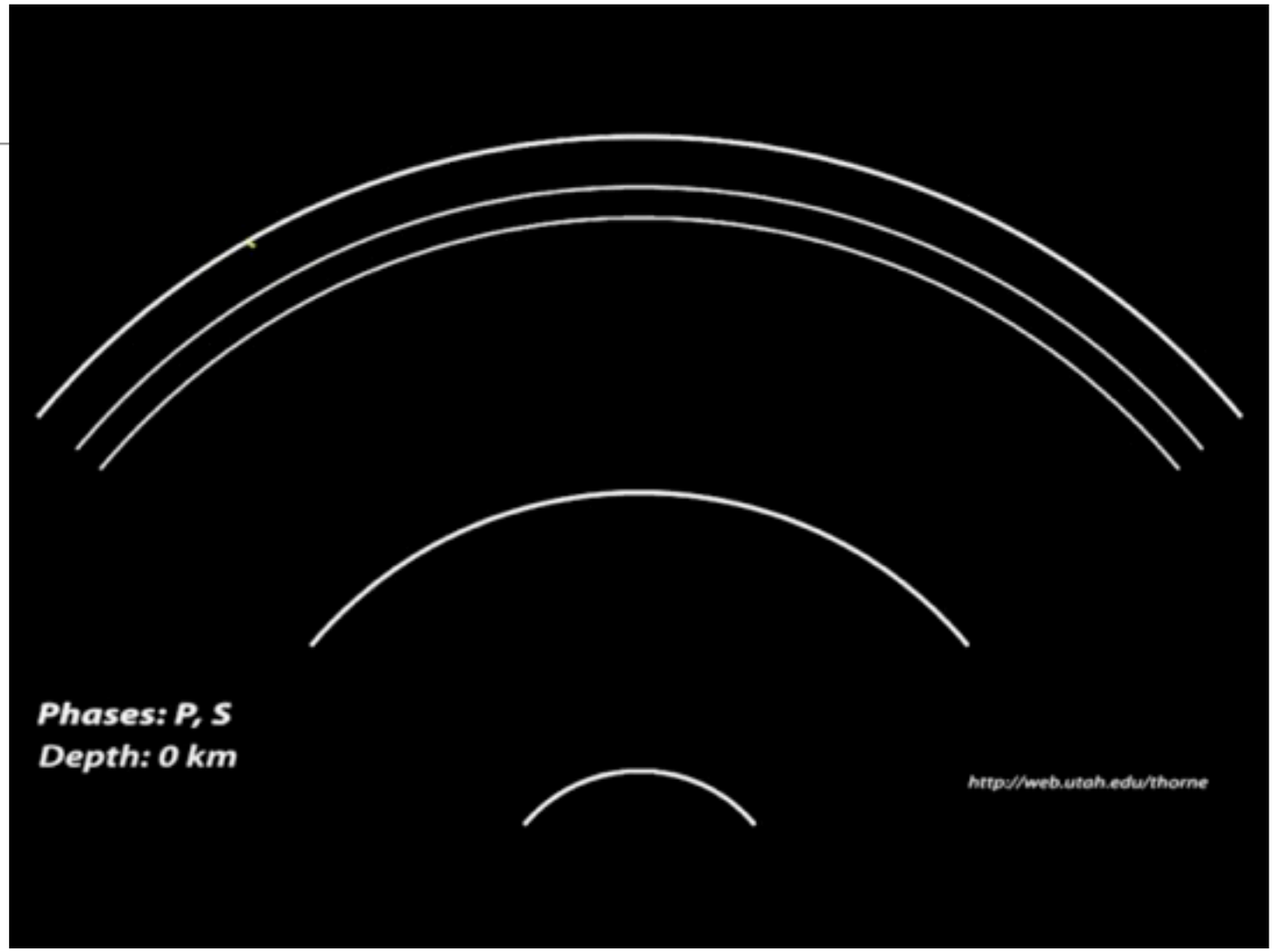


Body waves interacting with the Earth

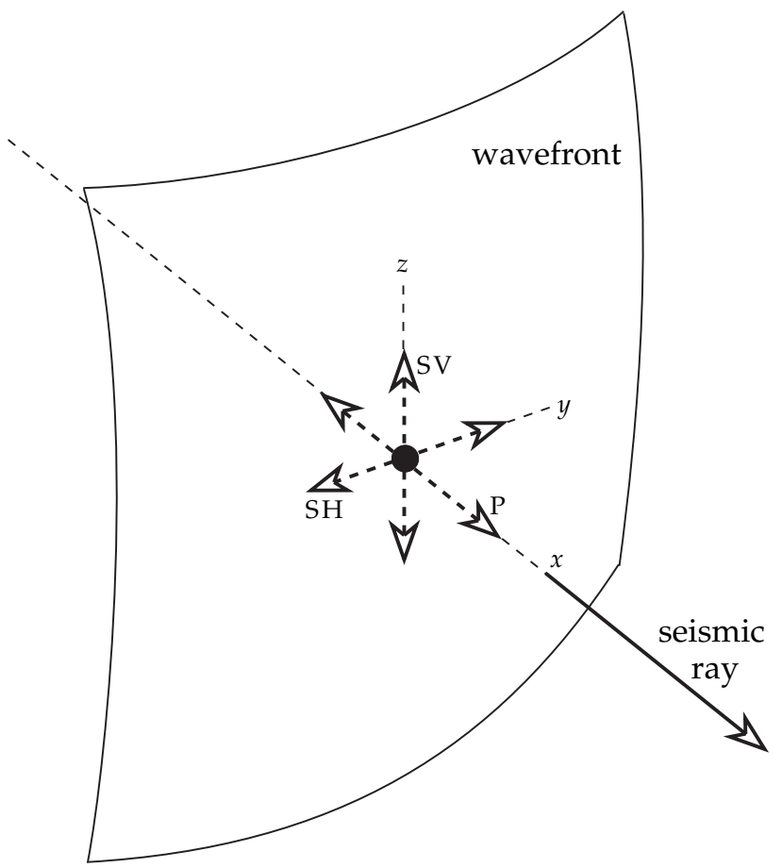
Some definitions and characteristics

Wavefront vs ray

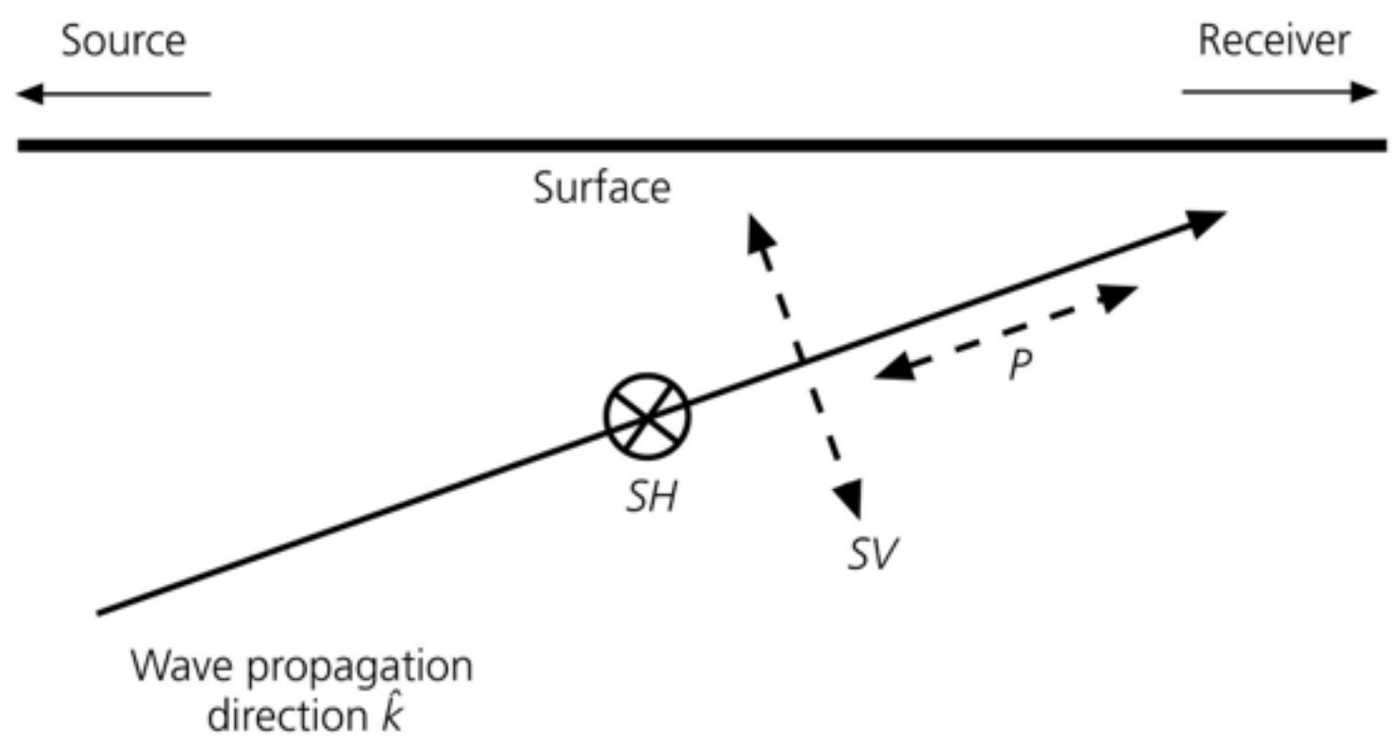




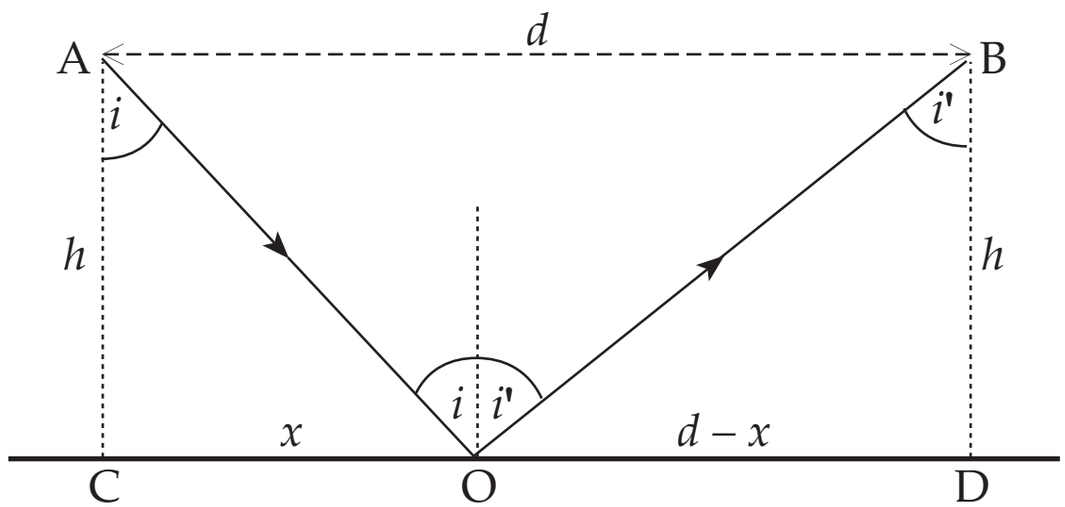
Wave components



- **P component** is in the direction of the ray path
- **SV component** lies in the earthquake-receiver plane.
- **SH component** is orthogonal to this.



Interaction with interfaces

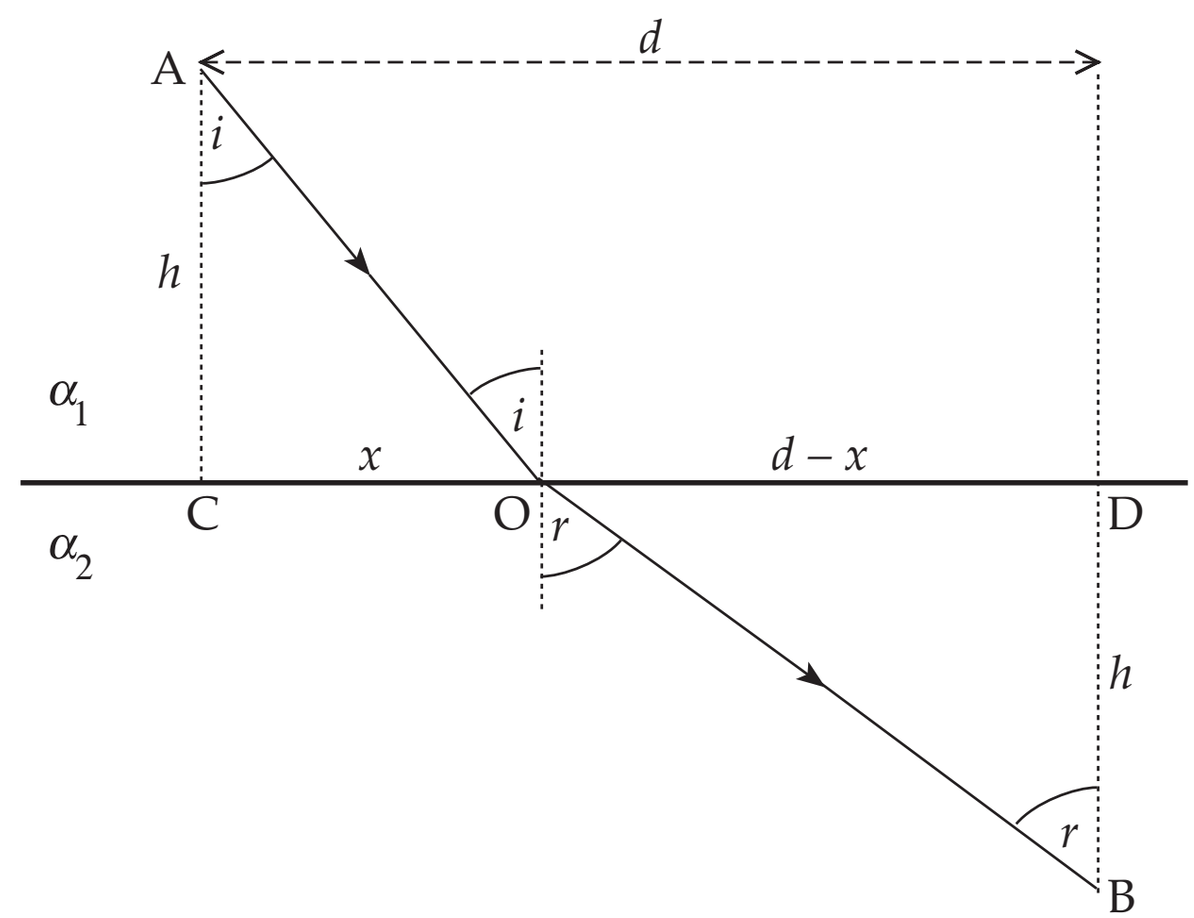


Reflection

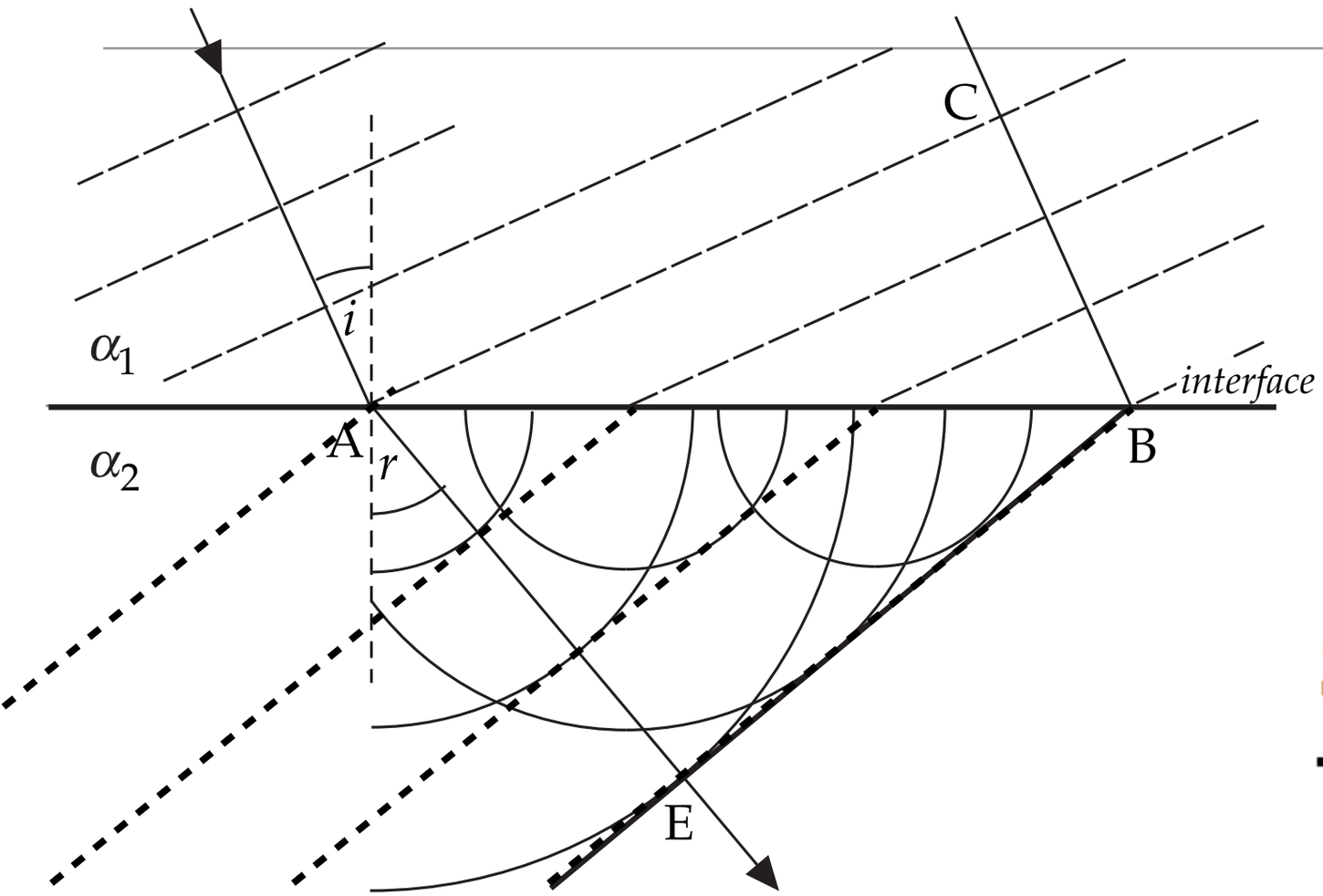
$$i' = i$$

Refraction

$$\frac{\sin i}{\alpha_1} = \frac{\sin r}{\alpha_2}$$



Snell's Law



Wavefronts align at the interface, here both wavefronts have the same horizontal velocity.

$$\frac{\sin i}{\alpha_1} = \frac{\sin r}{\alpha_2} = p$$

p is also known as the *ray parameter* and remains constant along a ray path.

Slowness vector decomposed

Slowness= 1/velocity= [s/km]

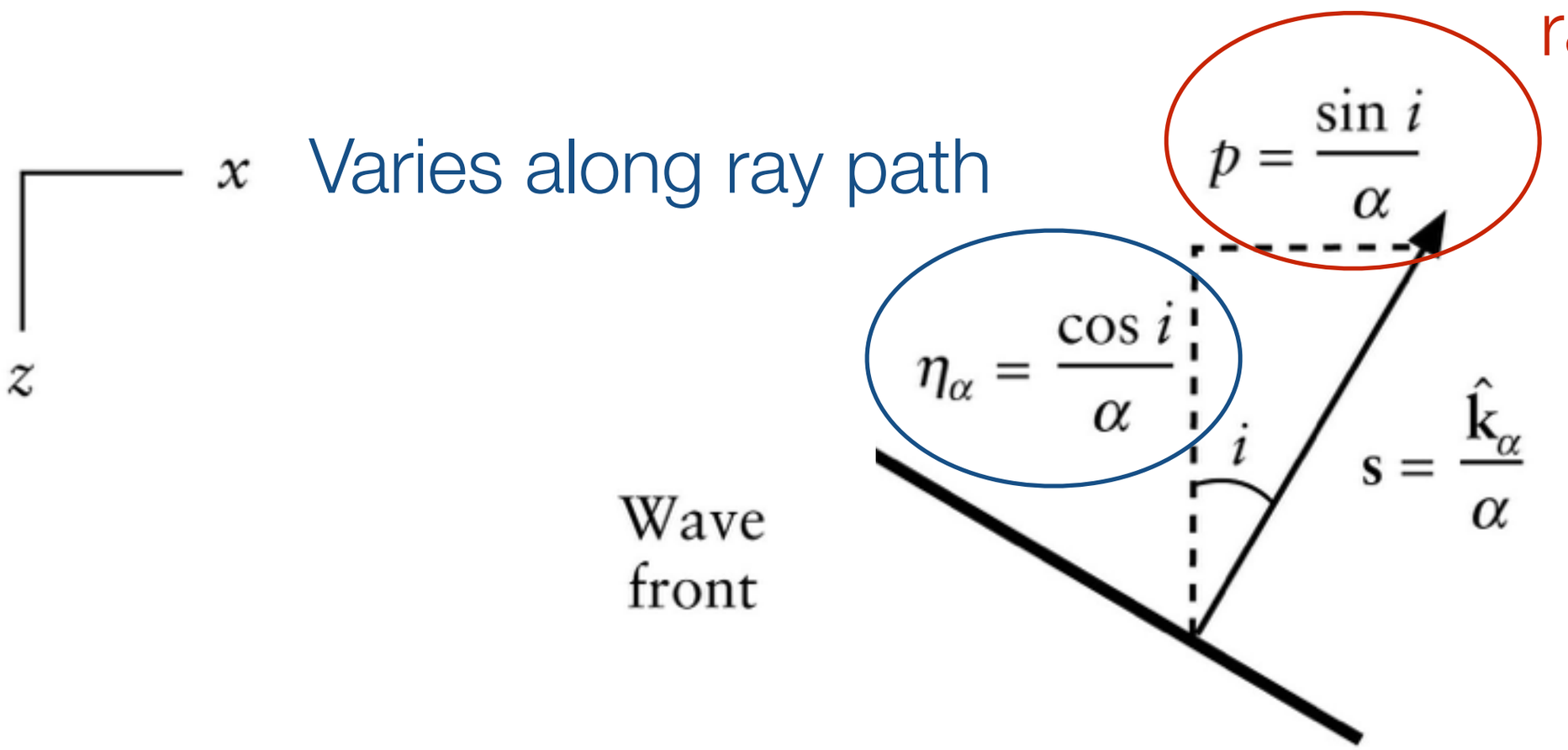
Half-space

p = horizontal slowness

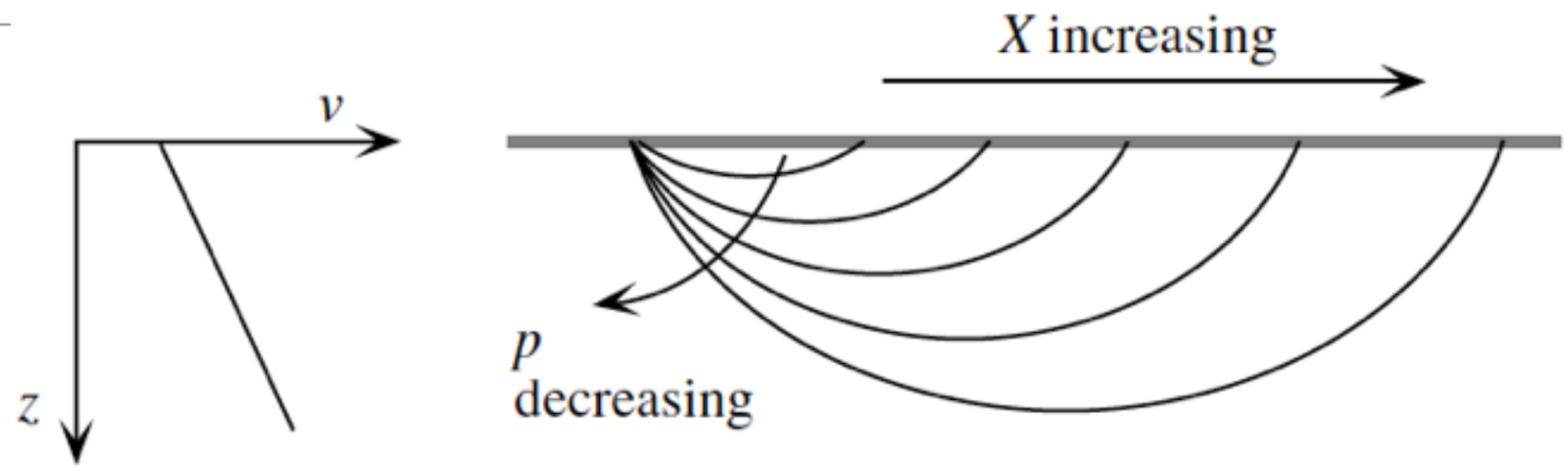
η = vertical slowness

Constant along ray path

Varies along ray path

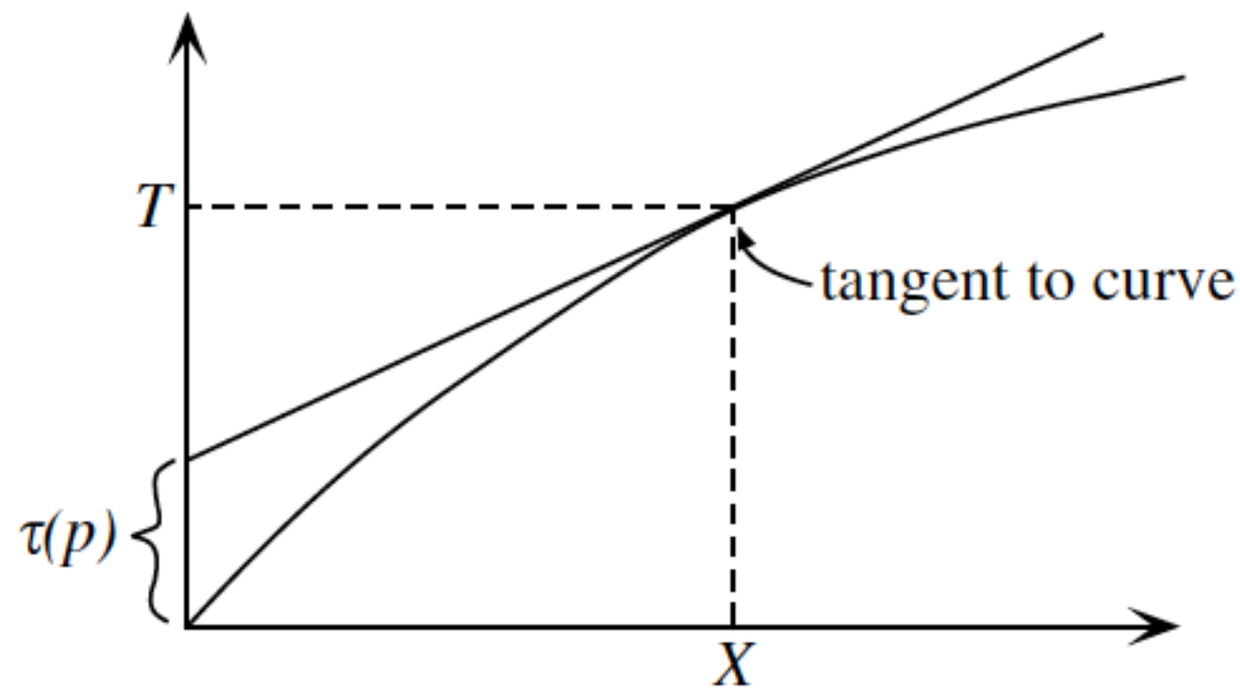


How to observe the ray parameter?



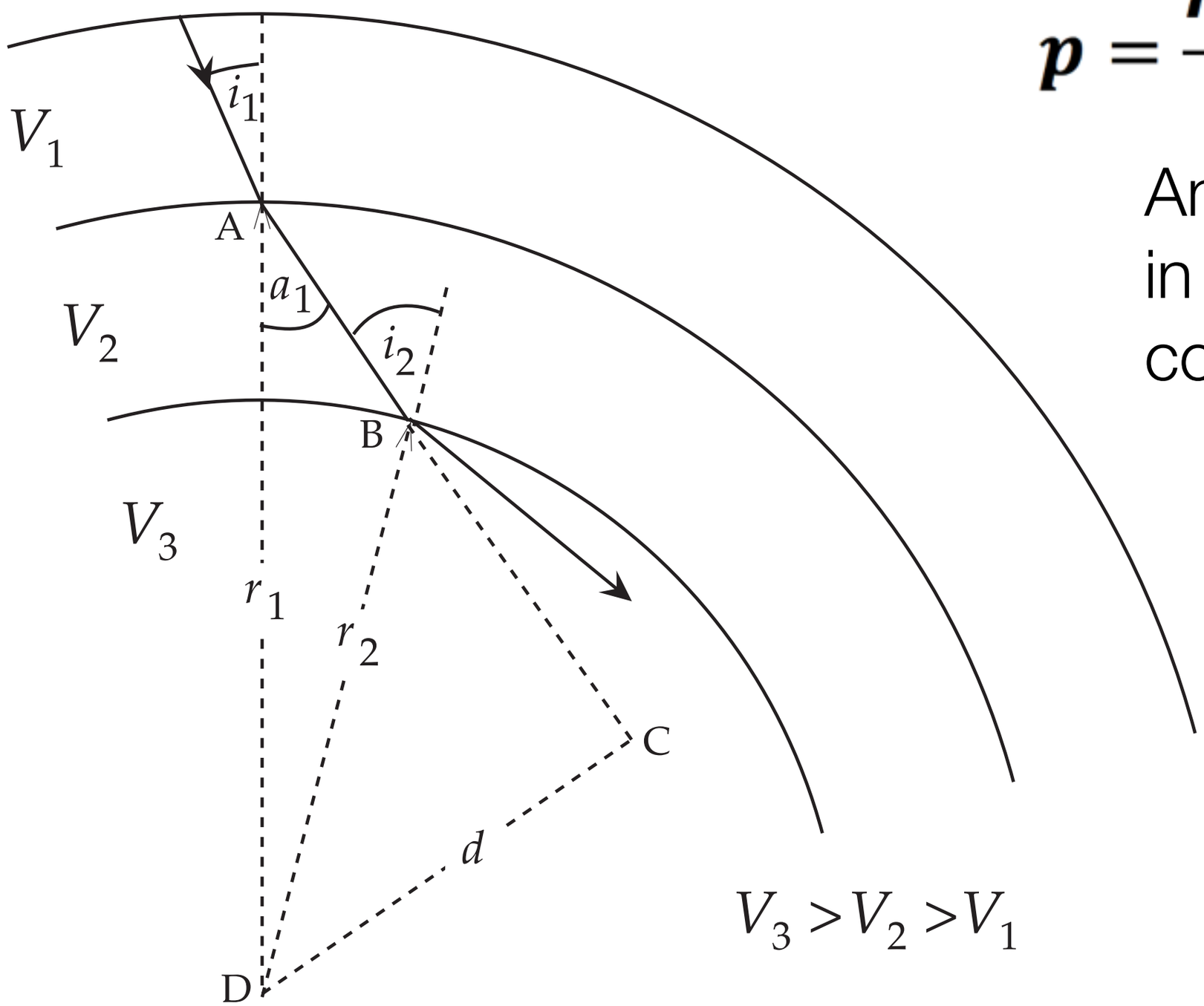
Ray parameter is the apparent horizontal slowness at the surface, i.e. ***the gradient to the travel time curve***:

$$p = dT/dX = [s/km]$$



Ray parameter decreases with distance for different ray paths if the velocity increases with depth.

Curvature in the Earth



$$p = \frac{r \sin i}{v}$$

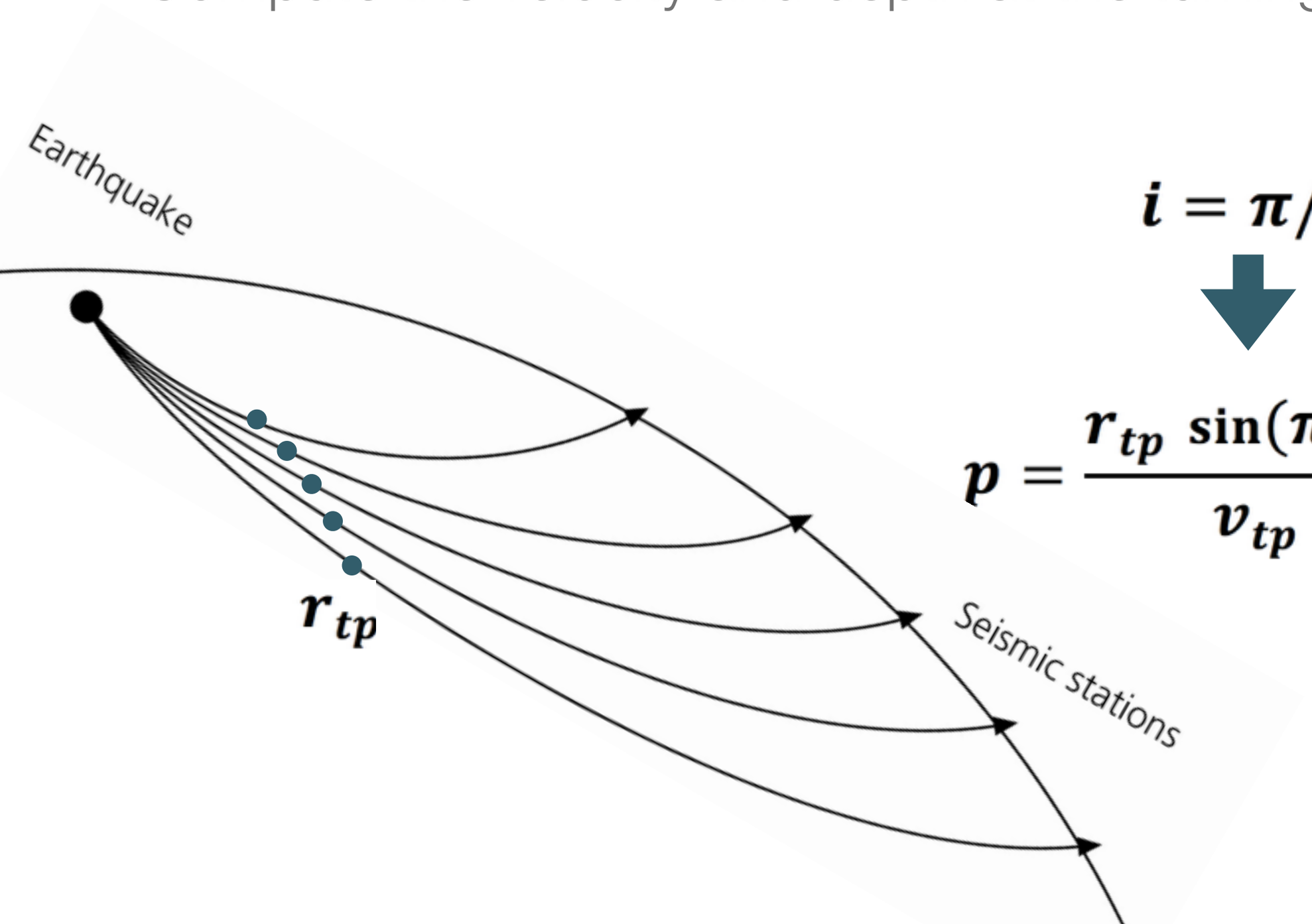
Angular slowness in s/rad remains constant.

v = [km/s]
r = [km/rad]
p = [s/rad]

r is equal to the radius
= 6371 - depth

What can you do with the ray parameter?

- Compute the velocity and depth at the turning point

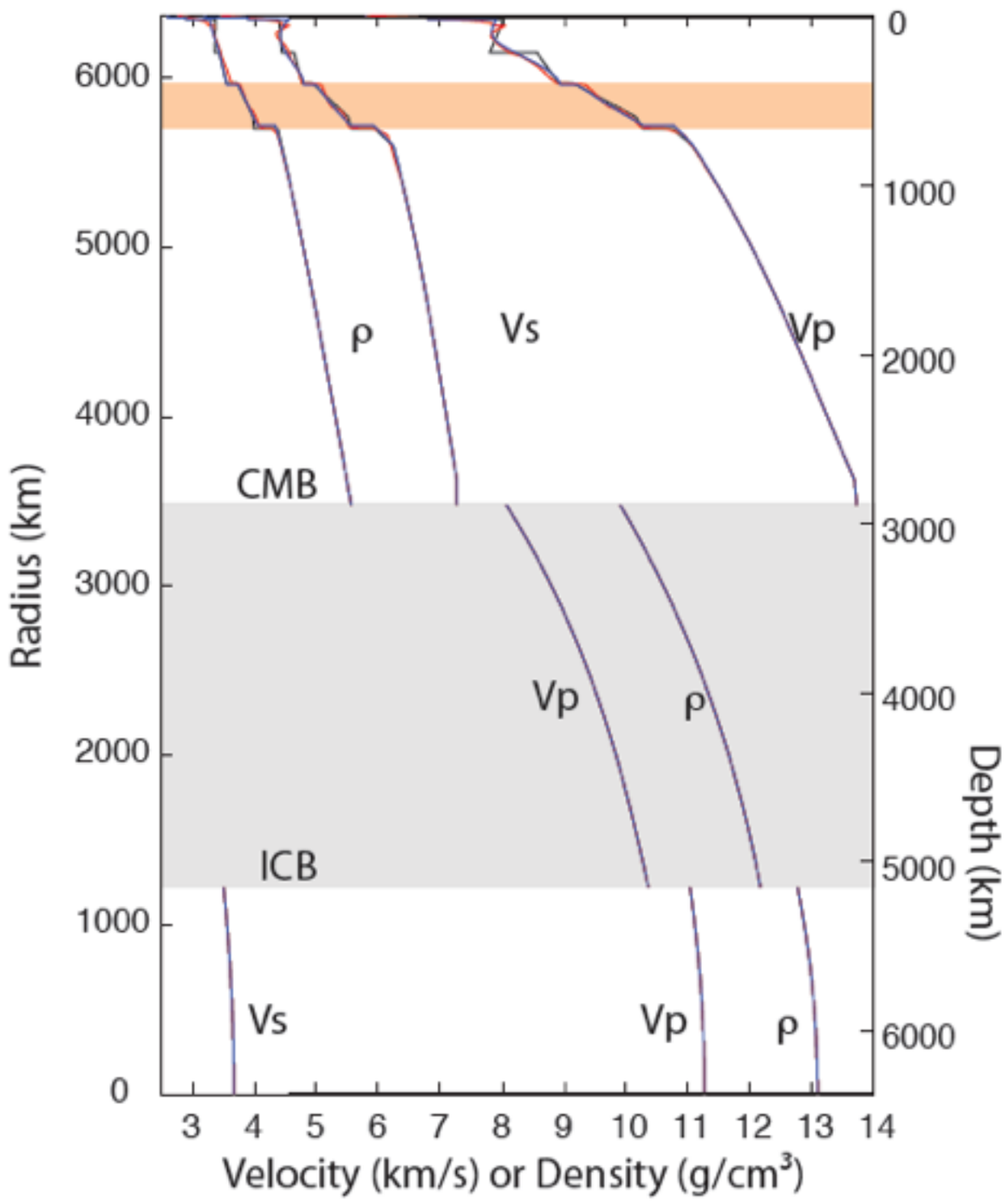


$$i = \pi/2$$

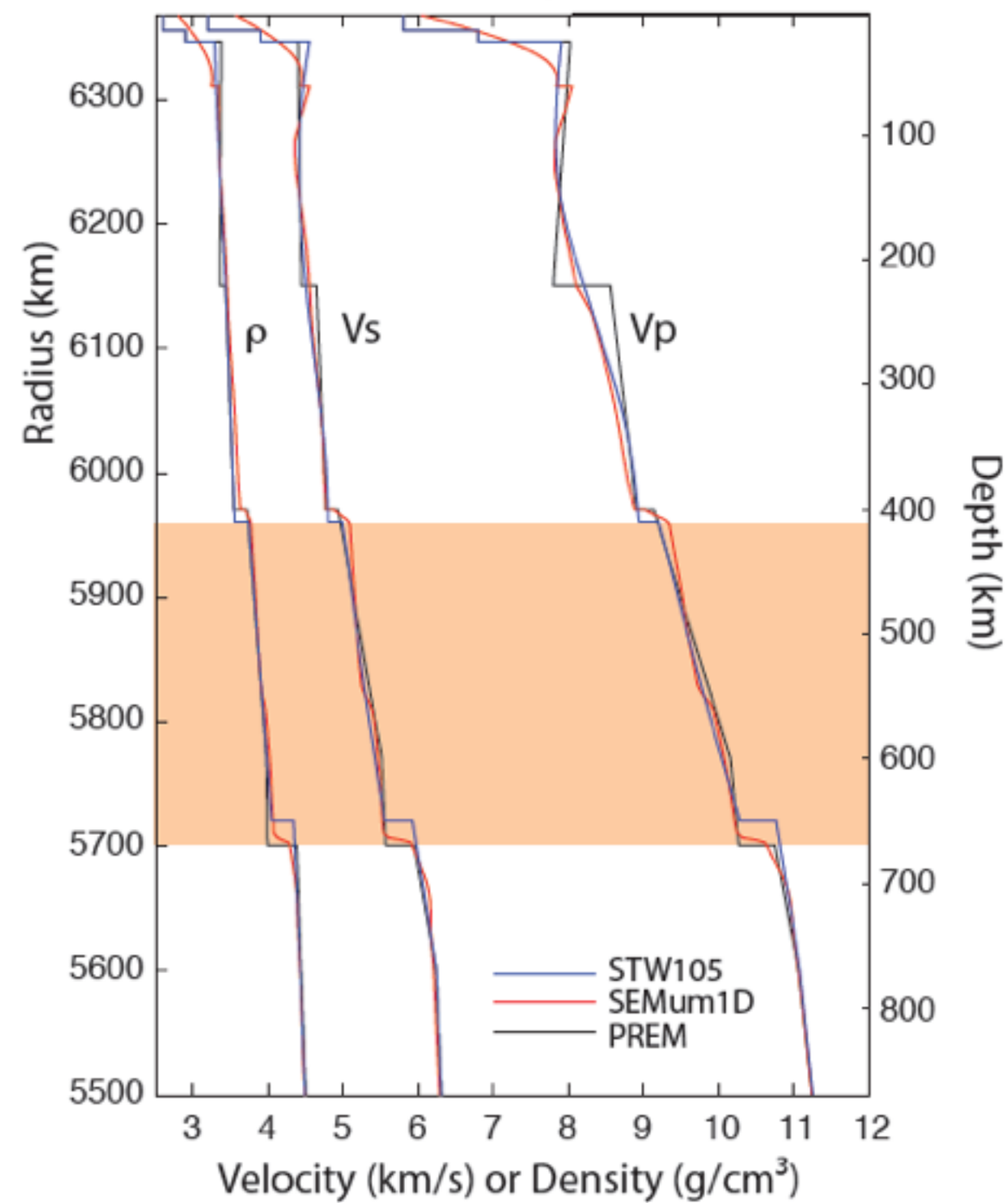
$$p = \frac{r_{tp}\ \sin(\pi/2)}{v_{tp}} = \frac{r_{tp}}{v_{tp}}$$

