Genesis, Preservation and Destruction of Mantle Heterogeneity
Reading Material

Michael Manga's 2012 lecture
Louise Kellogg's 2014 lecture

Two papers on Lagrangian Coherent Structures

Peacock and Haller, Physics Today, 2013
Haller, ARFM, 2015

http://shaddenlab.berkeley.edu/uploads/LCS-tutorial/overview.html
Creation, Destruction, Survival

Origin
Primordial
Convective/Tectonic Processes

Location of Heterogeneity
Where is the Physical Reservoir- Source region
Length-scales
What controls it?

Preservation, Destruction and Time Scales
Stirring and mixing processes
Controlled by mantle dynamics, material properties and Earth's history

What do we see seismically/geochemically?
Major element- seismically detectable, "easy" physical location, tied to genesis
Minor element- possibly seismic correlation (e.g. Dalton et al., 20xx)
Trace-only identifiable geochemically tied to:
genesis
source region vs entrained material
processes in melting region
sampling
Origin of Heterogeneity

Genesis:
- Primordial
- Via Convection/Plate Tectonics

[Labrosse et al., 2015]
Physical Reservoirs of Geochemical Signatures
Reservoir for $^{142}$Nd?

Labrosse et al., 2007

Signals from primordial magma ocean? Related to present LLVPs and ULVZs?
Origin of Heterogeneity

Generation (melting) and Destruction (subduction) of Oceanic Lithosphere

[Labrosse et al., 2015]
Convection creates and destroys heterogeneity

Melting, Subduction, Stretching

Plates ← Mantle Convection

Zhao et al., 1997

Continuous generation of dynamical (thermal) + geochemical (compositional) = seismic heterogeneity [including phase transitions (TZ!!)]
Mechanical Mixture

LLVPs - ancient basalt graveyards?
MANTLE-MM of eclogite and harzburgite?

Melting
Decompression Melting
Melting forms oceanic crust (basalt) and depleted residuum (harzburgite)

Densities
- basalt ~ 2.9 g/cm³
- harzburgite ~ 3.2 g/cm³
- lherzolite ~ 3.3 g/cm³

Oxburgh & Parmentier (1977)

Pervasive heterogeneity
Mechanical mixture

[Stixrude & Lithgow-Bertelloni, 2012]

eclogite ~ 3.5 g/cm³

Chemical Heterogeneity
Seismic
Models of chemical heterogeneity in the mantle. (a) Model represents the conventional view that much of the mantle is homogeneous and equal in composition to a fertile peridotite (pyrolite) that produces MORB on partial melting. This model requires that the heterogeneity in the subducted slab be rapidly destroyed by chemical diffusion. (b) Model with small amounts of subducted heterogeneity (plums) within a largely pyrolitic mantle. (c) Model of pervasive heterogeneity follows from the assumptions: (i) that the entire mantle has been processed through the mid-ocean ridge, (ii) that chemical diffusion is sufficiently slow that no heterogeneity has been destroyed, and (iii) that subduction has been operative for most of Earth’s history. Accumulation of basalt at the base results if it is denser than peridotite in the lower mantle and processed approximately once over the age of Earth so that some fraction of subducted crust likely remains from the beginning of plate tectonics (Allegre & Turcotte 1986, Davies 2009b, Korenaga 2008).

At the mantle’s base, the magnitude of the thermal and chemical boundary layers exceeds those at the surface: The density contrast between mantle and core is larger than that between crust and atmosphere, and the temperature contrast, though still poorly constrained, may exceed 1300 K (Jeanloz & Morris 1986, Knittle & Jeanloz 1991, Nimmo et al. 2004, Steinle-Neumann et al. 2002). The density contrast at the base of the mantle may serve, similar to the surface boundary, to accumulate heterogeneity. If subducted oceanic crust is denser than depleted residue at depth, and the basal temperature is high enough to allow the crust to delaminate, it may accumulate at the base of the mantle (Christensen & Hofmann 1994).

