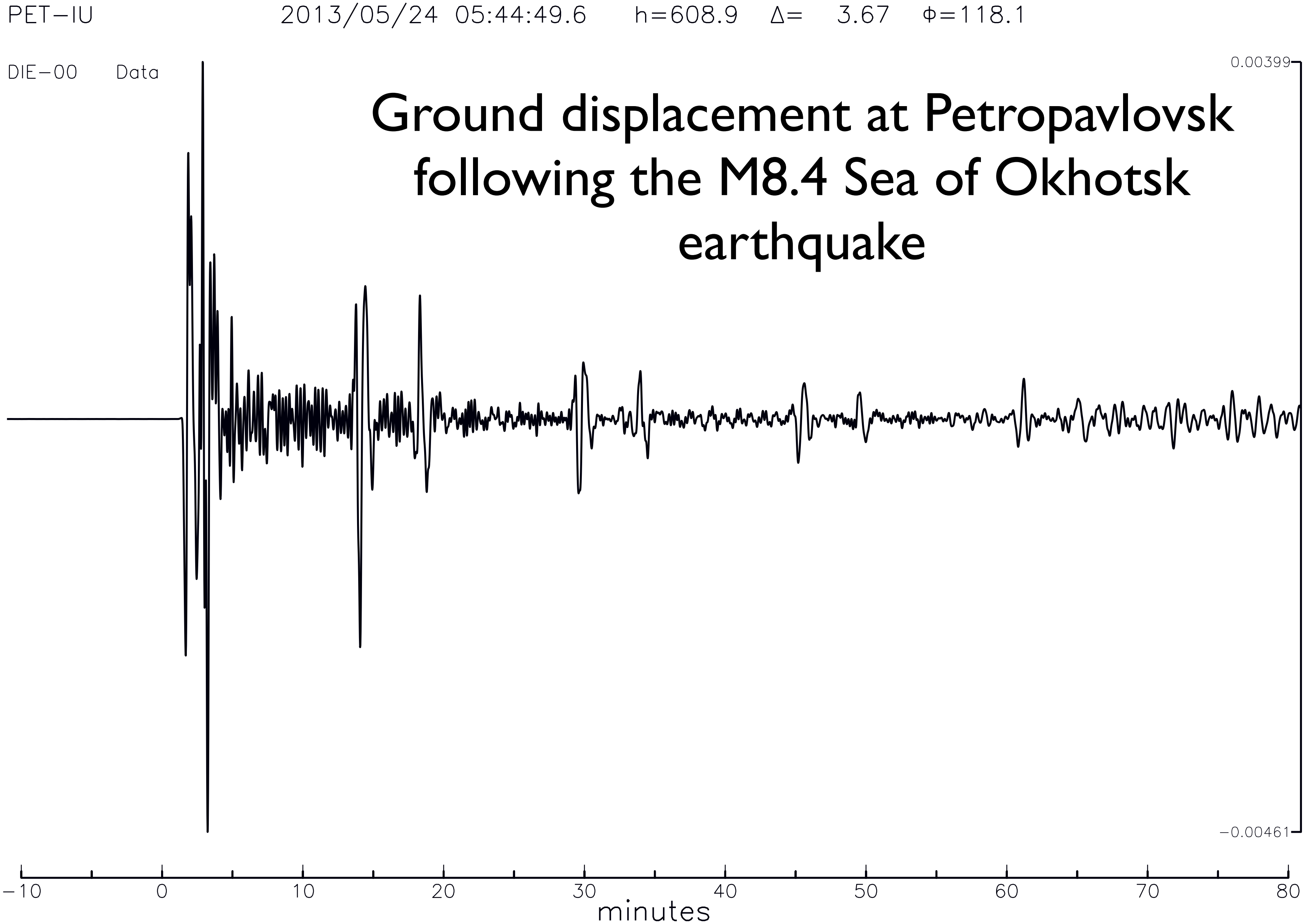
General Seismology
or
Basic Concepts of