1D Earth model

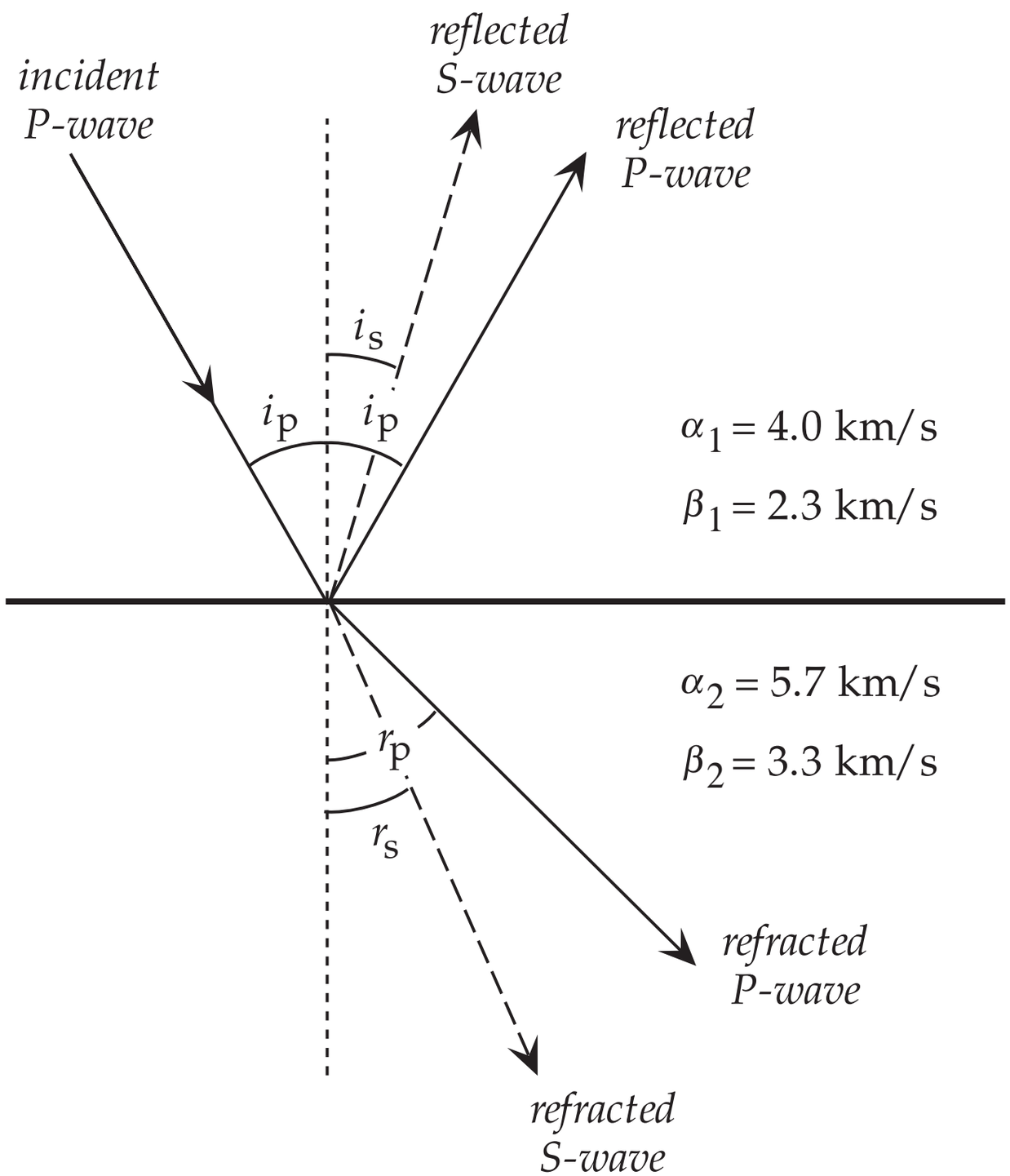
Entire Earth



Mantle Only



P and S conversions



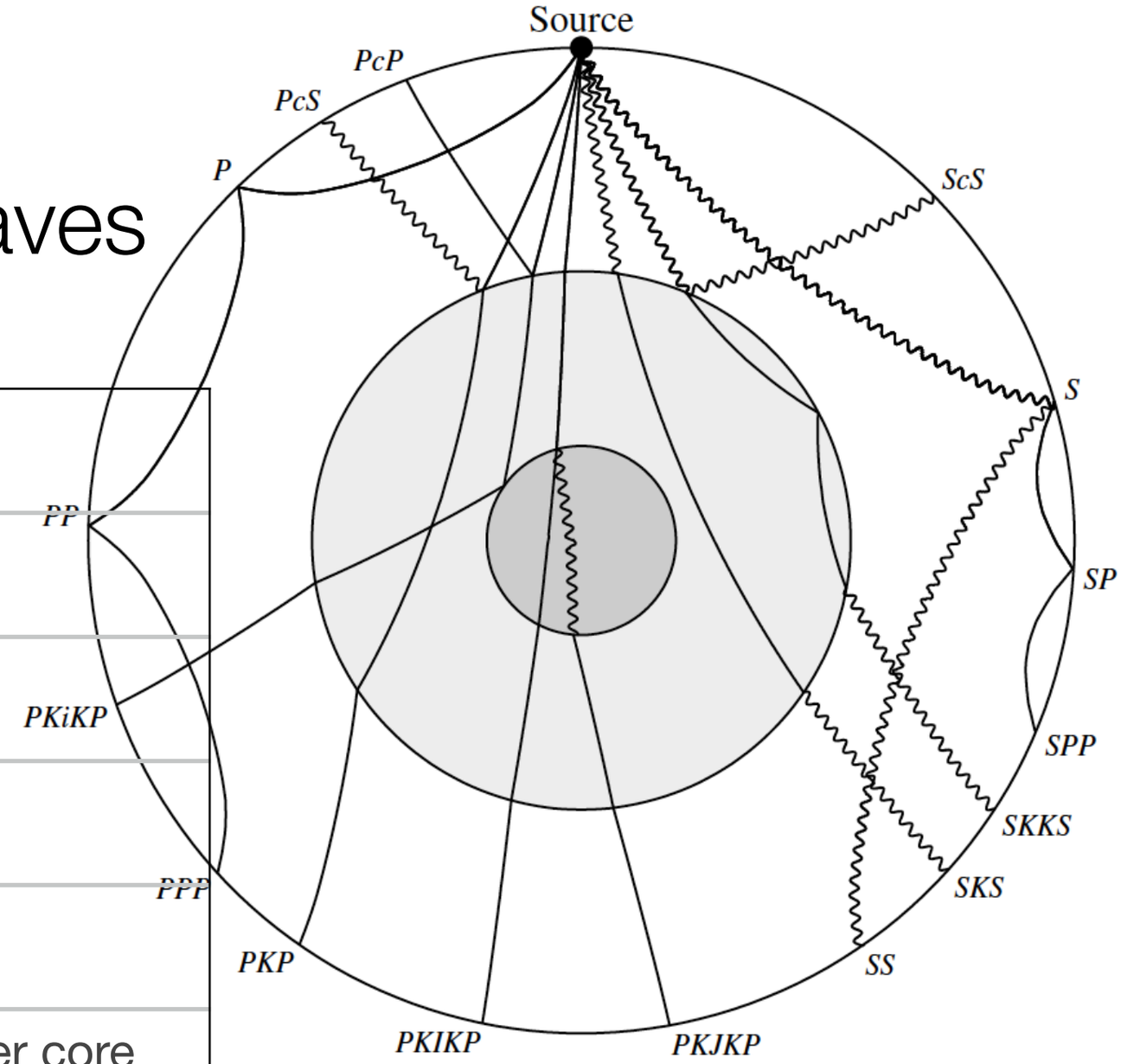
Besides *reflection* and refraction, energy at an interface also *converts.*

Conversions in layered media occur between P and SV energy, while SH is an independent system.

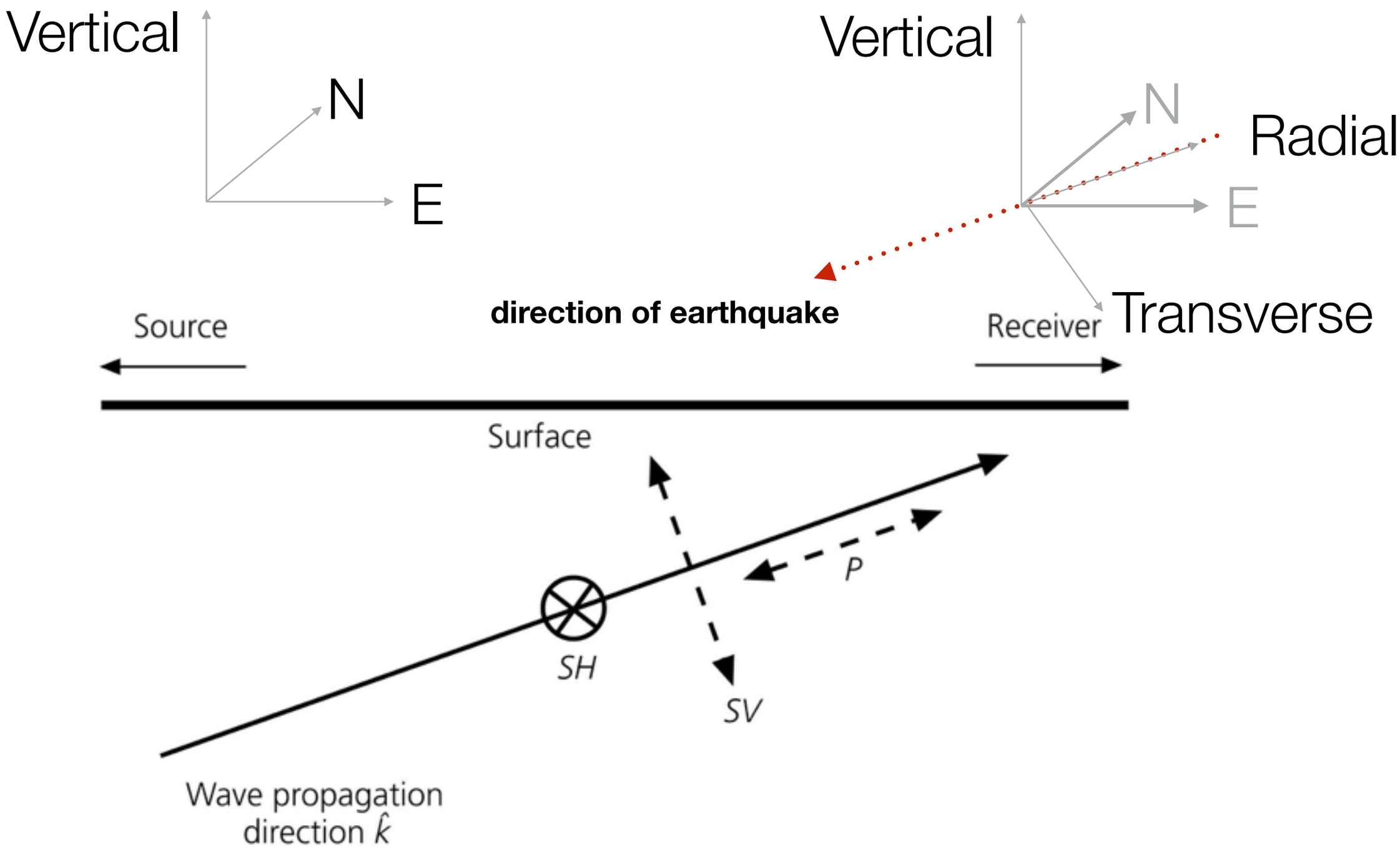
Amplitude depend on *impedance = velocity x density*

Nomenclature of body waves

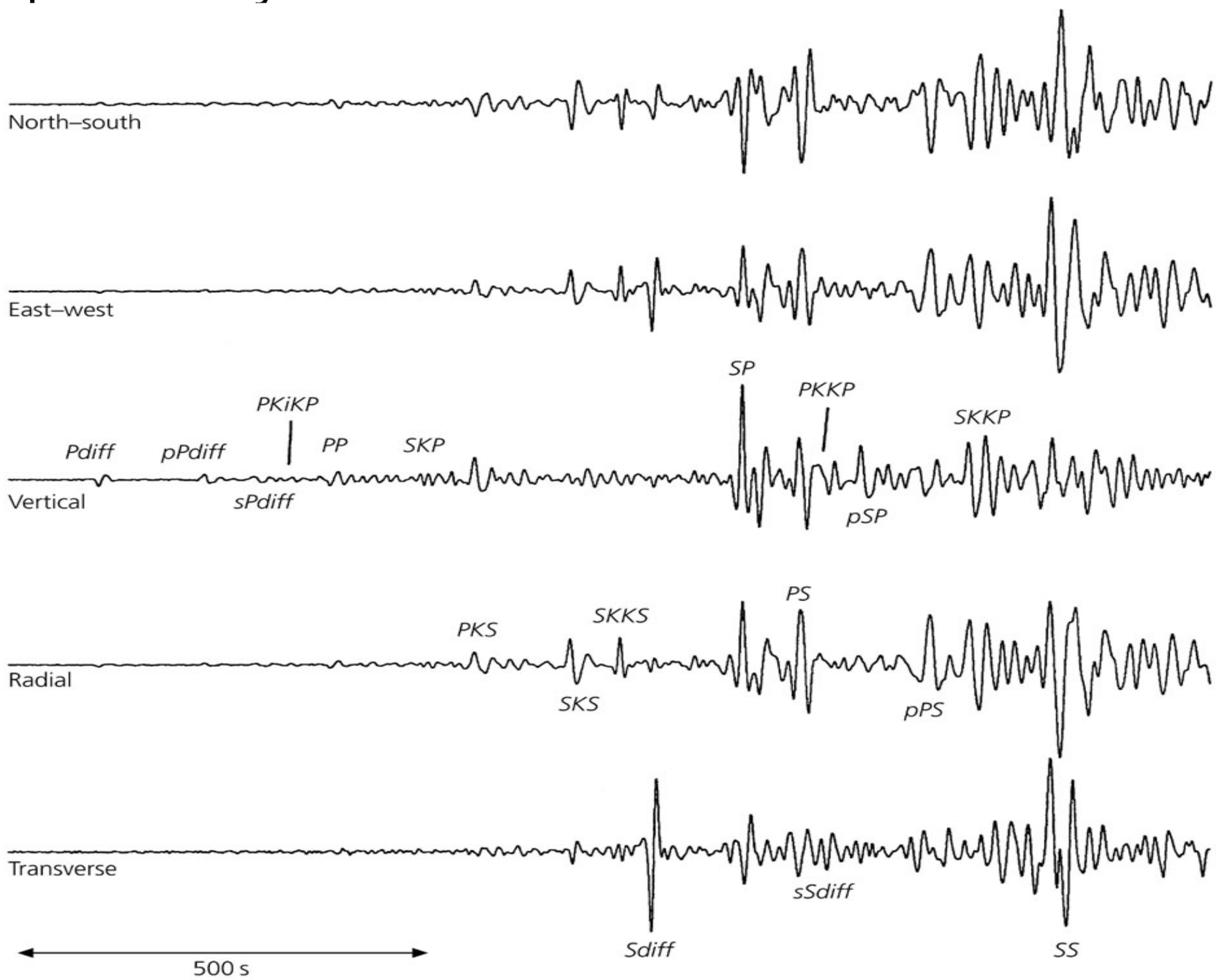
P	P wave in the mantle
S	S wave in the mantle
K	P wave in the outer core
I	P wave in the inner core
J	S wave in the inner core
c	a reflection from the mantle-outer core boundary
i	a reflection from the outer- inner core boundary
m	a reflection from the Moho
p	a reflection of the surface from an upgoing P-wave near the source
s	a reflection off the surface from an upgoing S wave near the source



Station components



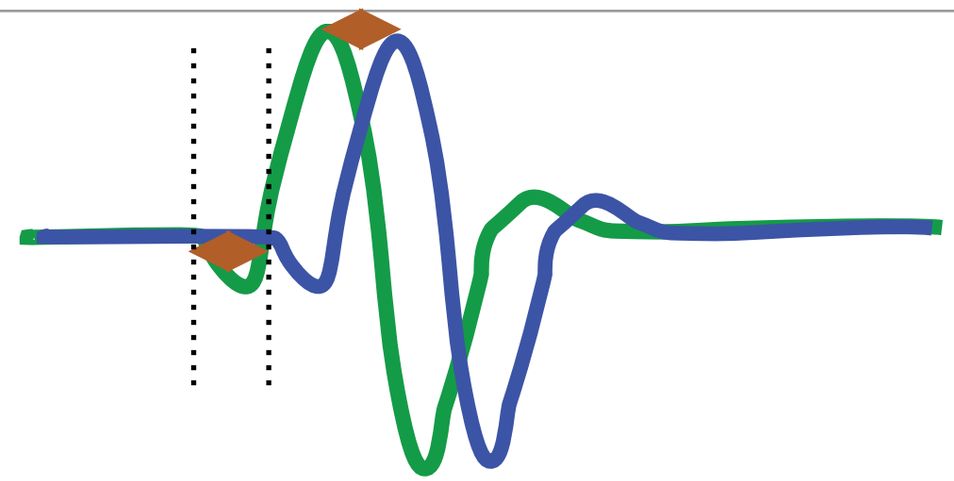
Example body wave arrivals



The contemplations of a body wave seismologists

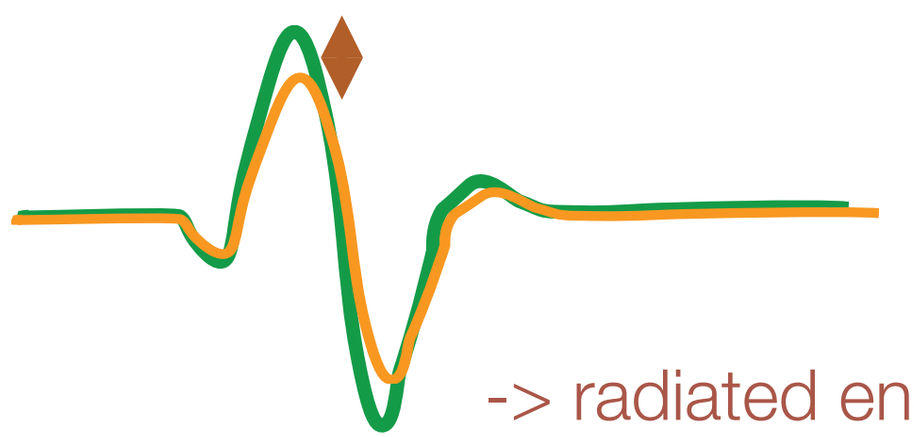
Signals of Interest

- travel times



-> velocity anomaly along the ray path
-> Incorrect ray path

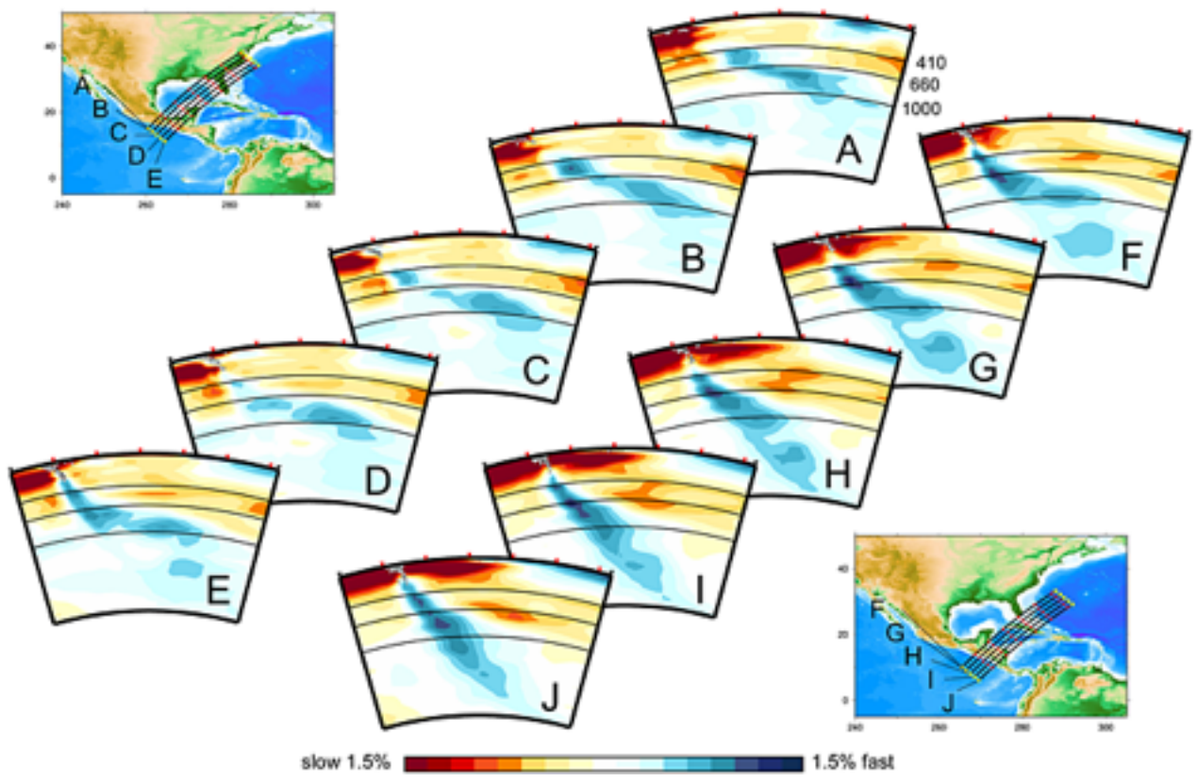
- amplitudes



-> radiated energy from earthquake
-> focussing and defocussing
-> energy scattered away - reflection and transmission coefficients
-> inelasticity

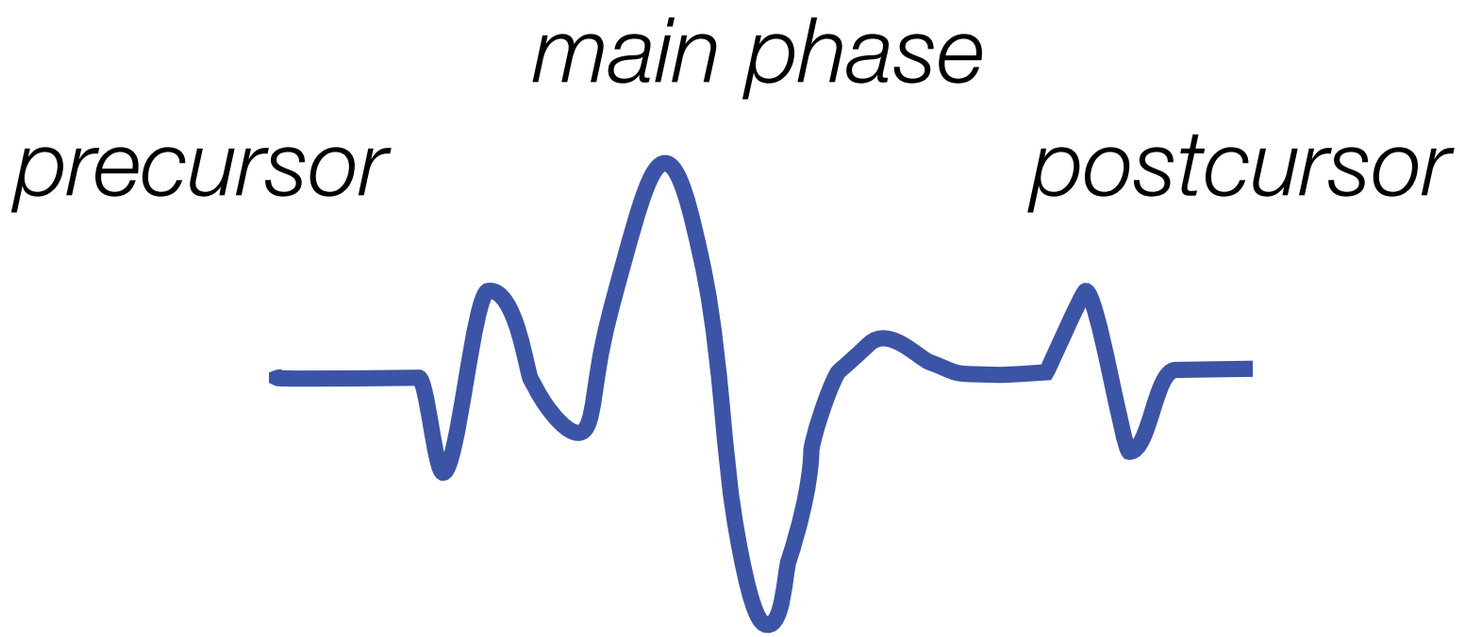
Use in tomographic models

- How to combine many, many observations to make an image of the variations in the Earth?



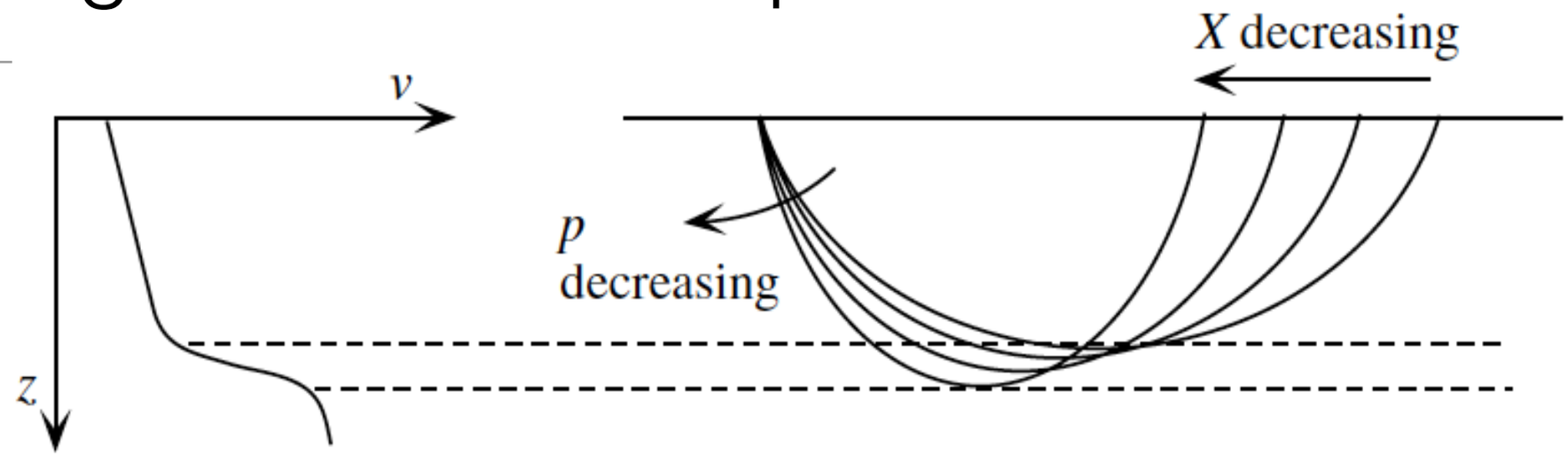
Ved Lekic's lecture on inverse theory
Seismology #3

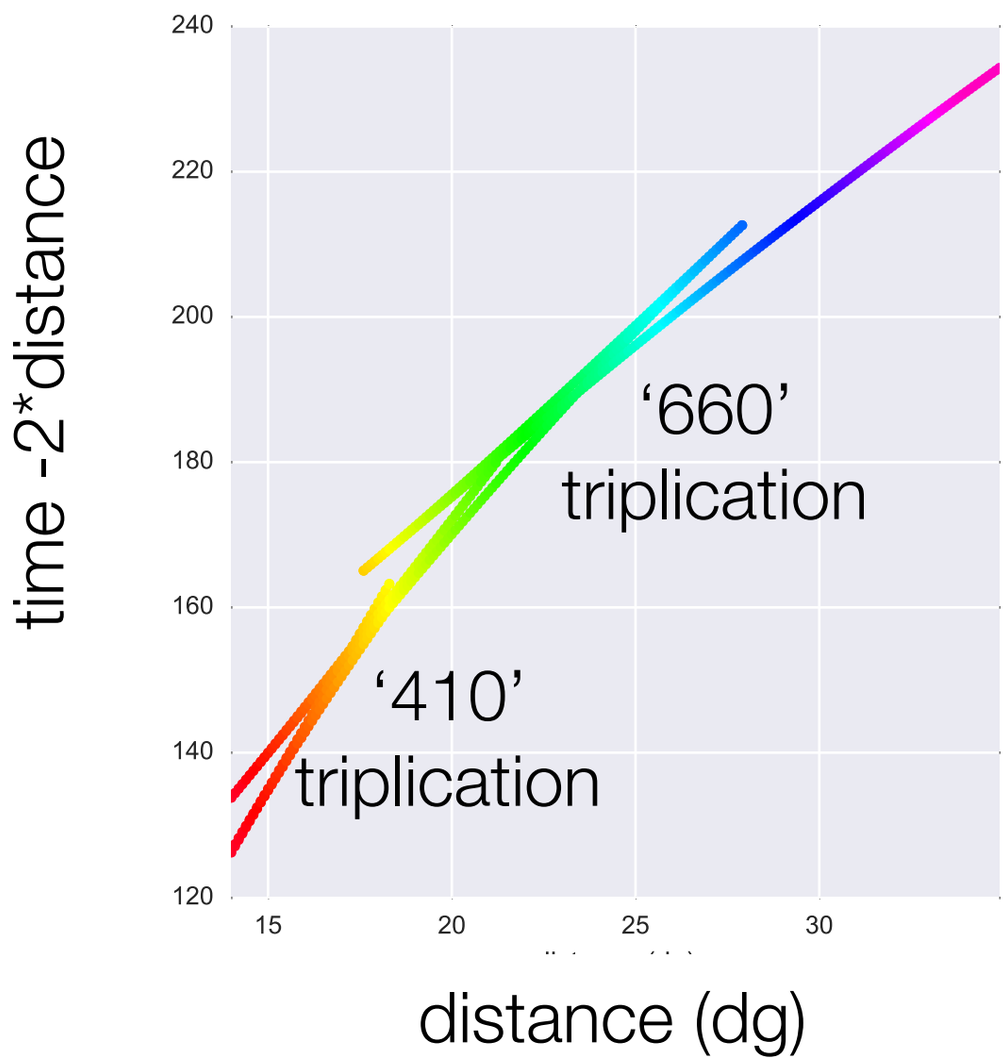
Waveform complexities

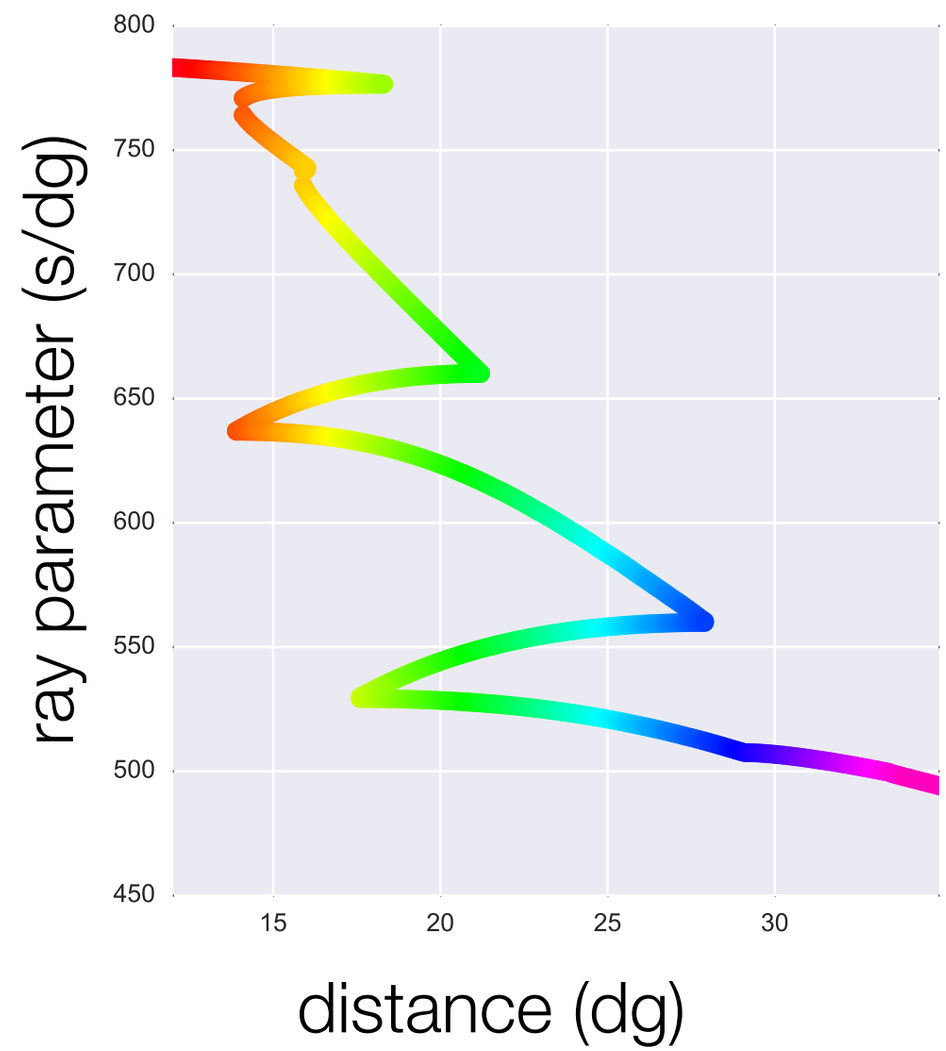


- triplications or multi-pathing
- Measure direction, differential travel-time, amplitude and frequency content of arrivals

Targeting boundaries - triplications

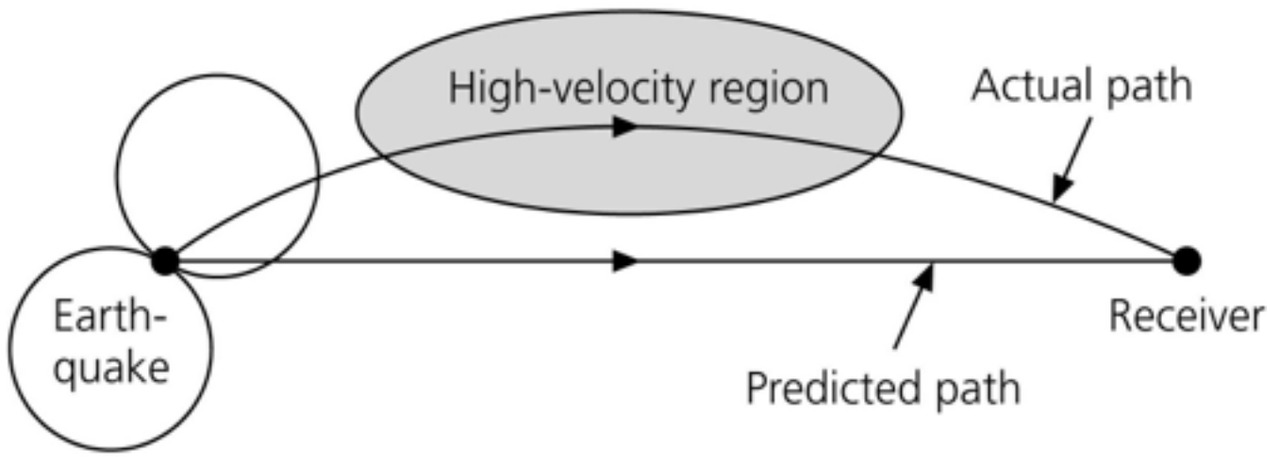




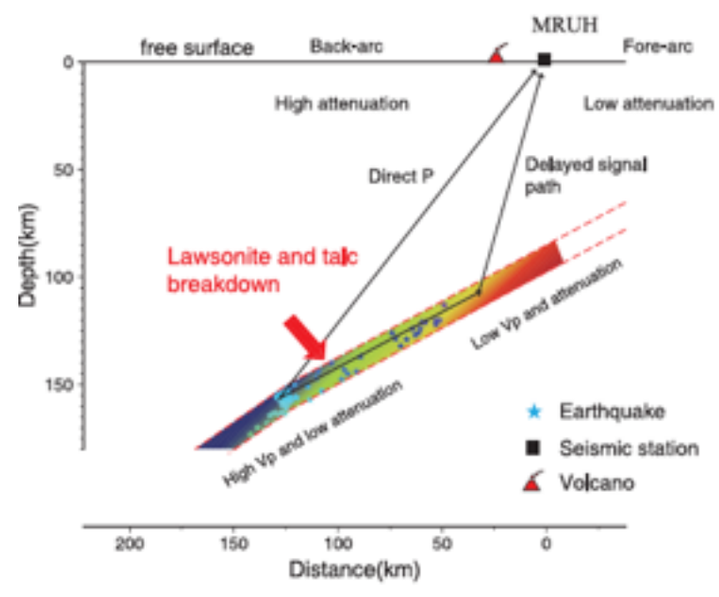


Targeting boundaries - multi-pathing

- multi-pathing



- guided waves

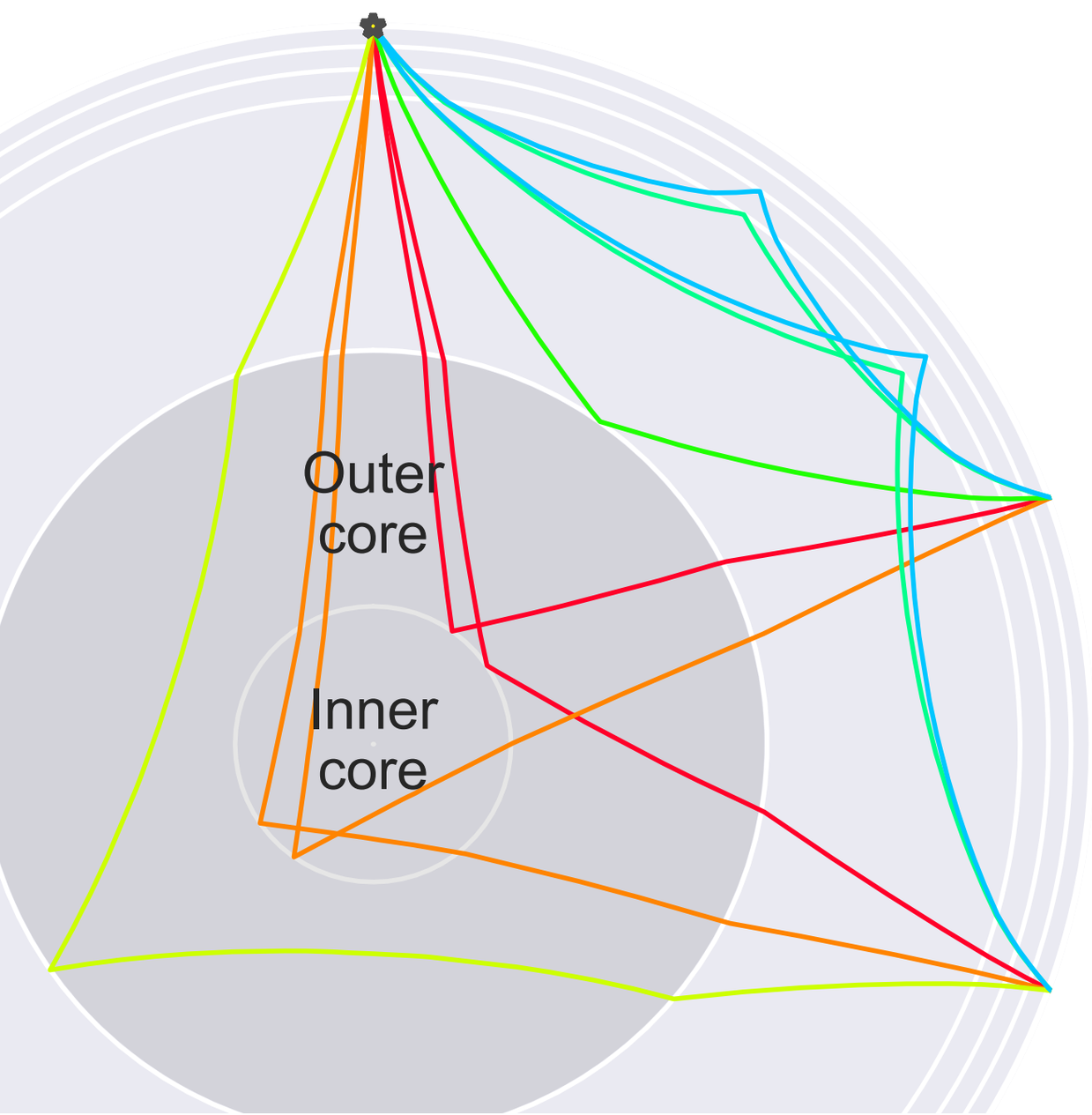


[Wu & Jessica Irving, 2018]

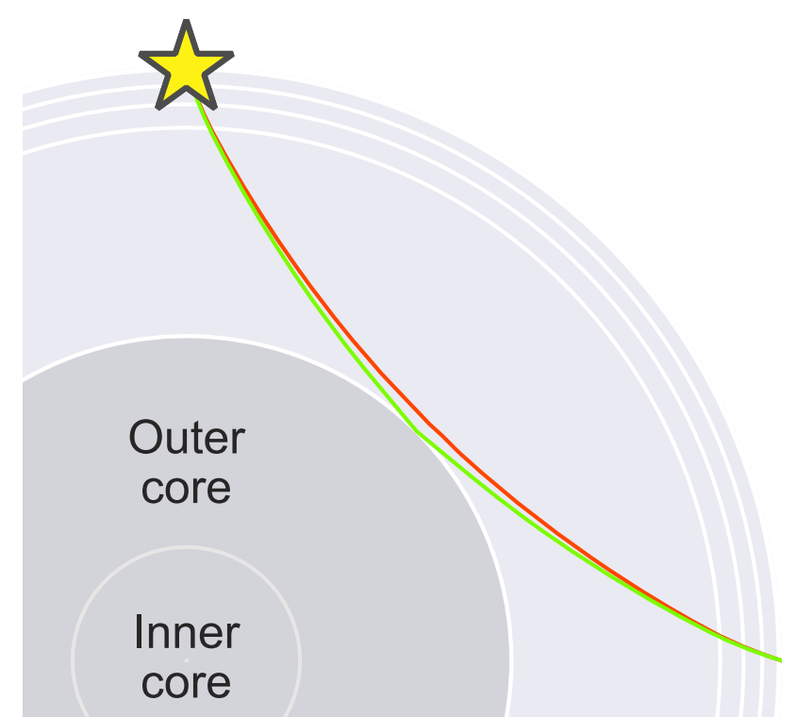
- Interpreting anomalies causing multi-pathing often takes initial knowledge from a tomographic model

-> What is the amplitude of the anomaly causing the multi-pathing?

