

Neil Cagney, †Carolina Lithgow-Bertelloni,
University College London
†University of California Los Angeles
Max Rudolph, Rene Gassmoeller
University of California Davis

Geodynamics Tutorial 2

Michael Manga's 2012 lecture
Louise Kellogg's 2014 lecture

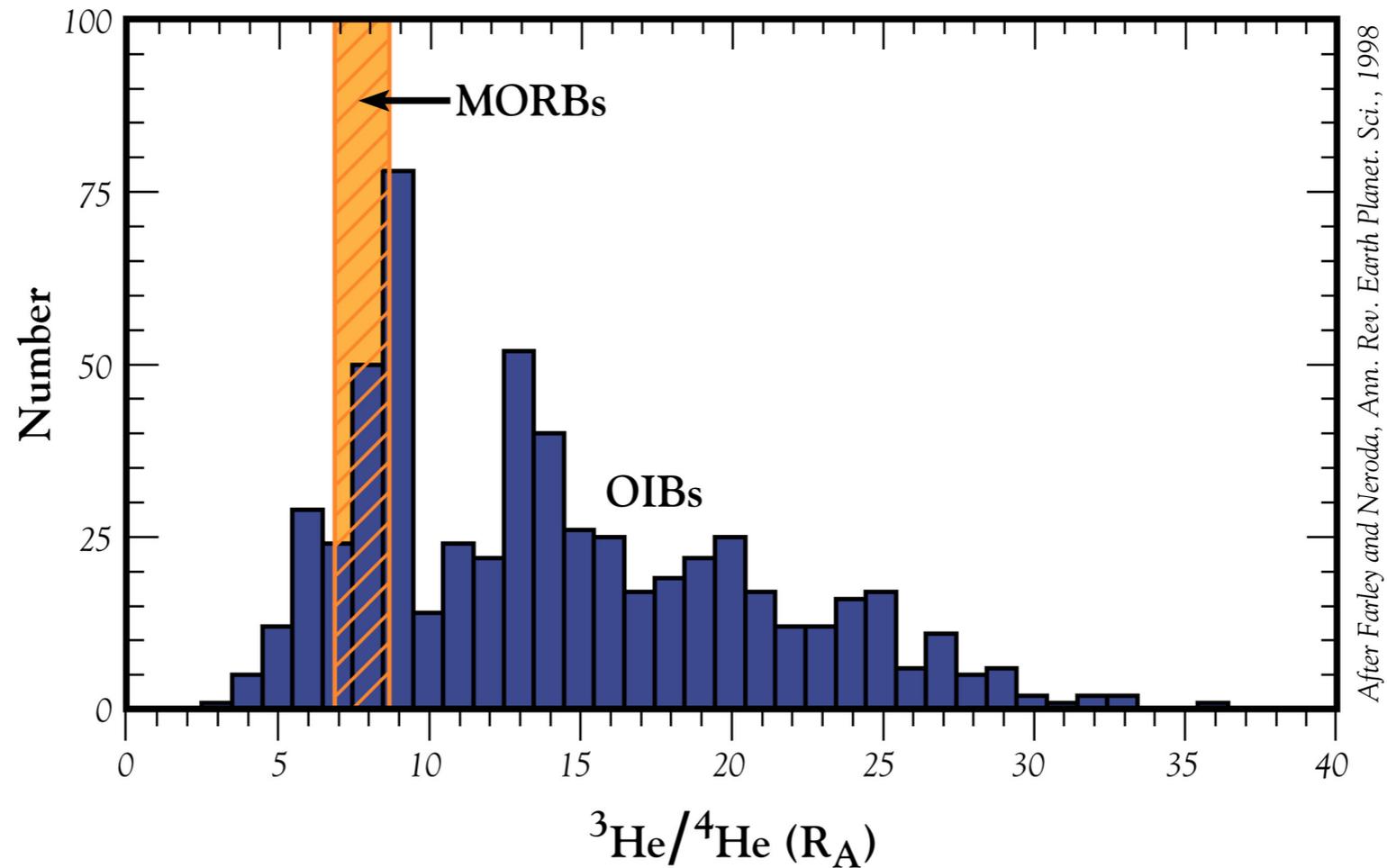
Two papers on Lagrangian Coherent Structures