Seismology

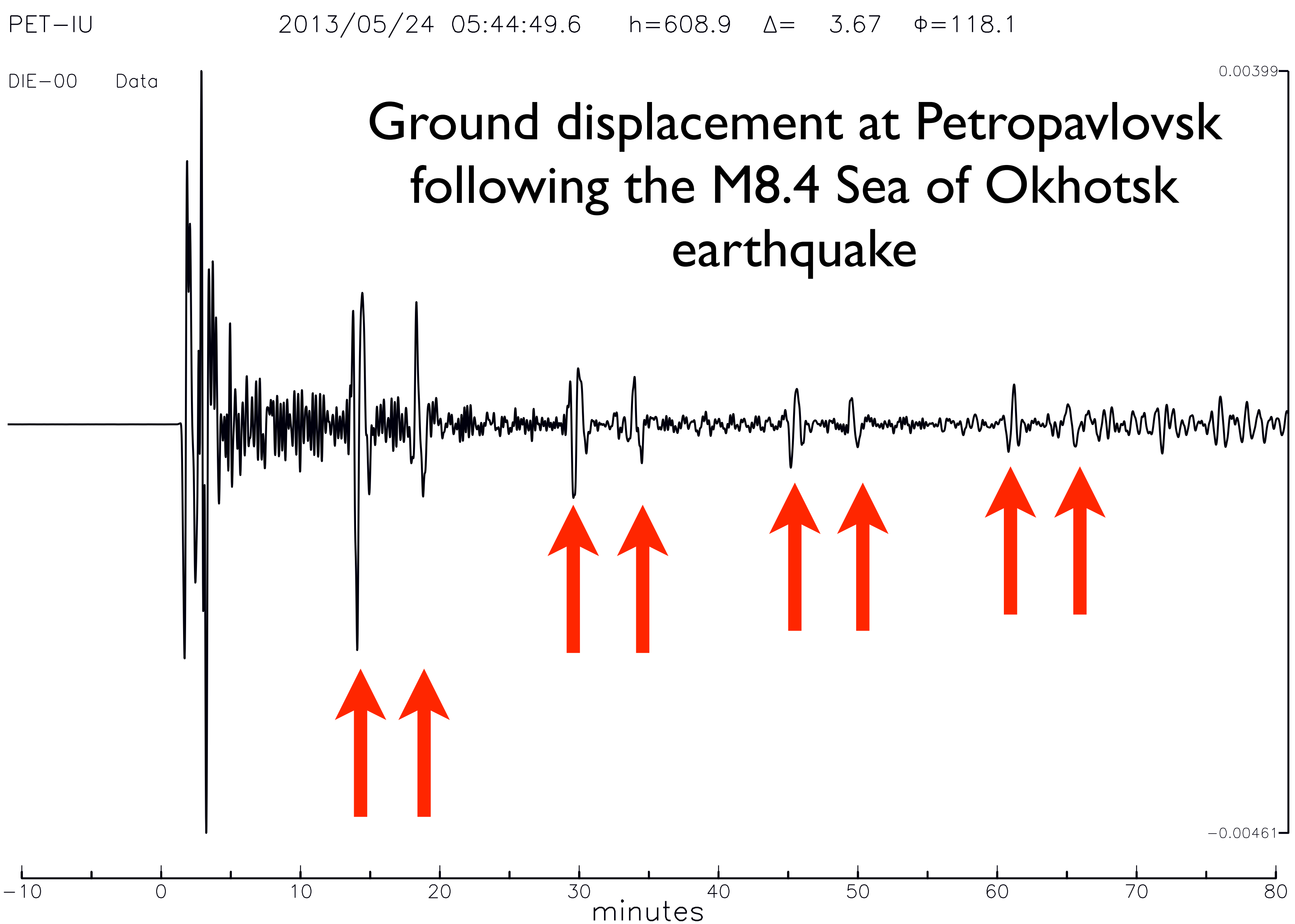
Göran Ekström