Targeting boundaries - impedance contrast

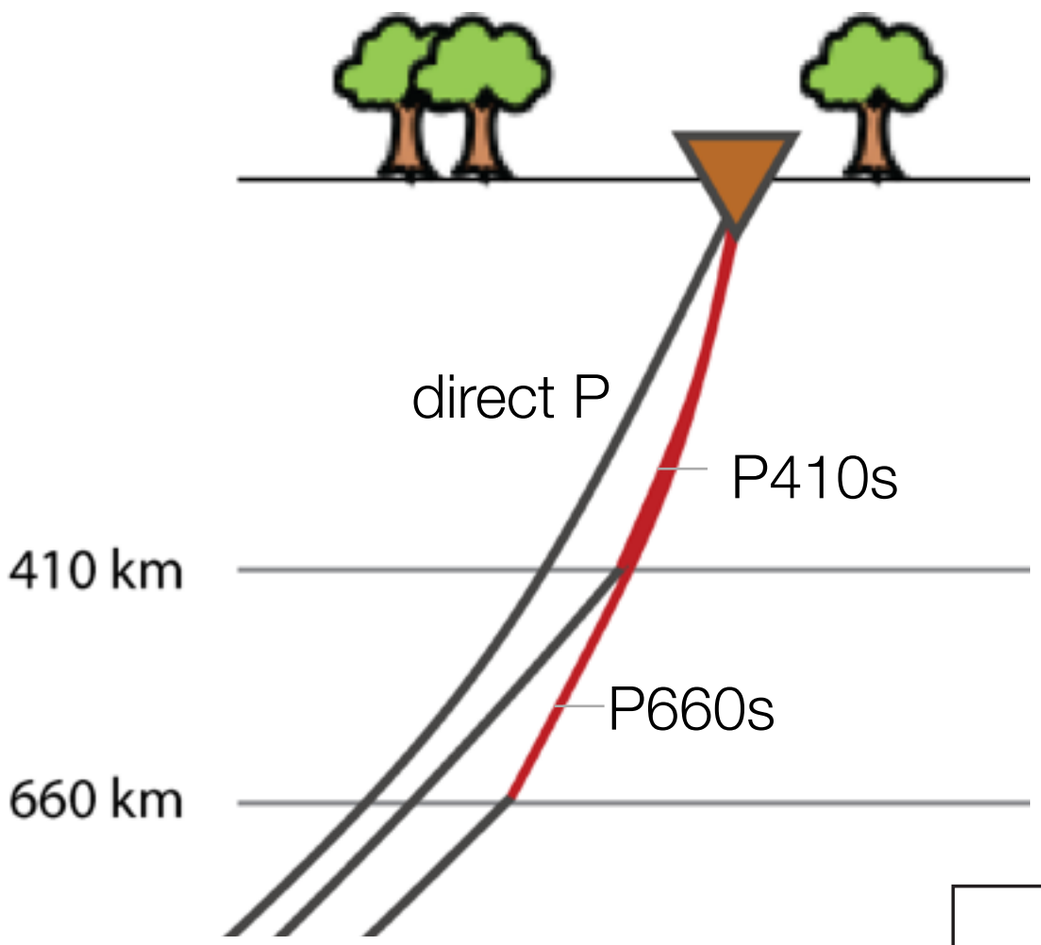


- What is the amplitude and sharpness of the seismic discontinuity? -> reflection coefficients
- What is the topography of the discontinuity? -> (differential) travel times
- Need reference phase not affected by boundary, e.g.:

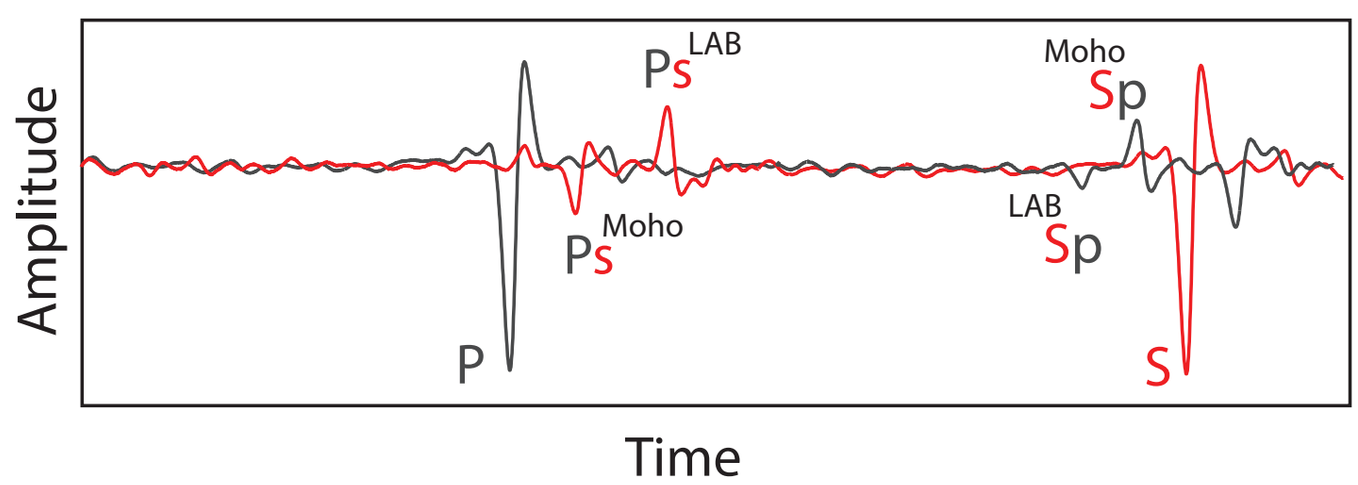


Targeting boundaries - converted phases

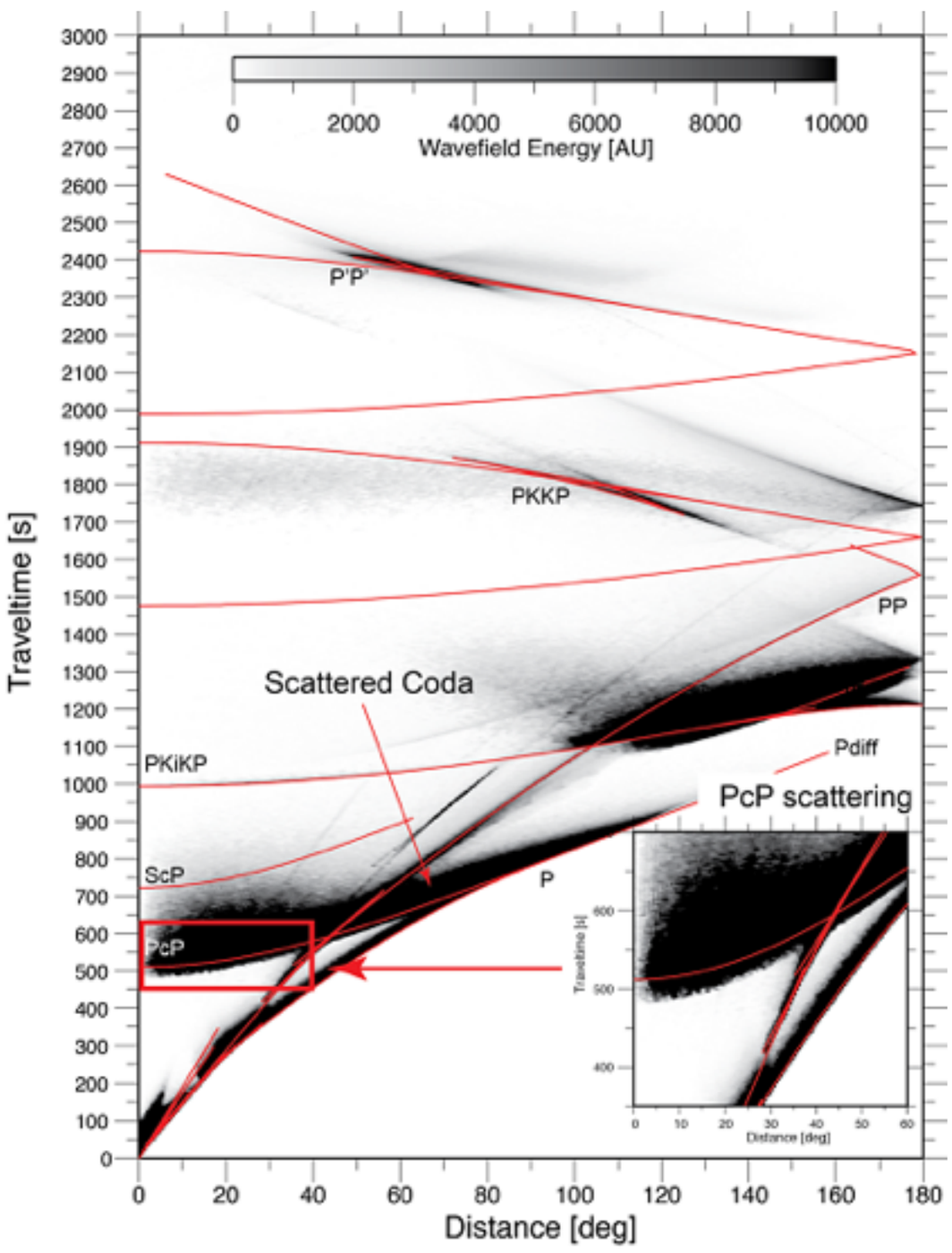
P-to-S conversions



- What is the amplitude and sharpness of the seismic discontinuity? -> measure frequency sensitive conversion coefficients
- What is the topography of the discontinuity? -> differential travel times



Scattering - or *Coda*

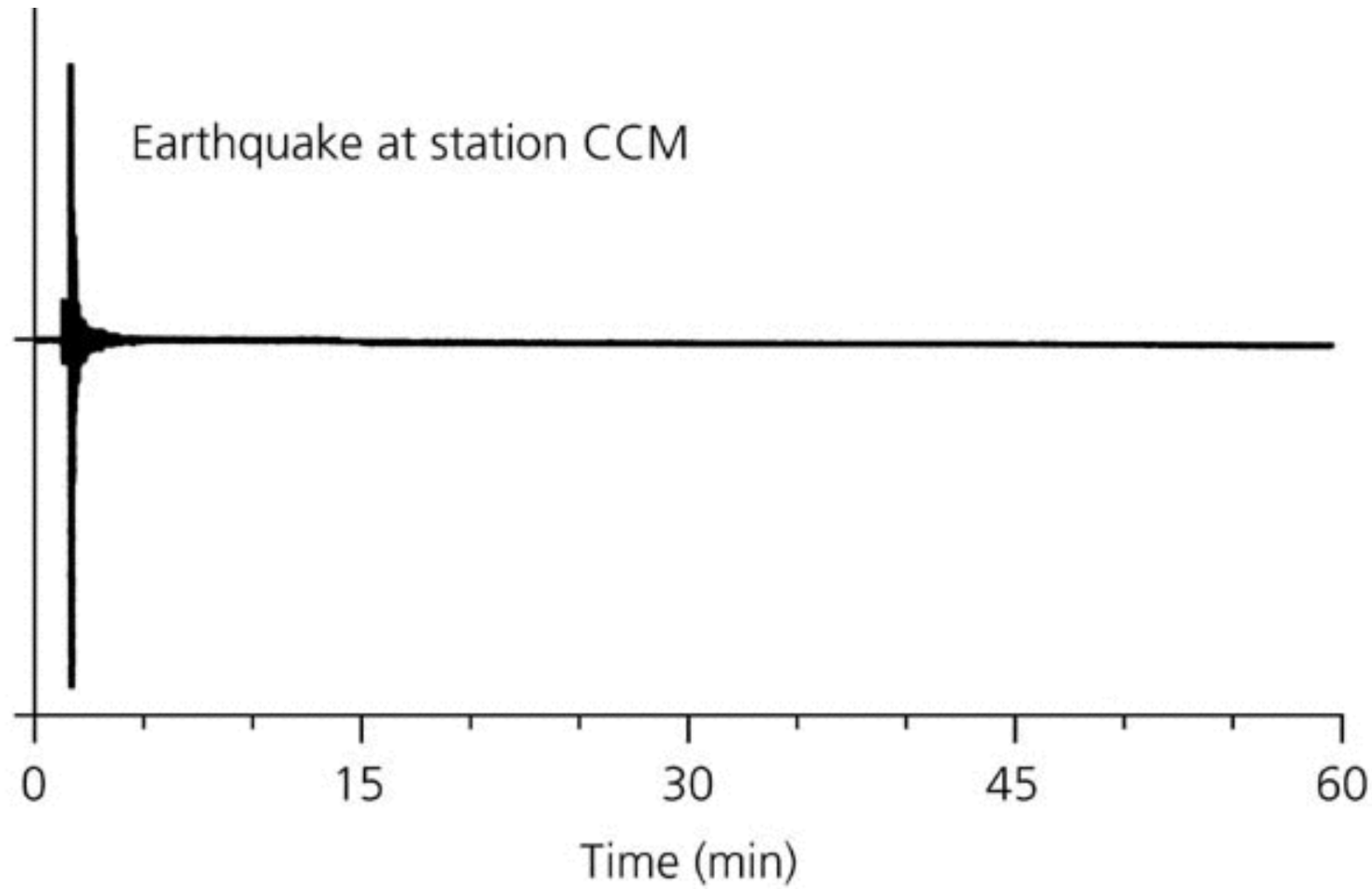


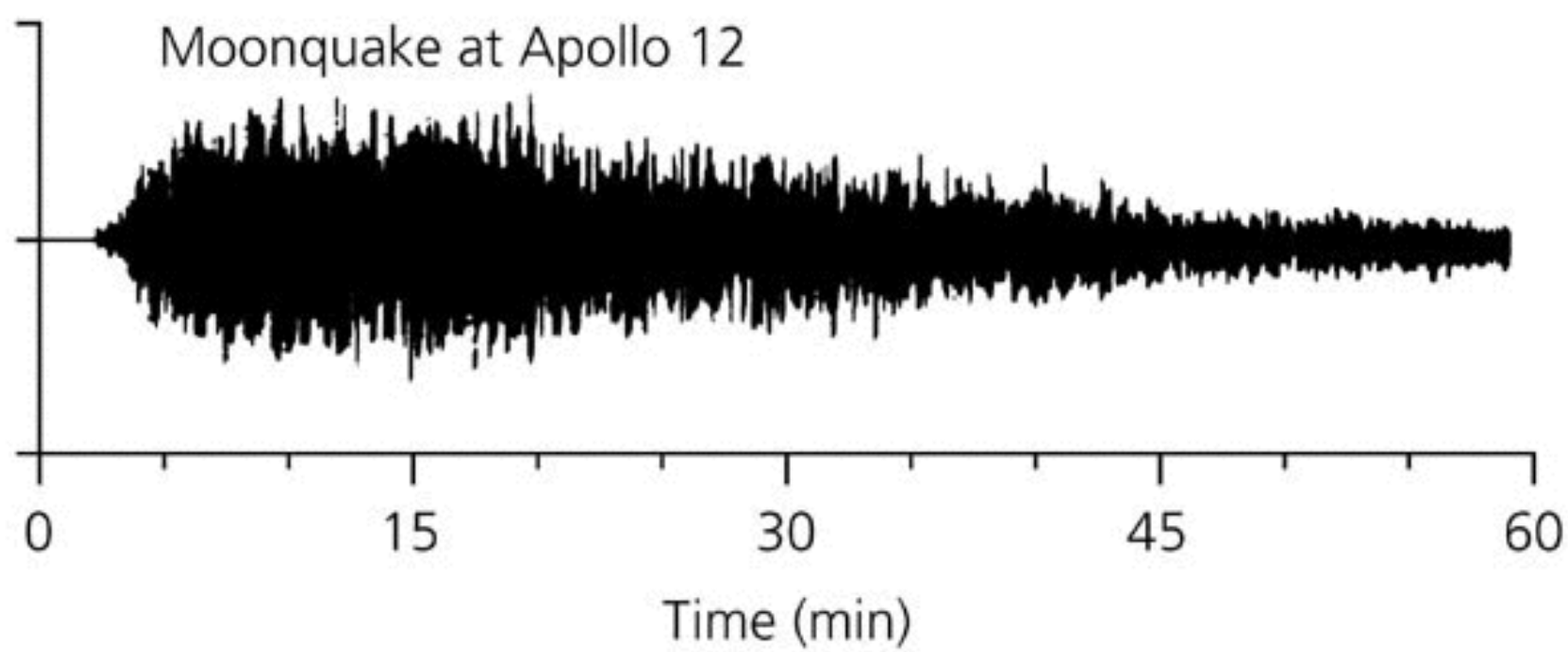
- What are the sizes of the heterogeneity?
  -> ~ Wavelength of scattered waves
- Where is this heterogeneity present?
  -> Incoming direction of waves
- What are amplitudes of the heterogeneity?
  -> Energy of scattered waves is sensitive to impedance contrast

  -> Sometimes individual scatterers can be constrained.

  -> Otherwise statistical properties of scatterers are studied

Scattering

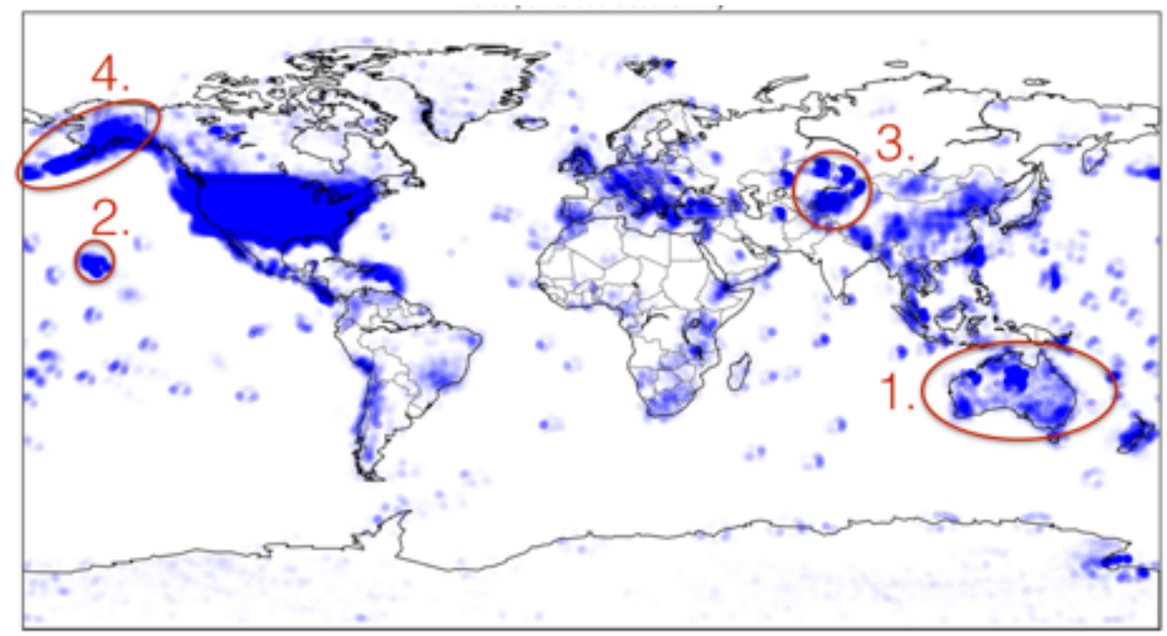

Earthquake at station CCM
0
15
30
45
60
Time (min)

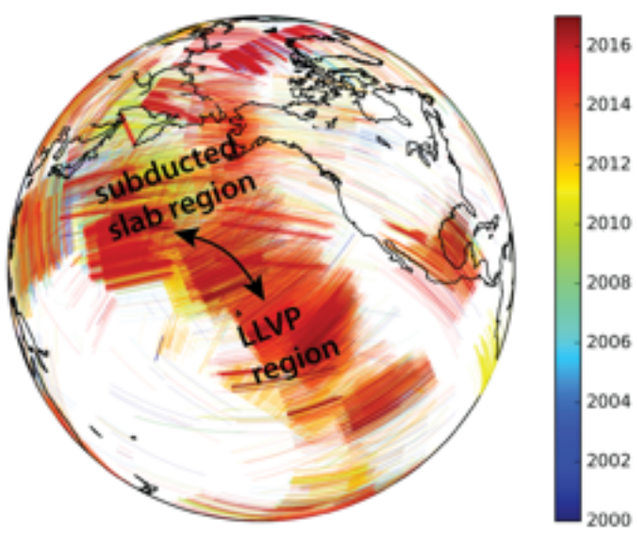

Moonquake at Apollo 12
0
15
30
45
60
Time (min)

Body wave seismologists: First contemplations

- What phase can be used for target area?
- What reference phase can be used?
- What event -station geometry works to target the area of interest? Sometimes an array of stations is needed

Pierce points at 660 km receiver functions



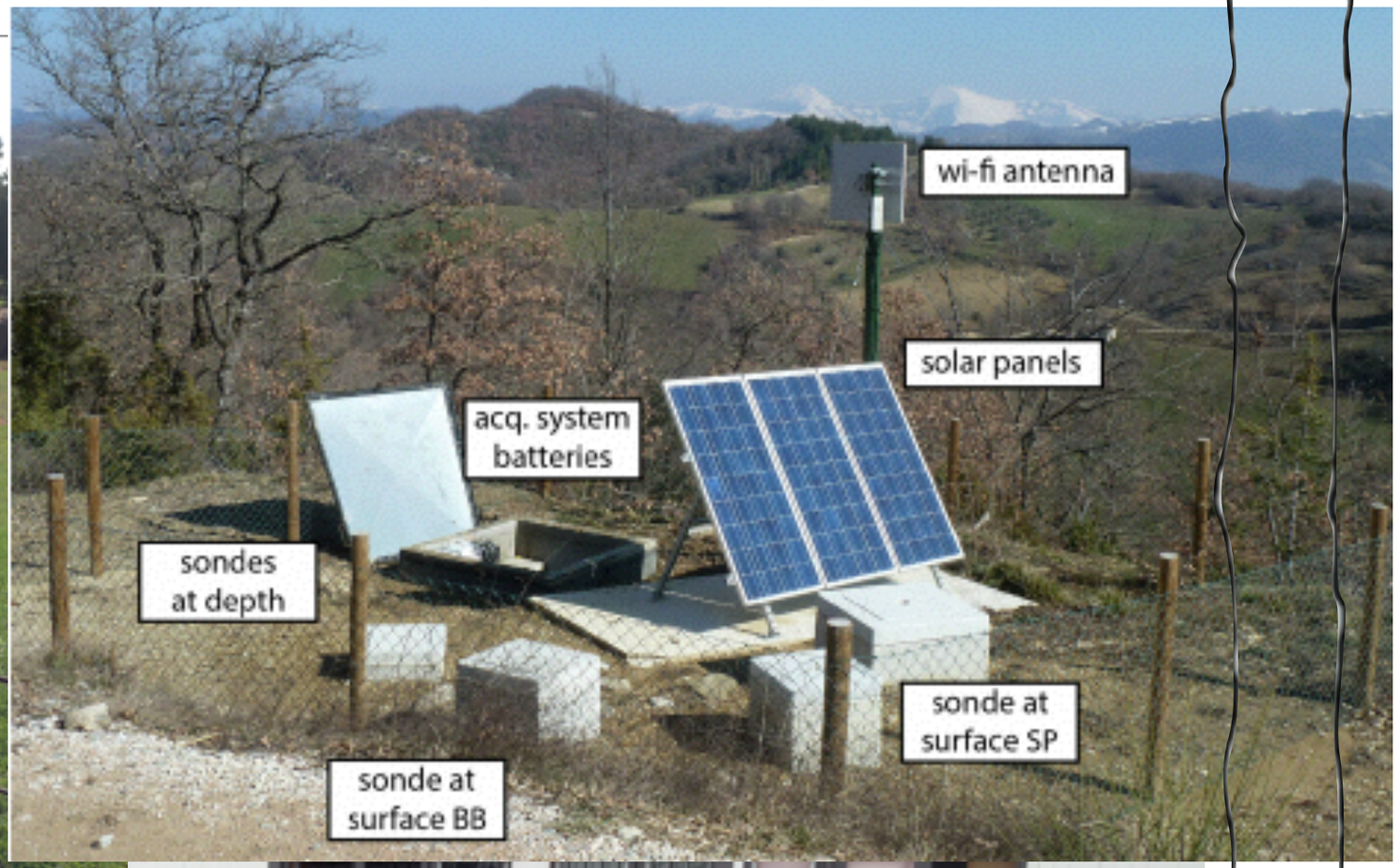


Diffracted paths at the core-mantle boundary across the Pacific

- Or

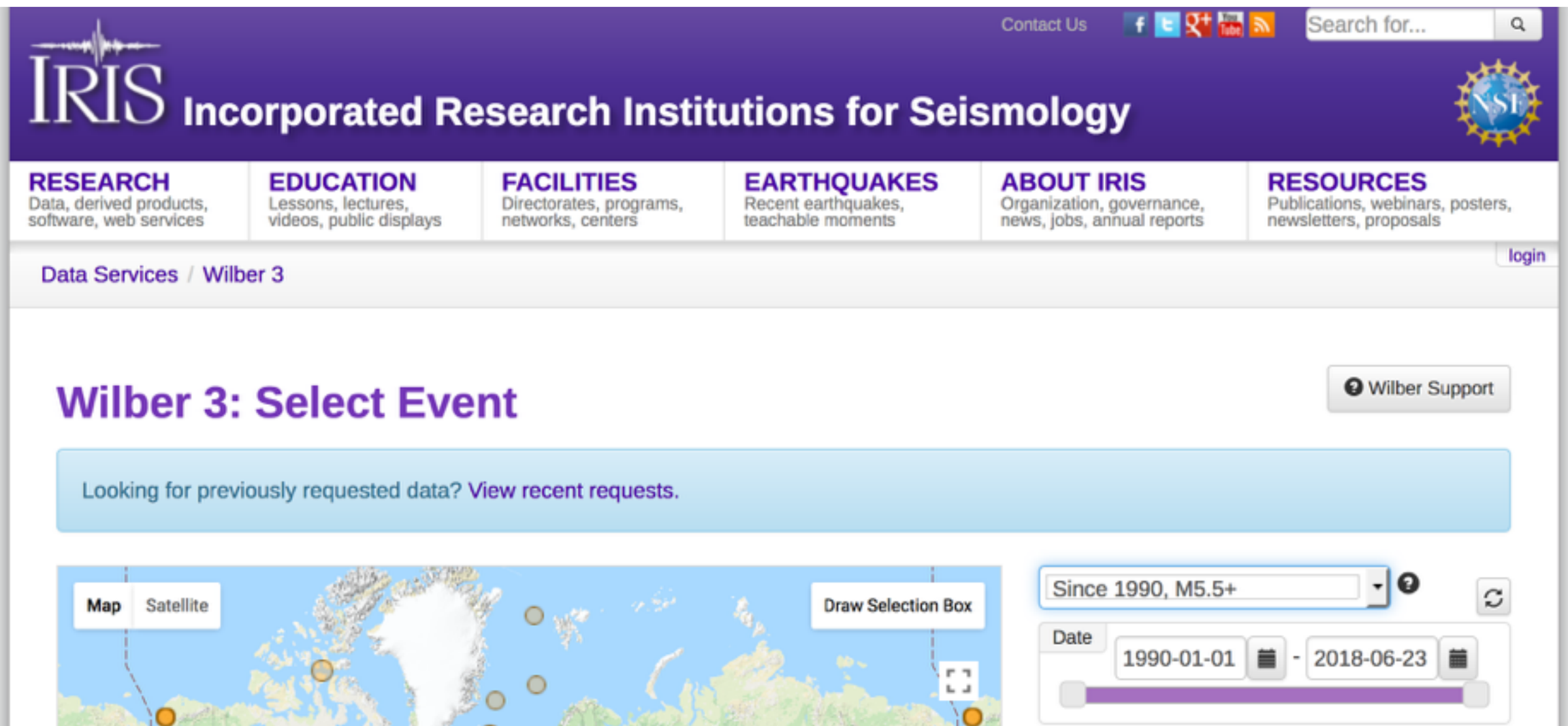
There is this new geometry available, I have the setup to look at and analyse specific seismic phases, why don't I get the data … I am sure there will be something interesting in the data set.

Body wave study 101: Seismic experiments



Body wave study 101: Downloading the data

- One database is www.iris.edu. Data available for Mw 5.5+ earthquakes since 1990.



Body wave study 101: Processing the data

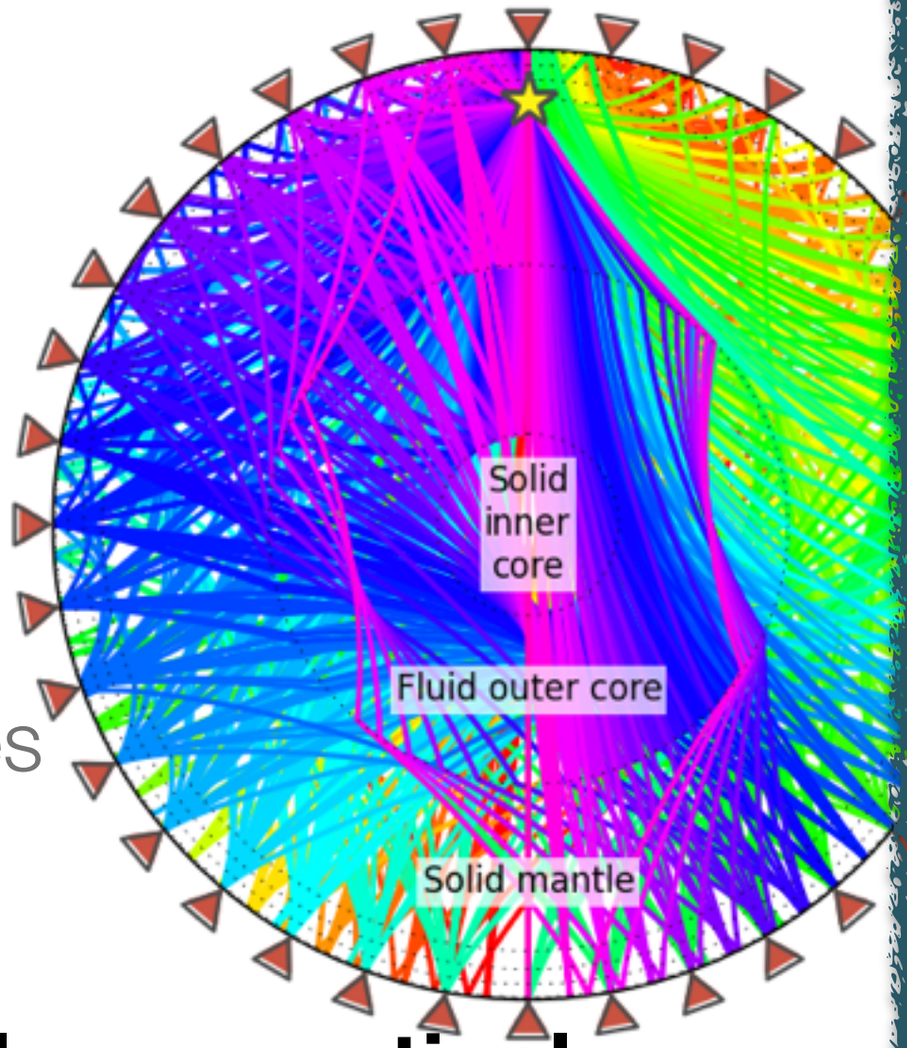
- Removing instrument response
- Rotation from North-East to Radial -Transverse
- Compute predicted arrivals to find phases of interest
- Potentially creating reference synthetics
- Applying some sorts of automated or *manual quality control*:

Body wave study 101: Software highlight

- Obspy -> www.obspy.org
- Open source python toolkit
- Handles
  - downloading
  - removing instrument response
  - general signal processing routines
  - plotting functions

ObsPy

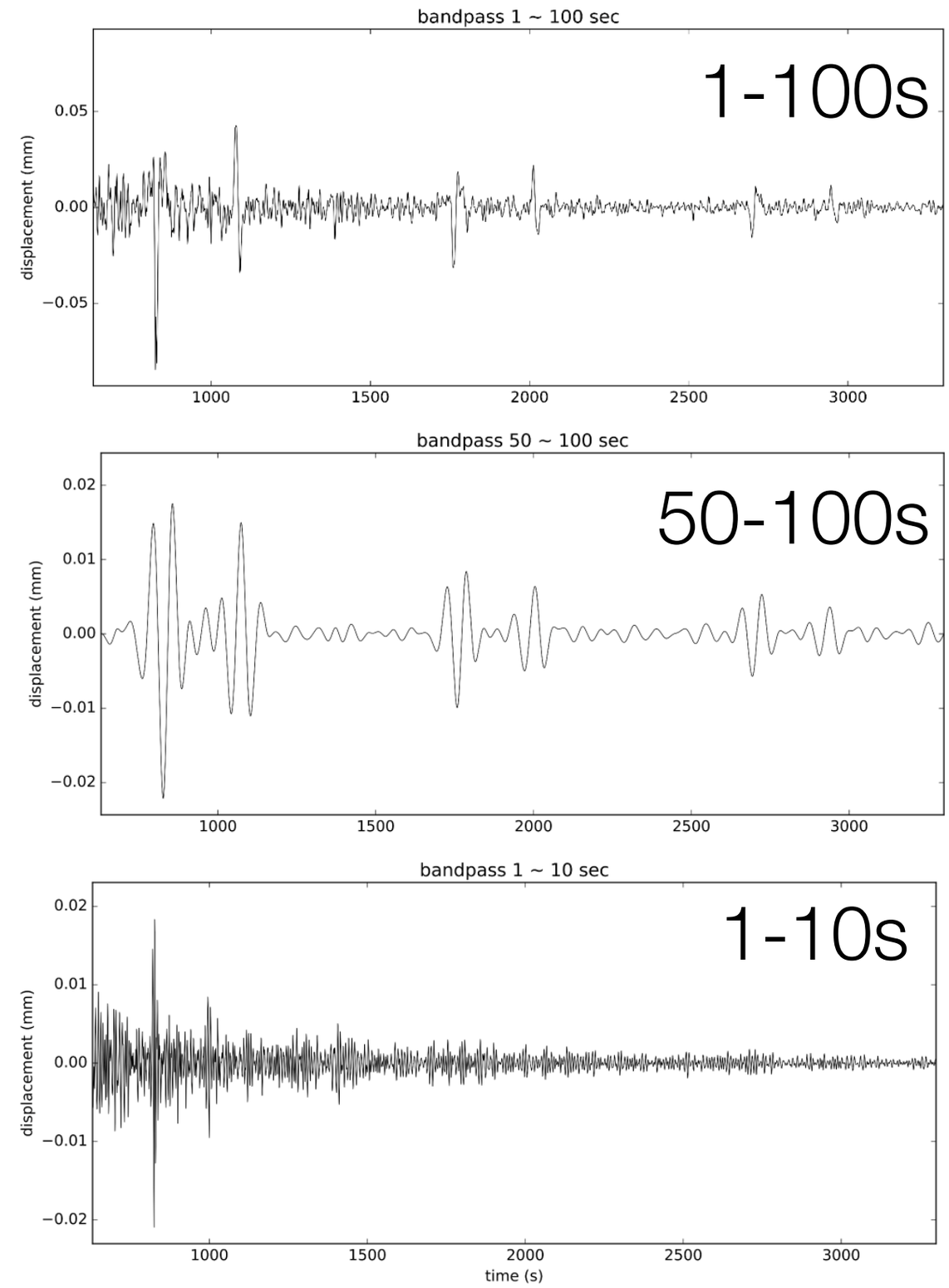
A Python Framework for Seismology



Will be used in the 2nd seismology practical

Body wave study 101: Data analyses

- Frequency band to choose resolution of interest
- Deconvolution of the assumed source (i.e. creating ‘receiver functions’
- Stacking to bring out smaller arrivals (array seismology techniques)



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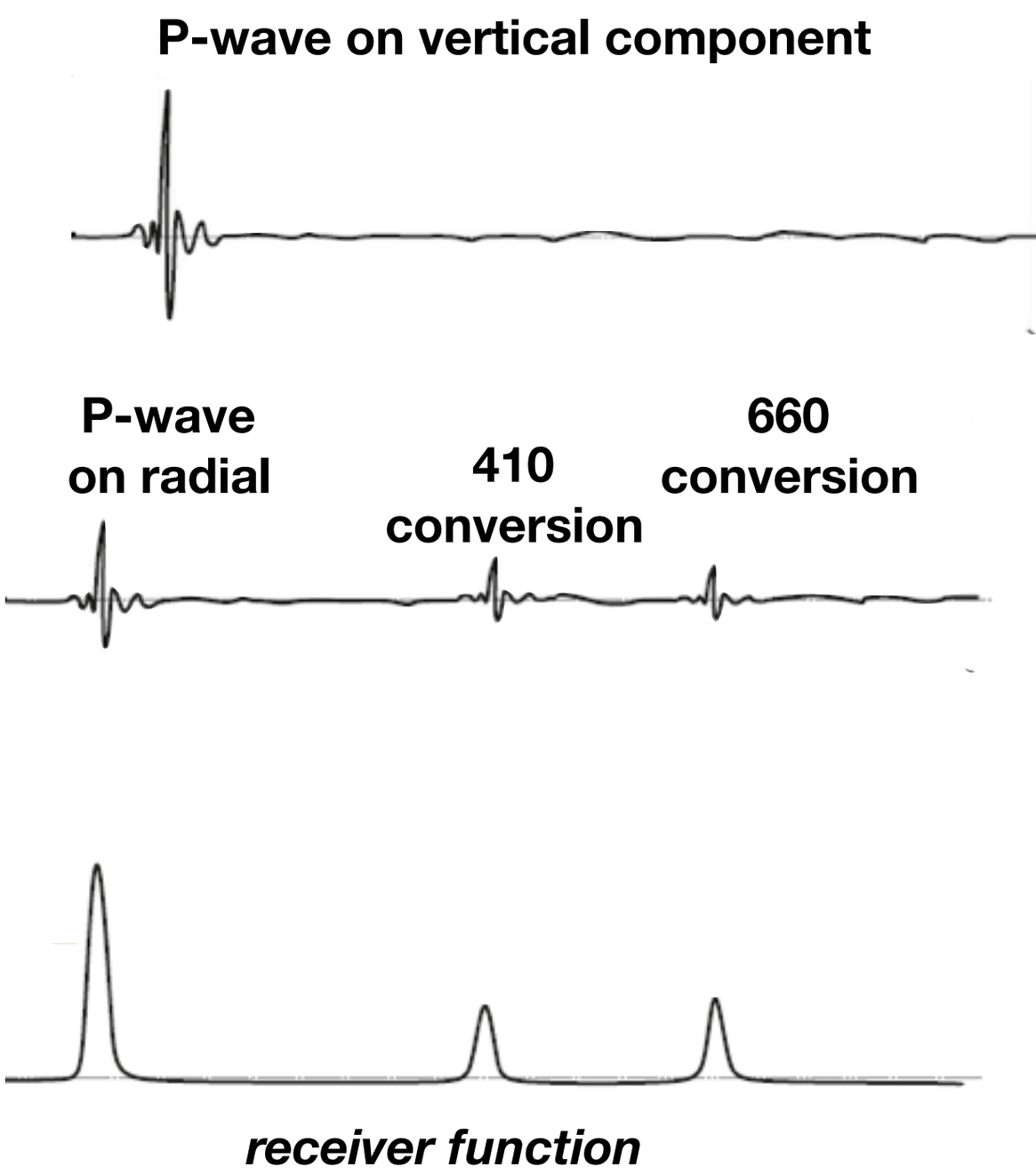
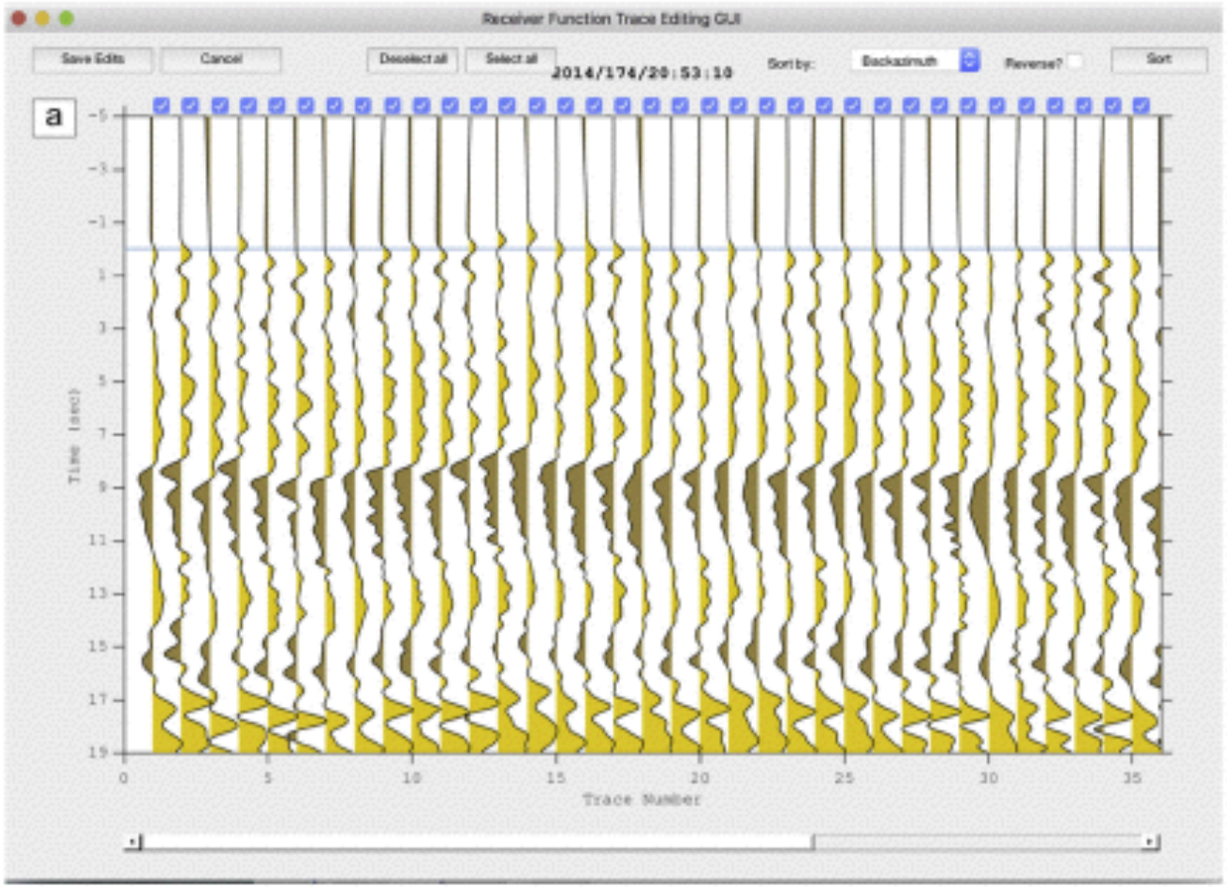


Software highlight: FuncLab

- Matlab GUI
- Receiver functions
  - download
  - processing
  - stacking



[https://seiscode.iris.washington.edu/, Porritt & Miller, 2018]

Body wave study 101: Data analyses

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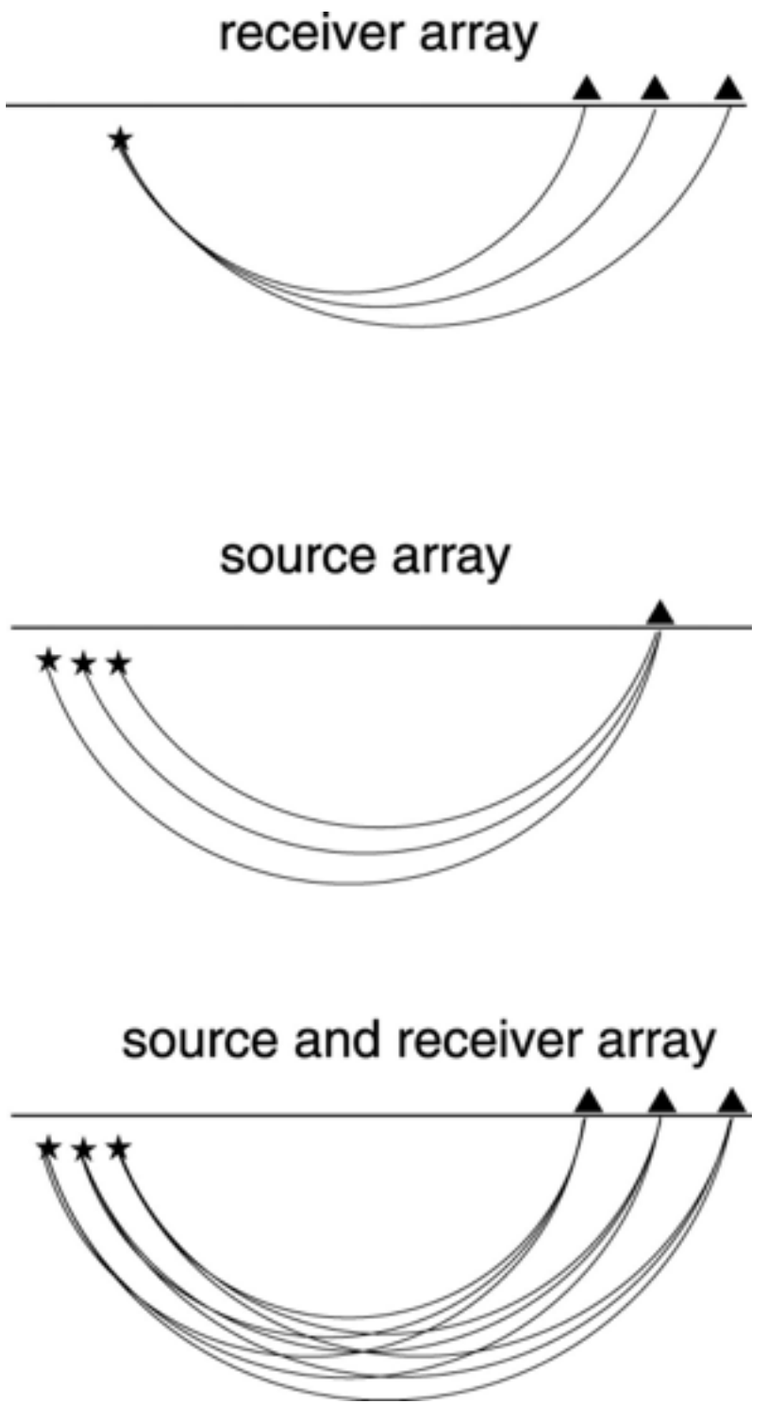