Peacock and Haller, Physics Today, 2013
Haller, ARFM, 2015

<http://shaddenlab.berkeley.edu/uploads/LCS-tutorial/overview.html>

Reading Material

1. Global scale: mantle contains well-mixed regions and heterogeneity

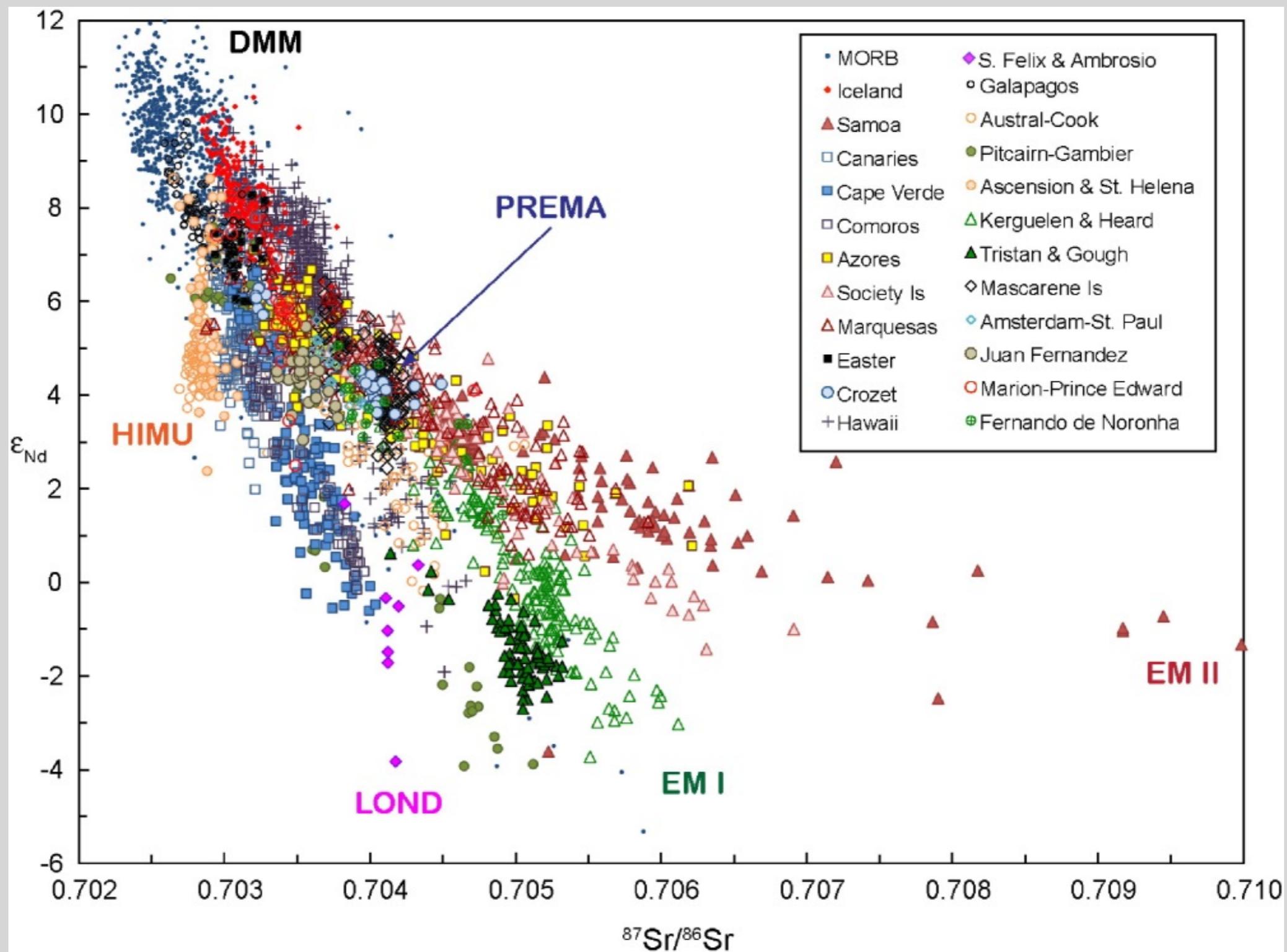


After Farley and Neroda, Ann. Rev. Earth Planet. Sci., 1998

From Michael Manga's 2012 lecture

Mixed vs Unmixed
Regions

Islands



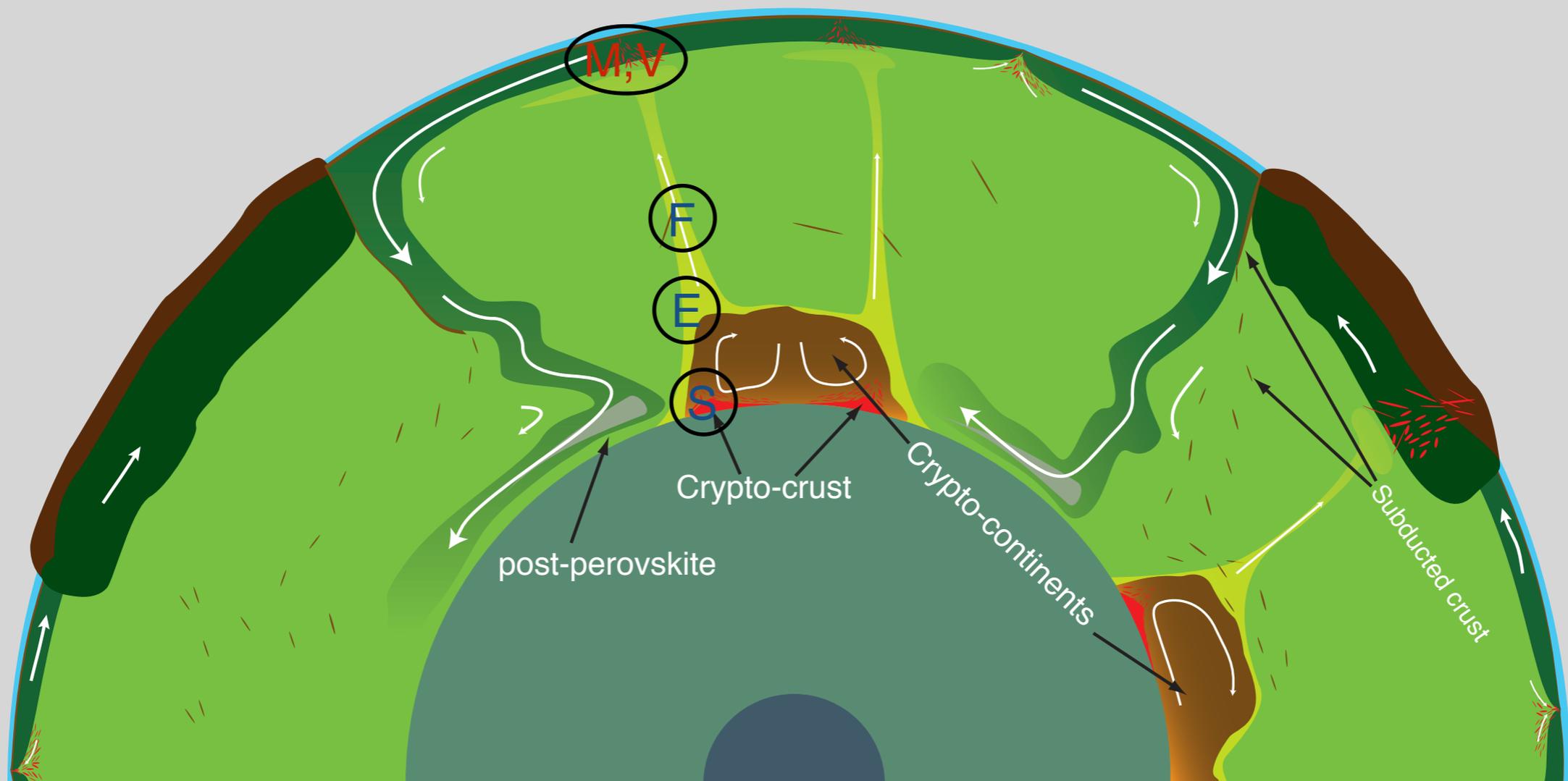
Isotope array

Interpreting requires understanding entrainment and mixing and sampling

How do we relate what we see geochemically to mantle heterogeneity?

- genesis
- source region vs entrained material
- stirring and mixing processes
(dependence on material properties)
- processes in melting region
- sampling

Geochemical Signals



[Labrosse et al., 2015]

Origin of Signal

Source region (ULVZ, LLVP)-S
 Entrainment (ambient, LLVP)-E
 Stretching, stirring-F
 Melting and Sampling region-M,V

What are we doing today?