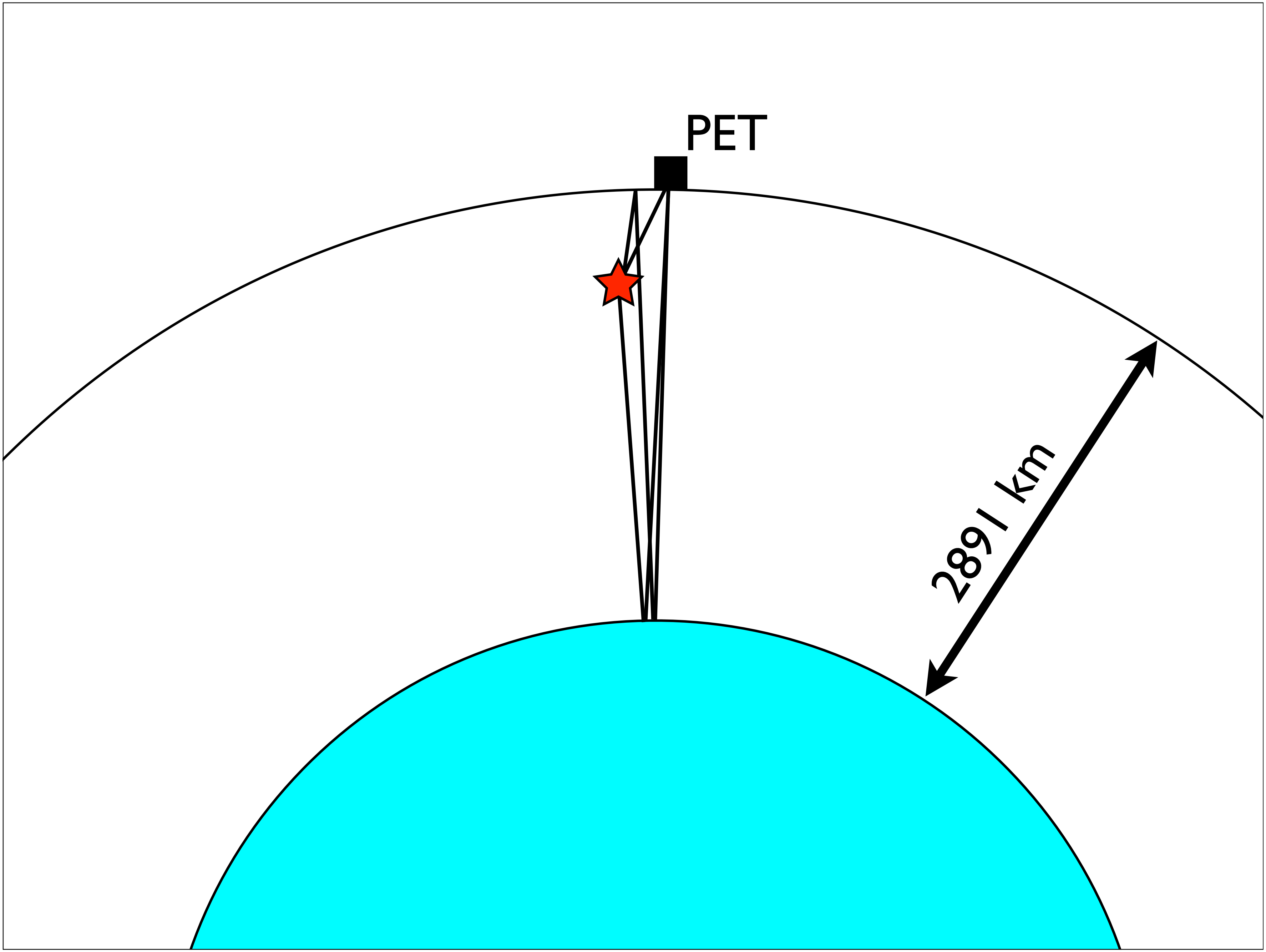
Basics Concepts of Seismology