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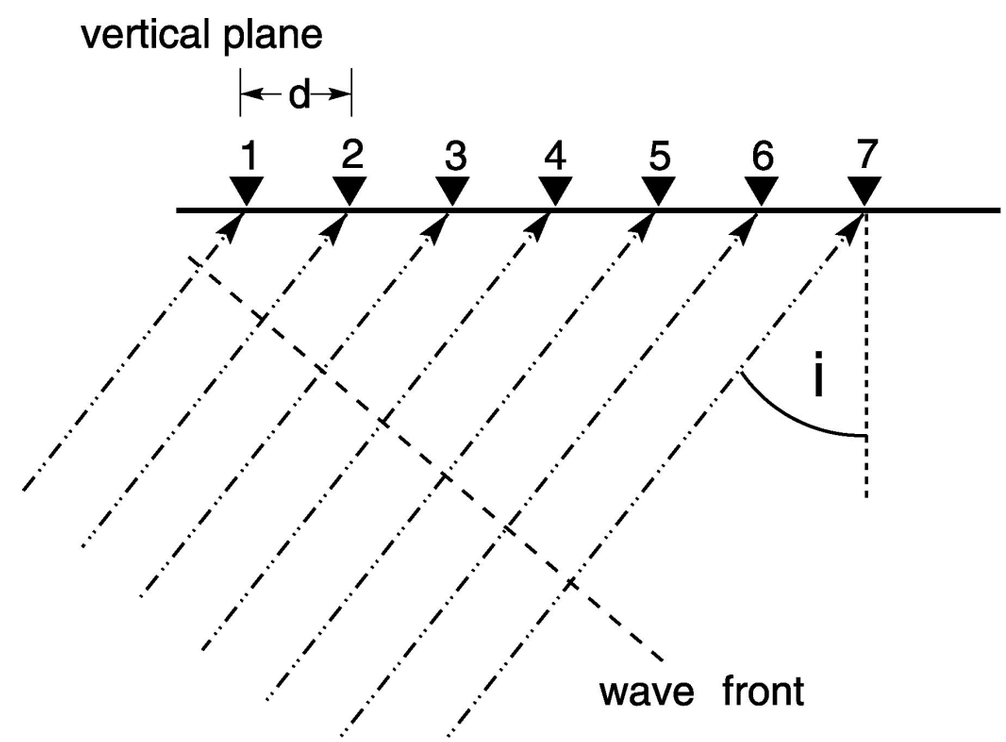

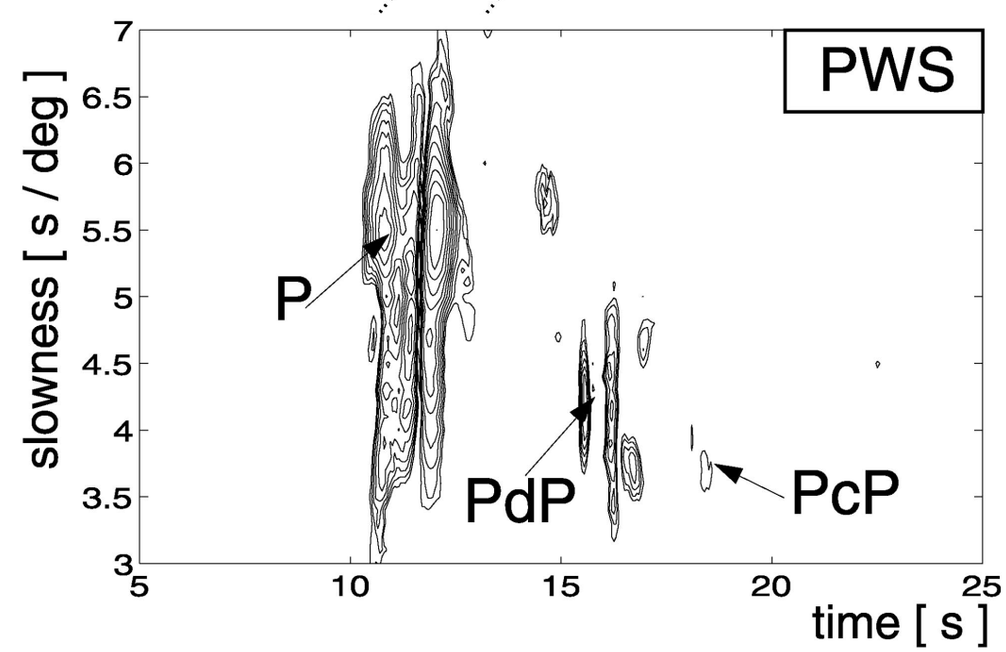

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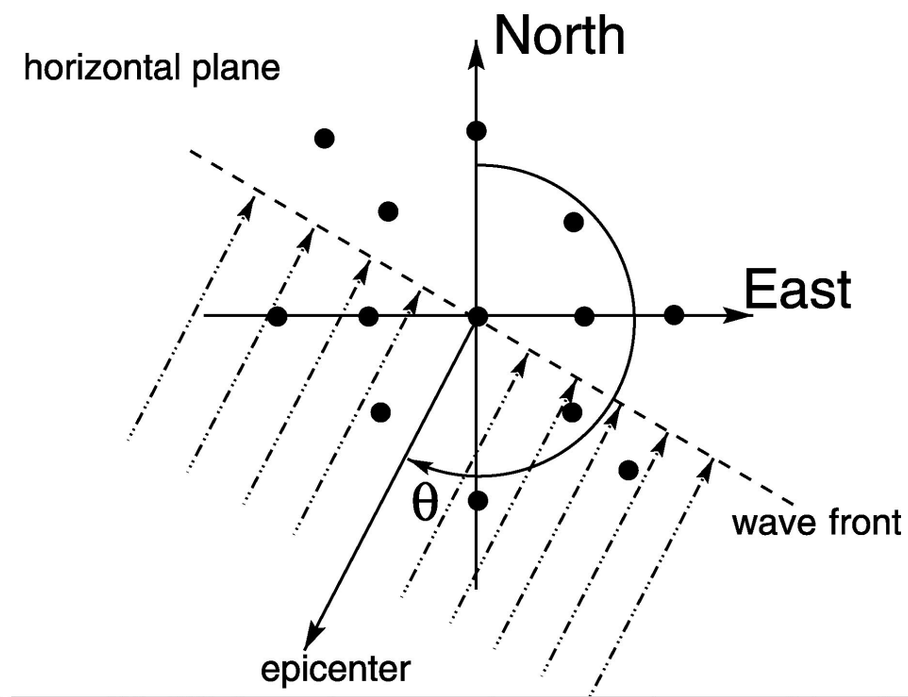

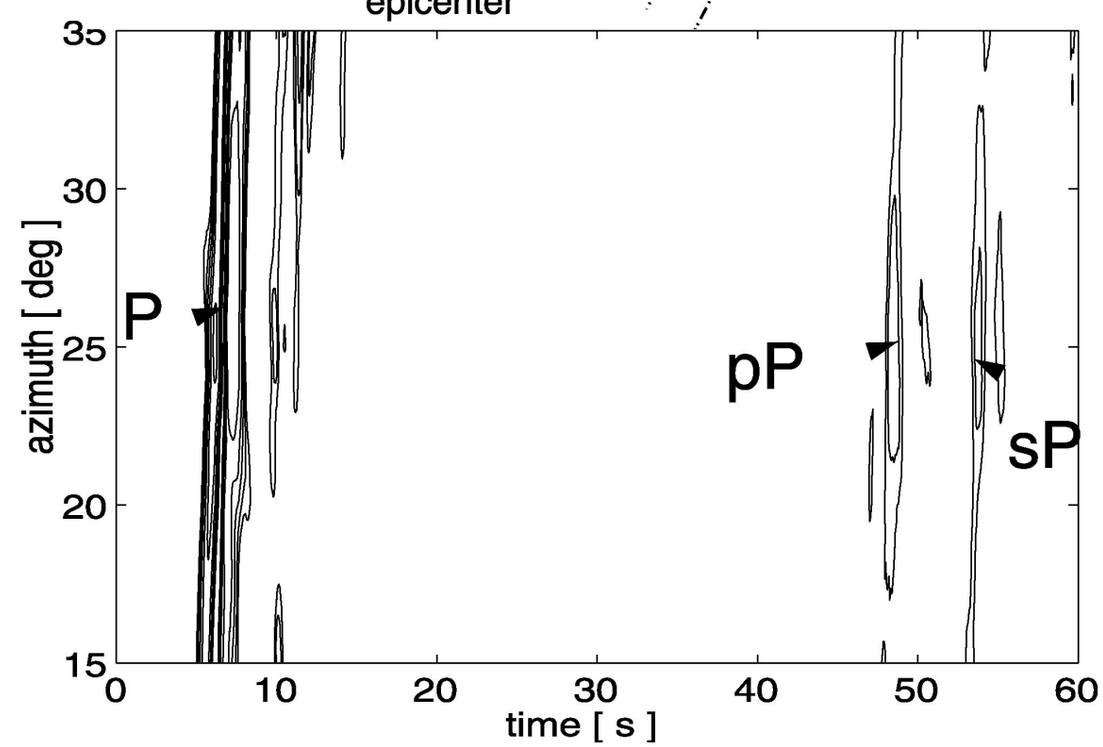

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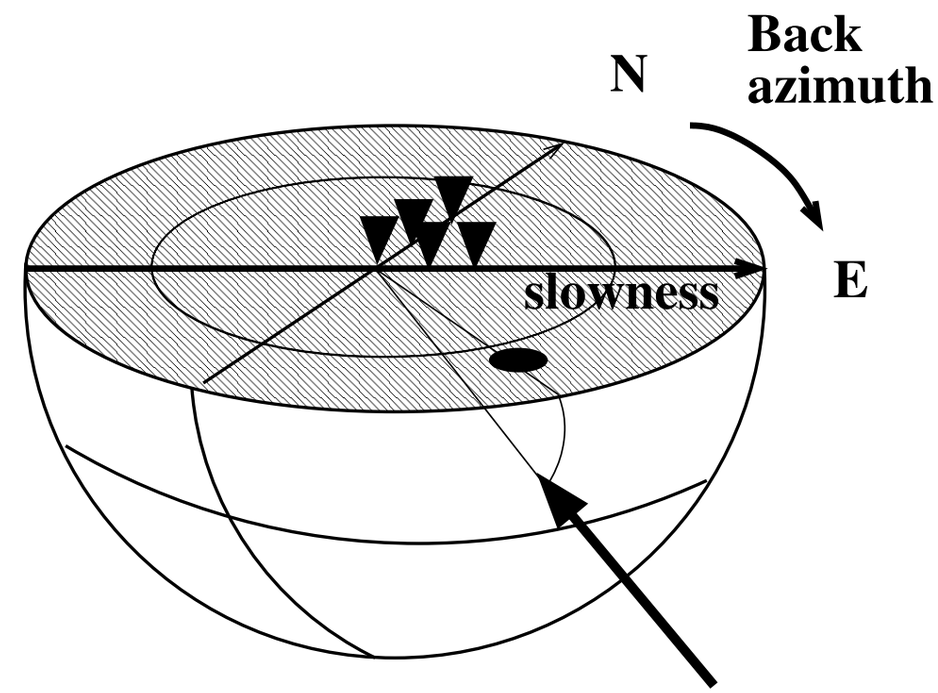
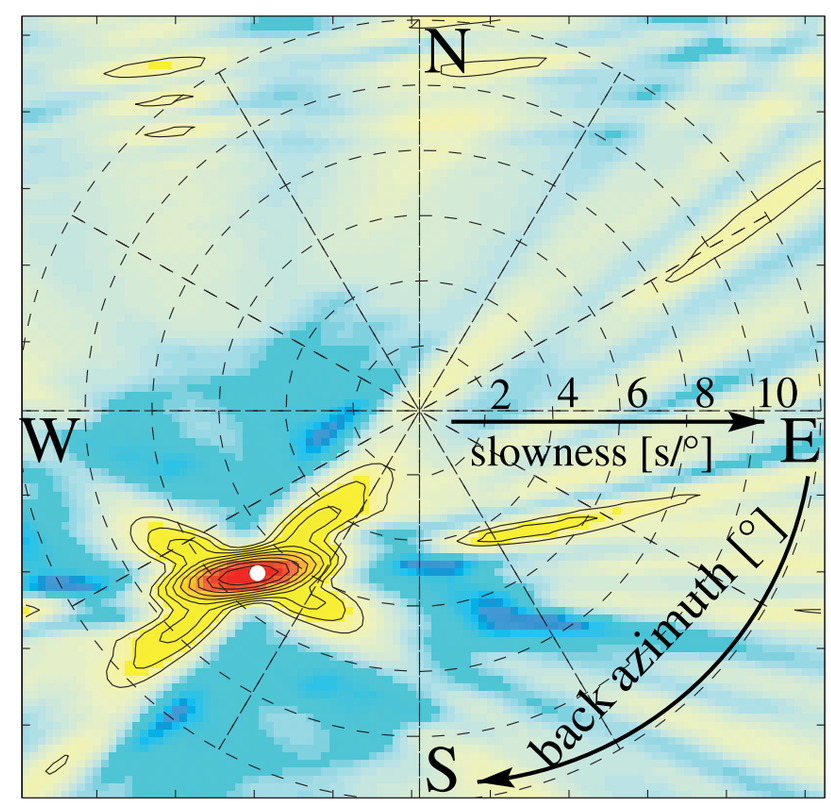
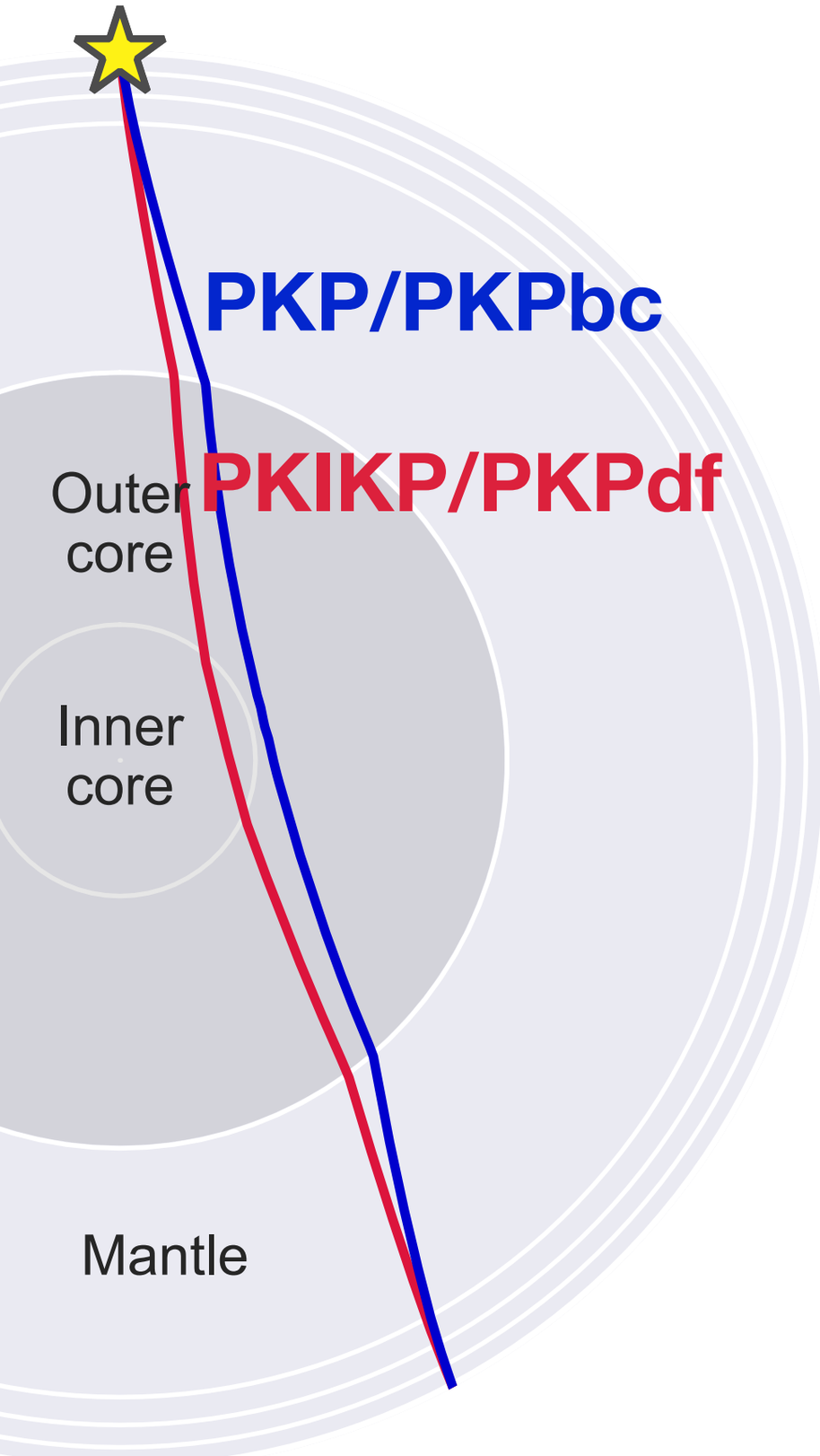
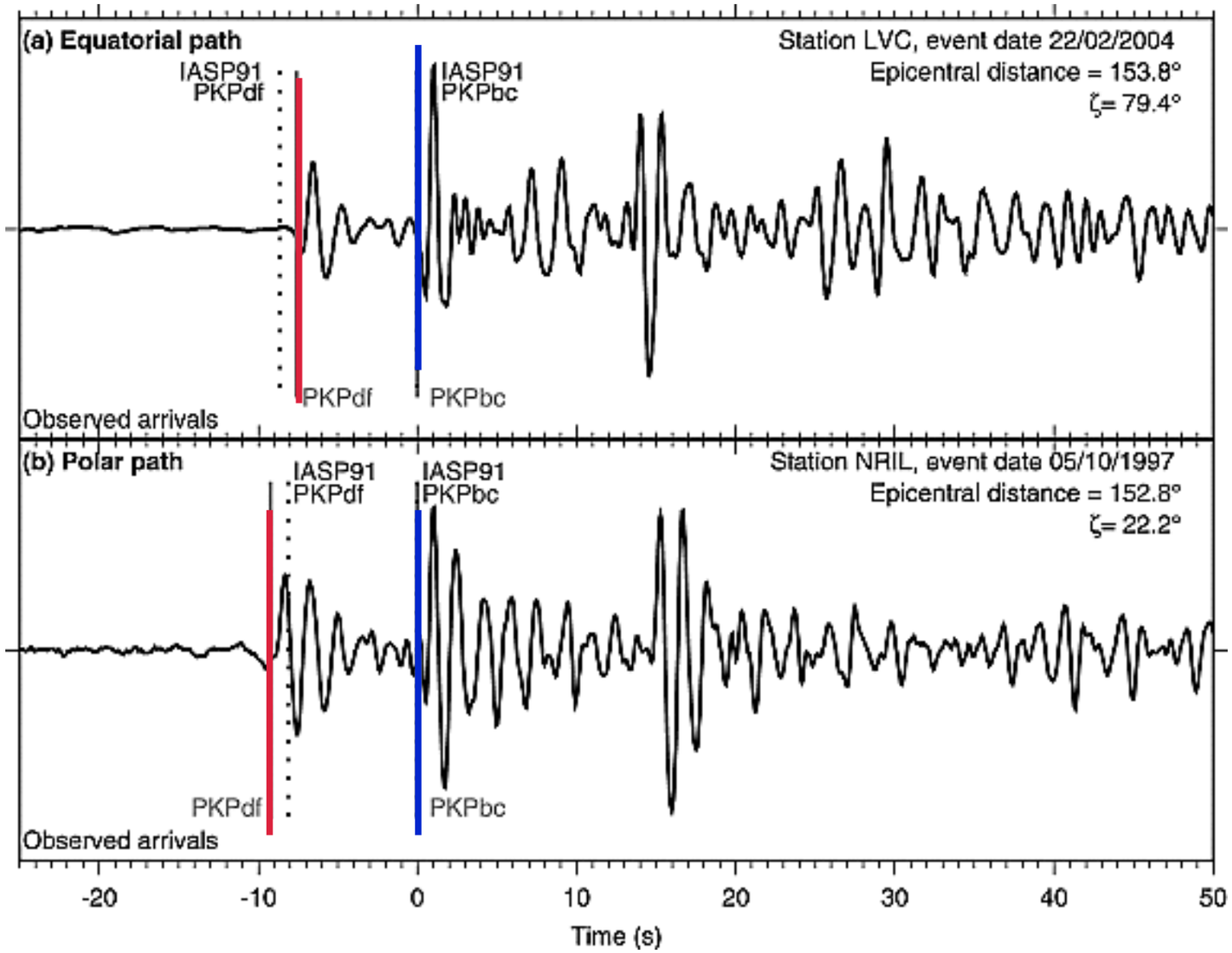




And now, on to some cool studies…

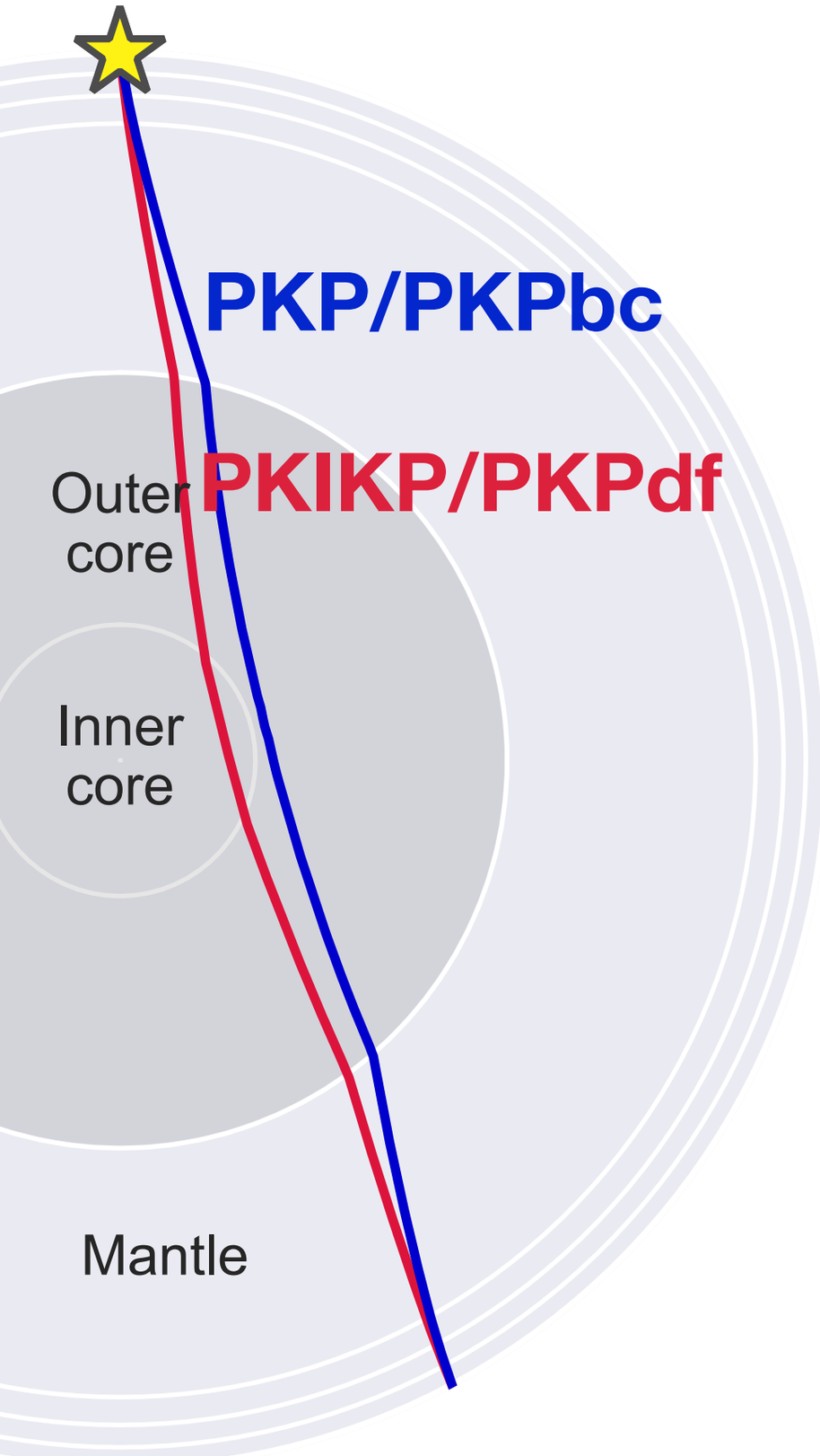
Inner Core phases

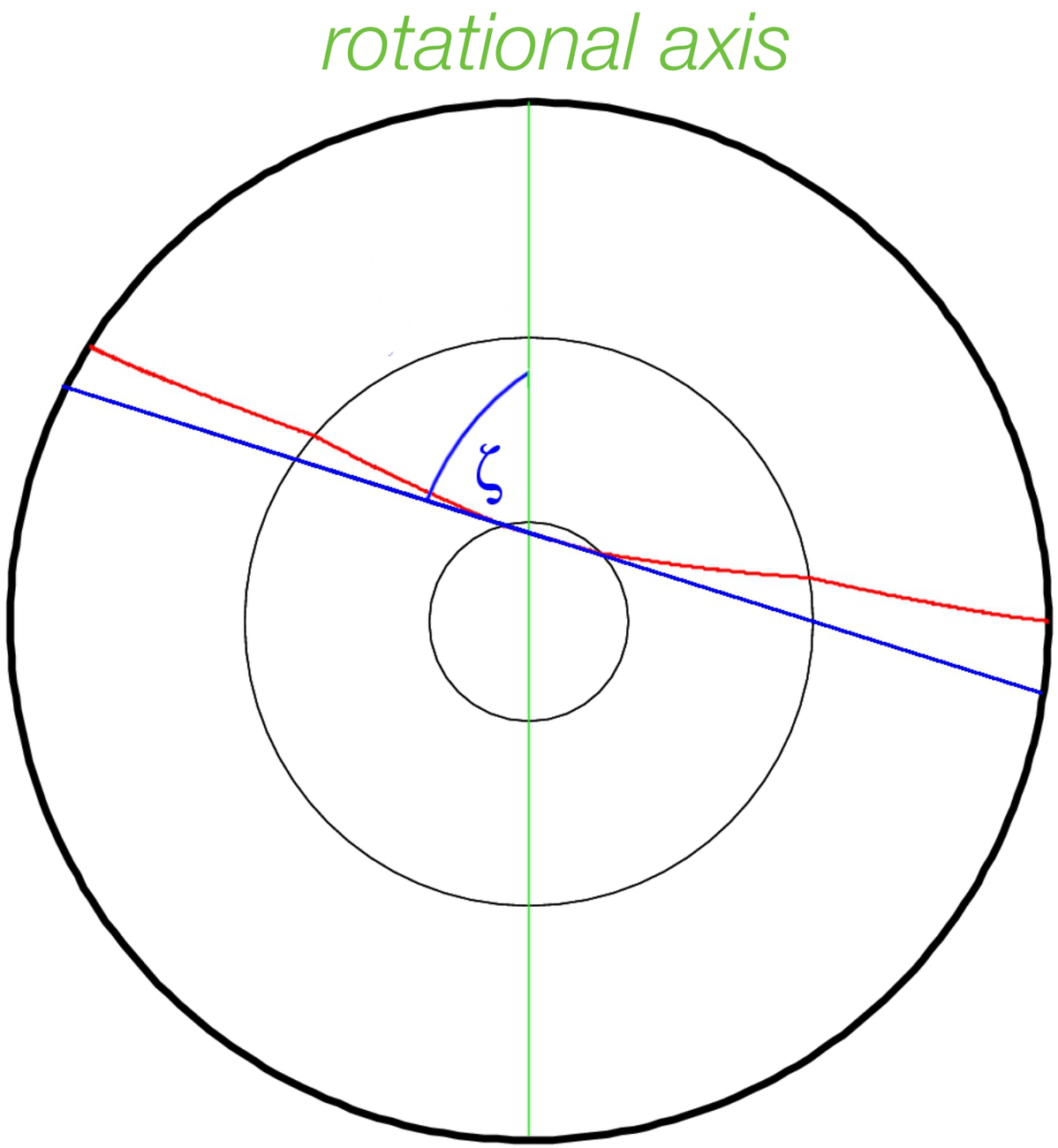




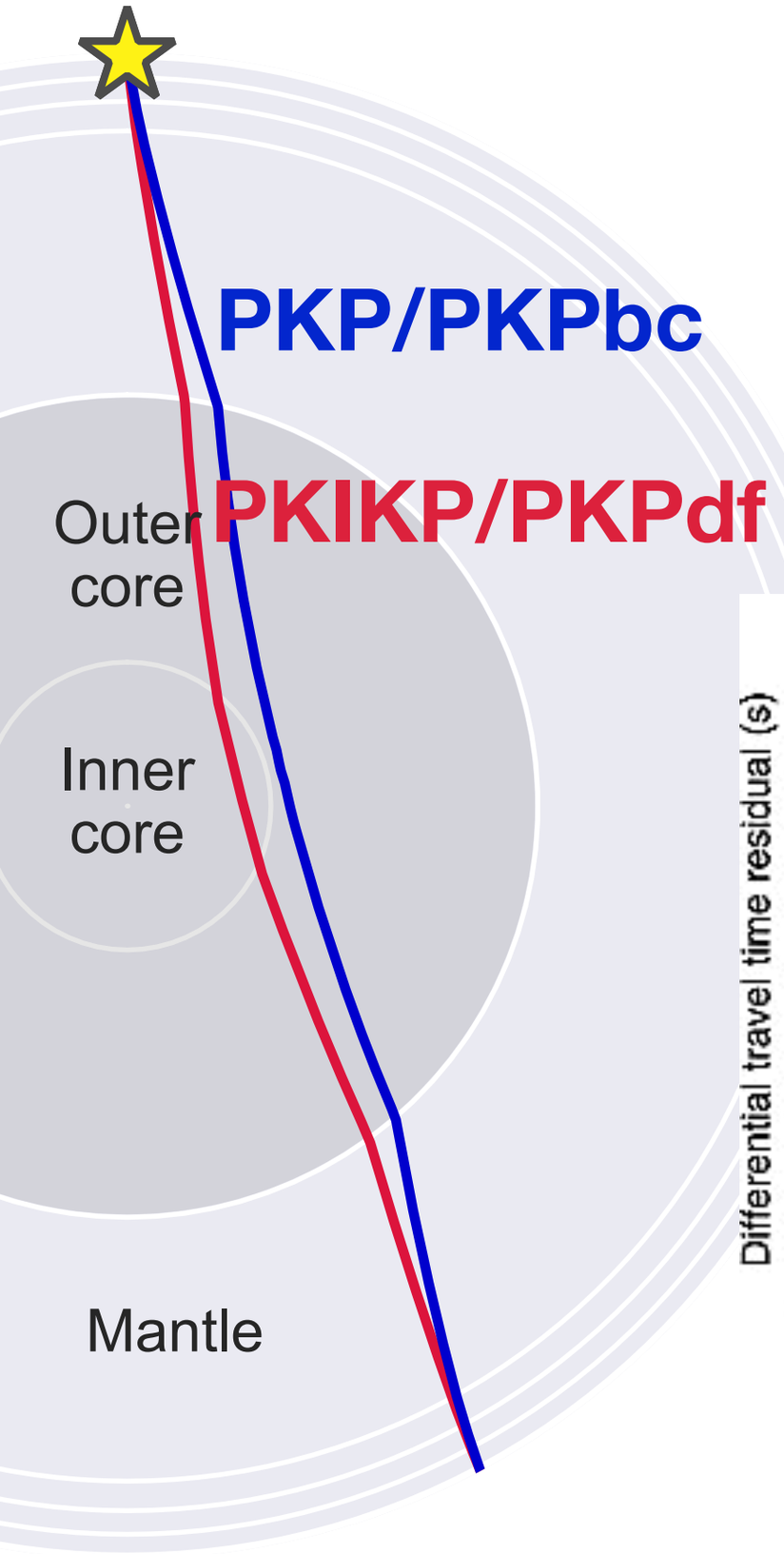
[Jessica Irving & Deuss, 2011]

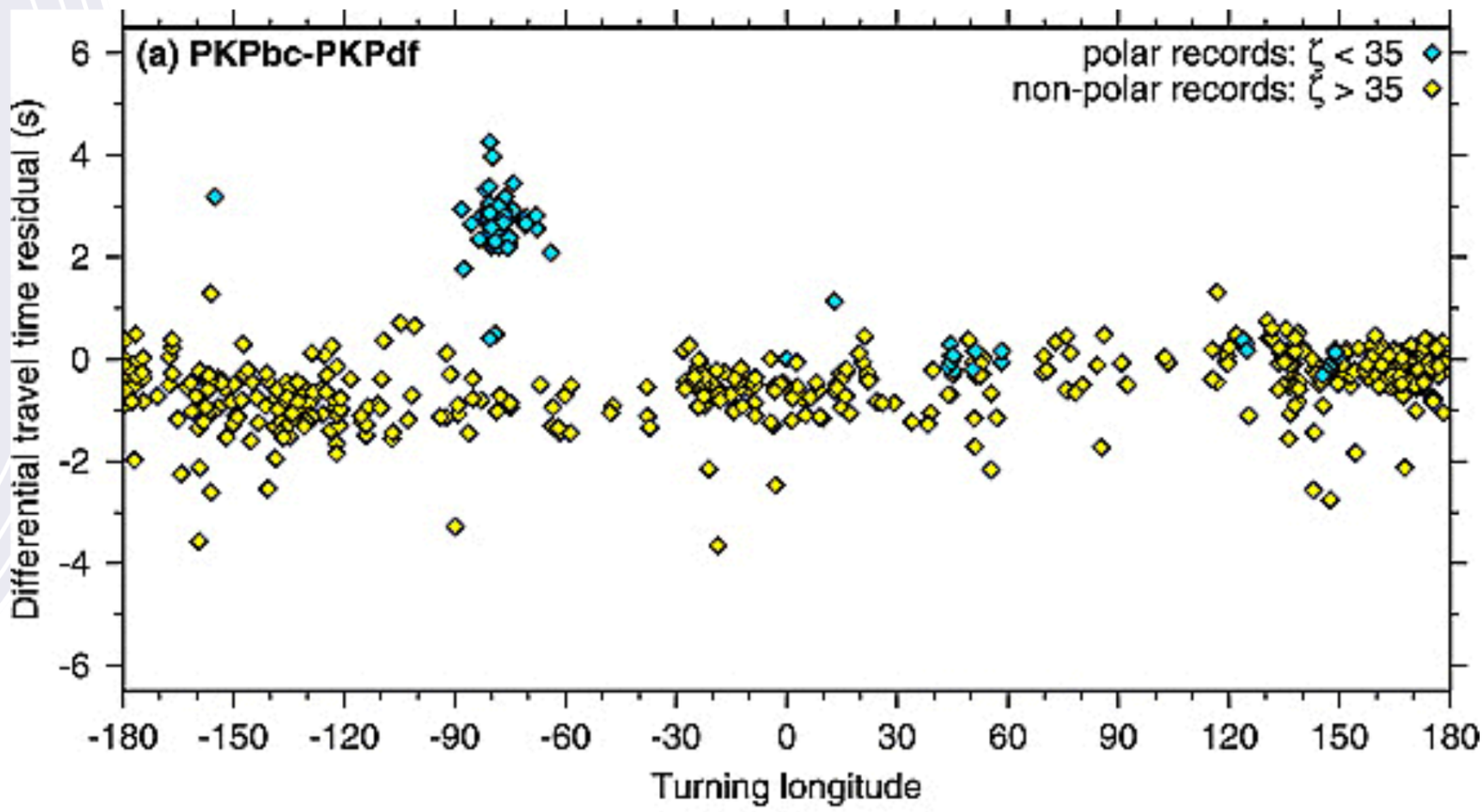
Inner Core phases





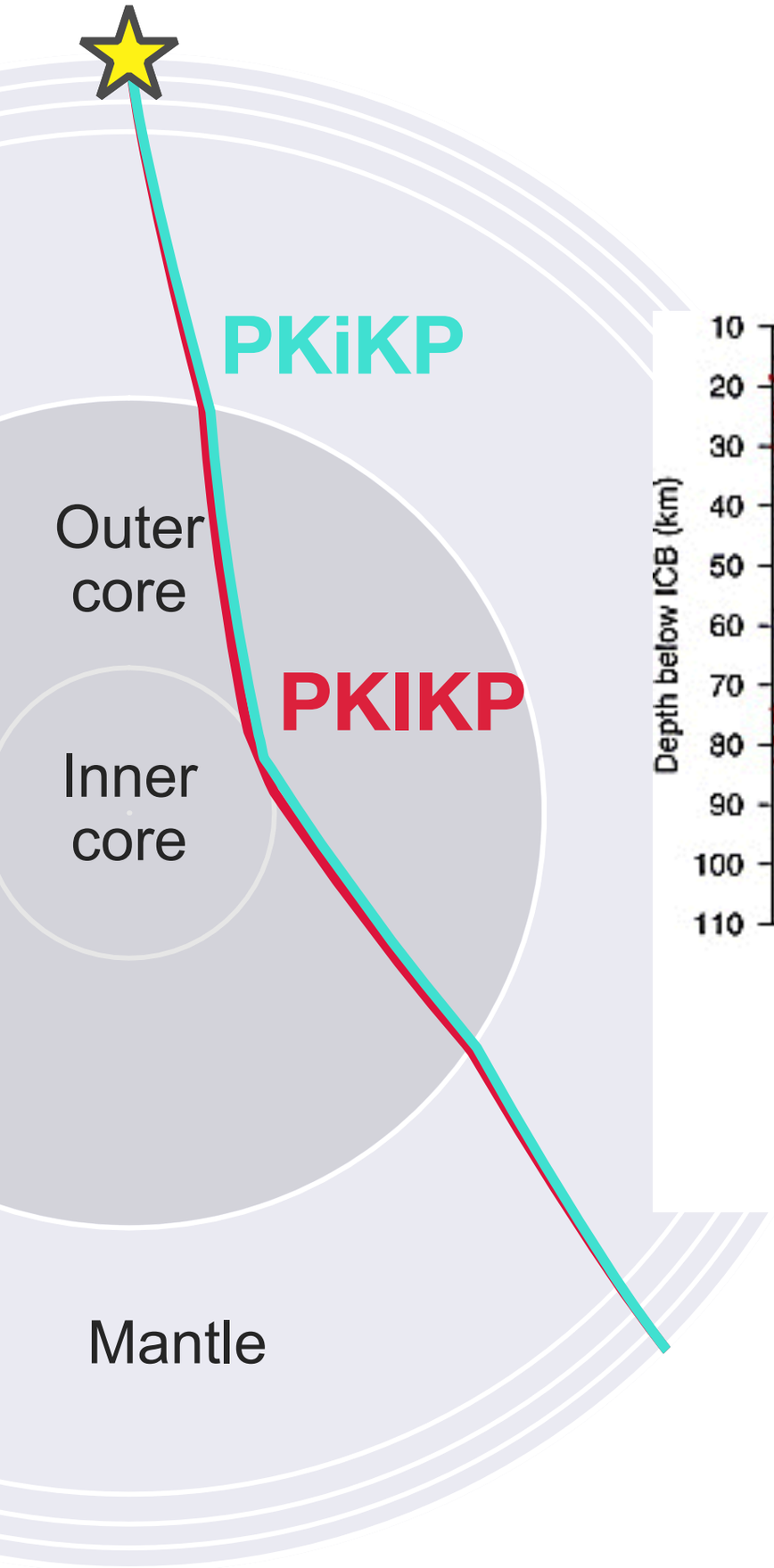
Inner core hemispherical anisotropy

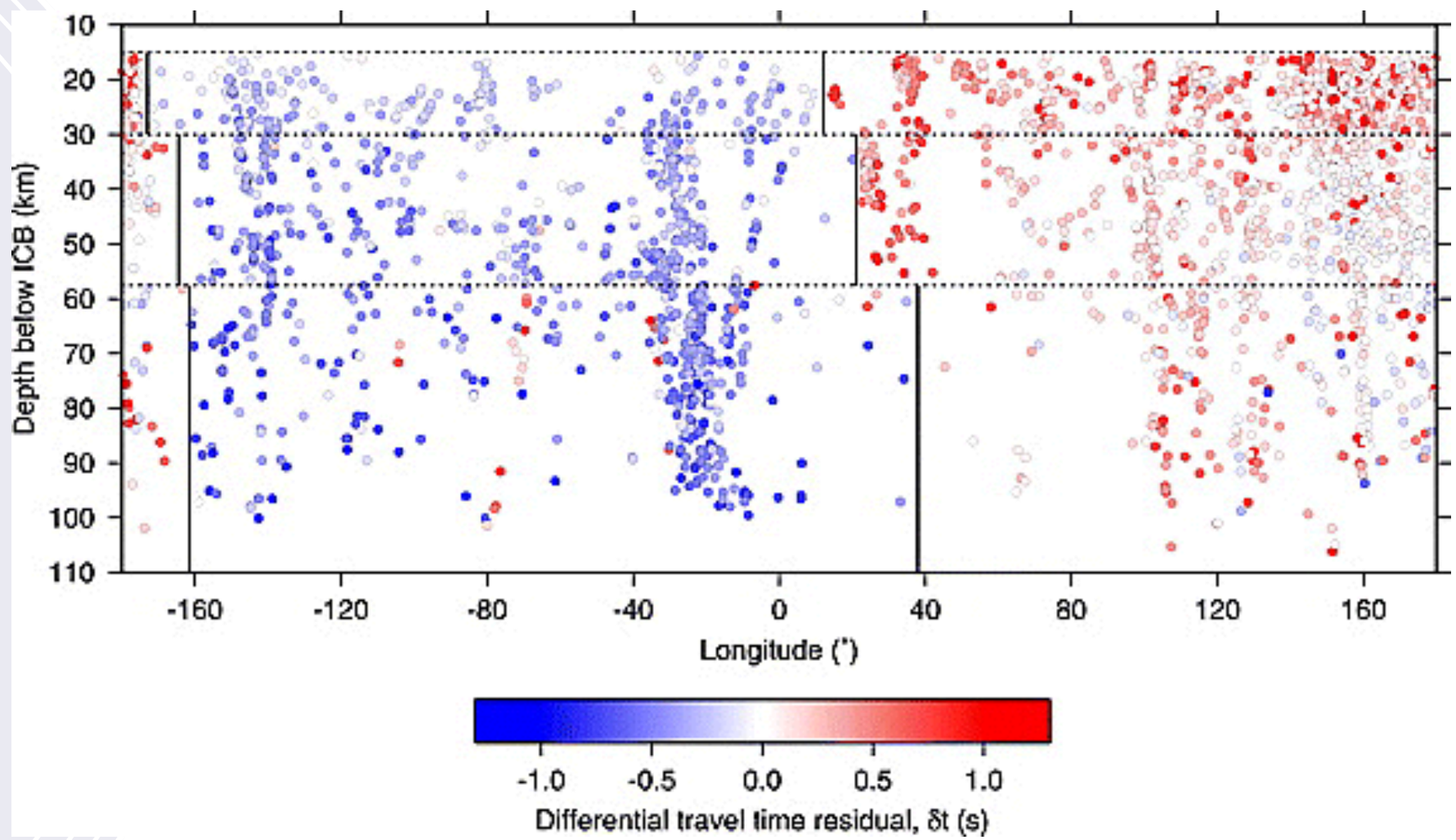




[Jessica Irving & Deuss, 2011]