- Pre-computed velocity and particle fields
 - Dependence on Rheology
 - Dependence on Composition
- Identify some source regions
- Define a function of entrainment (or absorption of signal)
- Quantify stretching (proxy for stirring) via FTLE [and hopefully a show and tell of the LCS]
- Examine effects of sampling region (proxy for melting region)

TUTORIAL GOALS

Entrainment, Stirring and Mixing: What's the difference?

Entrainment is when a fluid picks up and drags another fluid or a solid

Stirring is the mechanical motion of the fluid (cause)
stretching and *folding* of material surfaces to reduce length scales

Mixing is the homogenization of a substance
by *stirring* and *diffusion*

Entrainment, Stirring and
Mixing

Terminology

Finite Time Lyapunov Exponent

The average rate of stretching experienced by a region of fluid over the time-span t_0 to t is typically expressed using the Finite-Time Lyapunov Exponent (FTLE), σ_f (Shadden *et al.* 2005). This is found by first finding the flow map:

$$\phi(\mathbf{x}, t, t_0) = \mathbf{x}(\mathbf{x}_0, t, t_0), \quad (3.4)$$

which is the position of all tracers at time t which had the initial position, \mathbf{x}_0 at time t_0 (with $t_0 < t$). This is in turn used to find the Cauchy-Green deformation tensor

$$C = (\nabla\phi)^\top (\nabla\phi). \quad (3.5)$$

The largest real eigenvector of C , λ_{max} represents the maximum strain. Using this information, the FTLE can be calculated as

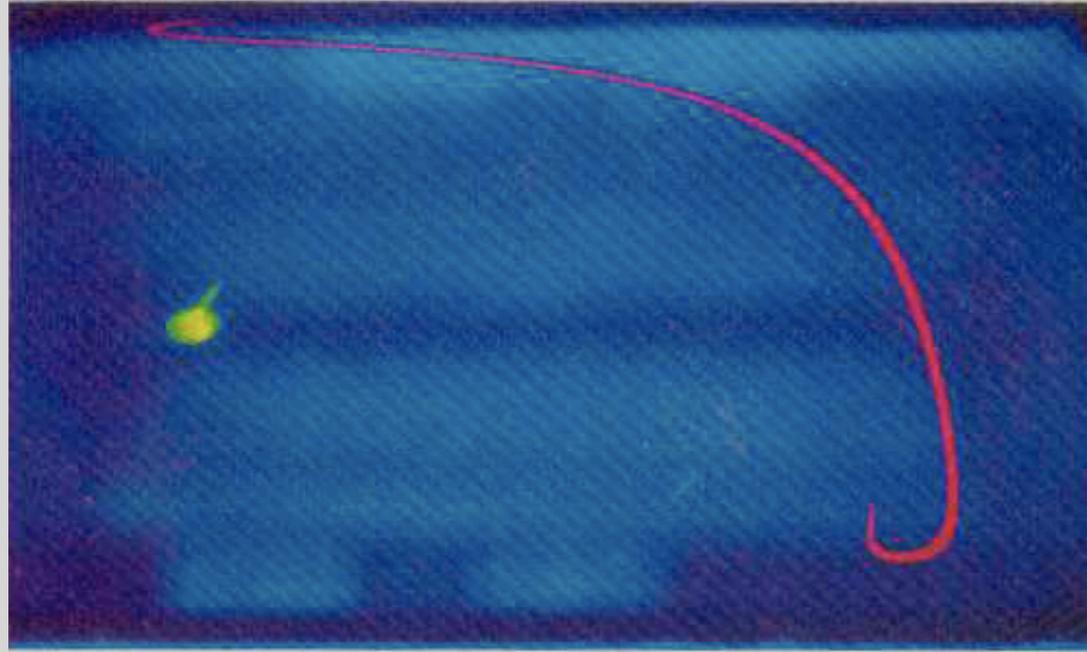
$$\sigma_f(\mathbf{x}, t, t_0) = \frac{1}{2(t-t_0)} \log \lambda_{max}. \quad (3.6)$$