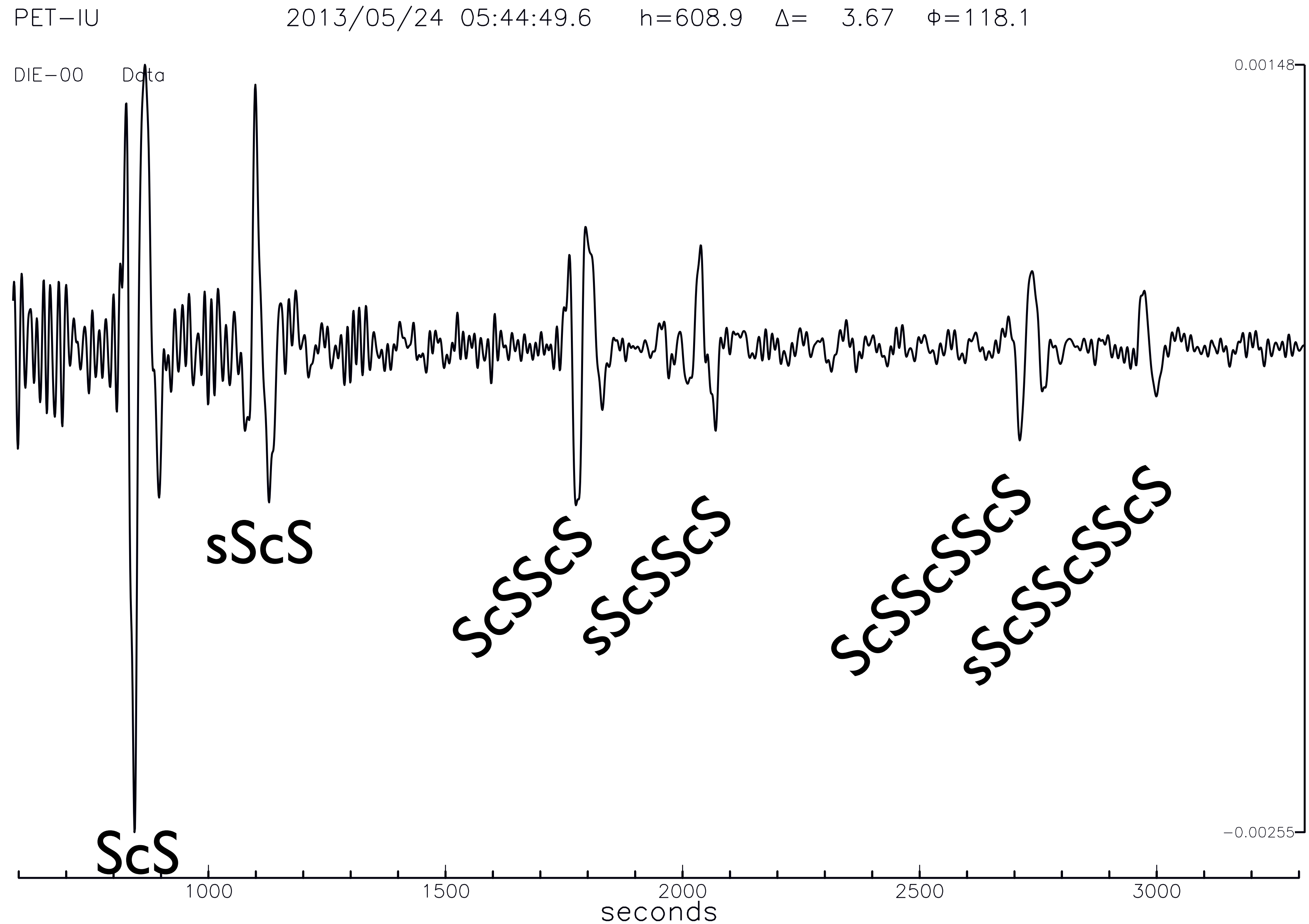
Travel Time

Ground displacement at Petropavlovsk following