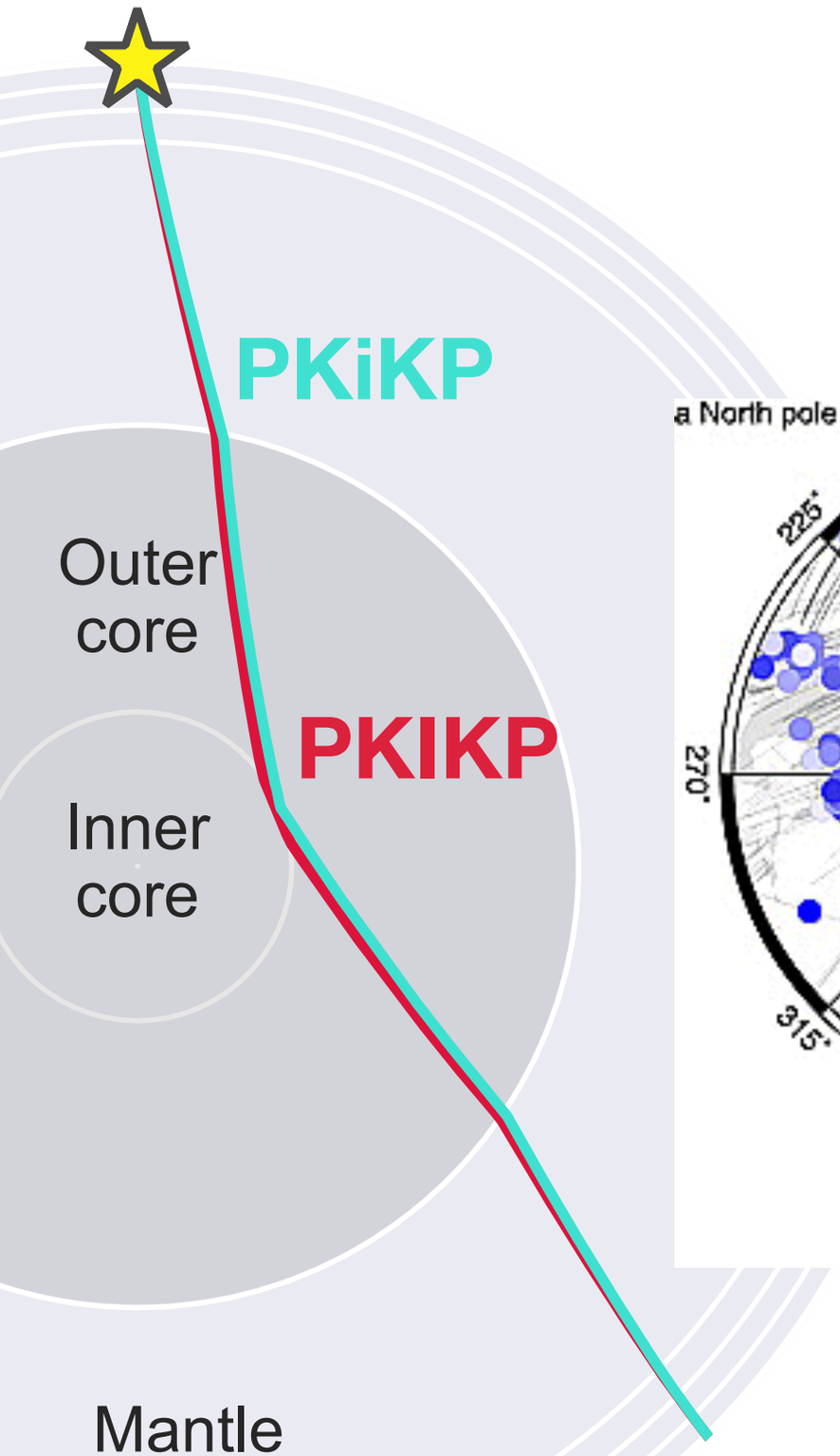
Inner core hemispheres

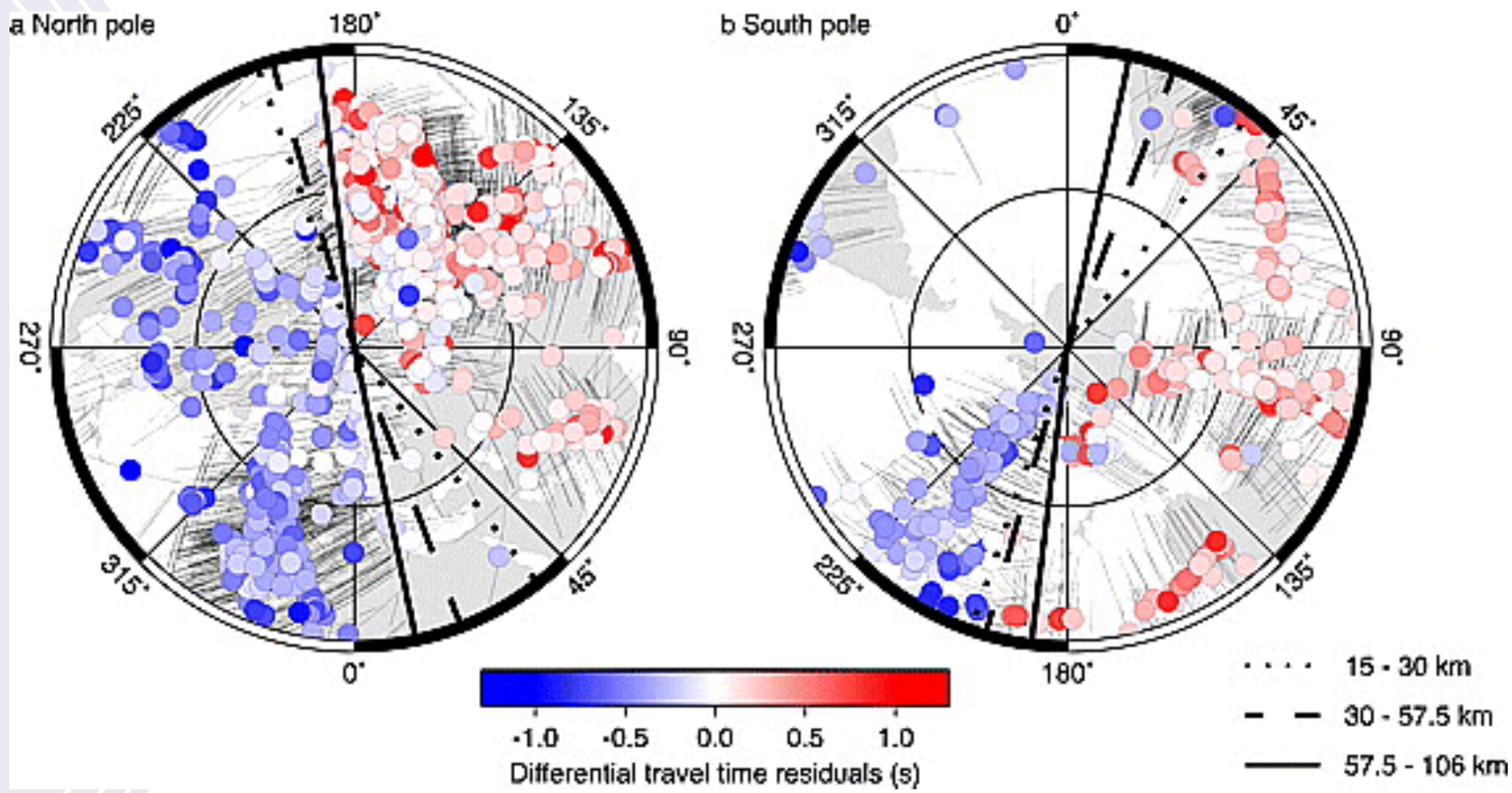




[Lauren Waszek & Deuss, 2011]

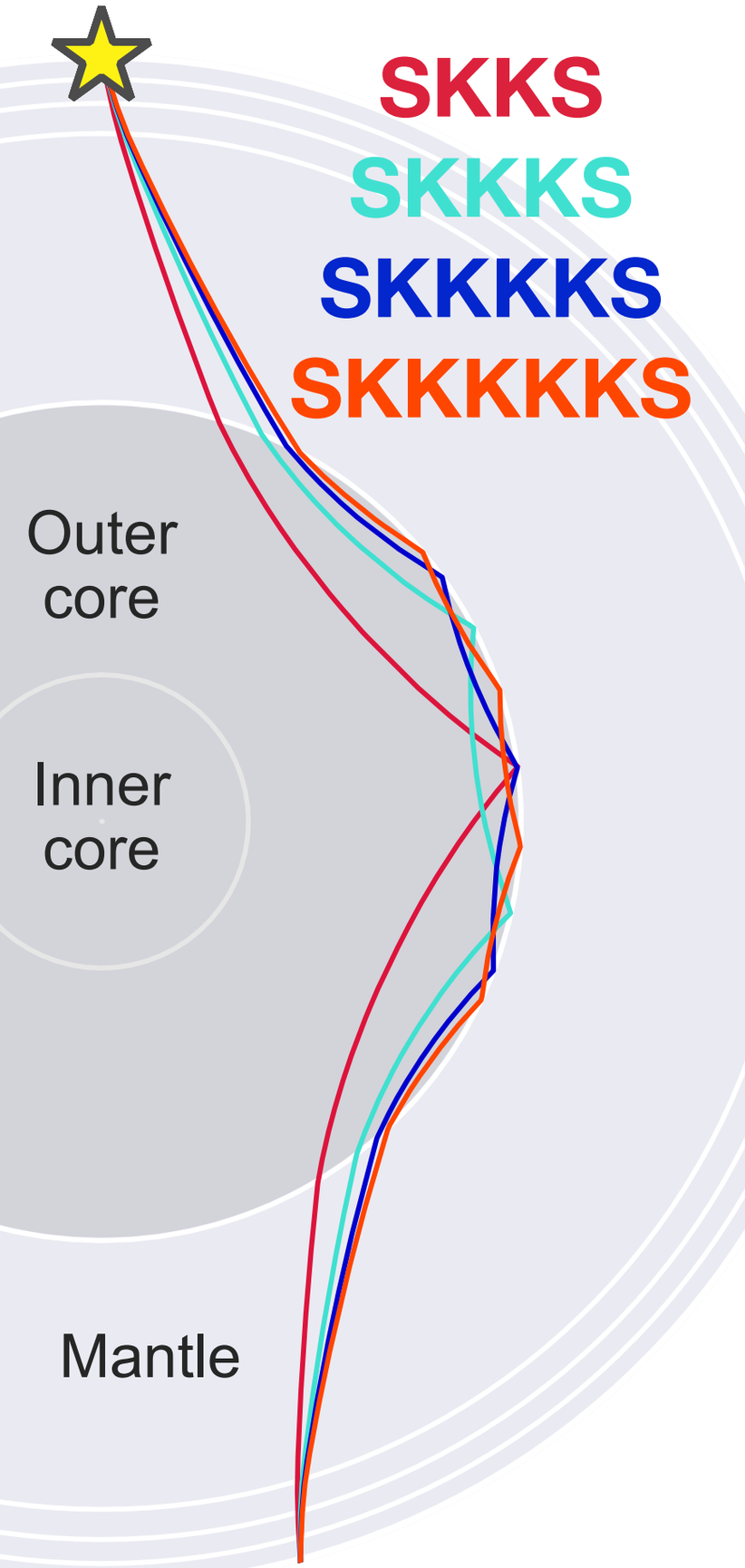
Inner core hemispheres

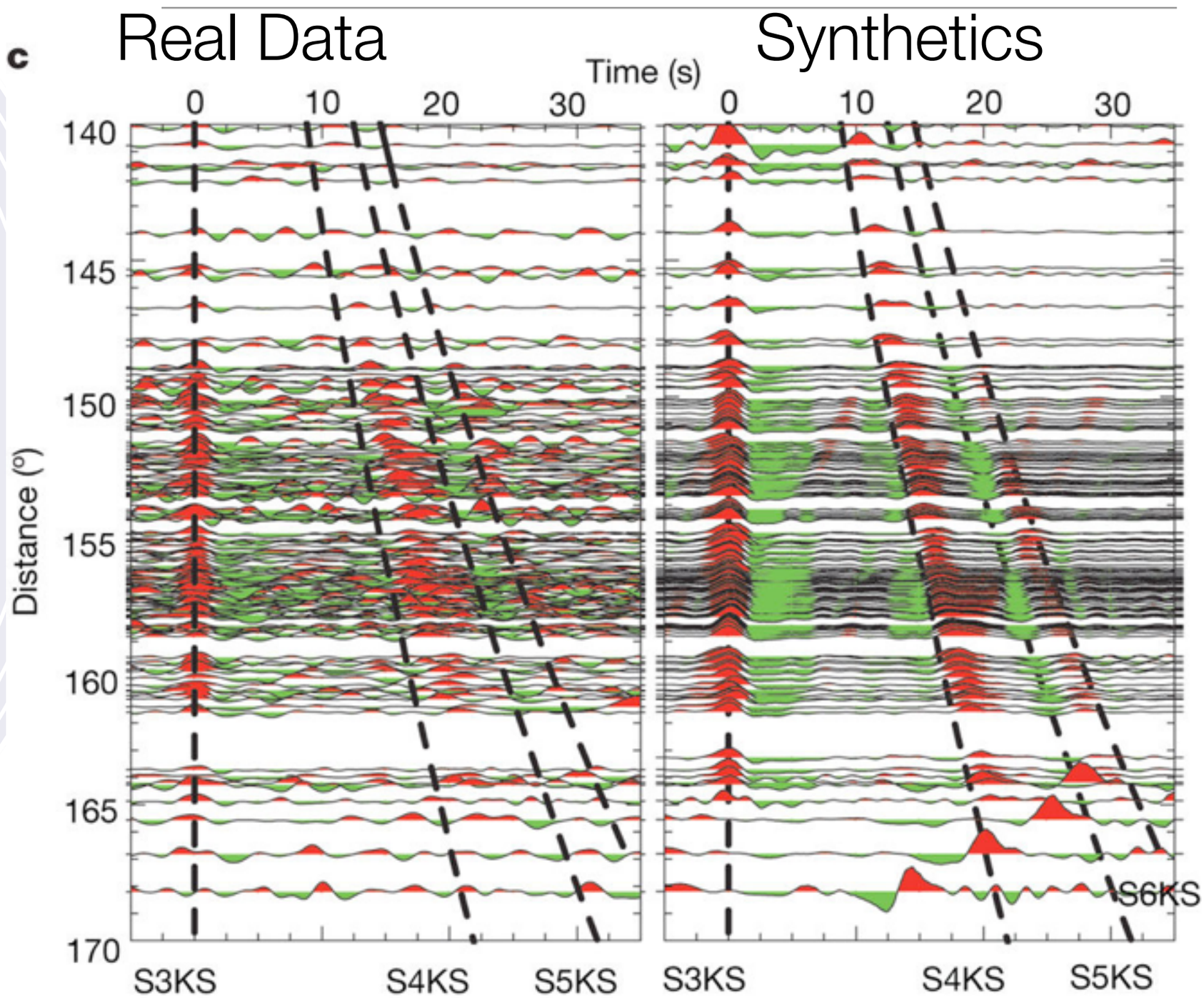




[Lauren Waszek & Deuss, 2011]

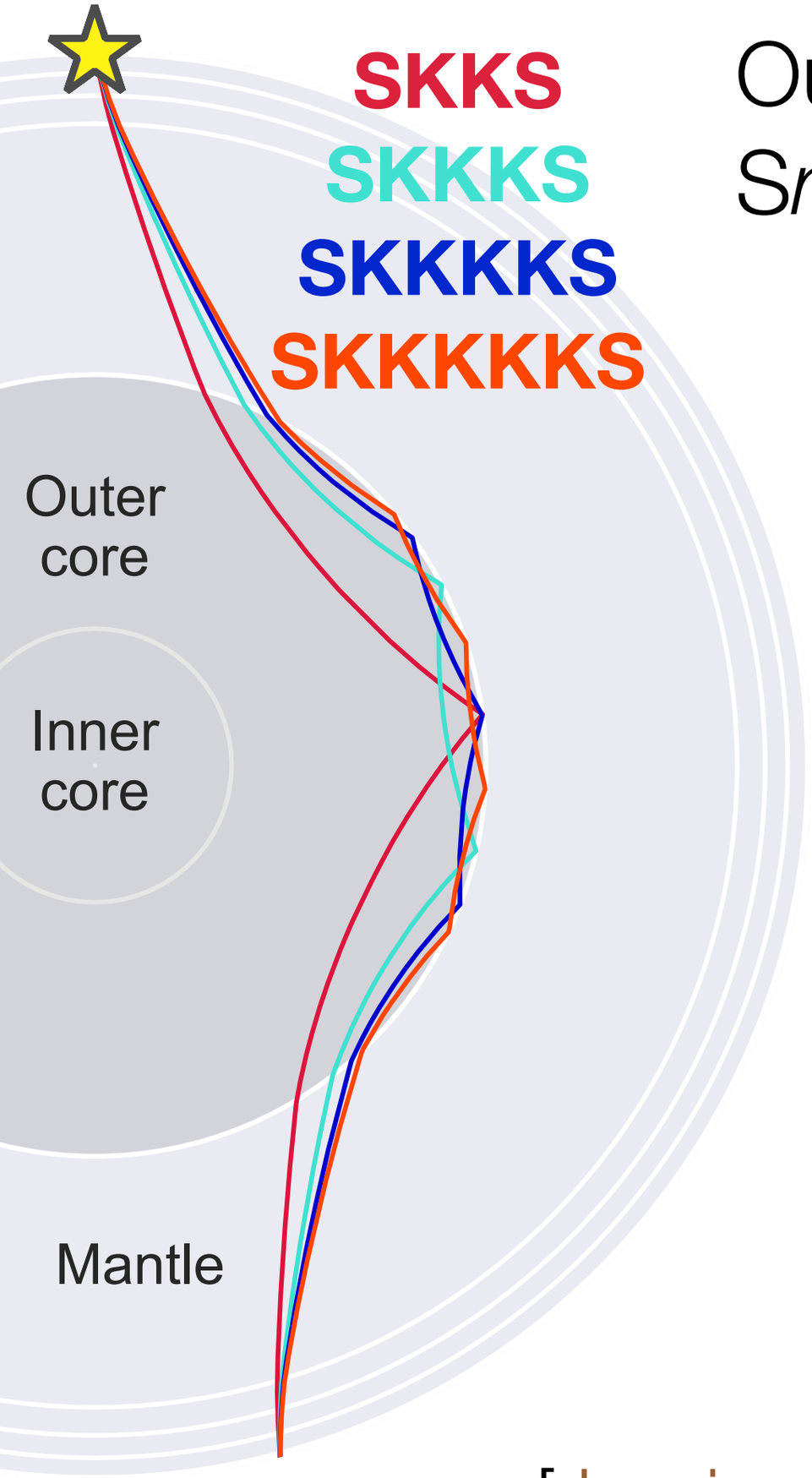
Outer Core anomalous layer?
S*m*KS data

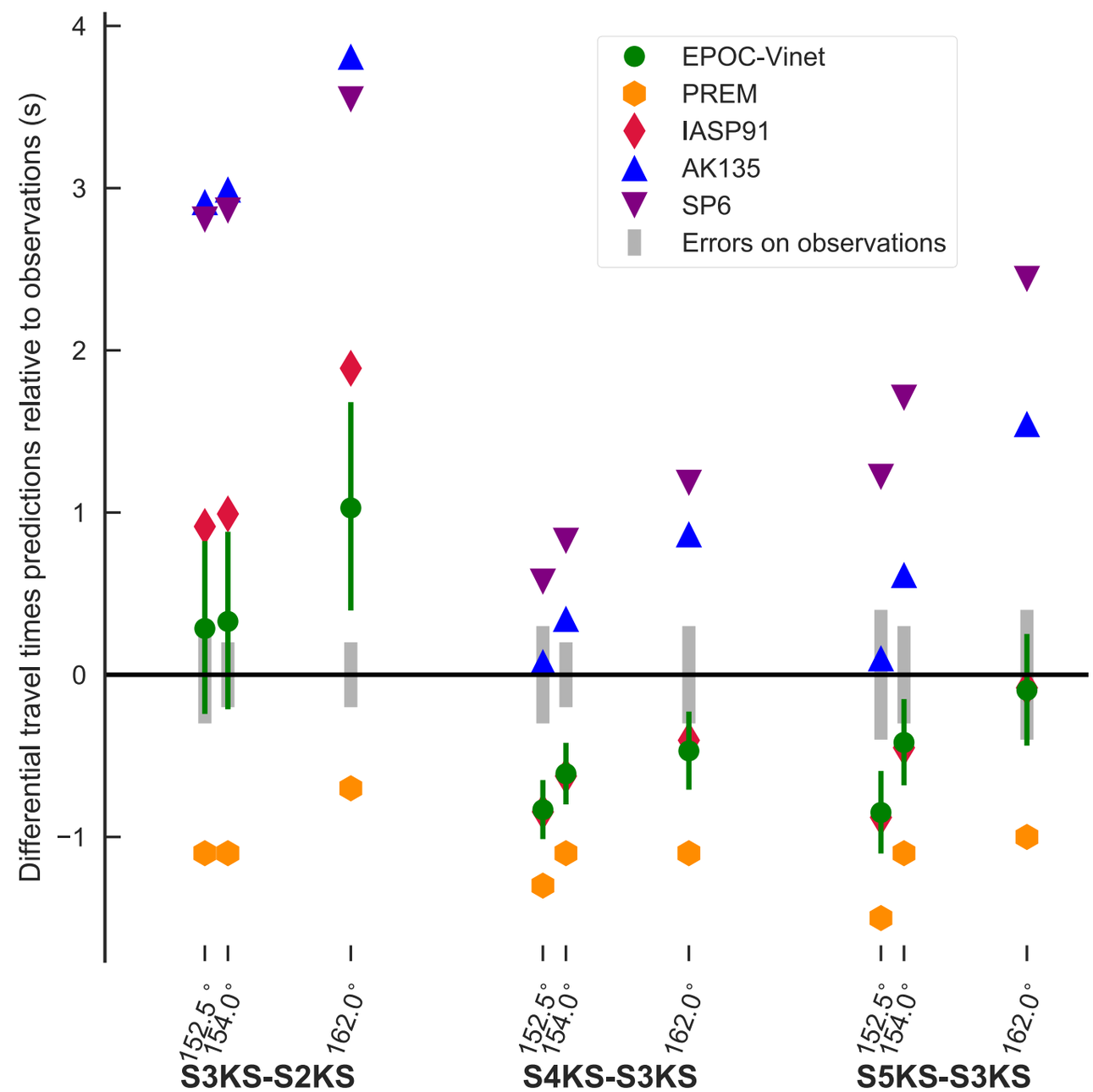




[Hellfrich & Kaneshima 2010]

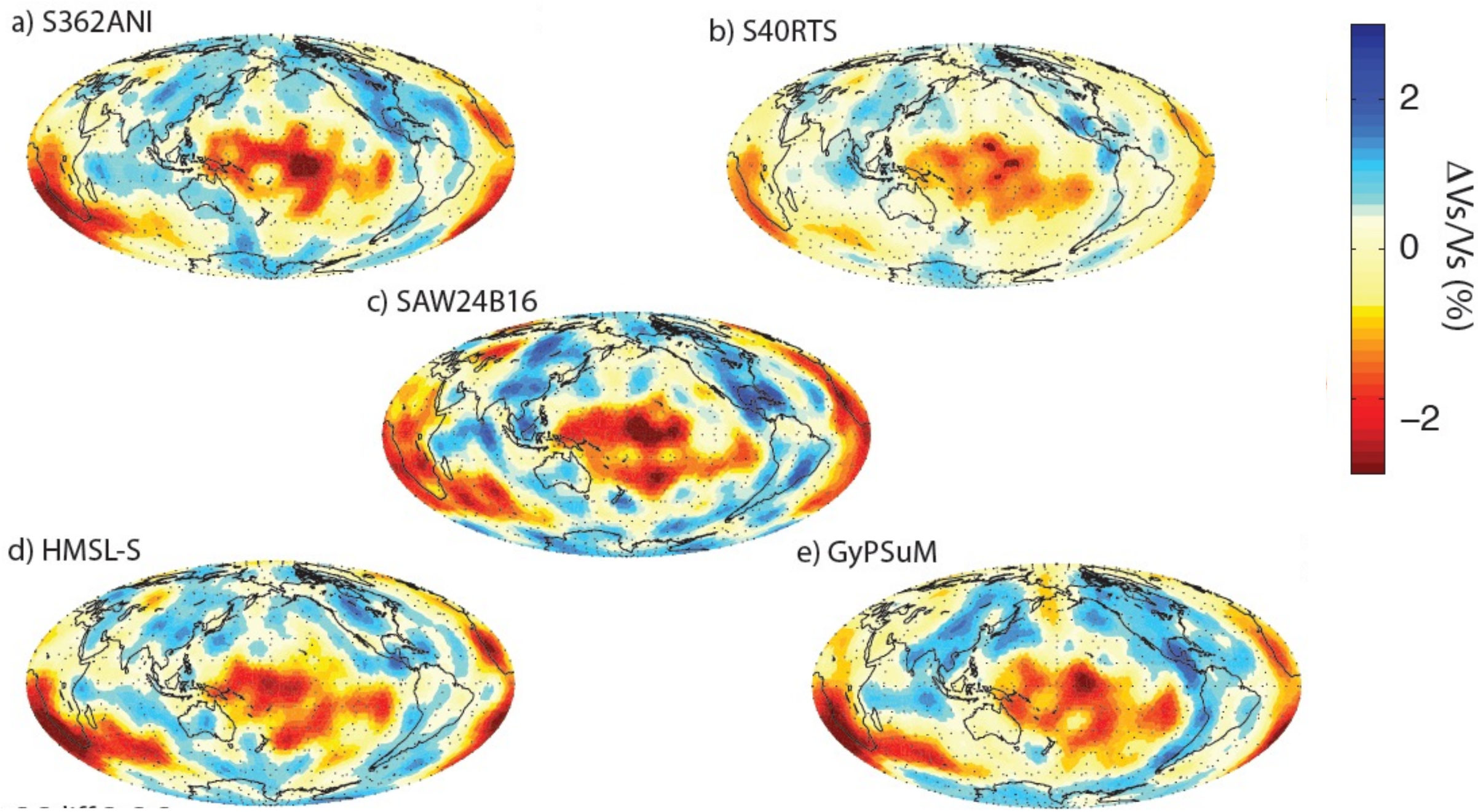
Outer Core anomalous layer?
S*m*KS data





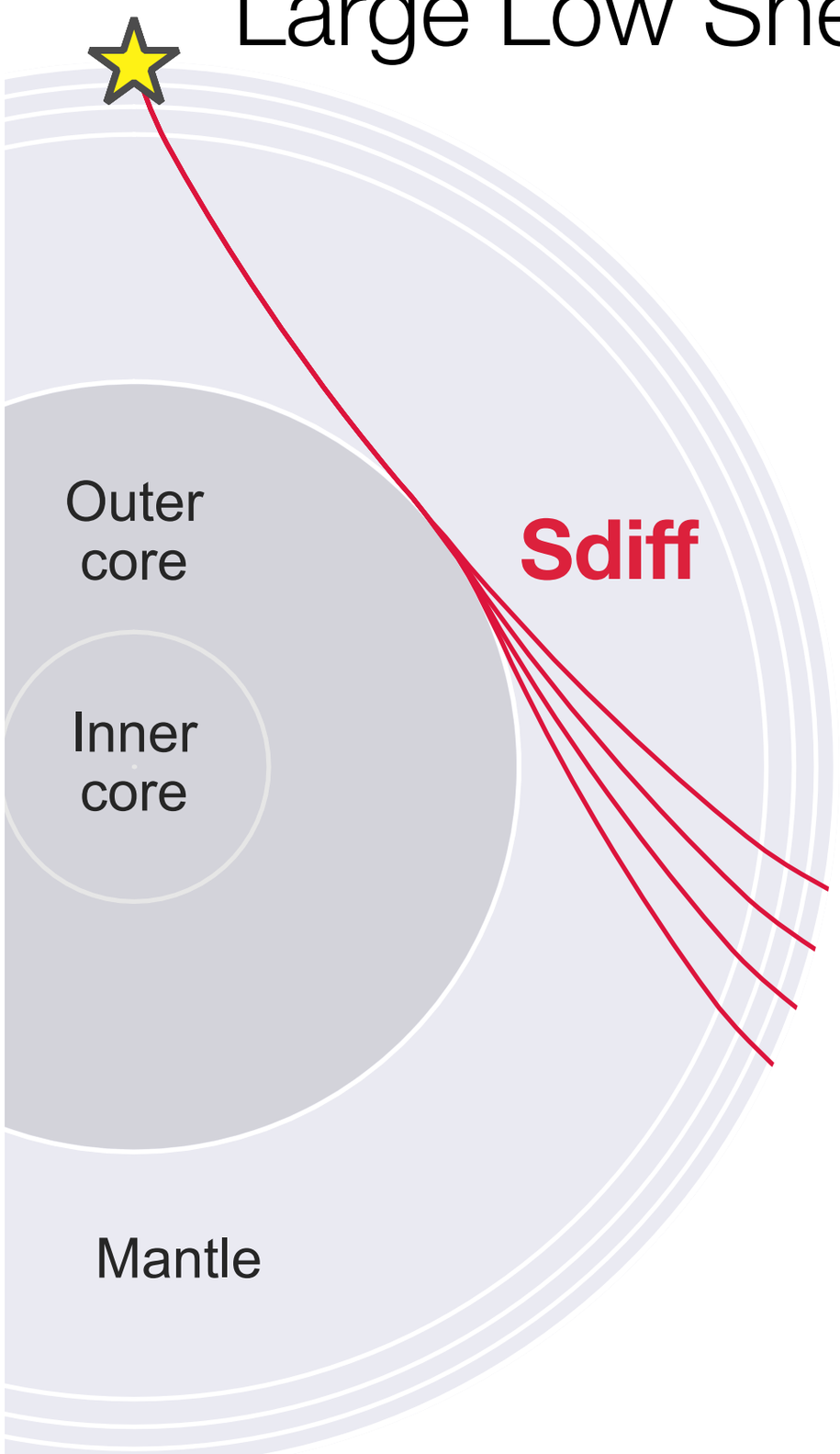
[Jessica Irving, Sanne Cottaar & Ved Lekic, 2018]

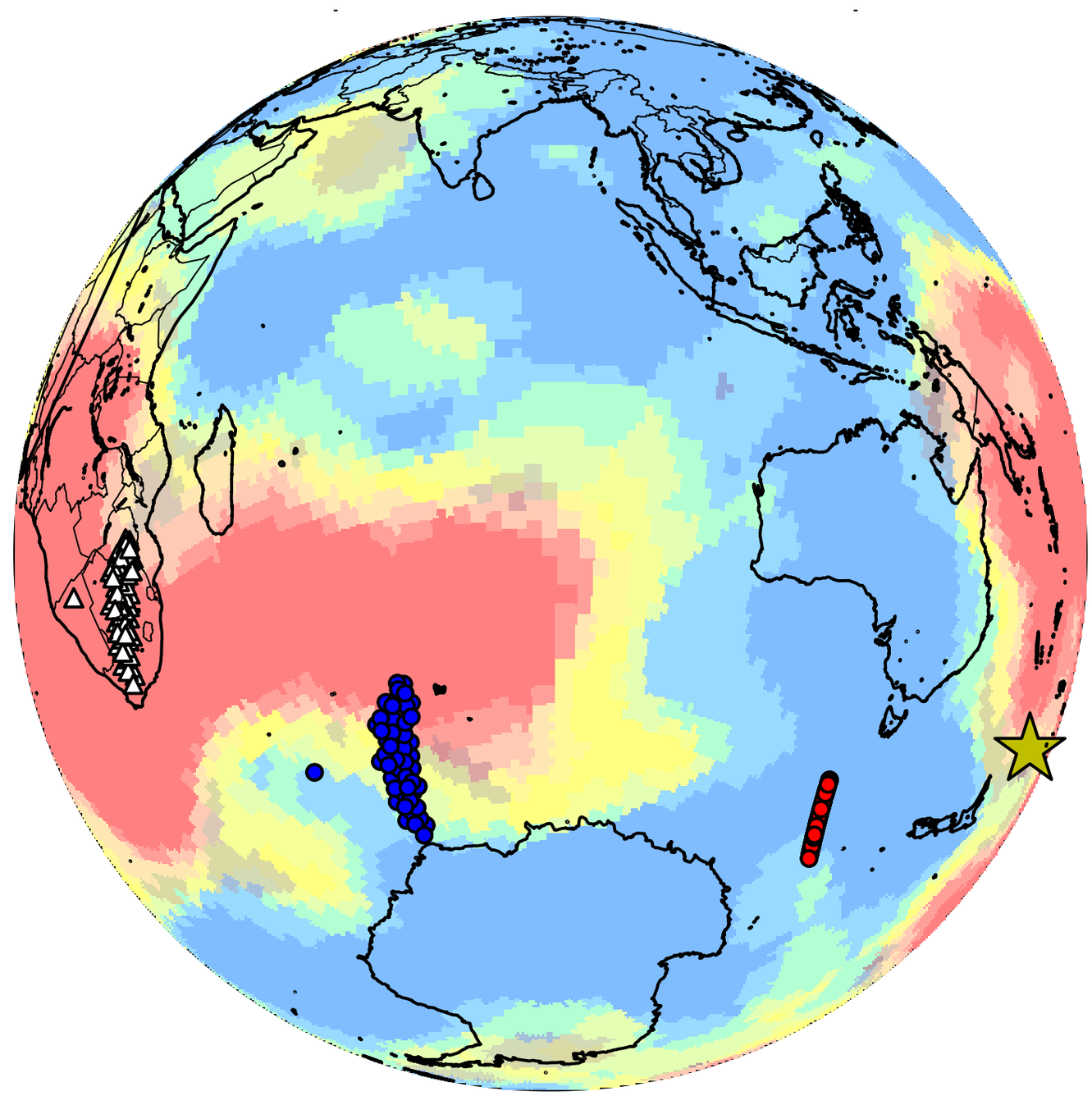
Large Low (Shear) Velocity Provinces LL(S)VPs



[Ved Lekic et al. 2012]

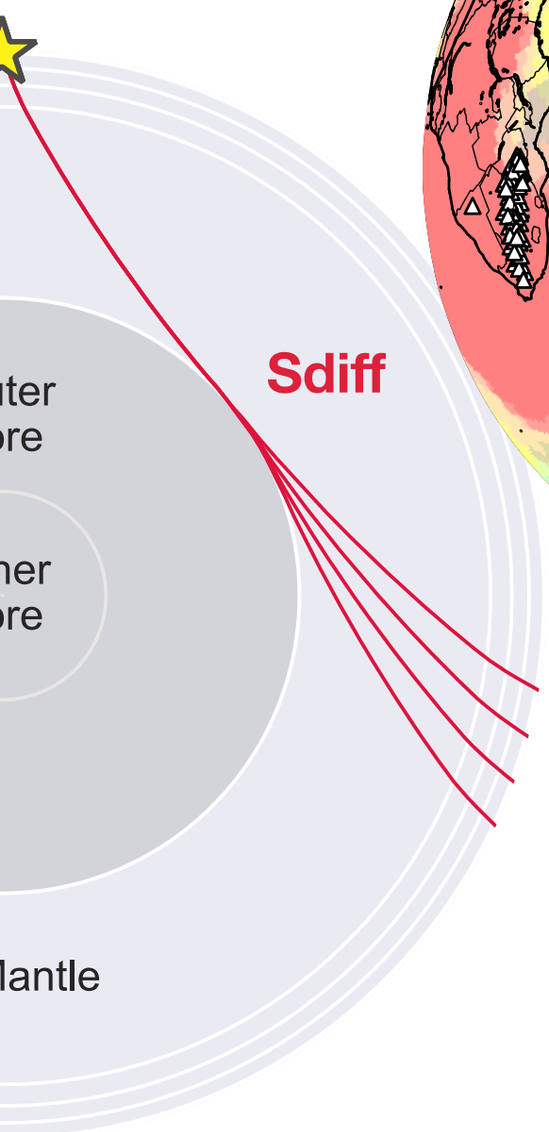
Large Low Shear Velocity Provinces: multi-pathing

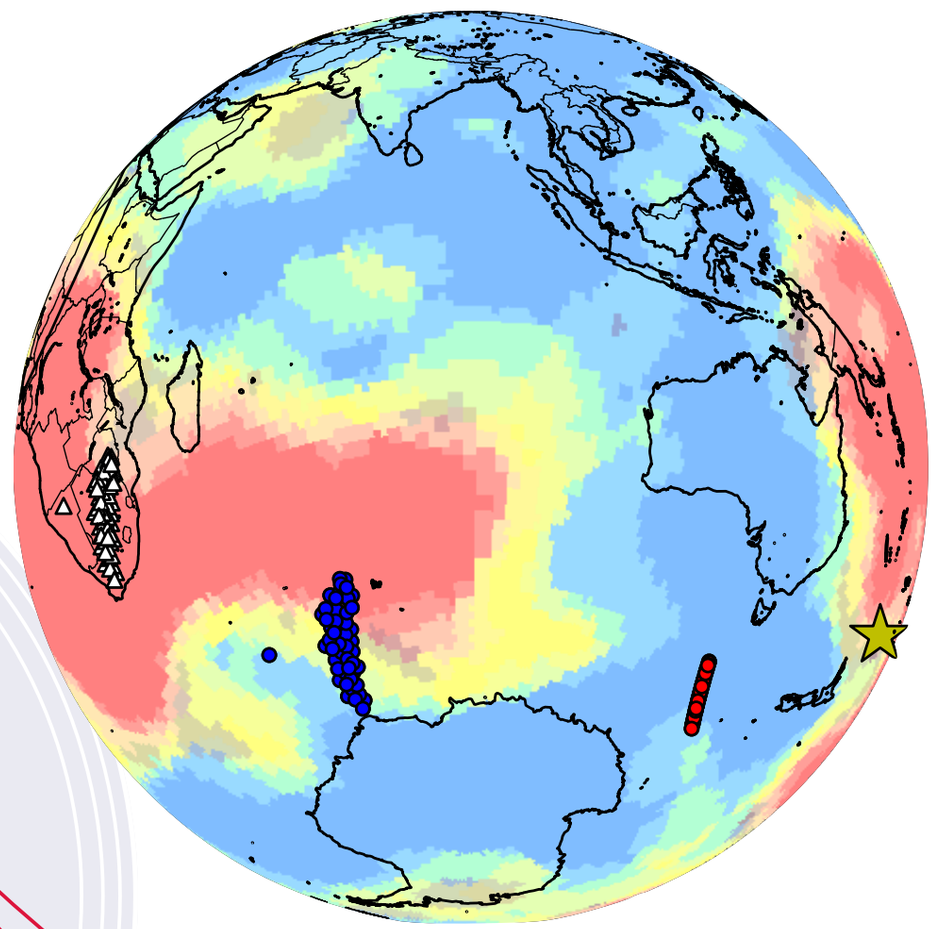




[Background: Sanne Cottaar & Ved Lekic, 2016]