$$\delta^* = \frac{\delta(t)}{\delta(t_0)} = e^{\sigma_f(t-t_0)},$$

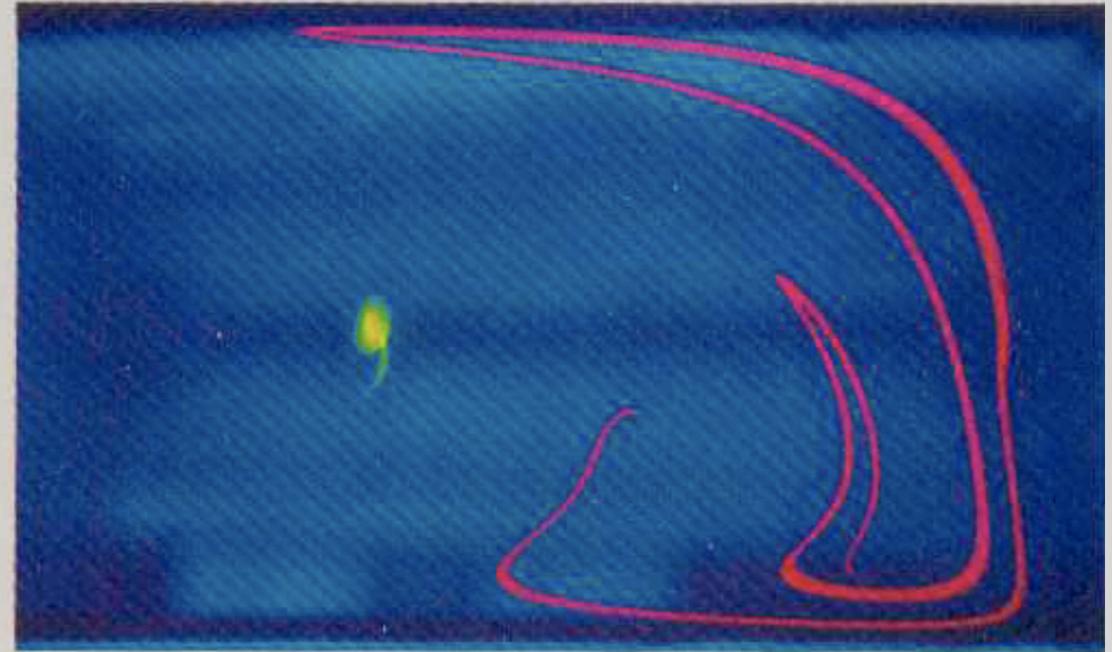
FTLE

How to compute

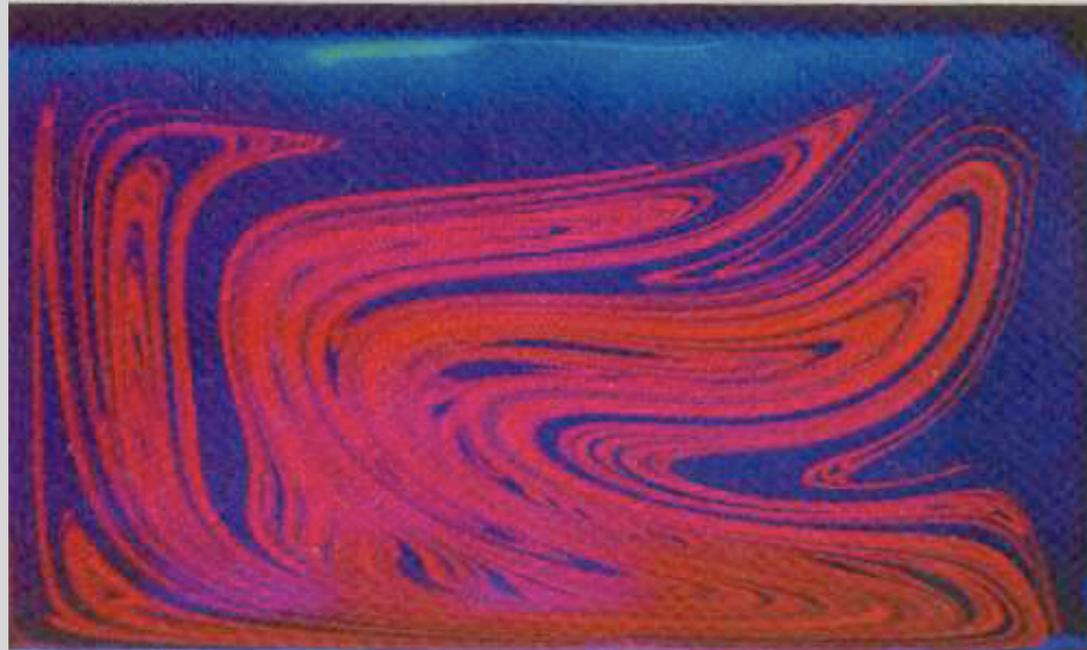
1 PERIOD