These fundamental considerations lead us to reconsider cartoons that have been used to represent chemical heterogeneity in the mantle (Figure 1). At one end of the conceptual spectrum (Figure 1a), we may place the pyrolite model and its descendants (Green & Ringwood 1967). This model views the MORB source or much larger portions of the mantle as being chemically homogeneous and similar in composition to a fertile peridotite, such as a garnet lherzolite. The pyrolite model has had considerable success in explaining the composition of mid-ocean ridge basalts, and the relationship between lava chemistry, crustal thickness, mantle temperature, and ridge height (Klein & Langmuir 1987, McKenzie & Bickle 1988, Workman & Hart 2005). The pyrolite model has served as the starting point for models of whole-mantle structure (Cammarano et al. 2005, Matas et al. 2007, Stixrude et al. 1992, Weidner 1985). And yet, the pyrolite model of the mantle cannot be physically correct. For the mantle to be homogeneous requires subducted heterogeneity to be destroyed on an unrealistically short timescale: one mantle overturn time (∼100–200 Ma). An intermediate model has been suggested by a number of studies (Figure 1b).
Evolution of length scales relevant to the survival of heterogeneity in the mantle. (Red) Flow-induced thinning of heterogeneity with initial thickness equal to that of present day oceanic crust (7 km) assuming exponential thinning (see text) and values of the strain rate ranging from $\epsilon = u/L = 6 \times 10^{-16}$ s$^{-1}$ to one-tenth this value to account for the possible effects of pure versus shear strain partitioning and more sluggish convection in the lower mantle, where $u$ is present-day average plate speed (5.5 cm year$^{-1}$) and $L$ is the depth of the mantle (2891 km). (Purple) Length scales over which chemical diffusion is operative with values of the effective diffusion coefficient appropriate to the lower mantle (0.4–7 $\times$ 10$^{-18}$ m$^2$ s$^{-1}$). The width of the envelope corresponds to a range in oxygen fugacity of six orders of magnitude (Holzapfel et al. 2005). (Blue) The maximum rate of accumulation of a cylindrical pile of basalt with radius $= \sqrt{vhc} t$, where $v = 1$–10 cm year$^{-1}$ is the sinking velocity, assuming that it separates from harzburgite efficiently at the core–mantle boundary and is not re-entrained in the overlying flow. The dashed gray line shows the radius of a Stokes blob that will traverse half the mantle depth in time $t_r = 9L\eta/4g\rho t$, where $\eta = 5 \times 10^{22}$ Pa s is the lower mantle viscosity (Lithgow–Bertelloni & Richards 1998), and $g$ is gravitational acceleration for a nominal density contrast $\rho = 100$ kg m$^{-3}$. The blue circle represents the thickness of the accumulated pile of basalt in the geodynamic model of Nakagawa et al. (2010). The gold band represents the range of thickness of pyroxenite bands measured in the Beni Bousera (BB) peridotite massif (Pearson & Nowell 2004) and the rhenium–osmium age of melt extraction from the Beni Bousera peridotite (Pearson & Nowell 2004).
Isotope array

Interpreting requires understanding entrainment and mixing and sampling
“Virtually everyone agrees that mixing is complicated”

J.M. Ottino, Annual Reviews of Fluid Mechanics, 1990

Also watch Michael Manga’s and Louise Kellog's excellent lectures on Fundamentals of Mixing at CIDER 2012, 2014
Entrainment, Stirring and Mixing

Figure 1 Models of chemical heterogeneity in the mantle. (a) Model represents the conventional view that much of the mantle is homogeneous and equal in composition to a fertile peridotite (pyrolite) that produces MORB on partial melting. This model requires that the heterogeneity in the subducted slab be rapidly destroyed by chemical diffusion. (b) Model with some small amount of subducted heterogeneity (plums) within a largely pyrolitic mantle. (c) Model of pervasive heterogeneity follows from the assumptions: (i) that the entire mantle has been processed through the mid-ocean ridge, (ii) that chemical diffusion is sufficiently slow that no heterogeneity has been destroyed, and (iii) that subduction has been operative for most of Earth's history. Accumulation of basalt at the base results if it is denser than peridotite in the lower mantle. processed approximately once over the age of Earth so that some fraction of subducted crust likely remains from the beginning of plate tectonics (Allegre & Turcotte 1986, Davies 2009b, Korenaga 2008).