Large Low Shear Velocity Provinces: multi-pathing

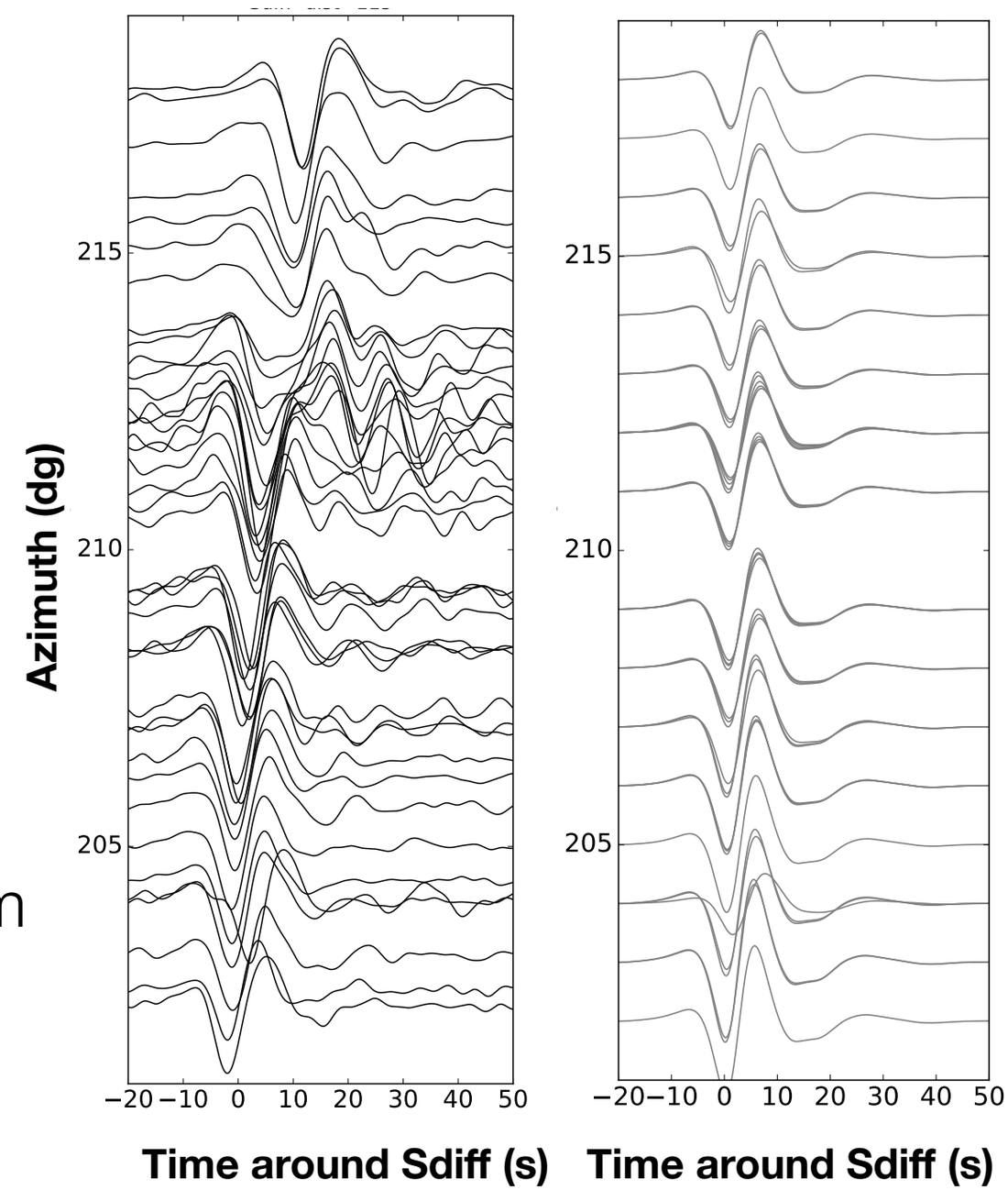




Pierce points at 2800 km

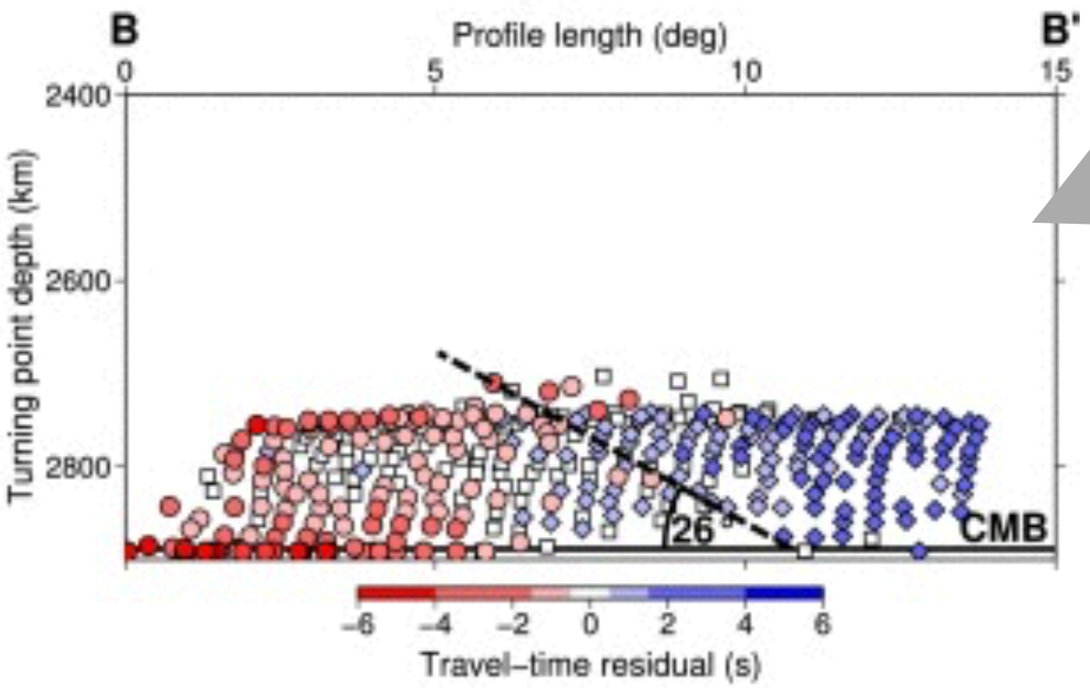
entry

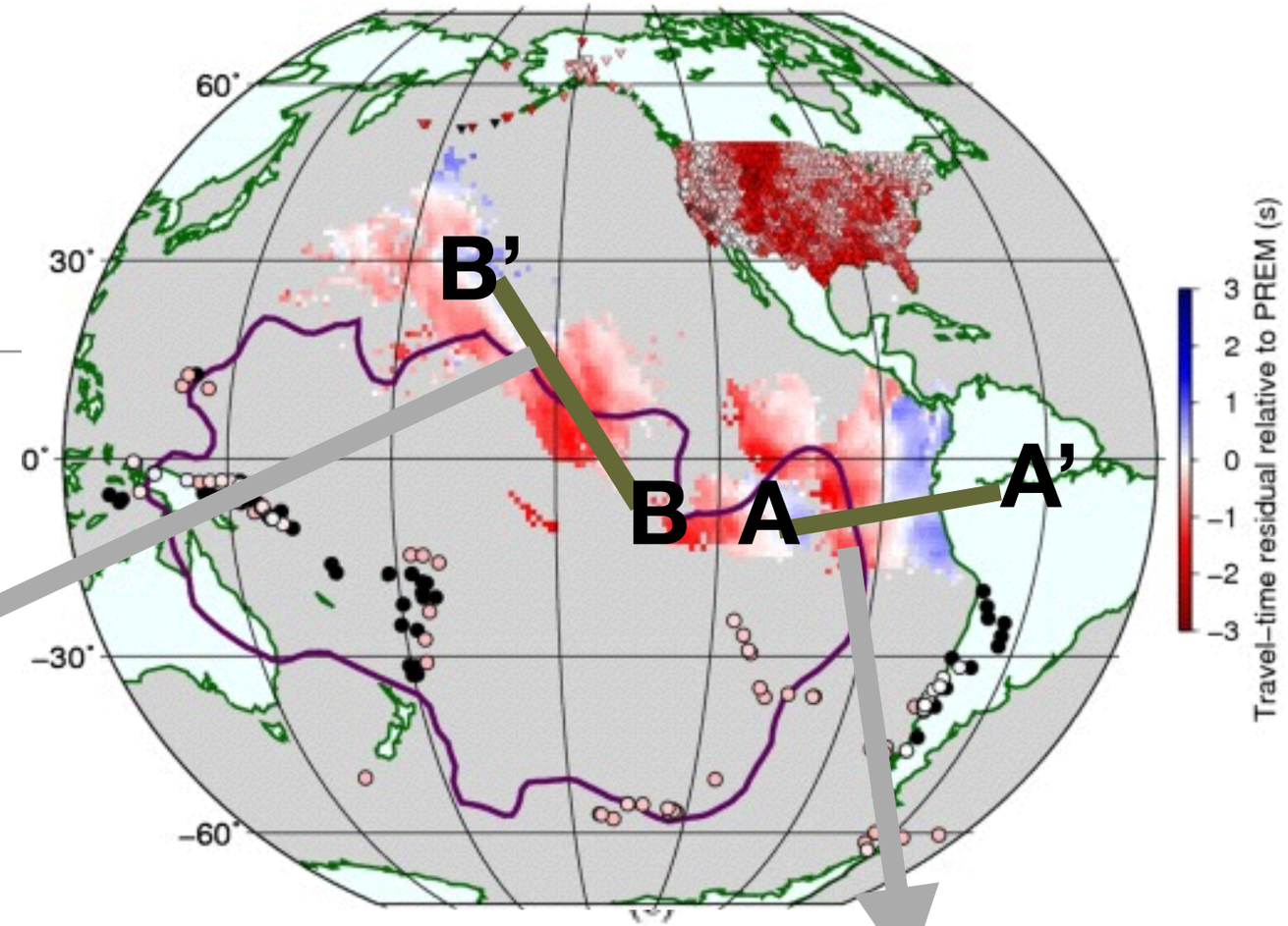
exit

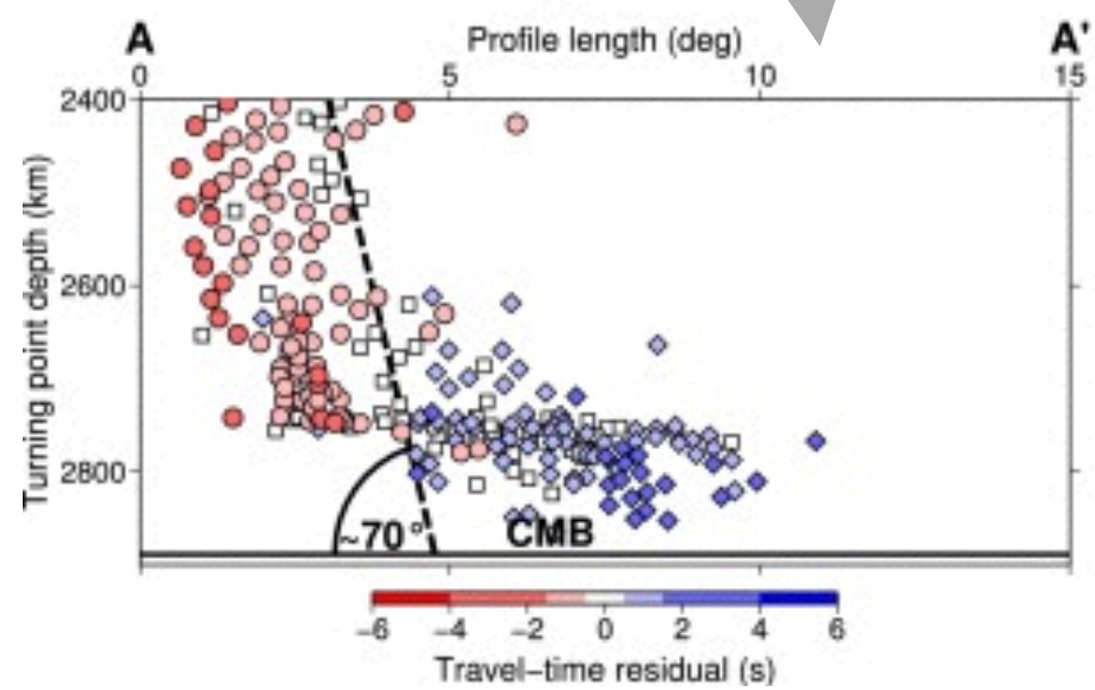


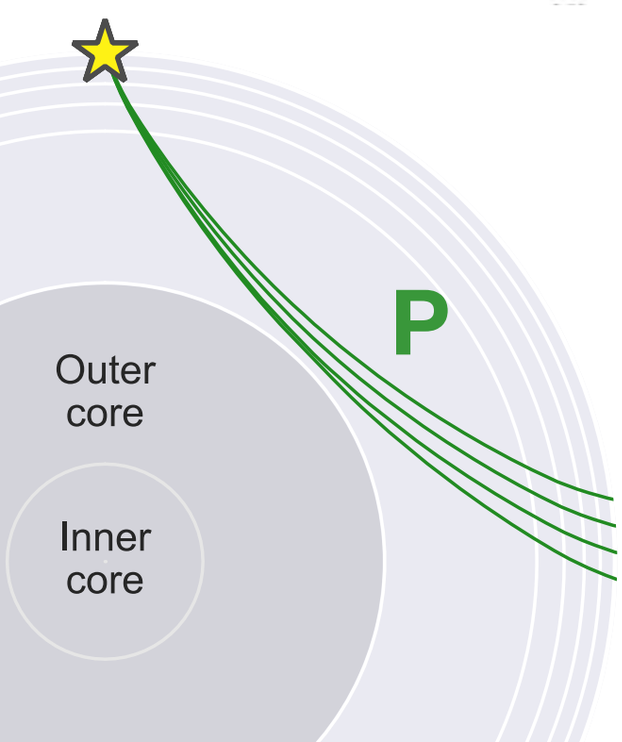
[To et al. 2005]

Large Low Velocity Provinces: P waves



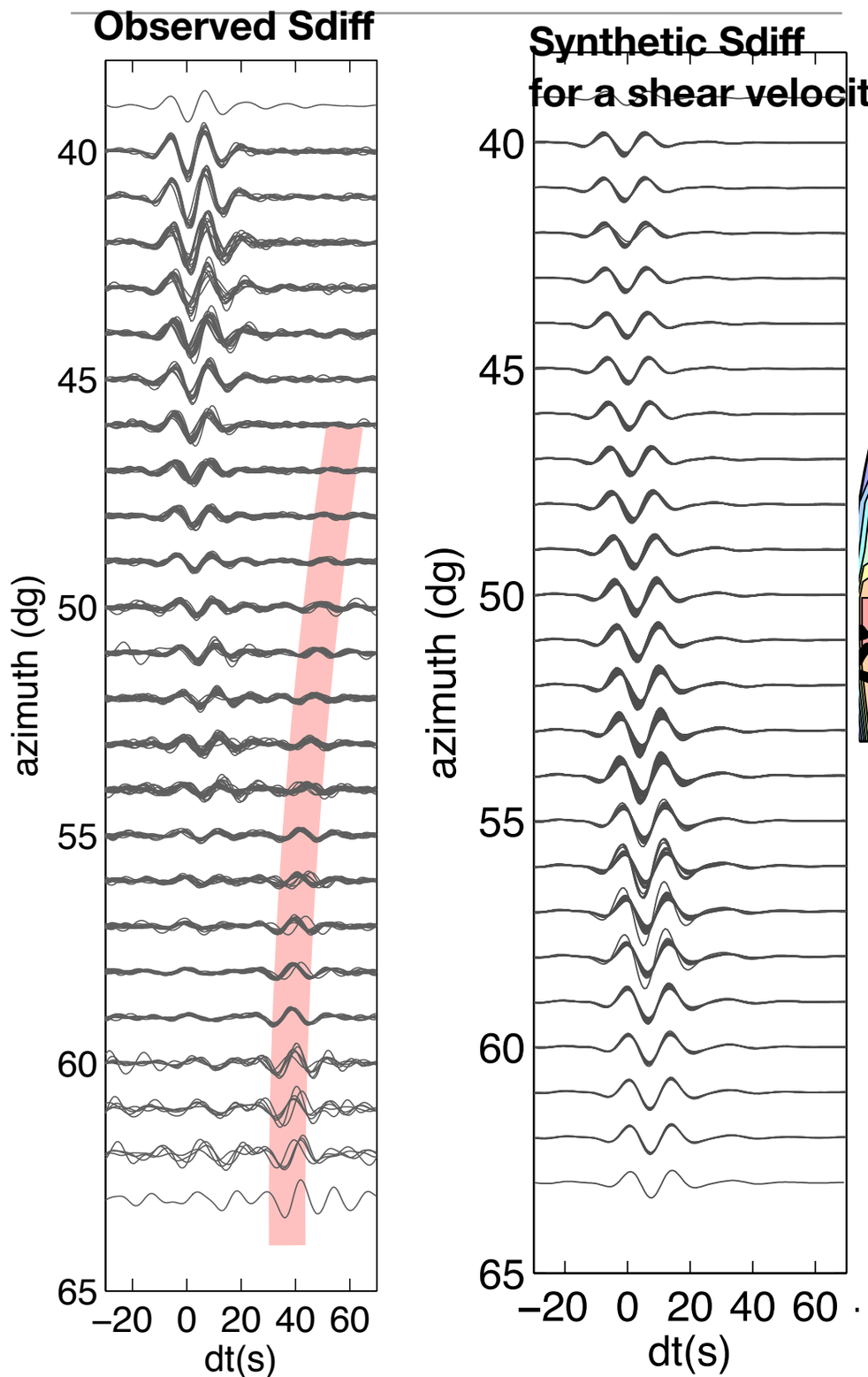


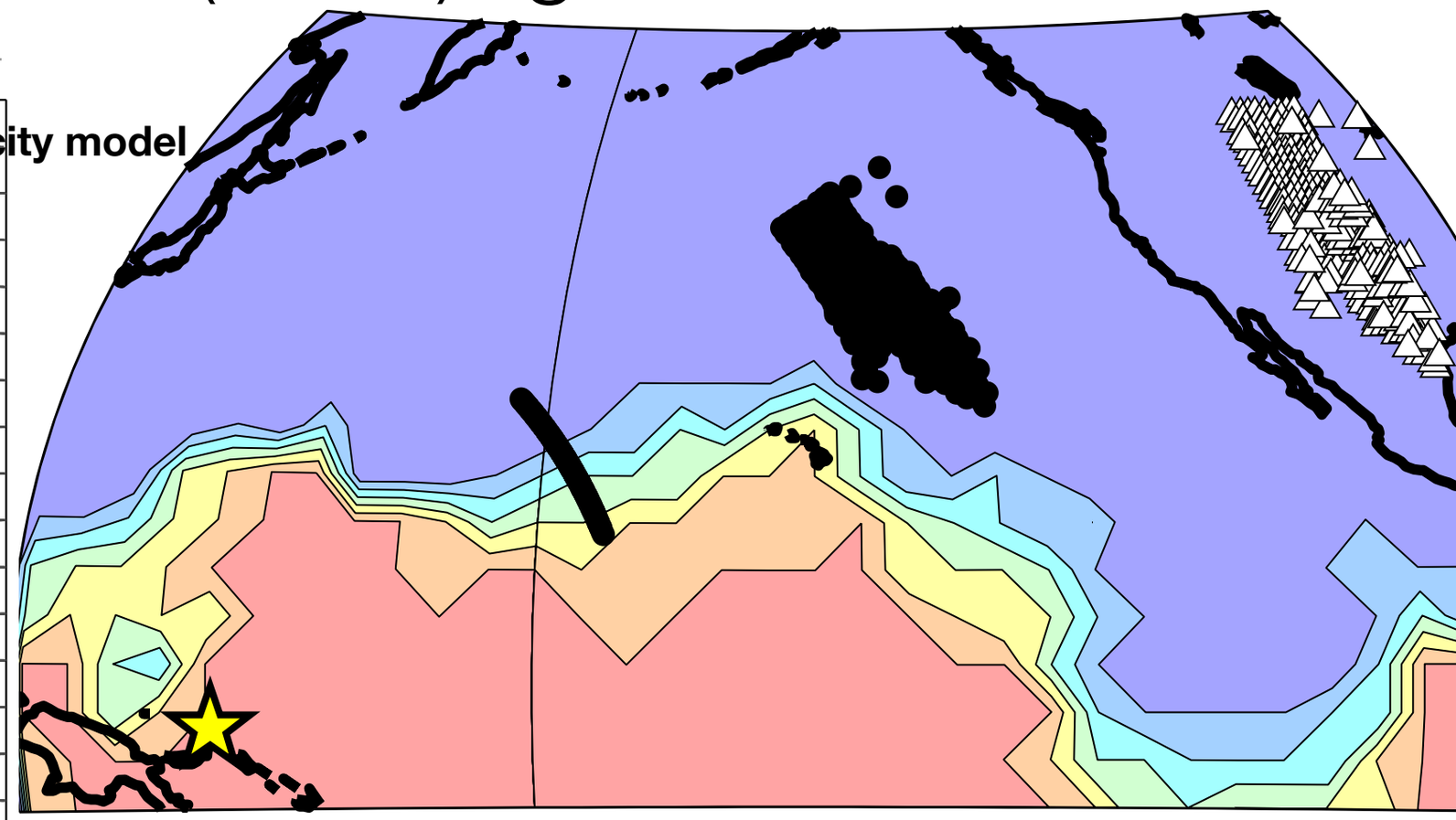




[Dan Frost & Rost, 2014]

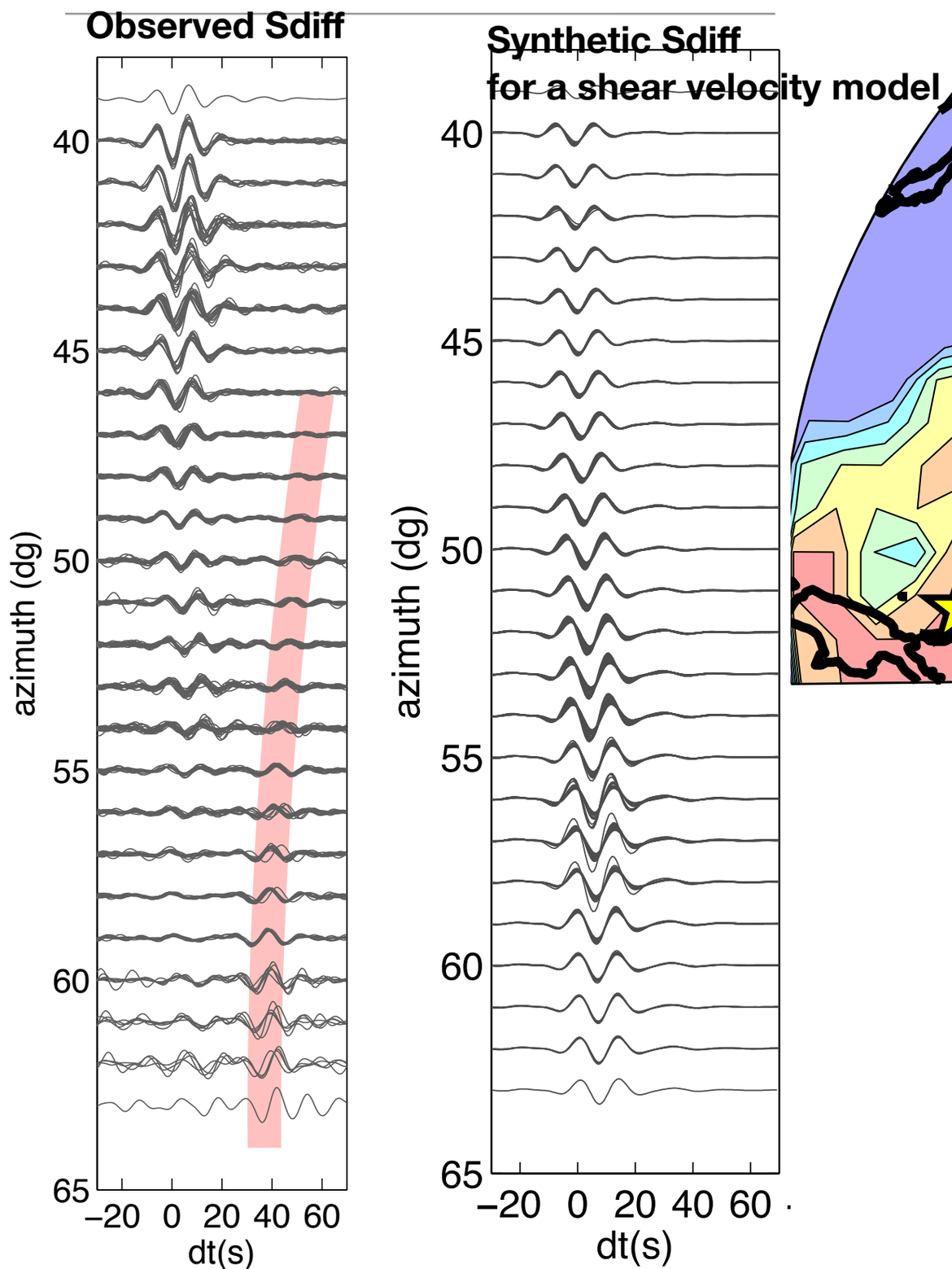
Ultra-low Velocity Zone (ULVZ)- guided wave

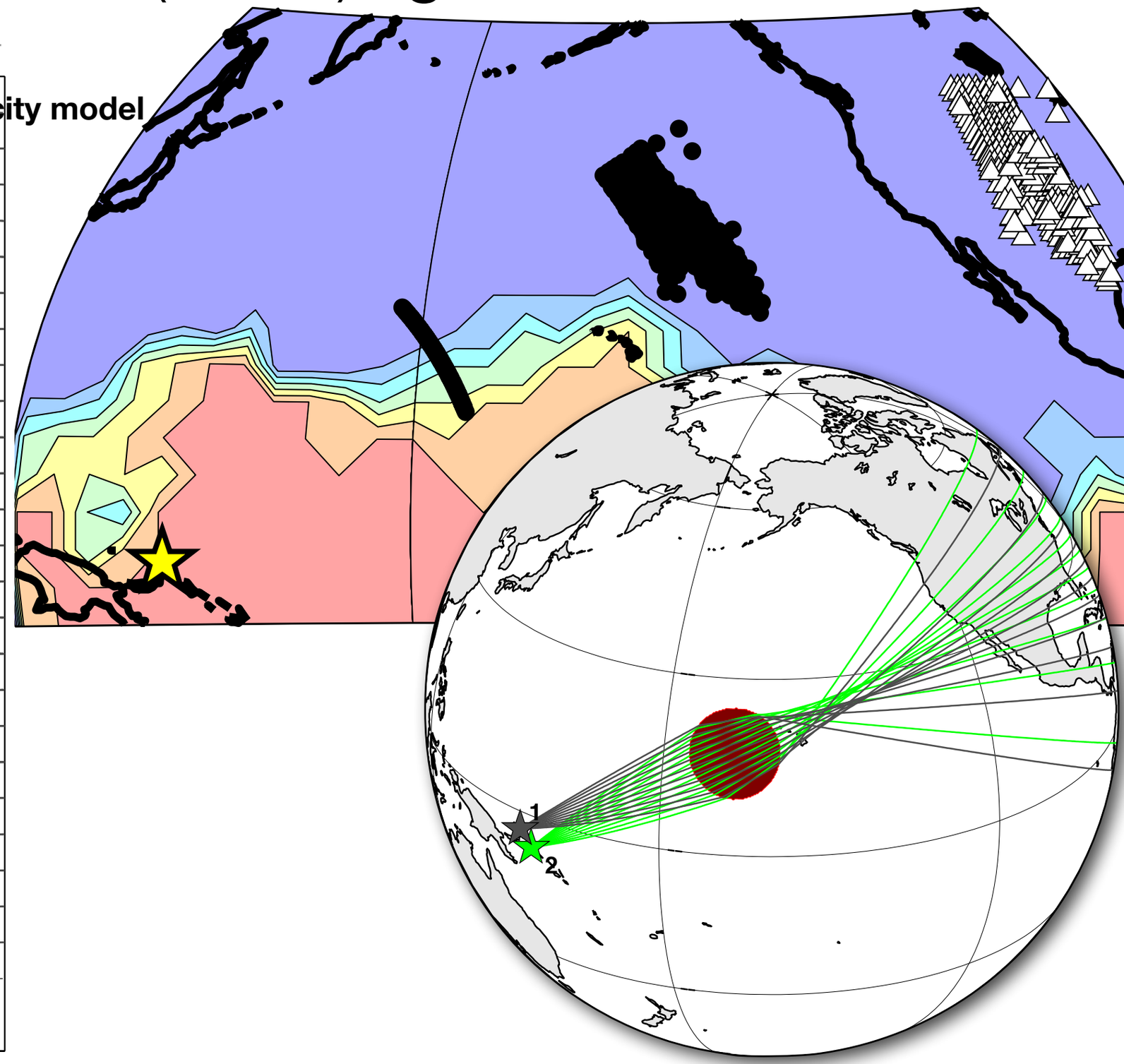




[Sanne Cottaar & Romanowicz, 2012]

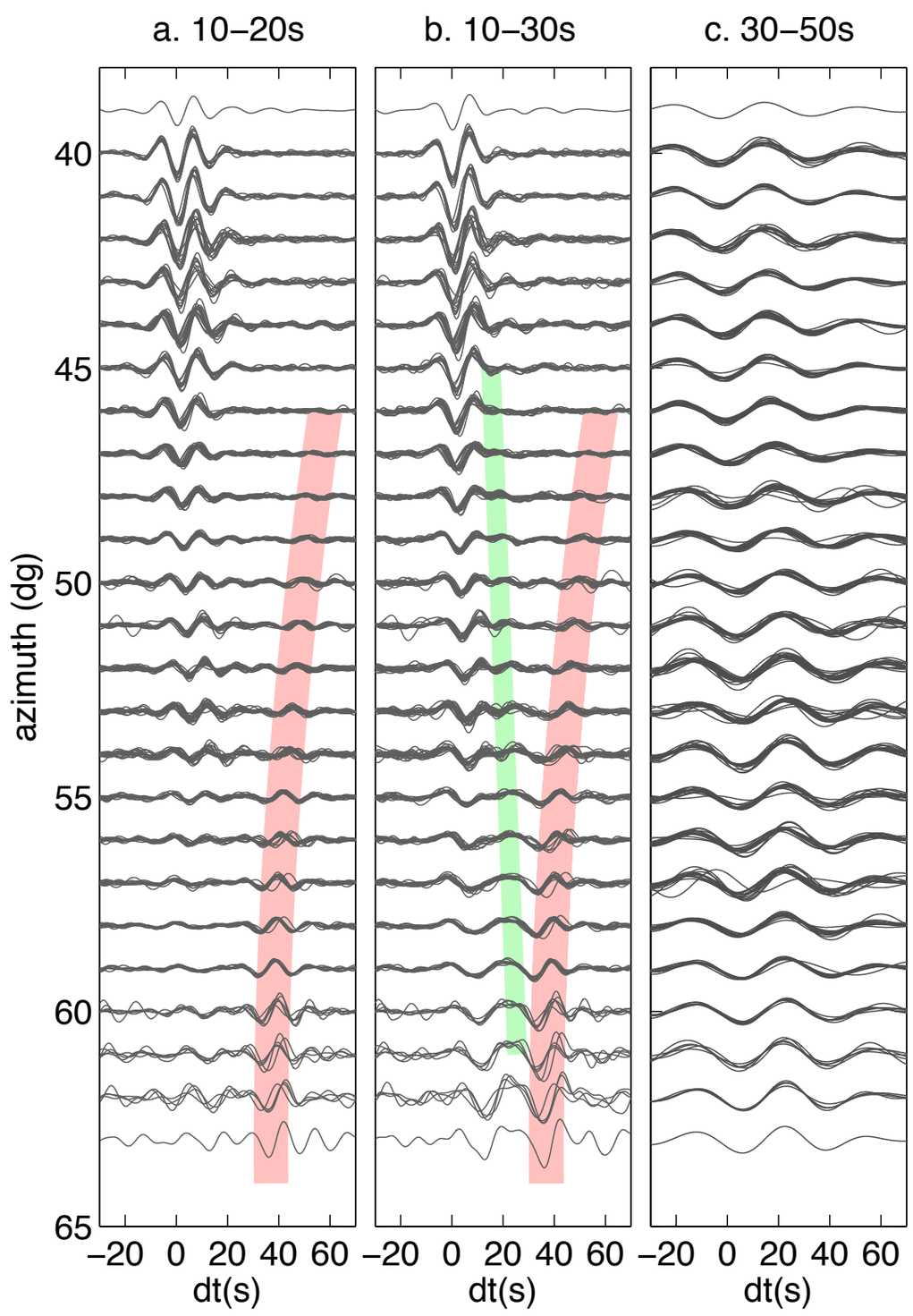
Ultra-low Velocity Zone (ULVZ)- guided wave

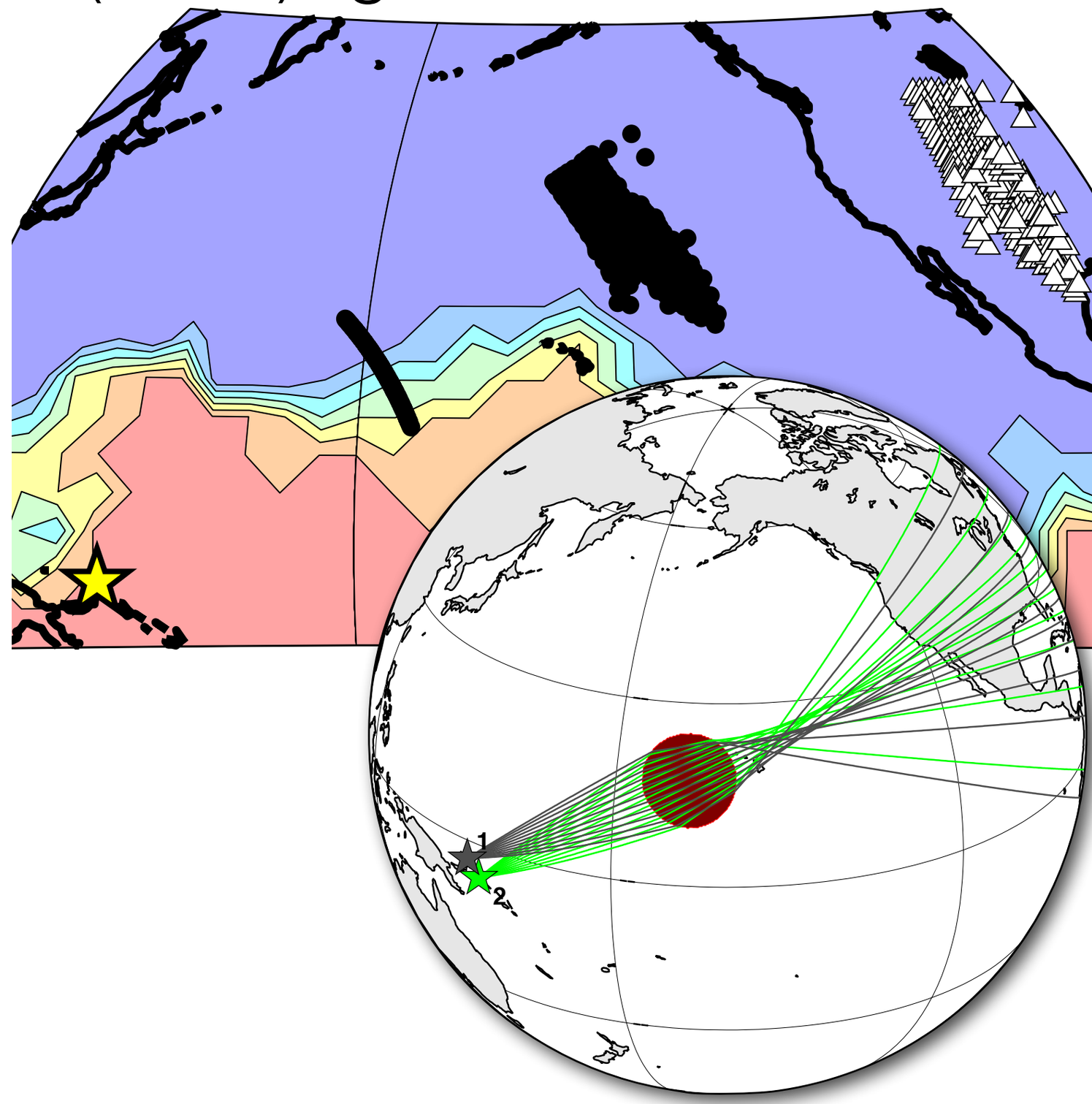




[Sanne Cottaar & Romanowicz, 2012]

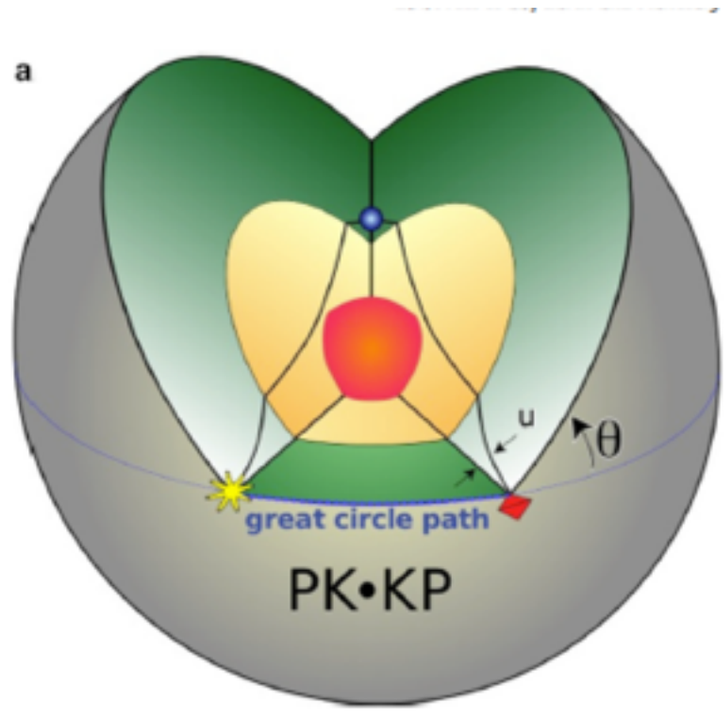
Ultra-low Velocity Zone (ULVZ)- guided wave

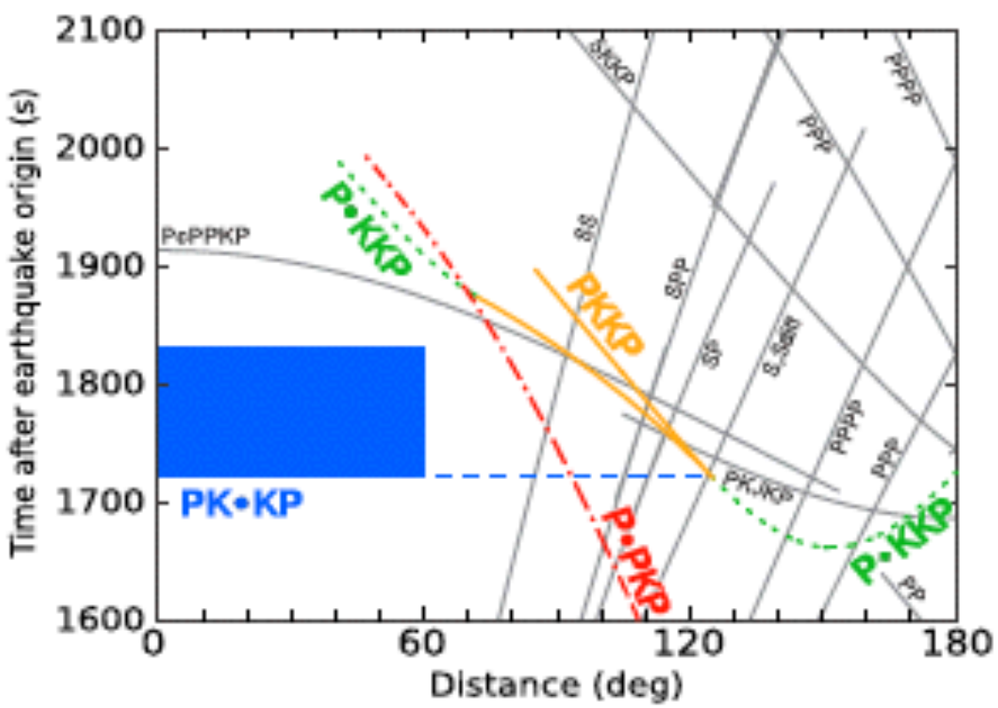


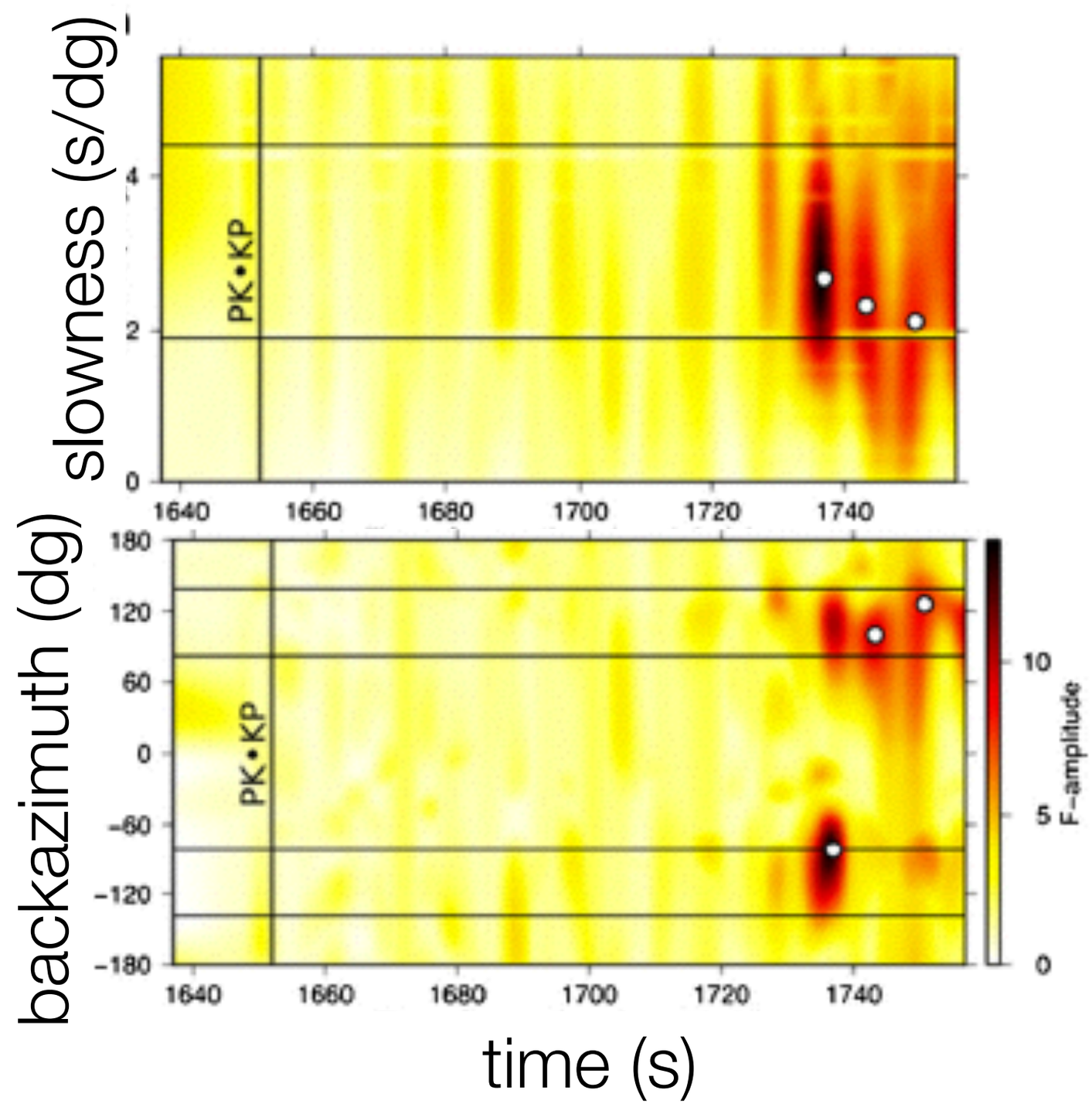


[Sanne Cottaar & Romanowicz, 2012]