3 PERIODS



3 1/4 PERIODS



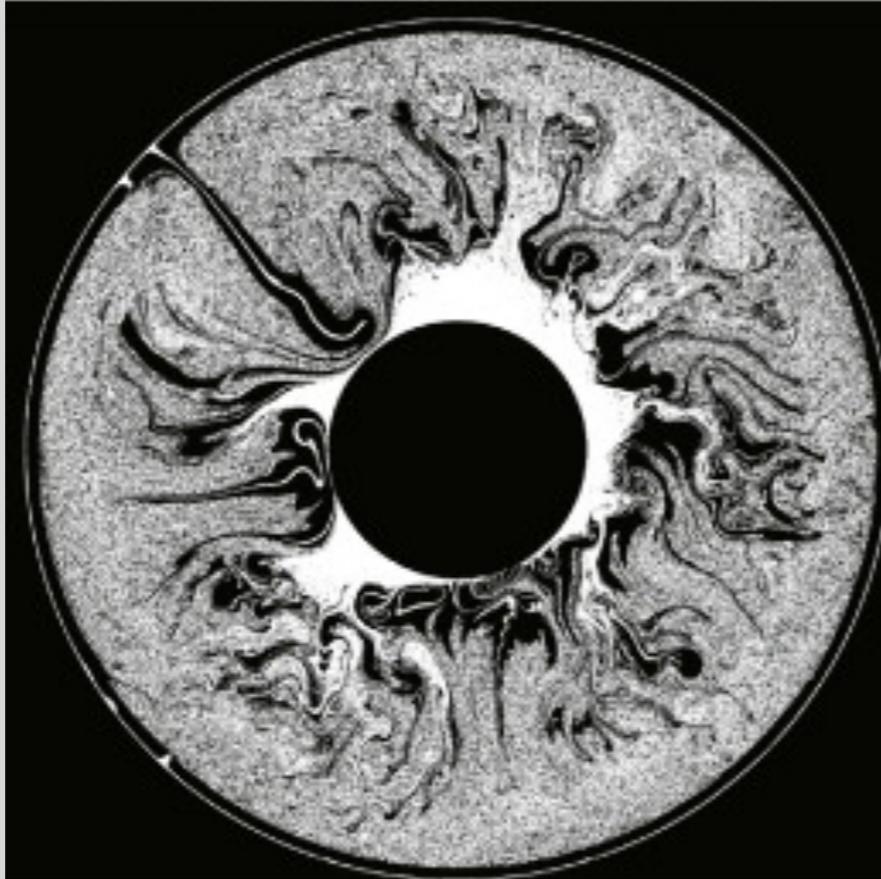
8 1/2 PERIODS



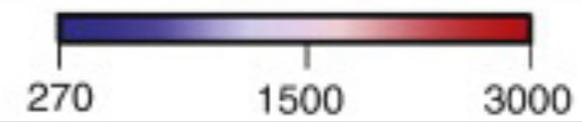
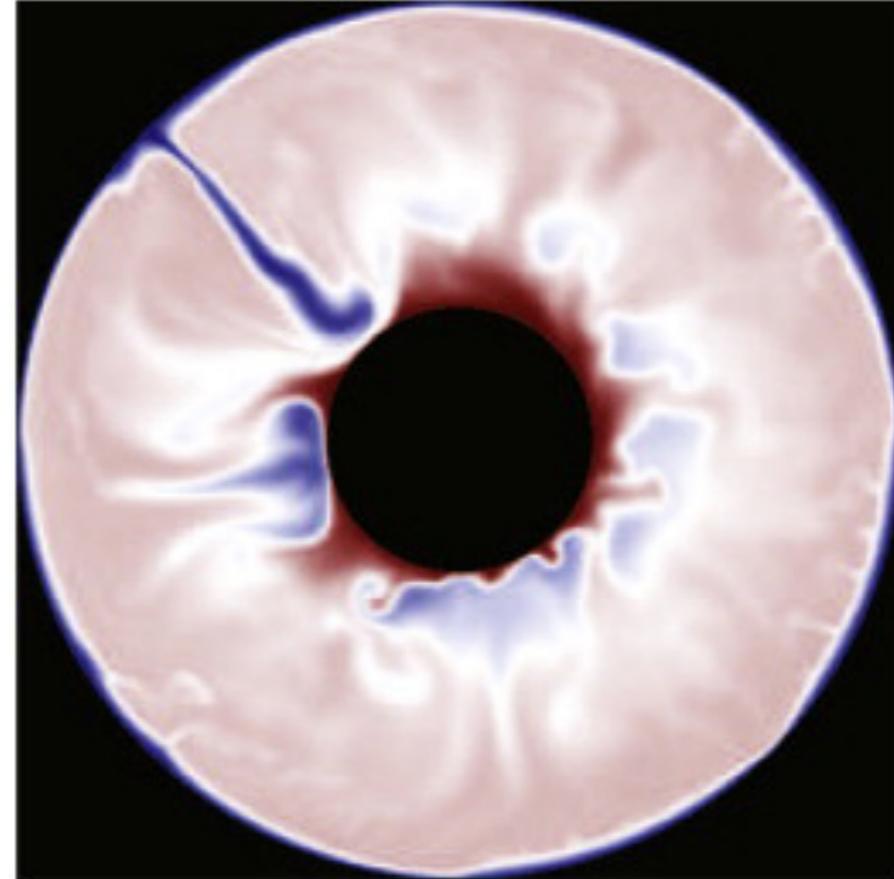
Time-dependence