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Entrainment is when a fluid picks up and drags another fluid or a solid.
Stirring and Mixing: What’s the difference?

**Stirring** is the mechanical motion of the fluid (cause) 
*stretching* and *folding* of material surfaces to reduce length scales

**Mixing** is the homogenization of a substance 
by *stirring* and *diffusion*

Two extremes: **Turbulent** and **laminar** mixing

For very viscous flows (mantle) use simple time-dependent flows to create **chaotic** mixing
Diffusion

The governing equation of the natural diffusion (dispersal) of a substance (heat, dye, chemical) is

\[
\frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta
\]

\(\theta(x,t)\) concentration of something we need to mix

\(\kappa\) is the diffusivity (thermal, chemical)

But natural or molecular diffusion is really slow... so homogenization hard
thermal diffusivity \(\sim 10^{-6} \text{ m/s}^2\)
chemical diffusivities \(\sim 10^{-18-20} \text{ m/s}^2\) (mantle)
\(\sim 10^{-11} \text{ m/s}^2\) (magma)

\(\ell = \sqrt{\kappa \tau}\)

\(~10-20 \text{ cm in a 1 Byr}\)
Evolution of length scales relevant to the survival of heterogeneity in the mantle. (Red) Flow-induced thinning of heterogeneity with initial thickness equal to that of present day oceanic crust (7 km) assuming exponential thinning (see text) and values of the strain rate ranging from $\varepsilon = u/L = 6 \times 10^{-16}$ s$^{-1}$ to one-tenth this value to account for the possible effects of pure versus shear strain partitioning and more sluggish convection in the lower mantle, where $u$ is present-day average plate speed (5.5 cm year$^{-1}$) and $L$ is the depth of the mantle (2891 km). (Purple) Length scales over which chemical diffusion is operative with values of the effective diffusion coefficient appropriate to the lower mantle (0.4–7 × 10$^{-18}$ m$^2$ s$^{-1}$). The width of the envelope corresponds to a range in oxygen fugacity of six orders of magnitude (Holzapfel et al. 2005). (Blue) The maximum rate of accumulation of a cylindrical pile of basalt with radius $= \sqrt{vhc/\eta}$, where $v = 1$–10 cm year$^{-1}$ is the sinking velocity, assuming that it separates from harzburgite efficiently at the core–mantle boundary and is not re-entrained in the overlying flow. The dashed gray line shows the radius of a Stokes blob that will traverse half the mantle depth in time $t = 9L\eta/4g\rho t$, where $\eta = 5 \times 10^{22}$ Pa s is the lower mantle viscosity (Lithgow-Bertelloni & Richards 1998), and $g$ is gravitational acceleration for a nominal density contrast $\rho = 100$ kg m$^{-3}$. The blue circle represents the thickness of the accumulated pile of basalt in the geodynamic model of Nakagawa et al. (2010). The gold band represents the range of thickness of pyroxenite bands measured in the Beni Bousera (BB) peridotite massif (Pearson & Nowell 2004) and the rhenium–osmium age of melt extraction from the Beni Bousera peridotite (Pearson & Nowell 2004).
Advection and Diffusion

Neglects effect of stirring which creates a flow $\mathbf{v}(\mathbf{x},t)$ giving the advection-diffusion equation

$$\frac{\partial \theta}{\partial t} + (\mathbf{v} \cdot \nabla) \theta = \kappa \nabla^2 \theta$$

**Advection** term massive effect by increasing spatial gradients of $\theta$

**Stirring** makes mixing much faster, because it reduces the length scales (stretching, folding)
How does mixing happen?

Role of stretching and folding

Ottino, Scientific American 1989

signature of chaos
Example: Rod moving in a figure 8 pattern

Stretching and Folding

Stirring and Mixing

Rod Stirring

Topology

Multiphase Flows

Conclusions

References

The Figure-Eight Stirring Protocol

• Circular container of viscous fluid (sugar syrup);
• A rod is moved slowly in a 'figure-eight' pattern;
• Gradients are created by stretching and folding, the signature of chaos.