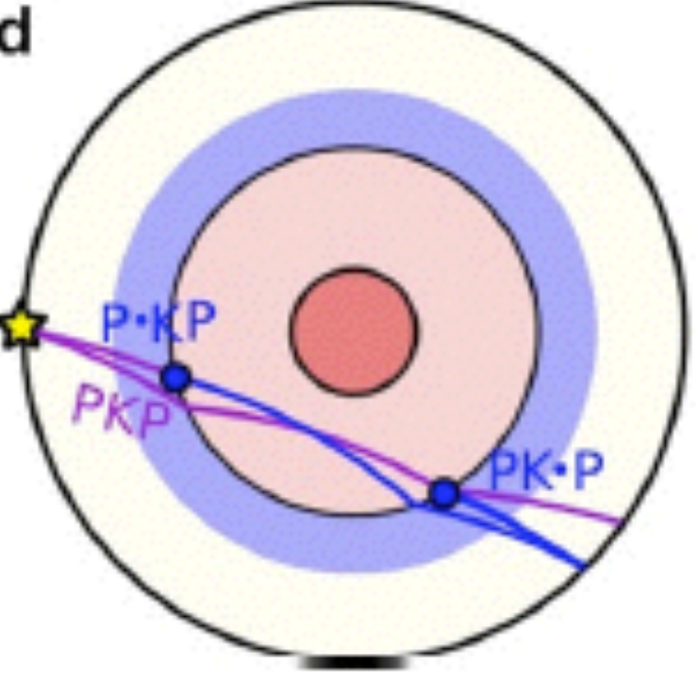
Post PKKP scatterers

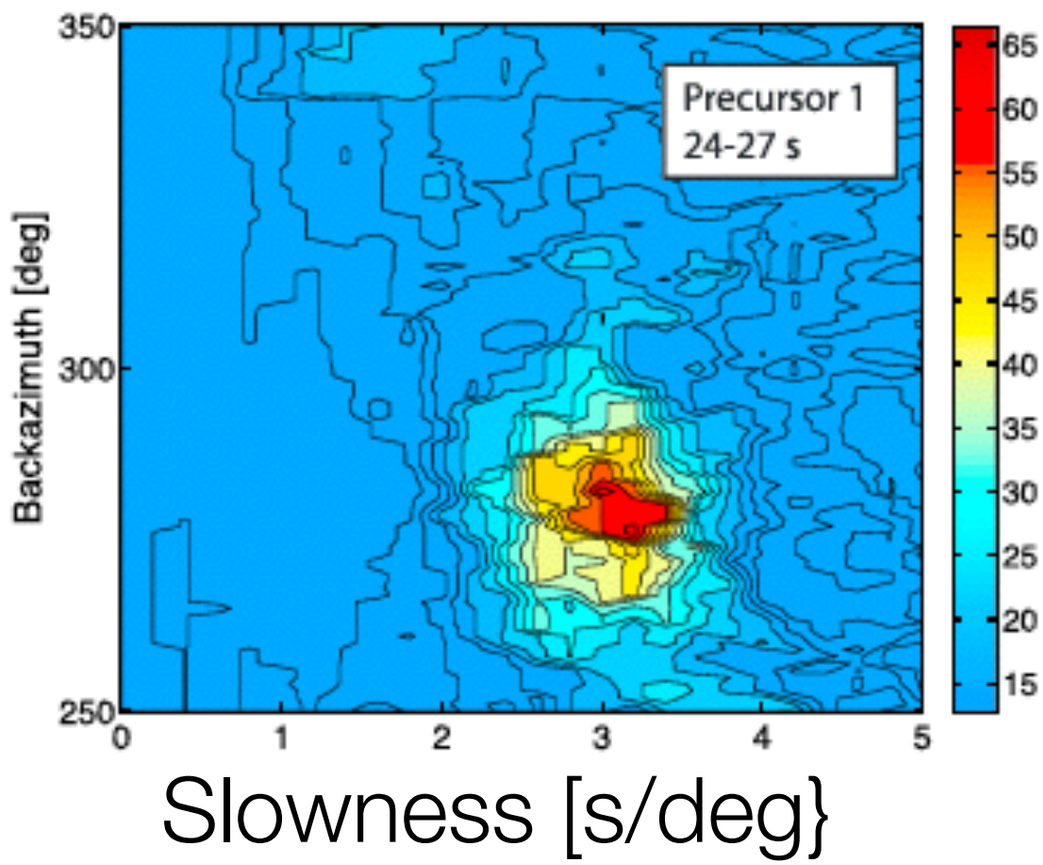


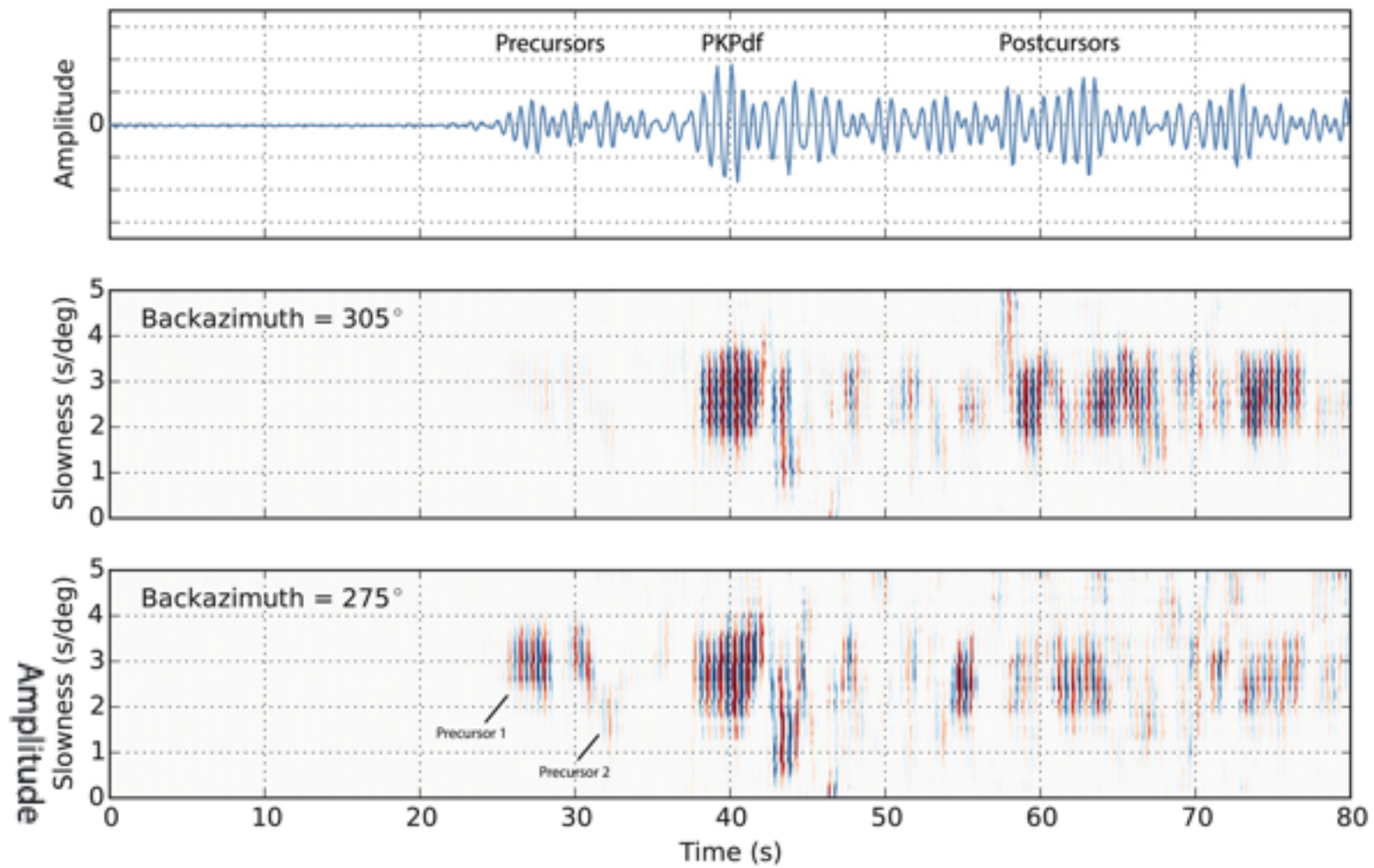




PKP scattering -array stacking

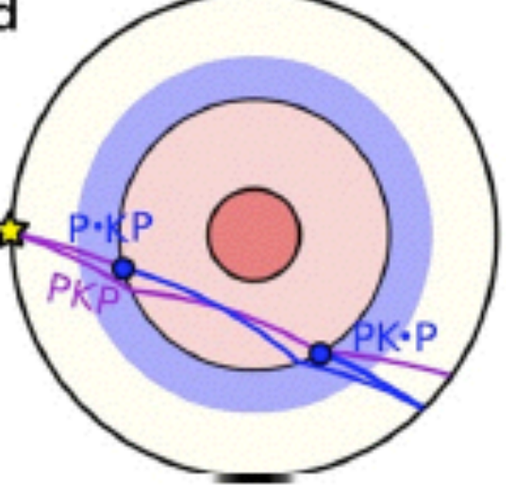


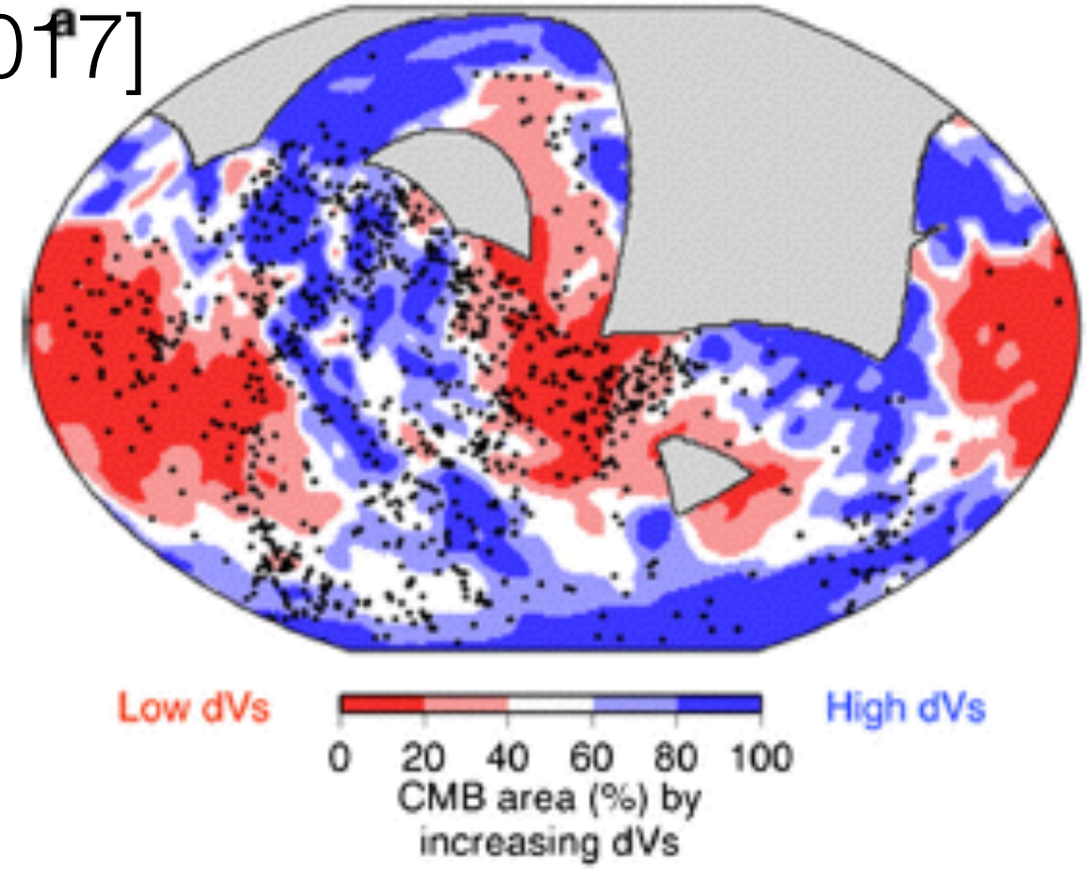


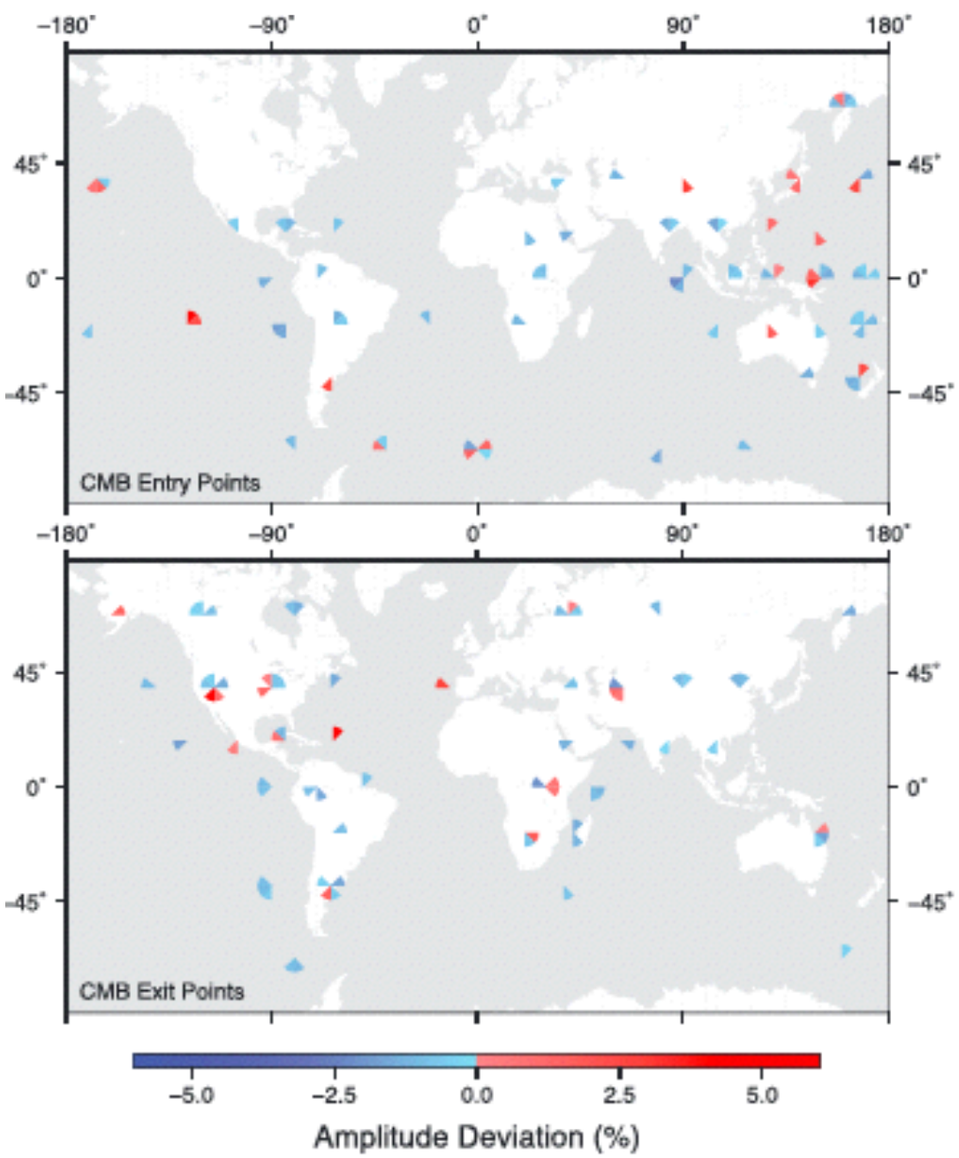


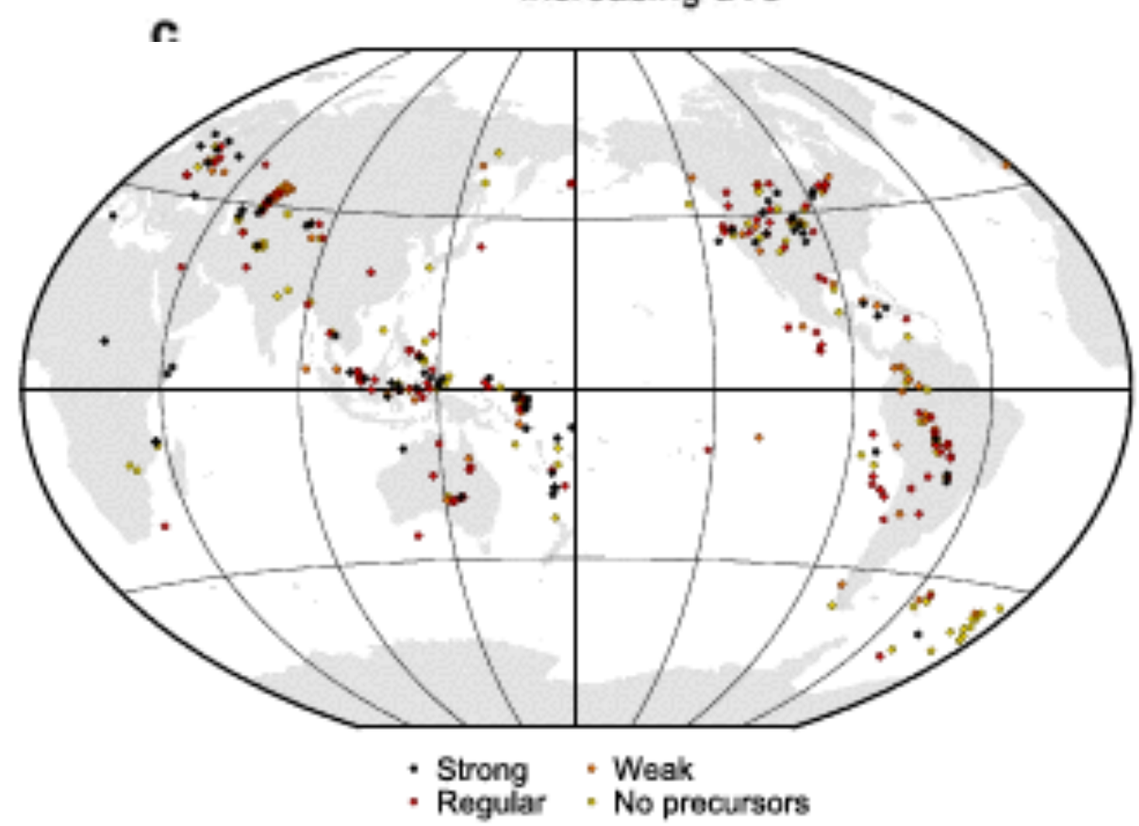
[Nick Mancinelli et al. 2016]

[Daniel Frost et al. 2017]





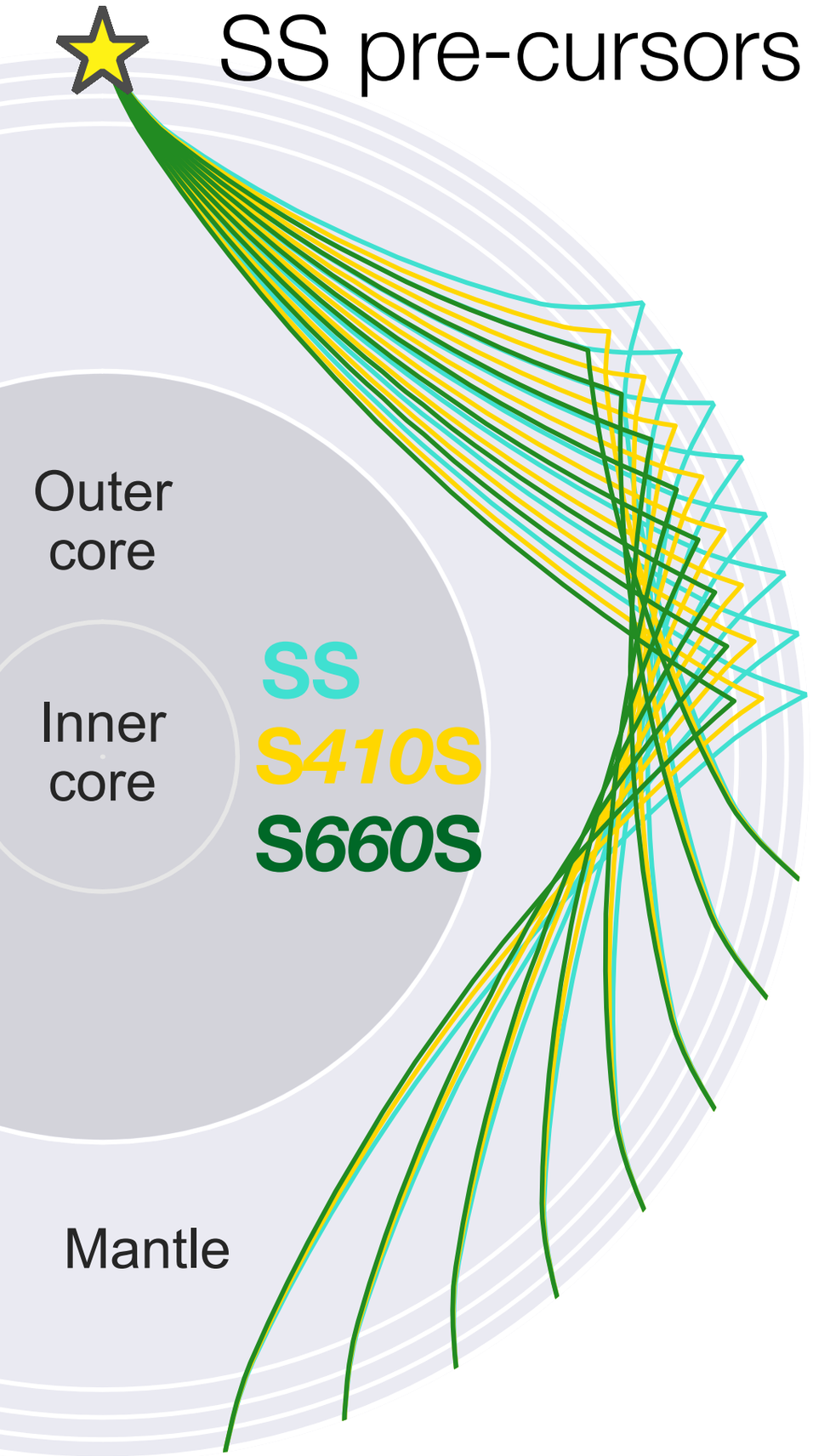


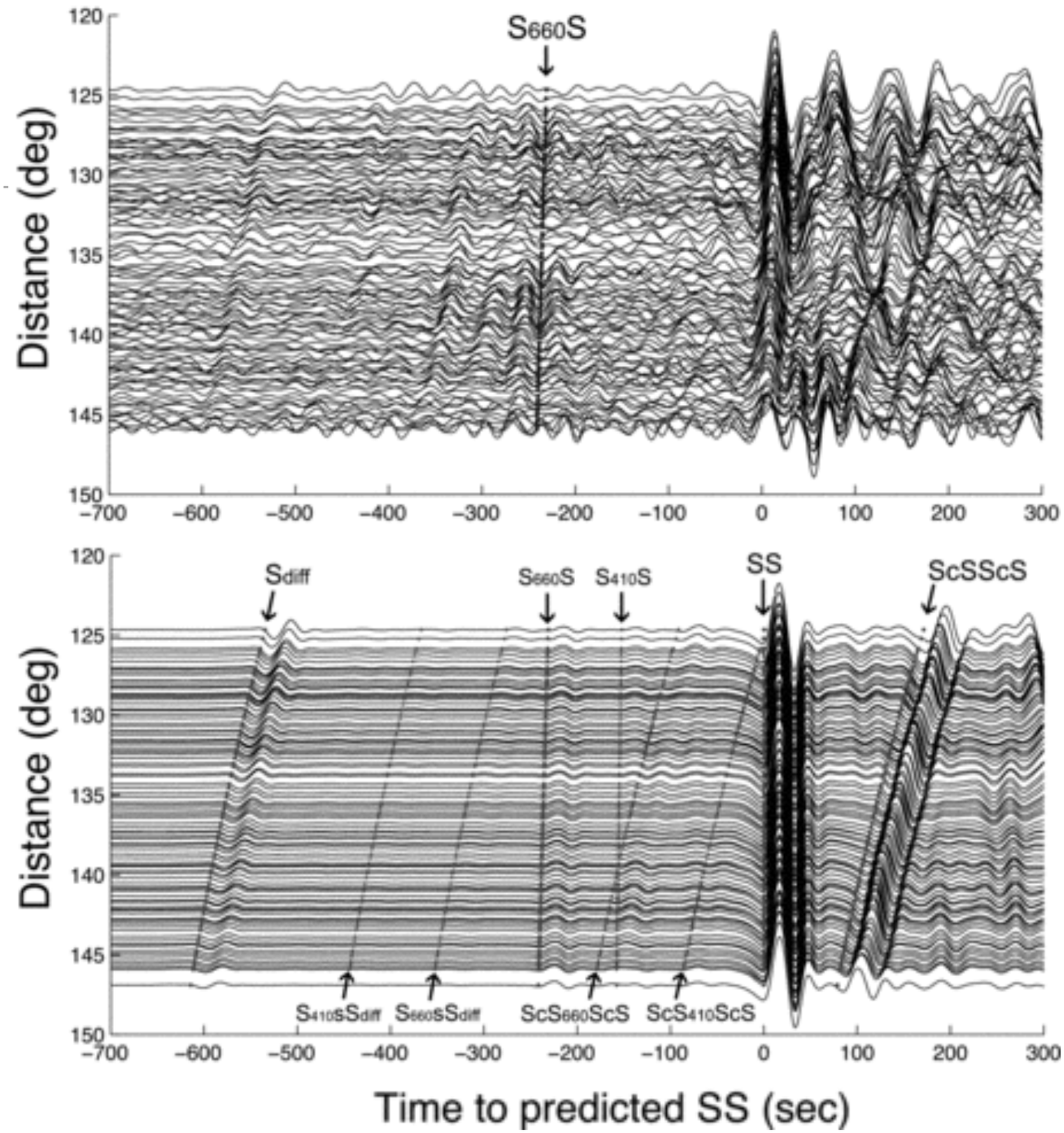


[Nick Mancinelli et al. 2016]

[Lauren Waszek et al. 2015]

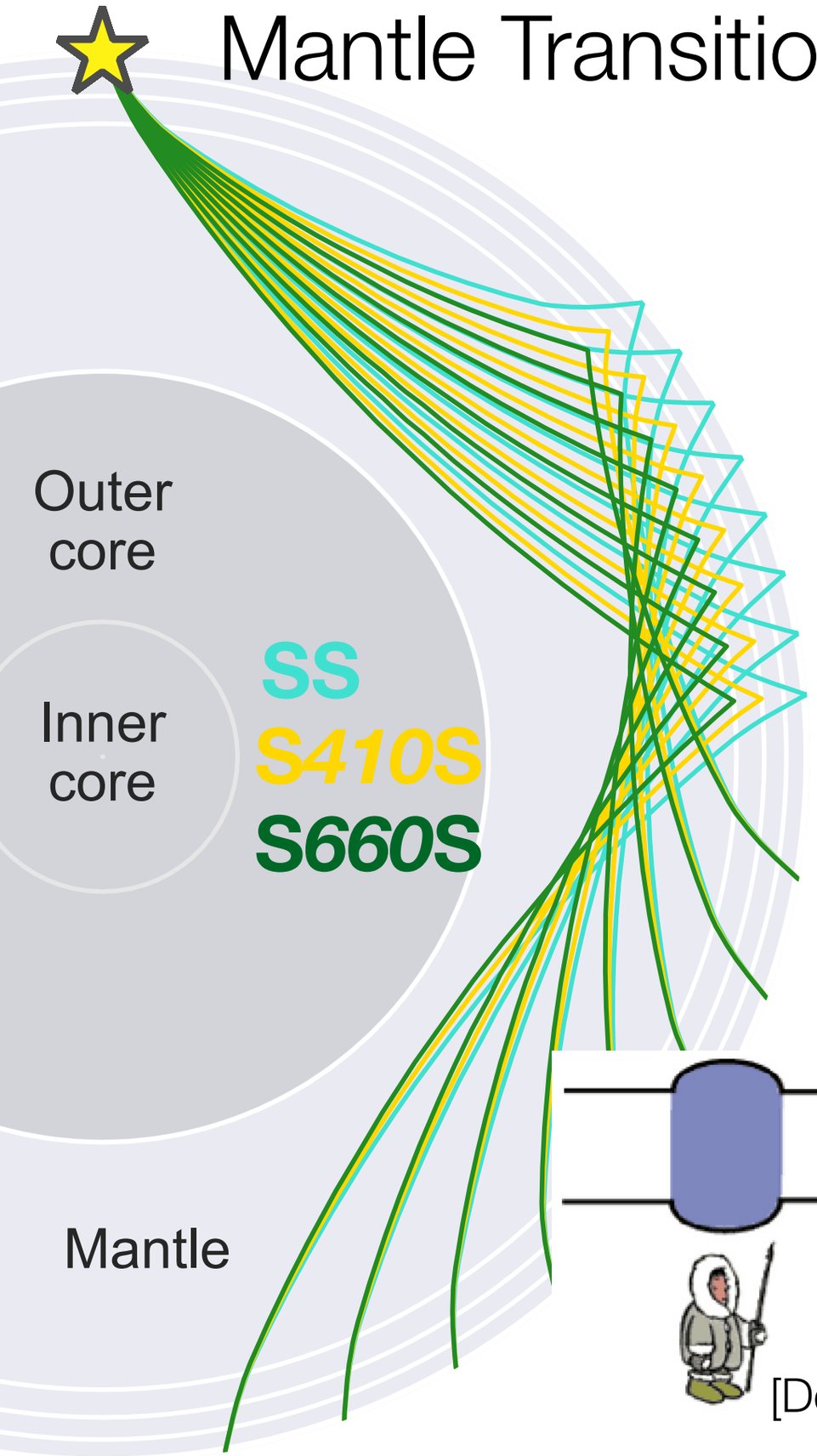
SS pre-cursors

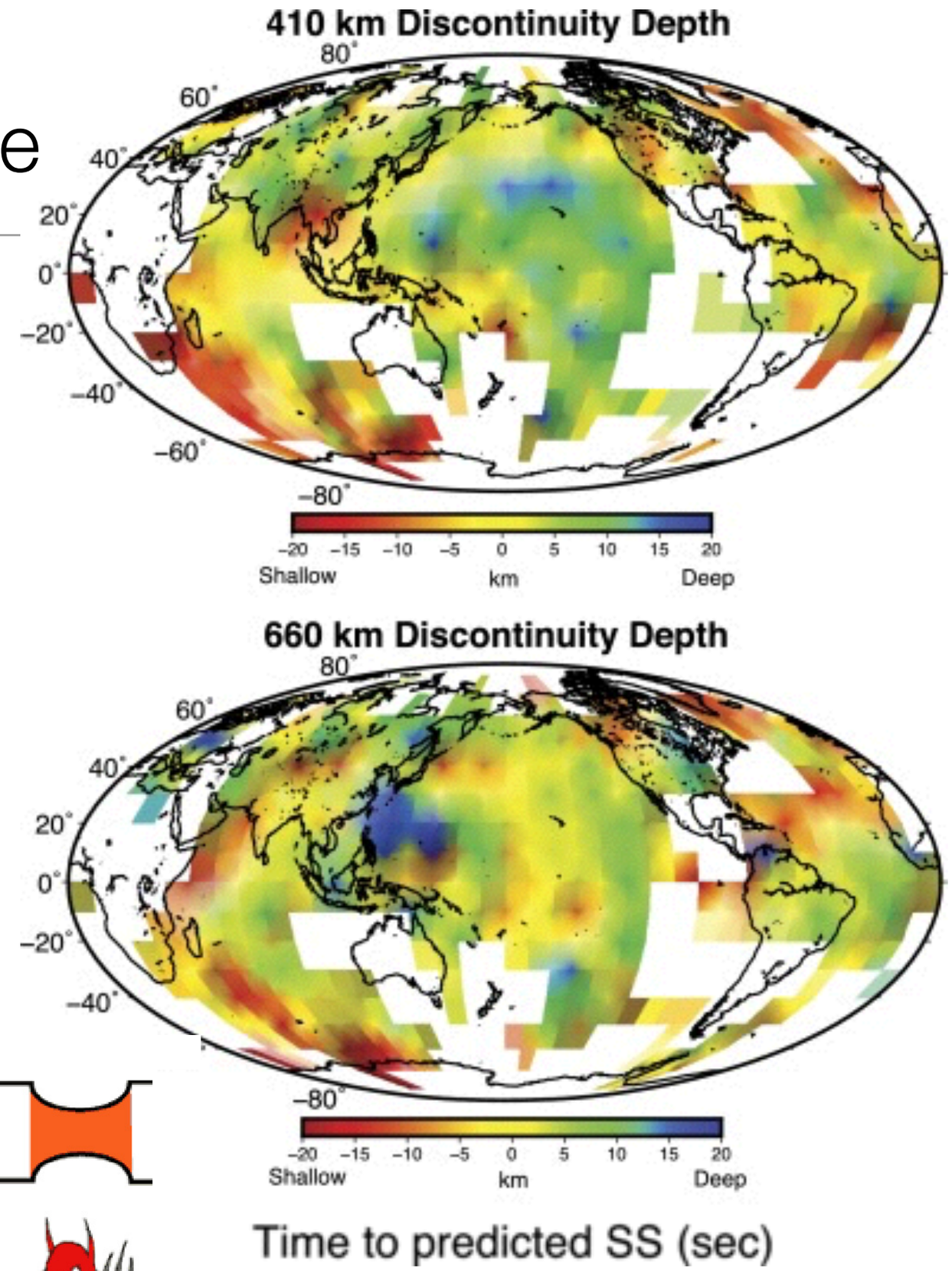


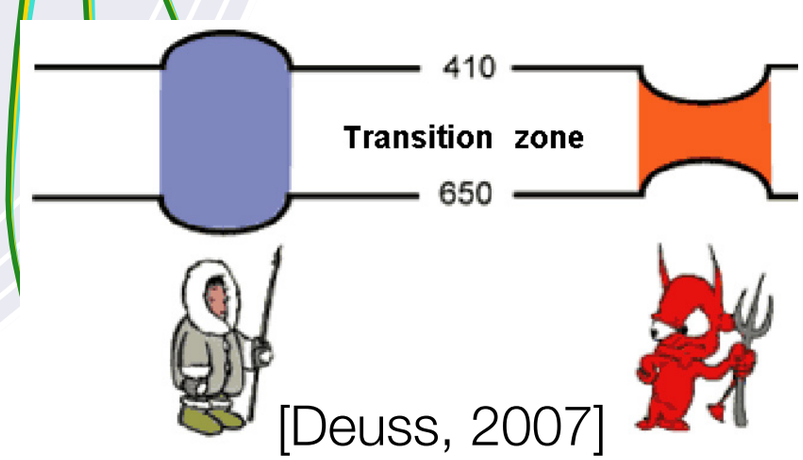


[Zheng & Romanowicz, 2012]

Mantle Transition Zone



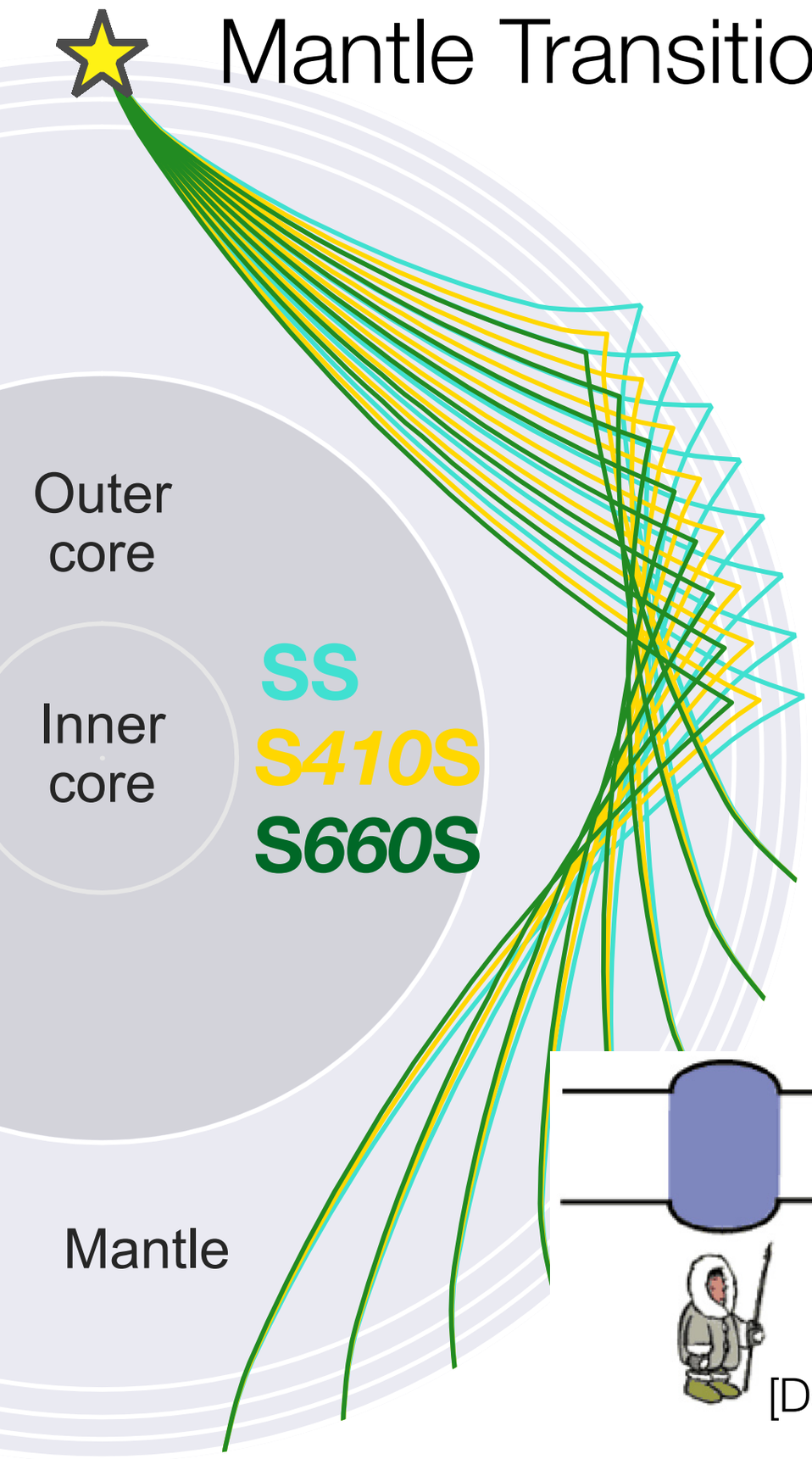


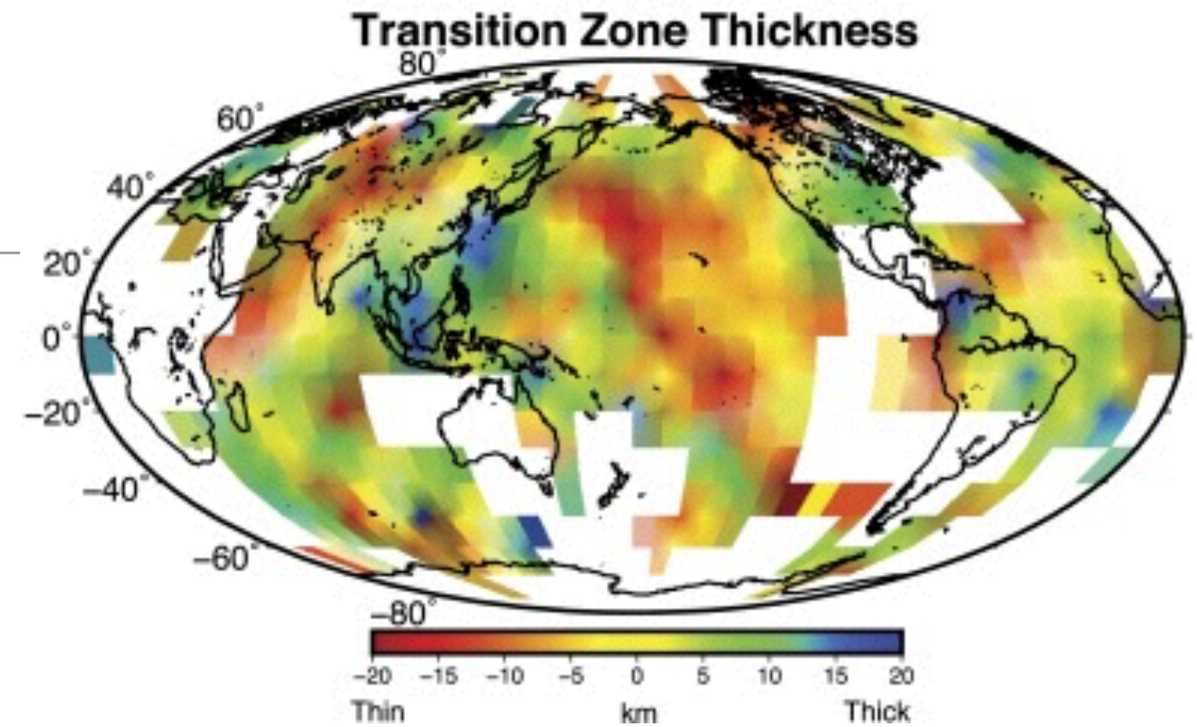


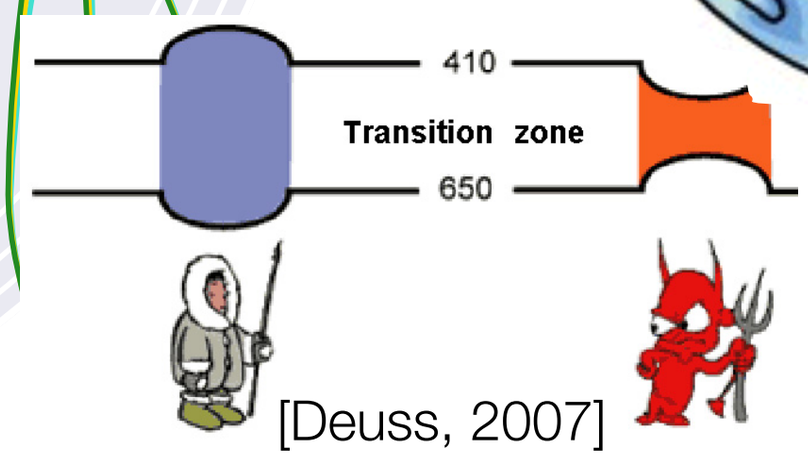
[Deuss, 2007]

[Christine Houser, 2016]