Well mixed and not well mixed regions coexist

9% Eclogite Excess Density, 4.5 Byr



Eclogite Tracers Shown



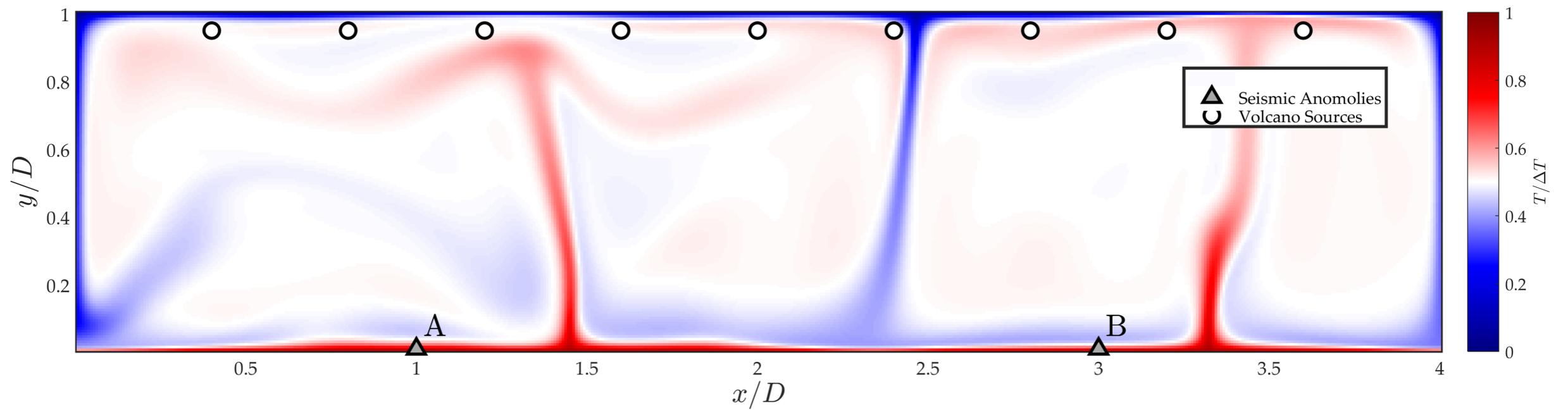
Advecting Tracers

CASES

- Isoviscous thermal case
 - $Ra=1.14 \times 10^6$ [Matlab version of Stag]
 - $Ra=2.28 \times 10^4$ [ASPECT]
- Increase in viscosity UM/LM
- Compositional layer
- Viscous Blobs
- Change # of sampling sites
- Change depth or radius of sampling region

CASES

Observations at surface



Source introduced at spatially fixed locations at the CMB

ASPECT Calculations

Case	Stokes BC	Viscosity	Thermal Ra	Buoyancy ratio
purely-thermal-isoviscous	Free slip	Uniform 3×10^{23}	1.14×10^6	0 (no chemical layer)
purely-thermal-isoviscous	Free slip	Upper half 3×10^{23} Lower half 1.5×10^{24}	2.8×10^4 *	0 (no chemical layer)
driven-thermochemical-4to1	Driven cavity	Isoviscous	1.14×10^5	1.0
viscous-blobs	Driven cavity	10X higher for blobs	0	0

*Defined based on lower mantle viscosity

All calculations are run in a 4:1 box.