Experiments by E. Gouillart and O. Dauchot (CEA Saclay).
1. Global scale: mantle contains well-mixed regions and heterogeneity

From Michael Manga's 2012 lecture
How to characterize mixing?

Stretching: deformation of material filament from $dL(0)$ to $dL(t)$ given by deformation tensor (related to the velocity)

\[ \Delta L/L(t) = m \cdot (\varepsilon \cdot m) \]

Length Stretching

\[ \lambda = \lim_{L(0) \to 0} \frac{L(t)}{L(0)} \]

Rate of Stretching

\[ \frac{D\lambda}{Dt} = \frac{L(t)}{L(0)} \]

\[ \frac{1}{\lambda} \frac{D\lambda}{Dt} = \frac{D\ln \lambda}{Dt} = m \cdot (\varepsilon \cdot m) \]

Strain-rate tensor: \( \dot{\varepsilon} = \frac{1}{2}(\nabla \mathbf{v} + (\nabla \mathbf{v})^T) \)

Stretching Tensor

Note: I’ll be using \( x \) instead of \( L \)
How to characterize mixing?

**Stretching Efficiency:**

\[ e_\lambda = \frac{D \ln \lambda / Dt}{(\dot{e} : \dot{e})^{1/2}} \leq 1 \]

For simple shear \( e_\lambda \rightarrow 0 \) at large times

For pure shear \( e_\lambda \rightarrow 2/3 \)

What does it mean?

**THAT IT DEPENDS ON THE FLOW TYPE!**

**Lyapunov Exponent:**

\[ \sigma_{max} = \lim_{t \to \infty} \frac{1}{t} \ln \left| \frac{\delta x}{\delta x(0)} \right| \]

Not the same as \( e_\lambda \) because the strain rate tensor varies in space and time. There is more than one \( \sigma \) at a given point we only worry about the largest one.

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**Stretching Efficiency**

**Depends on Flow**

---

**Average rate of stretching experienced by a region of fluid over time span to to t**

Lyapunov exponent is a quantity that characterizes the rate of separation of infinitesimally close trajectories. Quantitatively, two trajectories in phase space with initial separation diverge where \( \lambda \) is the Lyapunov exponent.
Dependence on Flow Type

In a more complex flows, regions with pure shear (hyperbolic streamlines) will cause most of the stretching.
Elliptic and Hyperbolic points

Steady 2D flows don’t mix well…
but add time-dependence

Ottino, *Scientific American* 1989
Time-dependence

Well mixed and not well mixed regions coexist
Stirring can produce complex structures and *mixed* and *unmixed* islands.

Chaotic Flow and Mixing by *stretching* and *folding* tracers *sensitive to initial conditions* presence of special points produces *horseshoe maps*.

Dependence on type of heterogeneity *active* (chemical, rheological).

Effects of stirring
Global vs Local Mixing

Effect of viscous increase
Characterization of Structure

“Important to distinguish between mixing measure and the process producing mixing… The measure should be selected according to the application, and the measurements should be related to the fluid mechanics.”
Ottino, Kinematics of Mixing, 1989

e and σ characterize effectiveness of a given flow at stirring

Other measures can be used to characterize observed structures
(fractal analysis, spectral)
Generally speaking, the LCS approach provides a means of identifying key material lines that organize fluid-flow transport. Such material lines account for the linear shape of the ash cloud in figure 1a, the structure of the oil spill in 1b, and the tendrils in the spread of radioactive contamination in 1c. More specifically, the LCS approach is based on the identification of material lines that play the dominant role in attracting and repelling neighboring fluid elements over a selected period of time.
Dynamical Systems
Attracting and Repelling points

- Basic premise
- Non-linear equations hard
- Ignore struggle for particular analytical solution
- Focus on behavior of all solutions
- Stable and unstable manifolds organize state space

Dynamical System
\[
\begin{align*}
\dot{x} &= x + y \\
\dot{y} &= 4x - 2y \\
\dot{x} &= Jx
\end{align*}
\]

Fixed Point
\[
\begin{align*}
x &= (0, 0) \Rightarrow \dot{x} &= (0, 0) \forall t
\end{align*}
\]