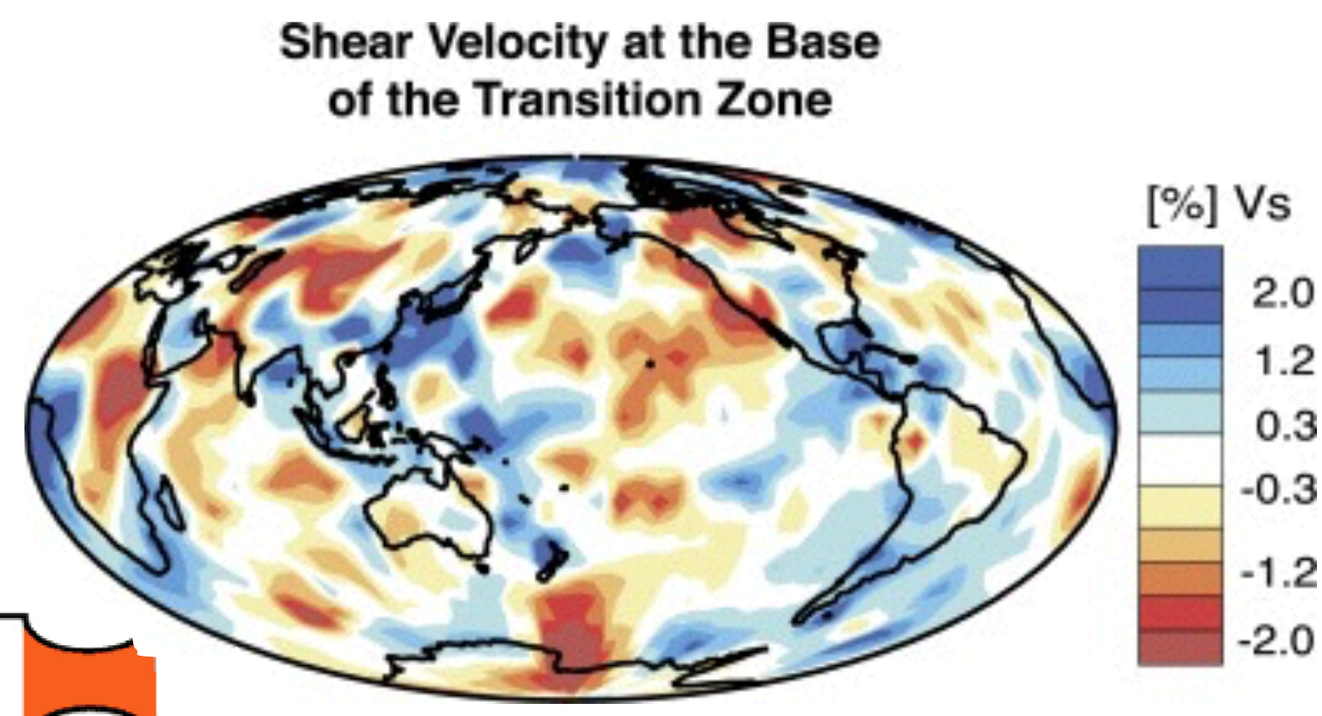
Mantle Transition Zone





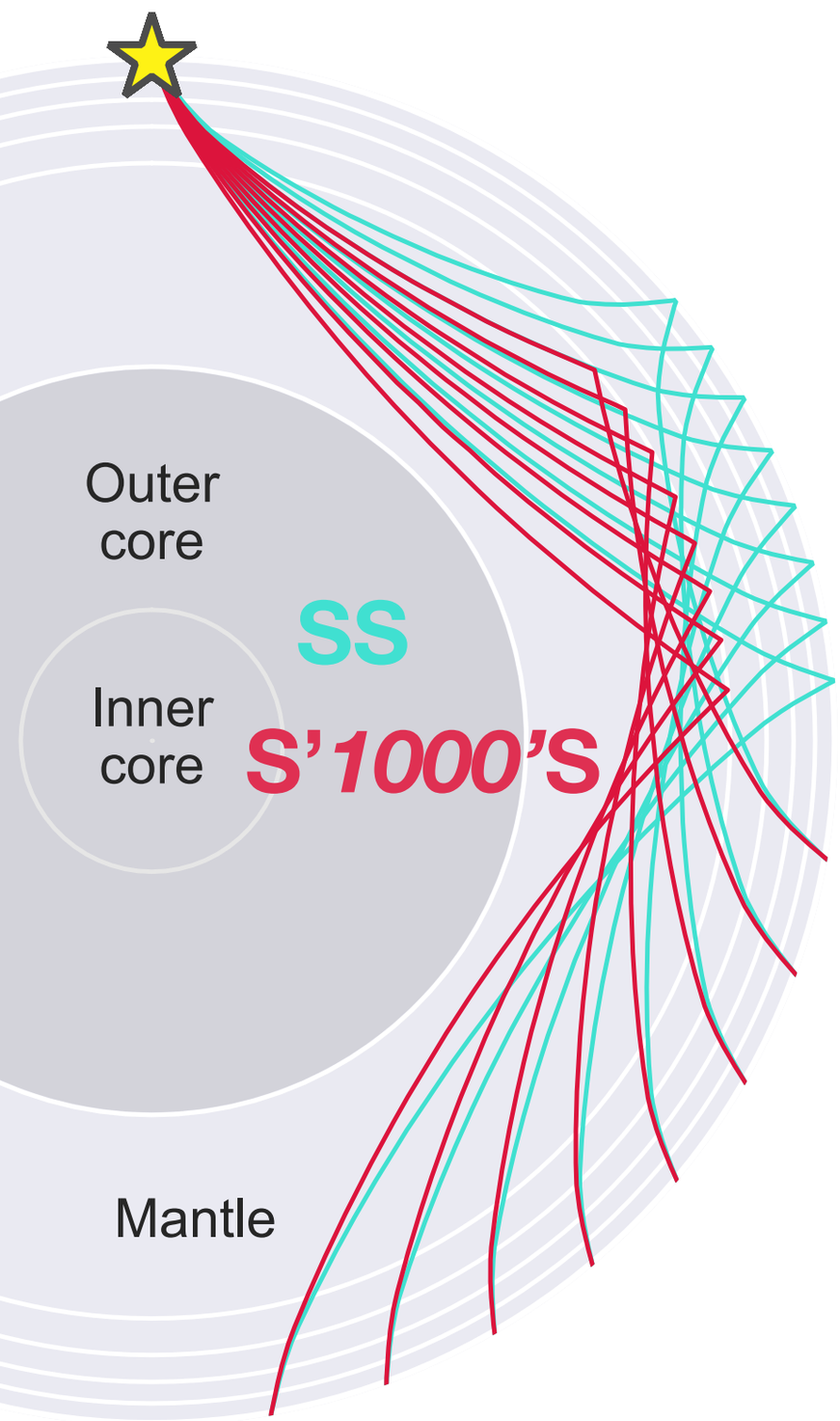


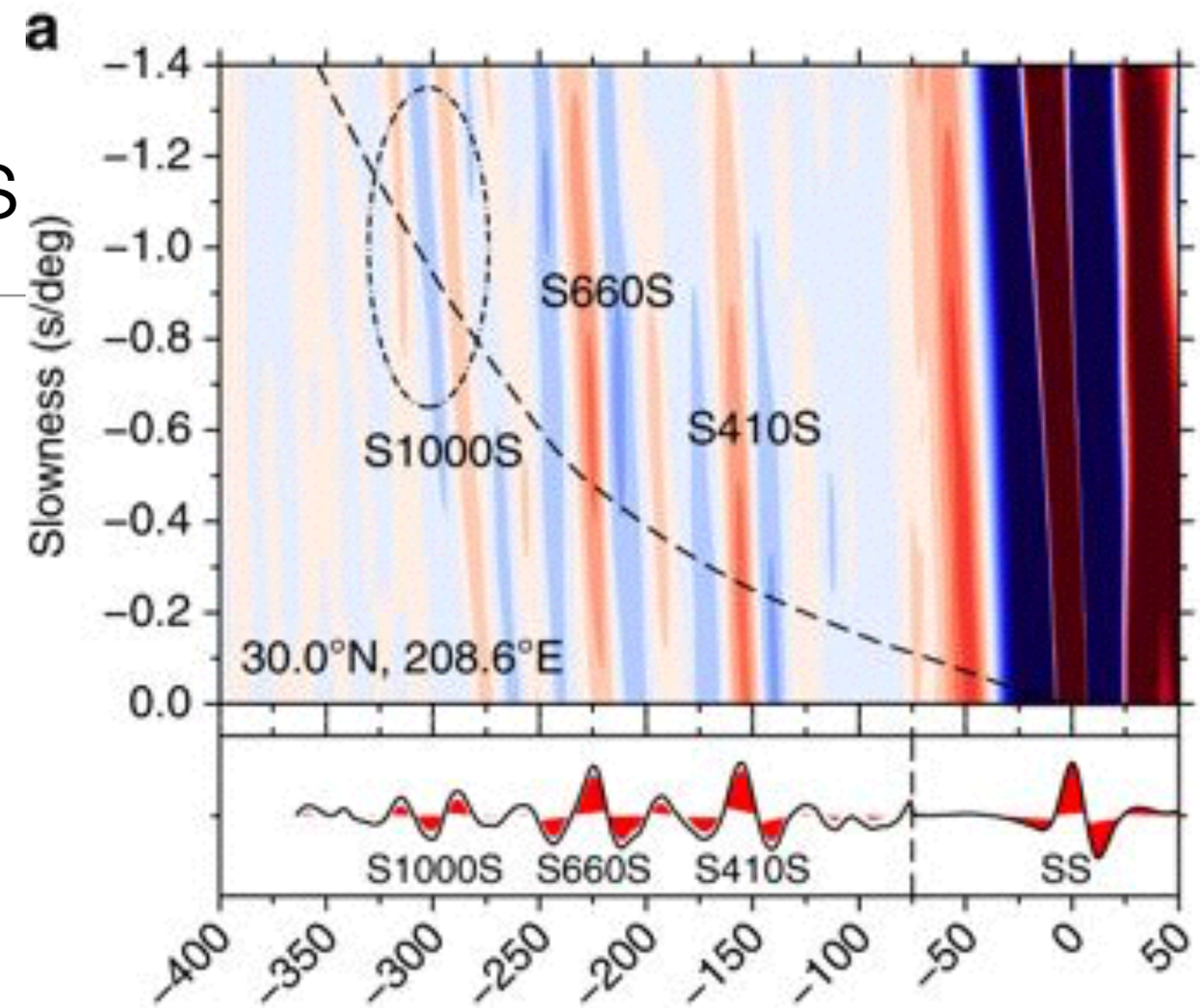
[Deuss, 2007]

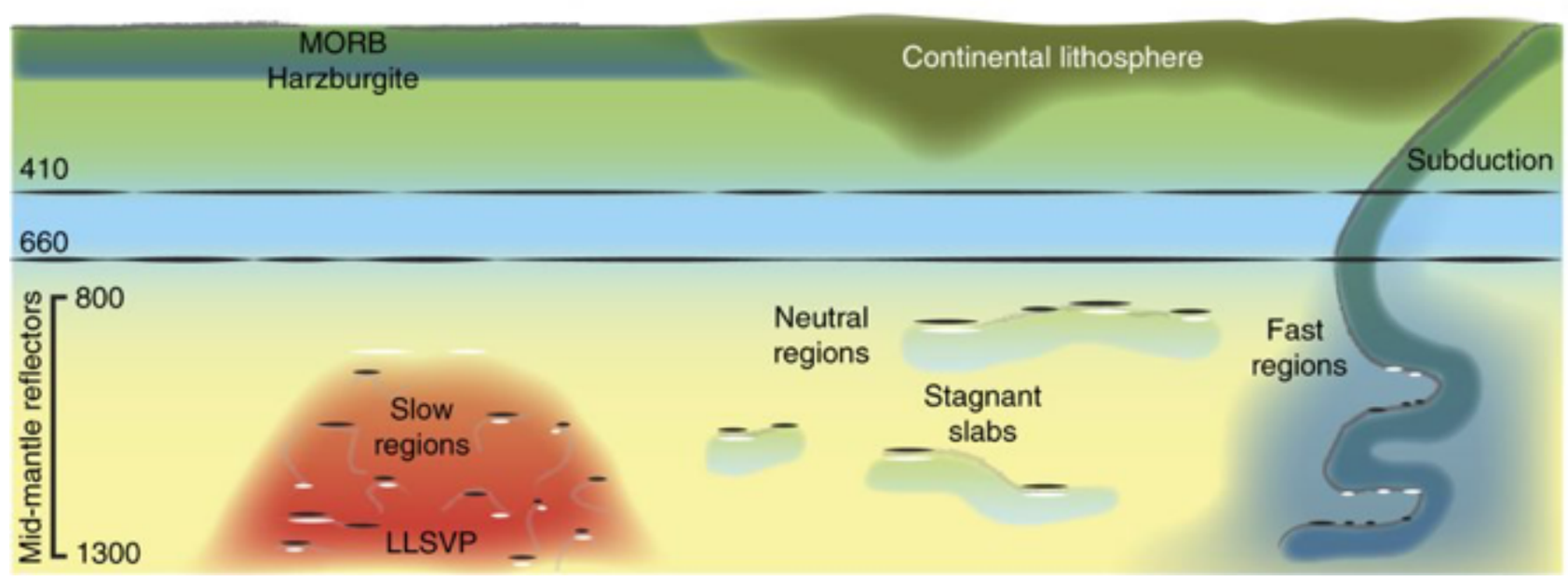


[Christine Houser, 2016]

Mid-mantle scatterers

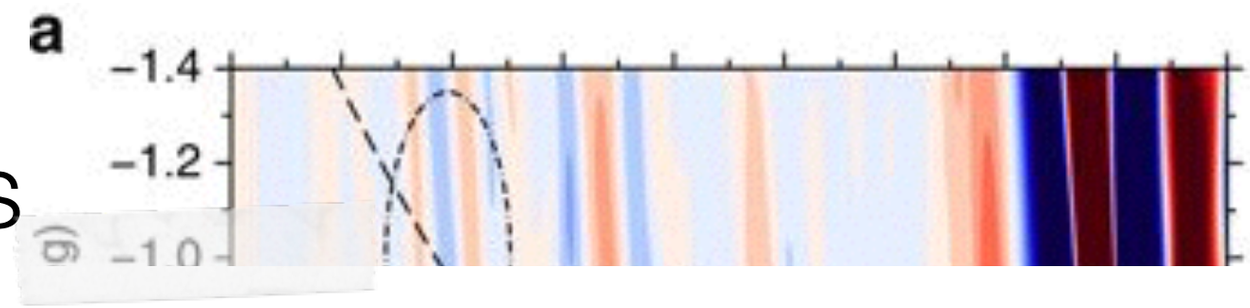






[Lauren Waszek, Schmerr, Ballmer, 2018]

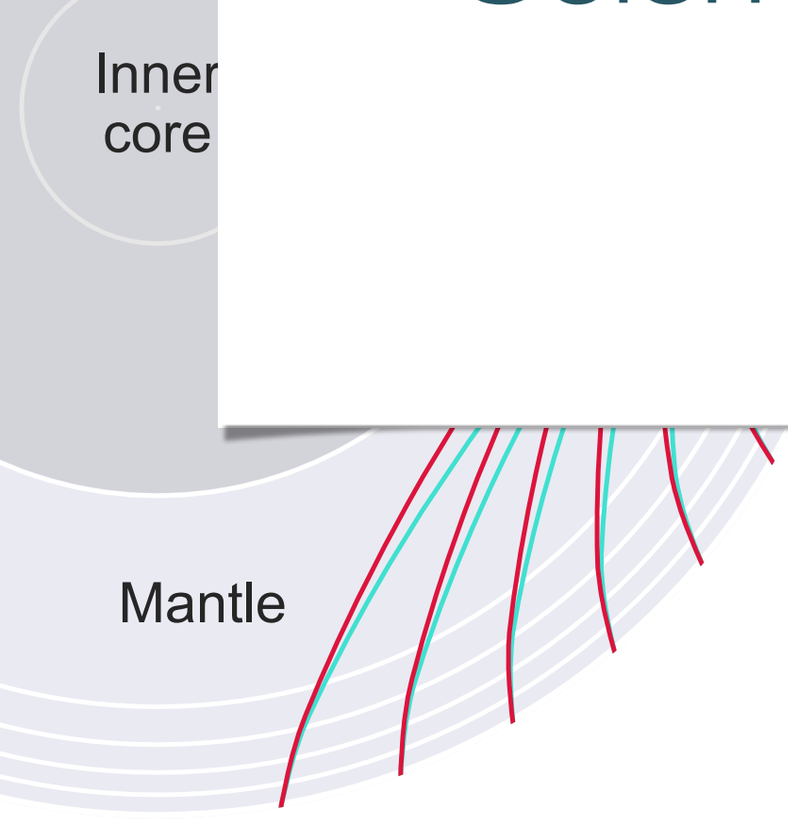
Mid-mantle scatterers

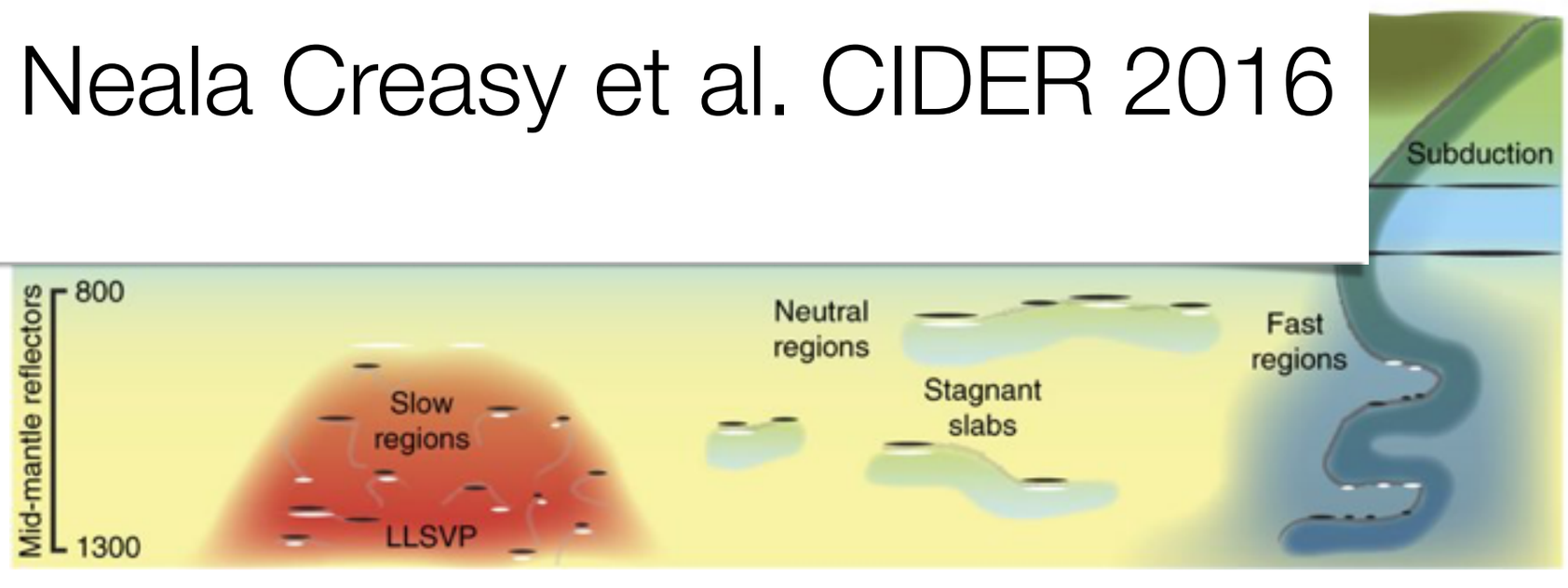


MiMOSA's

Mid-Mantle Observed Seismic Anomalies

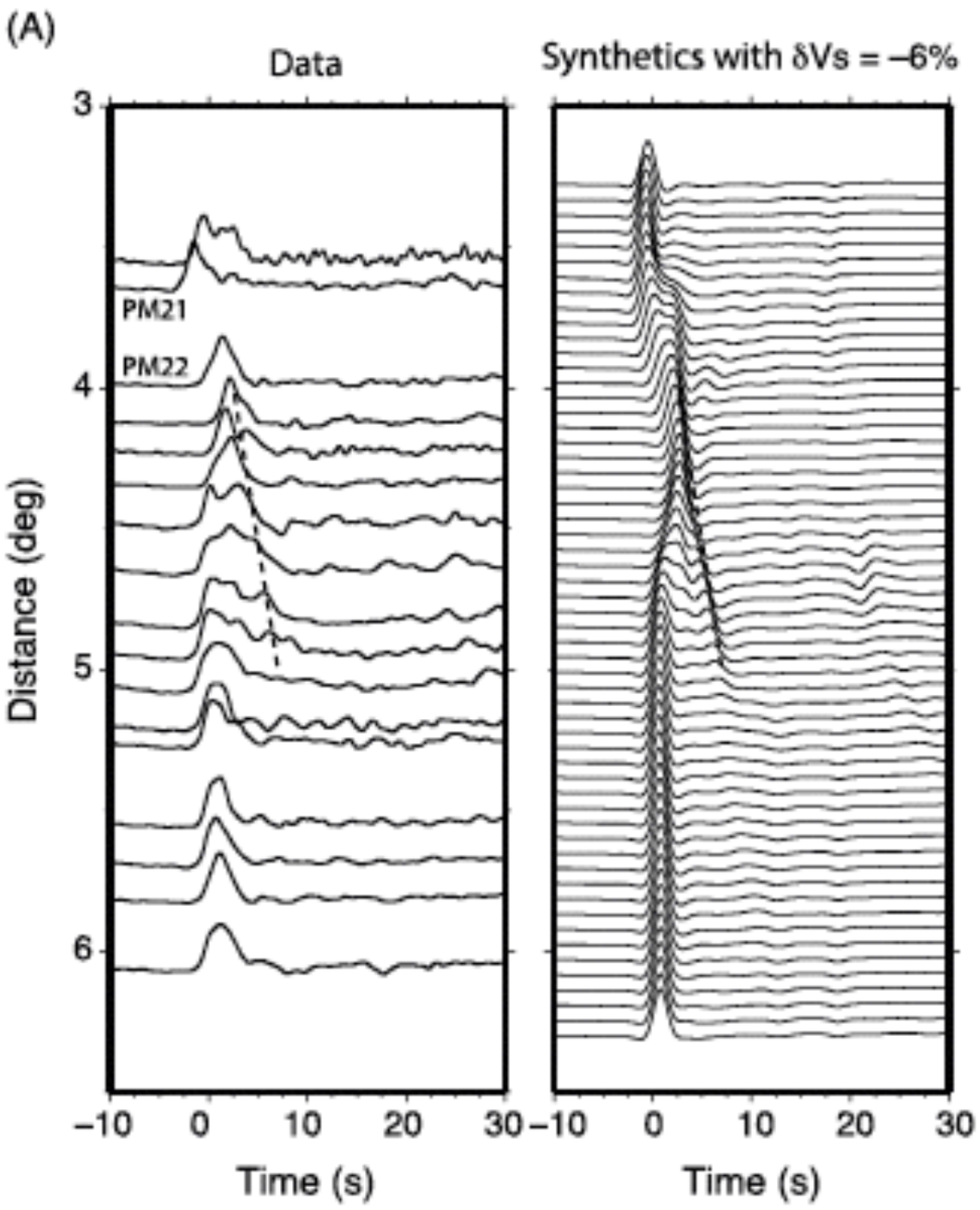
Neala Creasy et al. CIDER 2016

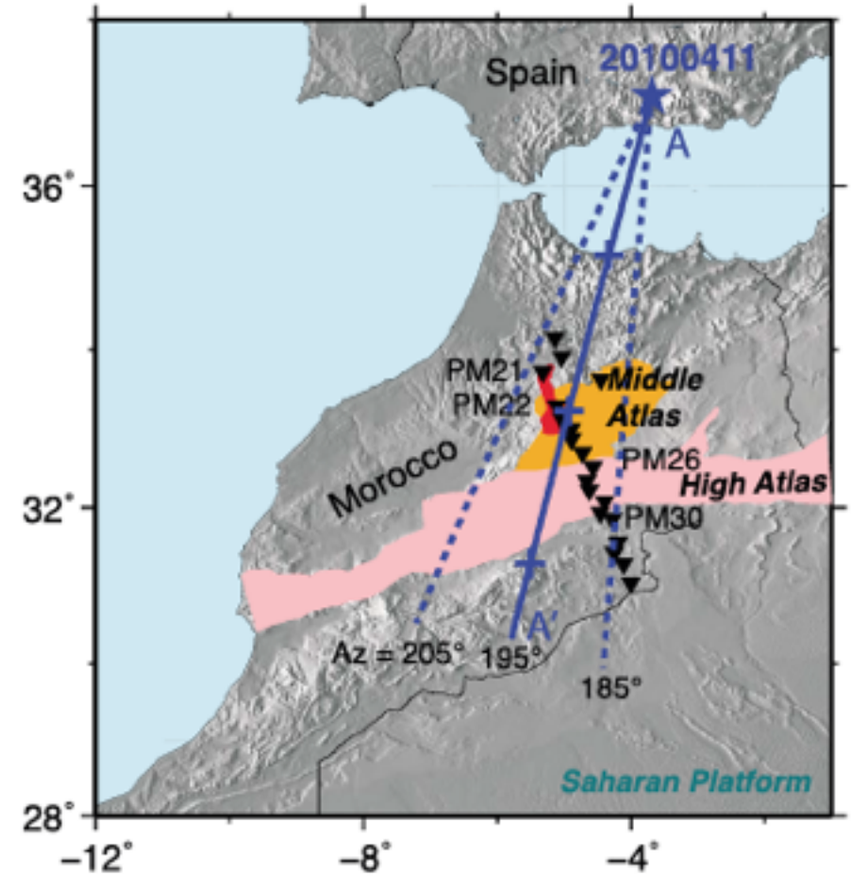


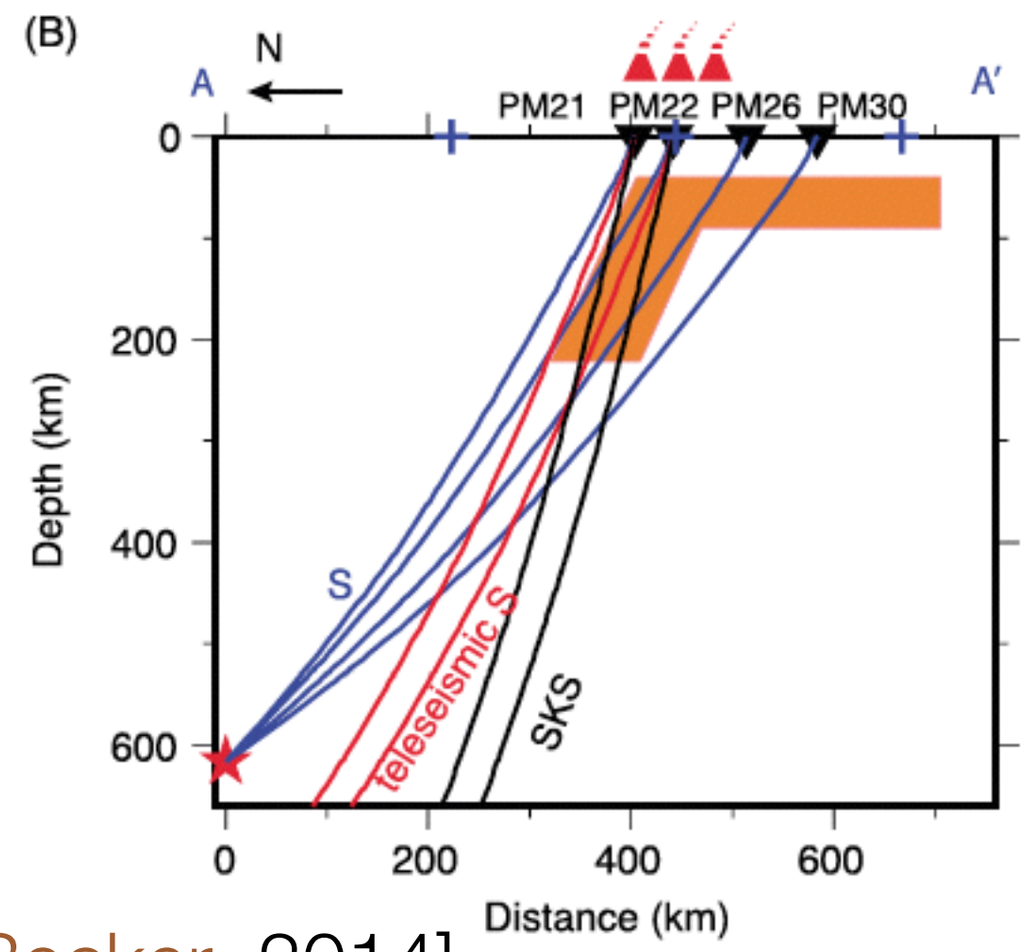


[Lauren Waszek, Schmerr, Ballmer, 2018]

Guided waves - slow material



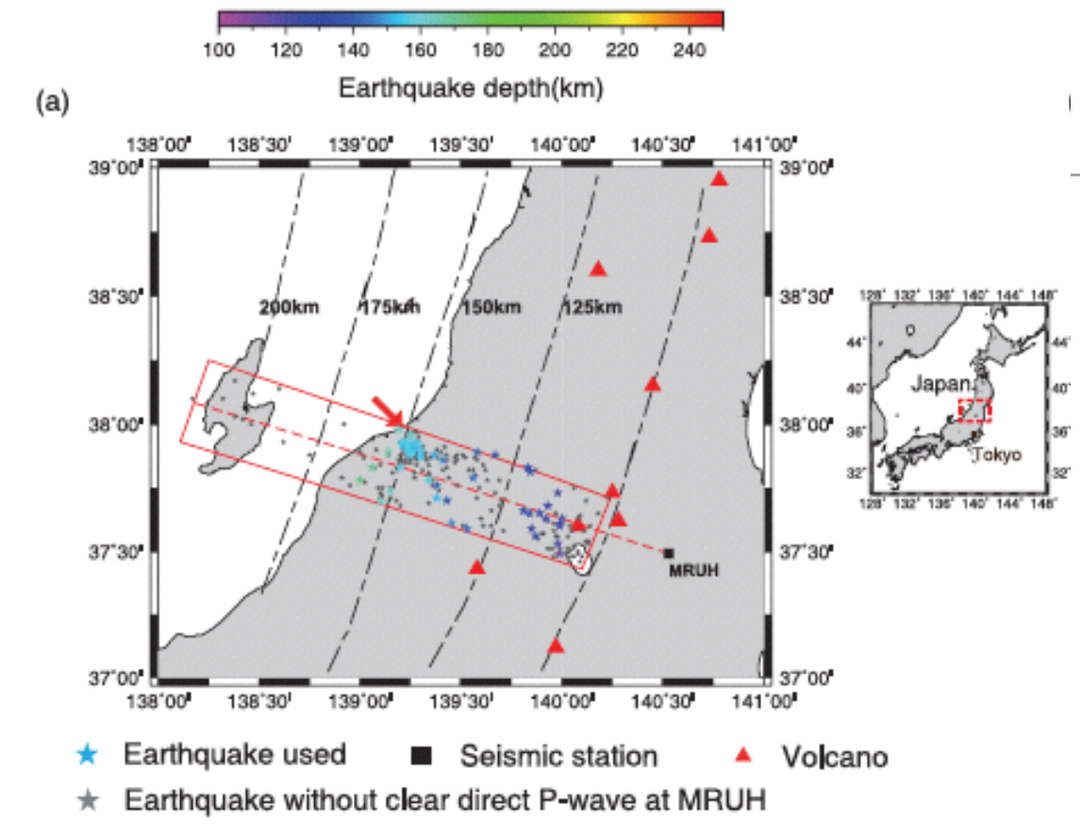


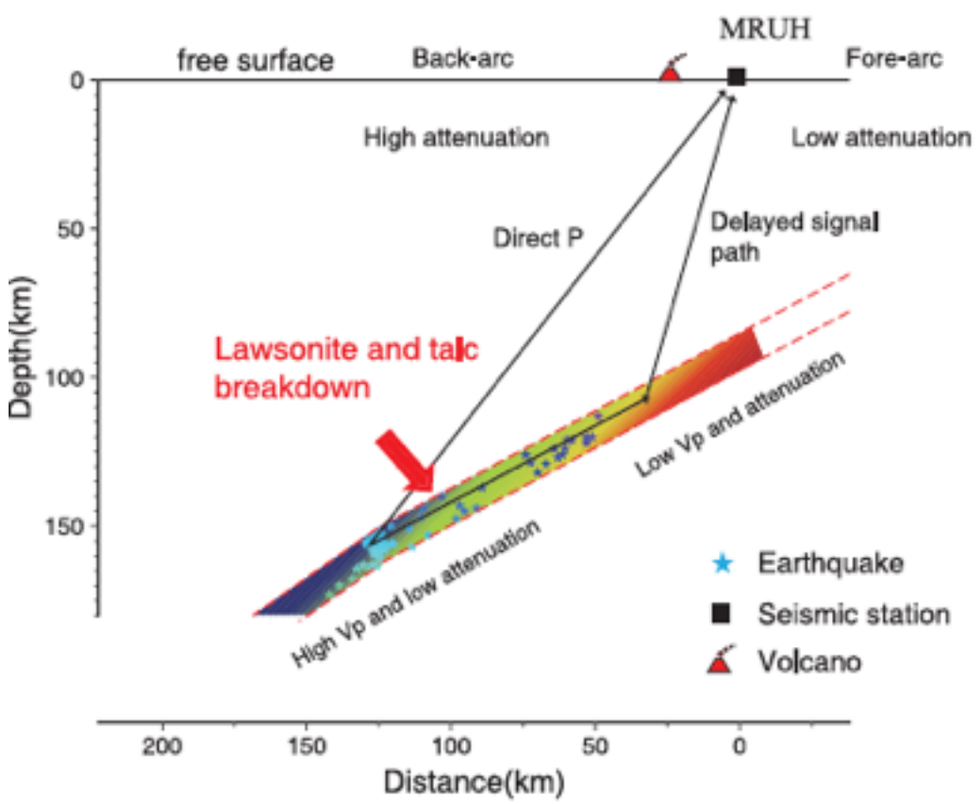


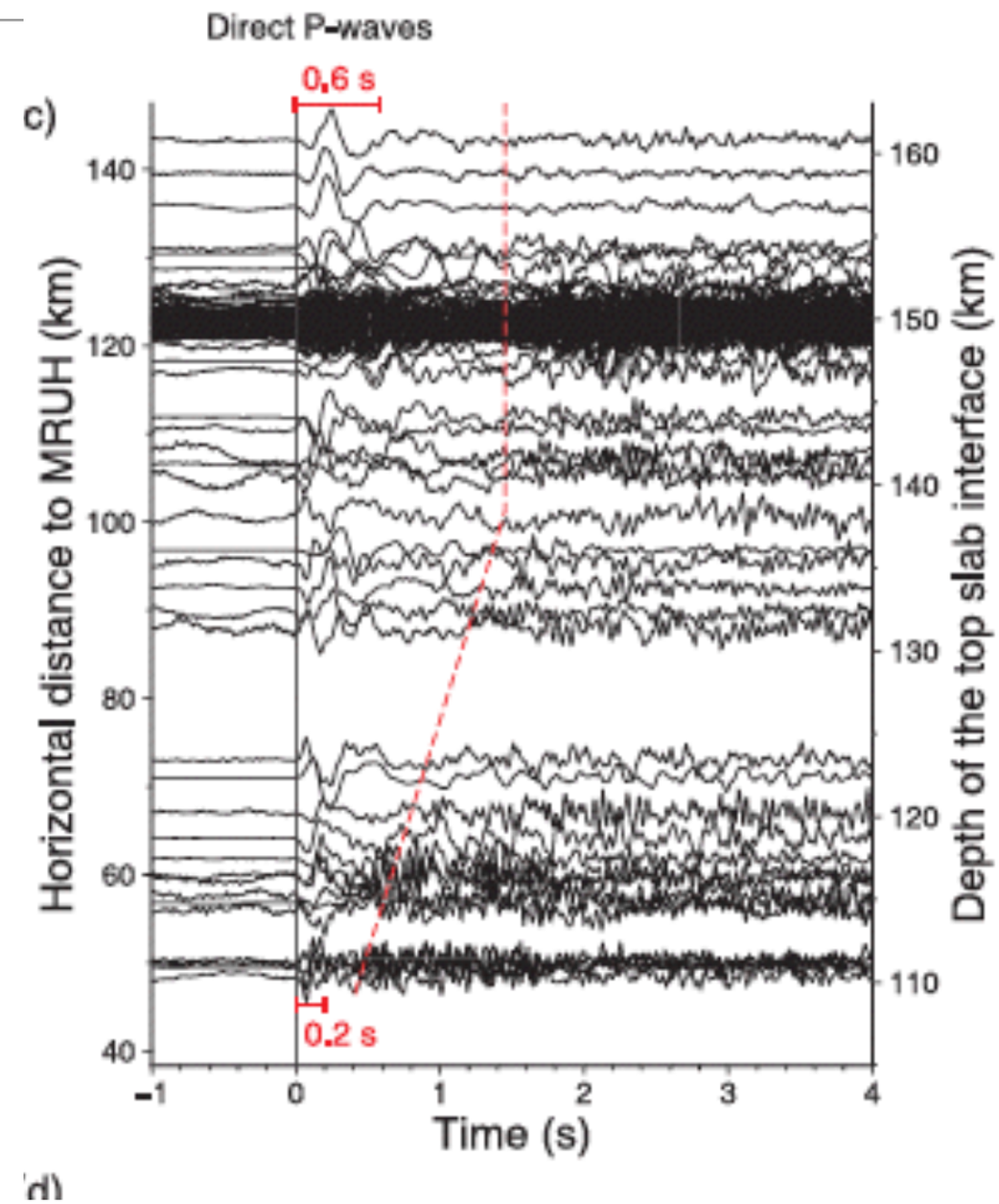
[Sun, Meghan Miller, Holt, Thorsten Becker 2014]

Guided waves - in a subducted slab

f > 1 Hz

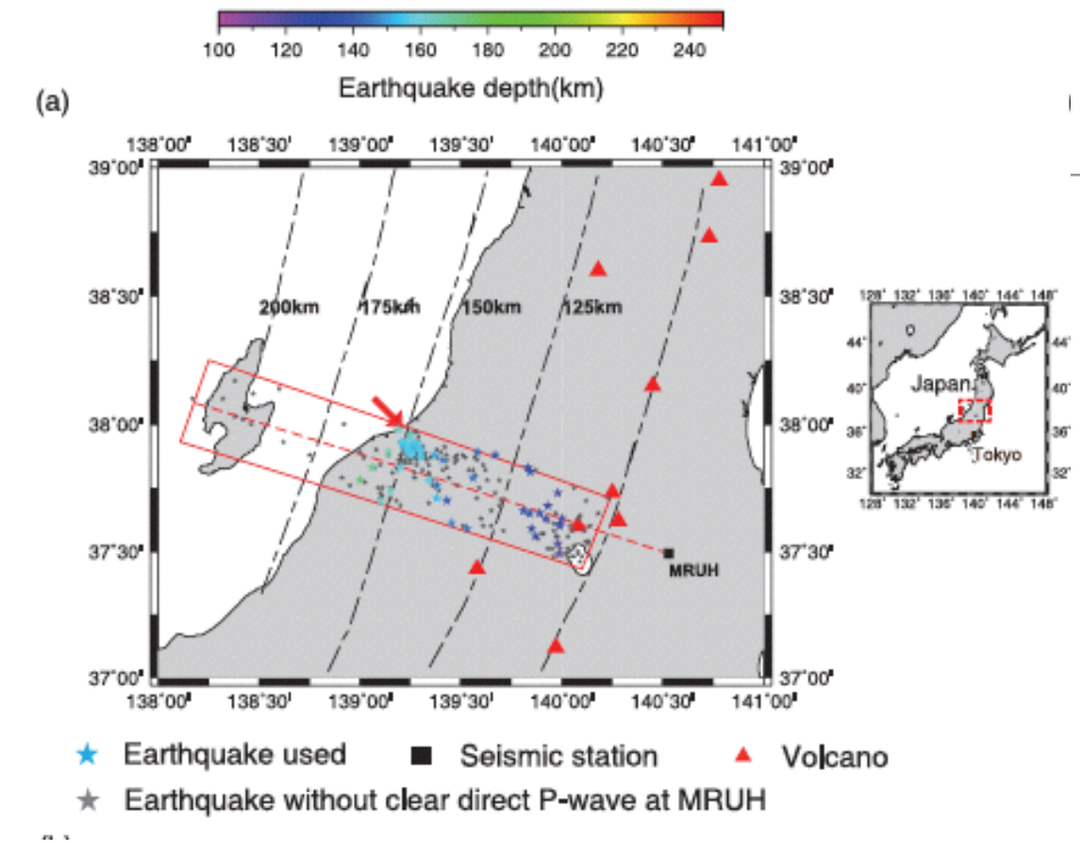


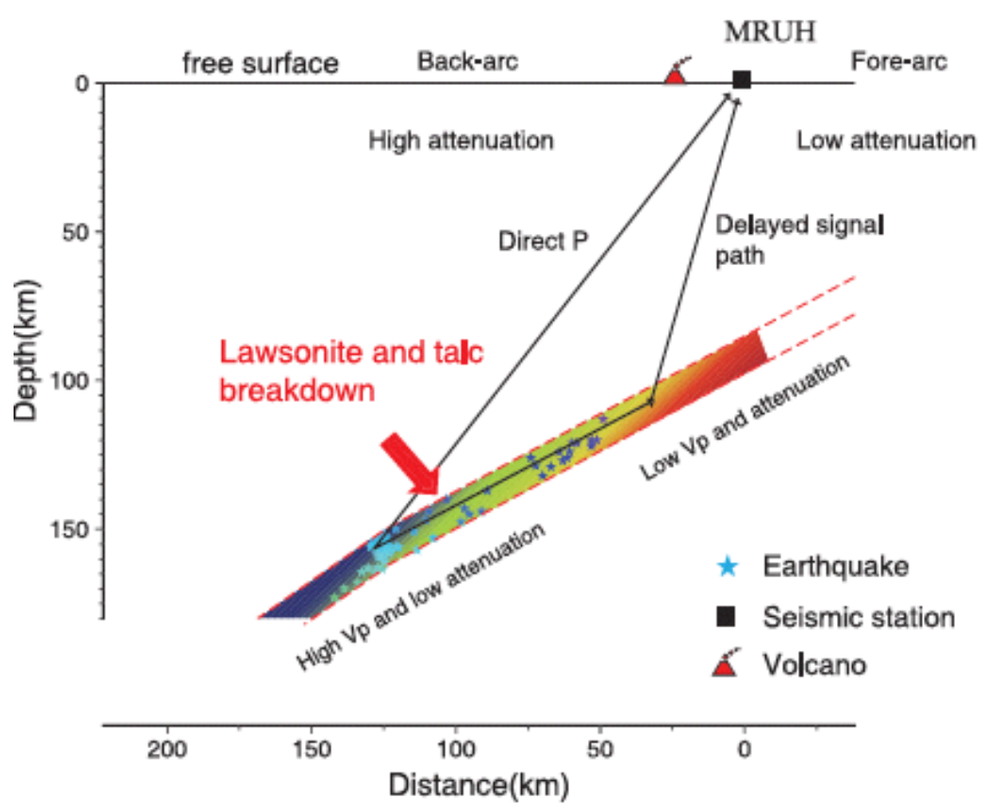




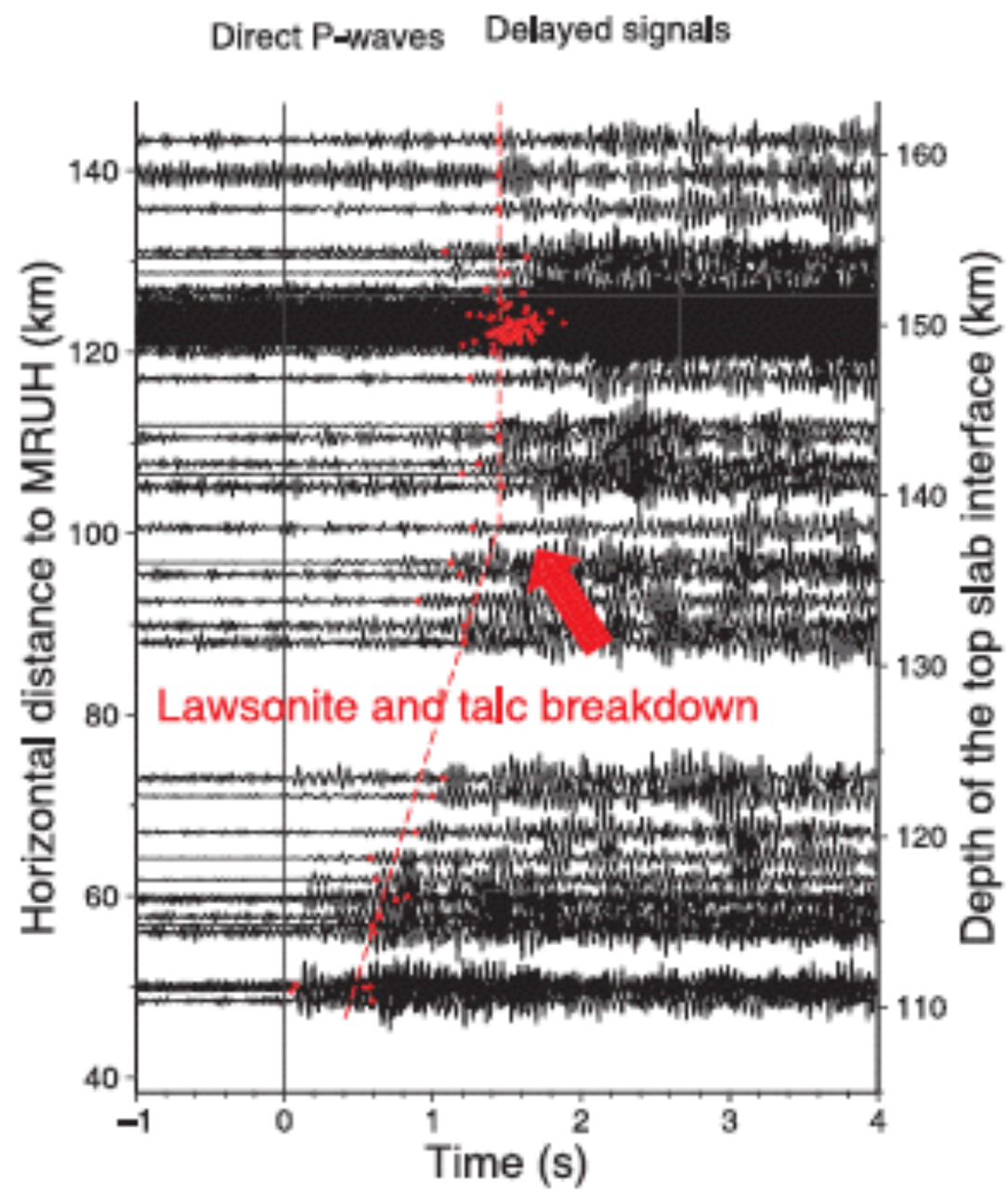
[Wu & Jessica Irving, 2018]

Guided waves - in a subducted slab





f > 20 Hz



[Wu & Jessica Irving, 2018]