All of the calculations have an active chemical tracer but the purely thermal calculations have the density difference set to 0.0. This was to ensure uniform output file format.

Nondimensionalization of ASPECT results

We scale position (x), time (t), temperature (T), and velocity (v) according to:

$$x' = Hx$$

Where H is layer depth
And κ is thermal diffusivity

$$t' = \frac{H^2}{\kappa} t$$

$$v' = \frac{\kappa}{H} v$$

Primed (') quantities are dimensional

$$B = \frac{\Delta\rho c}{\rho_0 \alpha \Delta T}$$

$$Ra = \frac{\rho_0 \alpha \Delta T g H^3}{\kappa \mu}$$

1. Look at output from each calculation

- What are the differences in the velocity and particle fields?
- What are they related to?
- Try changing the location of the source regions based on the velocity and particle fields.

2. Calculate FTLE

- Look at the FTLE fields
 - How do they differ per case?
 - Can you identify regions of high stretching efficiency? Low? Isolated regions?
 - What controls their presence?
 - Without looking at the actual concentrations can you predict which “volcano” might be getting more or less of each source region?
 - What happened when you changed the location of the source regions?

3. Calculate and visualize x_A and x_B

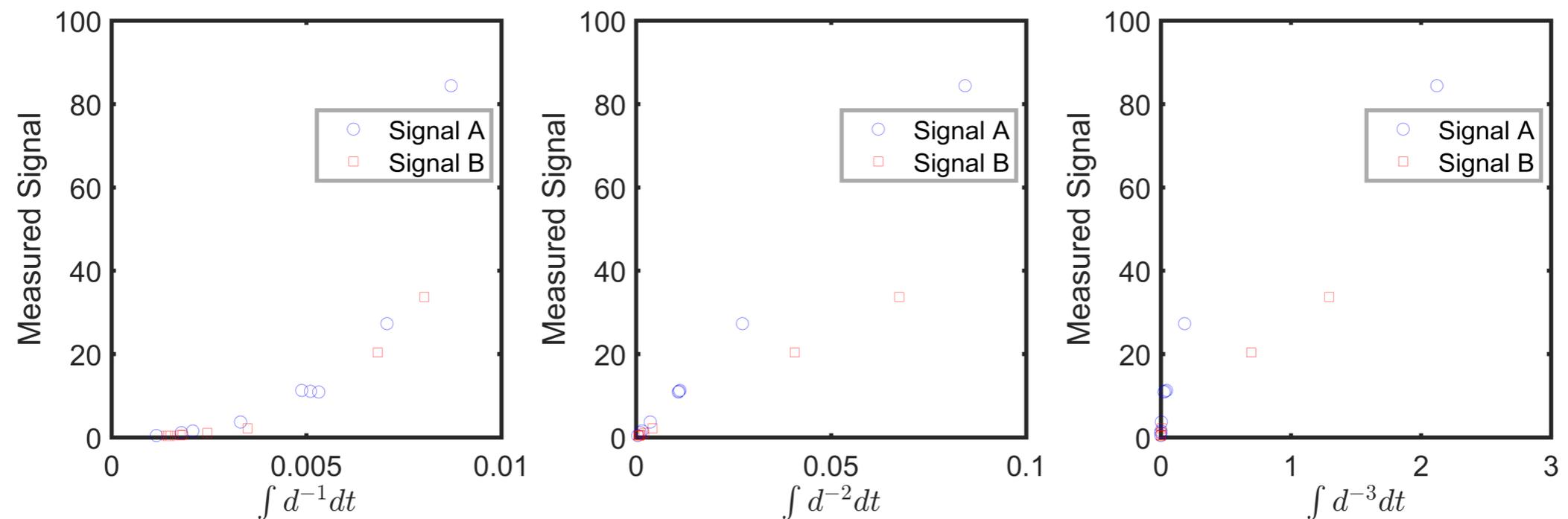
- These are the final concentrations of A and B sources at the end
- Any interesting patterns related to the flow? FTLE?
- If you were to predict the signal of A and B at the different volcanoes, what would you predict?

4. Look at effect of sampling region size

- What happens if the radius of the sampling region is smaller?
- What happens if the radius of the sampling region is larger?
- What happens if it shallower? Deeper?

- What happens if the number of sampling sites is less? More
- Distributed differently?

5. Look at function of entrainment



- What is the rate of “absorption” or entrainment of A that best explains the signals?
- What is that rate?