General Solution
\[
x(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t}
\]

Phase Portrait

A geometric-computational tool has been developed to locate Lagrangian coherent structures, or LCS. This is the concept that generalizes the notion of invariant manifold in dynamical systems to the time dependent case. As with separatrices, LCS are dividing lines between particles with different dynamical fates. An LCS (Lagrangian Coherent Structure) is a ridge in the FTLE field. LCS are time dependent curves in 2d flows and surfaces in 3d flows.
- Aperiodic systems have explicit time dependence
- Resulting dynamical system can be made autonomous by adding state variable for time
  - Fixed points vanish
  - Left with distinguished hyperbolic trajectories
    - Distinguished:
      - Behaves like moving fixed point (i.e., a parcel on this trajectory can’t get off)
      - Has invariant manifolds (aka, LCS)
      - Other trajectories exponentially approach or recede
    - Hyperbolic: non-zero Lyapunov exponents

\[ |\delta x| = |\delta x(0)| e^{\sigma t} \quad \sigma_{\text{max}} = \lim_{t \to \infty} \frac{1}{t} \ln \frac{|\delta x|}{|\delta x(0)|} \]

- Aperiodic systems theory is still under much development
Finite Time Lyapunov Exponent

The average rate of stretching experienced by a region of fluid over the time-span $t_0$ to $t$ is typically expressed using the Finite-Time Lyapunov Exponent (FTLE), $\sigma_f$ (Shadden et al. 2005). This is found by first finding the flow map:

$$\phi(x, t, t_0) = x(x_0, t, t_0),$$

(3.4)

which is the position of all tracers at time $t$ which had the initial position, $x_0$ at time $t_0$ (with $t_0 < t$). This is in turn used to find the Cauchy-Green deformation tensor

$$C = (\nabla \phi)^T \nabla \phi.$$

(3.5)

The largest real eigenvector of $C$, $\lambda_{\text{max}}$, represents the maximum strain. Using this information, the FTLE can be calculated as

$$\sigma_f(x, t, t_0) = \frac{1}{2(t-t_0)} \log \lambda_{\text{max}}.$$  

(3.6)

$$\delta^* = \frac{\delta(t)}{\delta(t_0)} = e^{\sigma_f(t-t_0)},$$

$$\cdots \cdots \cdots \cdots \cdots \cdots$$

In chaotic systems (as occurs in the mantle), a local region of heterogeneity will be stretched as where delta is the length of the filament of fluid.

Thus represents the average rate of stretching experienced by a region of fluid which originated at position $x$ over the time-span $t_0$ to $t$.

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FTLE | How to compute
Advecting Tracers
Thermal plumes can develop, and 2) a Rayleigh number for the fluid layer involved in convection exceeding $10^6$. Just below the core-mantle boundary, the D00 layer, with a temperature contrast ranging between 1000 and 1500°C, is a good candidate for a hot TBL.TBL showed to exist at the 660-km phase transition or in the mid-mantle if the mantle were strongly or even locally stratified (for a review, see Tackley [2000]).

Figure 1 shows the convective regime of a mantle layer as a function of its thickness and viscosity. Plumes can develop in the upper mantle (of thickness 660 km) only if its viscosity is lower than $10^{21}$ Pa.s. For an average mantle viscosity of $10^{22}$ Pa.s, plumes will be generated only if the mantle layer thickness exceeds 2000 km. Convective motions originating at the CMB and developing over the whole mantle thickness should therefore take the form of plumes.

2.2. Evolution of a Hot Thermal Instability

Experiments were performed with isoviscous fluids (silicone oils or aqueous polymeric solutions) and sugar syrup with temperature-dependent viscosity, for $5/\text{C} < Ra < 5 \times 10^7$ and Prandtl number $Pr > 10^3$. We developed a new experimental technique using liquid crystals and glass particles (<40 μm in diameter), to visualize simultaneously the thermal and velocity fields in a planar vertical cross-section of our tank (A. Davaille et al., Imaging isotherms in viscous fluids, submitted to Experiments in Fluids, 2005).

Figure 2 shows the development of convective instabilities in a layer of sugar syrup initially at uniform temperature $T_m$, suddenly heated from below to a constant temperature $T_m + \Delta T_m$ for $Ra = 1.7 \times 10^6$. At first, the isotherms (bright lines) remain horizontal and the temperature front moves away from the boundary by conduction (Figure 2a): there is no motion in the fluid. Then the conduction layer (or TBL) contained between the moving front and the outer boundary suddenly becomes unstable (Figure 2b) and breaks up (Figure 2c). It produces a mushroom-shaped plume (Figures 2c, 2e, and 2g) with a relatively small head (radius less than twice the stem radius; Figure 1.

Convective regime developing in a mantle layer as a function of its thickness and viscosity. The pattern is cellular for $Ra/\nu^2 1 = 650$, and if plumes for $Ra/\nu^2 1 \geq 10^6$.

The grey band shows the average viscosity inferred for the whole mantle [e.g., Ricard et al., 1989]. There the rectangle at the upper left delimits the upper mantle. The calculation has been done with equation (1) and $k = 10^{-6}$ m$^2$/s, $\alpha = 2 \times 10^{-5}$/C$176$ K and $\Delta T = 3000K$.
What is a plume?

- To formalize dynamical interaction of plume with surroundings, we need
  - Definition of plume
  - Definition of surrounding
- Our approach
  - Define plume head using dynamical systems concepts
  - Lagrangian Coherent Structures (LCS)
    - Stable and Unstable

Degree to which LCS approximate invariant manifolds is still under development. Is known that regions of high shear will produce high FTLE fields.
What is a plume?

- To formalize dynamical interaction of plume with surroundings, we need:
  - Definition of plume
  - Definition of surrounding
- Our approach:
  - Define plume head using dynamical systems concepts
  - Lagrangian Coherent Structures (LCS)
    - Stable and Unstable

**WHY BOTHER?**

**OBJECTIVE** definition of plume head and conduit-UNSTABLE

Define ALL material that will ever be entrained-STABLE

Degree to which LCS approximate invariant manifolds is still under development. Is known that regions of high shear will produce high FTLE fields.
Lagrangian Coherent Structures and Manifolds

Dynamical systems theory provides a robust definition of plume head and source material. The unstable invariant manifold of the distinguished hyperbolic trajectory at leading edge of the upwelling. The stable manifold defines all material that will ever be entrained (source)

- Separates flow field into regions with distinct dynamics
- Attract and stretch along manifold (unstable) or repel (stable) nearby fluid elements.
Lagrangian Coherent Structures and Manifolds

- Separates flow field into regions with distinct dynamics
- Attract and stretch along manifold (unstable) or repel (stable) nearby fluid elements.
**Injection plume vs localized heat source**

- Comparison of numerical models
- Both use 80 °C syrup
- Same initial conditions
- Same boundary conditions except
  - G6 upper surface permits outflow on perimeter
  - G6 upper surface temperature floats
- G6
  - Injection rate: 0.3 cm³/s
  - Nozzle: Ø 20 mm

\[
Ra = \frac{\alpha g Q d^3}{k^2 \rho c_p}
\]
\[
Ra(R6) = 5 \times 10^5
\]
\[
Ra(G6) = 2 \times 10^7
\]

G&C injection rate, Peter’s nozzle size (G&C used Ø 0.6 cm)

Very distinct differences: v_l 10x, head vorticity 20x greater, head is hotter (my head 33 degC)

Centerline velocity is not monotonic above 30 mm for G6
The LCS

- Two cases are very different
- G6 has
  - Bwd-time LCS surrounding injected mass
  - Fwd-time LCS shielding head
Injection vs Thermal

Figure 4.9: FTLE fields for the numerical models of starting plumes generated with a localized heat source (a, b) and via injection of hot syrup (c, d). (a, b) Case R6 backward and forward-time FTLE fields, respectively, at $t = 1050$ s following heater activation. (c, d) Case G6 backward and forward-time FTLE fields, respectively, at $t = 130$ s following start of injection. Integration times used to compute the FTLE fields (ref. Eq. 4.12) are (a) $t = 1050$ s, (b) $t = 910$ s, (c) $t = 130$ s, and (d) $t = 130$ s. The 25.3°C contour is shown in all figures as a solid black line.

Figure 4.14: Vertical and radial origin of mass contained within (a) the plume generated via a localized heat source in case R6, and (b) the injection plume of case G6. The image of (a) is provided at $t = 3250$ s following heater activation, while the image of (b) corresponds to $t = 300$ s after the start of injection. Mass contours are shown at 15 mm intervals. Radial contours are represented as solid blue lines while the vertical contours are red. The solid black line and shaded region enclose all material that has a temperature of at least 25.7°C.

• Mantle plumes operating similarly to the injection plume have acquisition flexibility
• Mantle plumes operating similarly to experimental cases are images of the source

Mantle plumes akin to our cases N1–N6
Head composition dominated by mass from lowermost 600 km
Head inefficient at incorporating overlying material
Overlying heterogeneities confined to outer perimeter of head
If mechanical mixture of eclogite and peridotite required for partial melting, must be characteristic of source
Mantle plume akin to injection plume
Head very efficient at incorporating overlying material
Can acquire signature needed during ascent
Material well-stirred
40% of total head mass derived from depth $\leq 1300$ km.
How does the flow transform the unit sphere

- Motivation: Can we use the spatial arrangement and shape of surface observations to constrain
  - transport mechanism
  - mass origin
- Small scale variation in Pb isotope data
  - 350 - 550 kyr Mauna Kea lavas are isotopically similar to present day Kilauea
  - Pancakes or filaments

3 Ma Hi started producing lavas with two distinct isotopic trends (Kea and Loa)
HSDP data indicates that small scale variation also exists
Launching spheres is tedious and we may miss a lot
Can we construct a predictive map of how the flow transforms all fluid elements
Transforming the unit sphere

- Dimensionality of LCS provides some insight, but
  - Not all fluid elements interact with LCS
  - Not all interacting fluid elements are transformed the same
- Shape metric $\Psi$ classifies dimensionality of deformed sphere
  - 1D line
  - 2D sheet
  - 3D sphere
- Take $s_i$ as principal stretches

$$\Psi = \frac{r^2}{s_1^2} \quad r = \sqrt{s_1^2 + s_2^2} \quad s_1 \geq s_2 \geq s_3$$

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\Psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere (3D)</td>
<td>2</td>
</tr>
<tr>
<td>Sheet (2D)</td>
<td>1</td>
</tr>
<tr>
<td>Pencil (1D)</td>
<td>0</td>
</tr>
</tbody>
</table>
Shape metric for plumes N6 and R6

- Computed using integration time $\tau = 2000$ s
- Predicts how 2000 s of flow will transform infinitesimal spherical mass

Plume state at $t = 2000$ s

Transformation of 4 macroscopic inclusions

Notice the pockets, there seems to be structure to the field.
Evolution of material pockets extracted from metric

A. Separates mass brought in from above vs below
B. Conduit core
C. Outer conduit
D. Plastered on bottom of LE LCS
E. Head entrain phase 1: Scroll core
F. Head entrain phase 2
G. Head entrain phase 3

Metric
- Elucidates hidden structure
- Permits segregation of head mass entrained from above vs. below

Dotted line encloses max principal stretch perpendicular to view
Lower section segregates mass brought in above vs below
Evolution of material pockets extracted from metric

Outer perimeter of rotation

Green rotates
Reflection: plume spatial/temporal variation

- Production of 2D structures limited to mass above the rising plume
- Conduit and head are 1D dominant
- Producing horizontal pancakes in the conduit is difficult

Abouchami et al. 2005

Blichert-Toft et al. 2003

Farnetani & Hofmann 2010

A. Separates mass brought in from above vs below
B. Conduit core
C. Outer conduit
D. Plastered on bottom of LE LCS
E. Head entrain phase 1: Scroll core
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G. Head entrain phase 3
Extricate Meaning

How might we use FTLE and LCS
Origin of Heterogeneity

Genesis:
- Primordial
- Via Convection/Plate Tectonics

[Labrosse et al., 2015]
We assume that material absorbs a 'A' signal according to one of the three following models:

1. \( \frac{dC_A}{dt} = \frac{r_A}{d_A(t)} \)
2. \( \frac{dC_A}{dt} = \frac{r_A}{d_A^2(t)} \)
3. \( \frac{dC_A}{dt} = \frac{r_A}{d_A^3(t)} \)

where \( \frac{dC_A}{dt} \) is the rate of change of the 'A' component of the geochemical signal, \( r_A \) is the rate of absorption of the 'A' signal, and \( d_A(t) \) is the distance of a fluid element from Source A. The equivalent models hold for Source B.

The participants are asked to identify which model is correct, and estimate the absorption rates (\( r_A \) and \( r_B \)), which are not necessarily equal.

To do this, they have use different functions provided, including getinfo.m, which returns the previous trajectory of a tracer starting at a given location \((x_o,y_o)\) that the user inputs, and writedataexcel.m, which converts the output of getinfo.m into an Excel spreadsheet with headers.
Active heterogeneity density differences influences velocity field (and, of course, amount entrained)

Kumagai et al., GRL 2008

 effects of composition affects entrainment and mixing
FTLE field
Demonstration of shutting off convection

Layered Convection

[Image: two circular diagrams labeled A and B]

[Tackley et al., 1993]
Phase Transitions

\[ \rho C_p \left( \frac{\partial T}{\partial t} + u \cdot \nabla T \right) - \nabla \cdot k \nabla T = \rho H \]

\[ \quad + 2\alpha \left( \varepsilon(u) - \frac{1}{3} (\nabla \cdot u)^2 \right) \cdot \left( \varepsilon(u) - \frac{1}{3} (\nabla \cdot u)^2 \right) \]

\[ \quad + \alpha T (u \cdot \nabla \rho) \]

\[ \quad + \beta \Delta S \left( \frac{\partial X}{\partial t} + u \cdot \nabla X \right) \]

- Material Properties
- Thermodynamics (Static Properties)
Many Phases

- Equilibrium thermodynamics of multi-component systems
- Differentiation
- Affects physical properties
Thermodynamic Model

- Bulk composition
- Pressure
- Temperature
- Phase Equilibria
- Physical Properties
- Self consistent

Bulk Composition

\[ \text{Mg}, \text{Na}, \text{Fe}, \text{Si}, \text{Ca}, \text{Al}, \ldots \]

Phase Equilibria

\[ P, T \]

Physical Properties

\[ \rho, \alpha, C_P, V_p, V_s, \ldots \]

\((X, P, T)\)

[Stixrude and Lithgow-Bertelloni, 2005; 2011]
Based on Fundamental Thermodynamic Relations

Minimize Gibbs free energy over the amounts of all species \( n_i \)

\[
G(P,T,n_i) = \sum_{i=1}^{\text{species}} n_i [\mu_i(P,T) + RT \ln a_i]
\]

Subject to constraint of fixed bulk composition

\[
s_{ij} n_j = b_i
\]

Full Anisotropic Generalization

\[
c_{ijkl} = \frac{1}{V} \left( \frac{\partial^2 F}{\partial E_{ij} \partial E_{kl}} + P \left( \delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{jl} \delta_{ik} \right) \right)
\]

Many previous efforts, however

- Full self-consistency between phase equilibria and physical properties (not only one or the other)
- Anisotropic generalization and robust thermal extrapolation for shear properties
Self consistent calculation of physical properties and phase equilibria and here it's what happens. Here is a cold adiabat in blue and a hot adiabat in red and mostly density decreases on heating, but here because of the phase transitions they are crossing in the wrong direction, and this negative thermal expansion shuts off convection.
Demonstration of shutting off convection
What about other geotherms in the past, look at it in a different way, look at all the negative thermal expansion.
EVOLUTION

Initial $T_{P0}=2000\ K$

Geotherm at $t=0$ passes through region of negative $\alpha$
EVOLUTION

Initial $T_{P0}=2000\ K$

Shallow mantle rapidly cools
Rising $T$ causes $\alpha$ to become negative-Lower mantle is insulated
Once geotherm can bypass this region—heat flow is rapid.
Roughly steady after this point (including present-day)
Layered state about \(~80\%\) of the time