the M8.4 Sea of Okhotsk earthquake

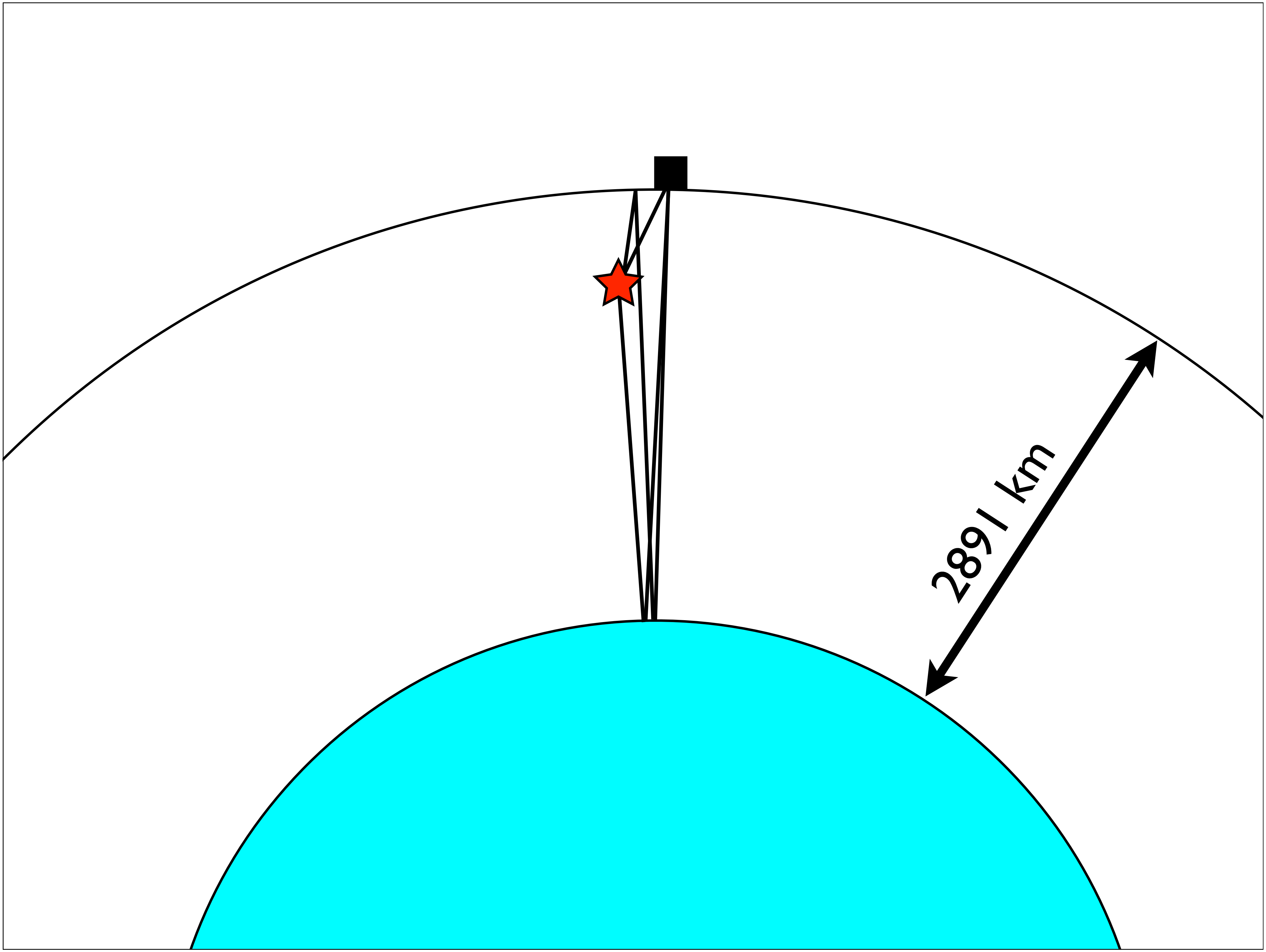


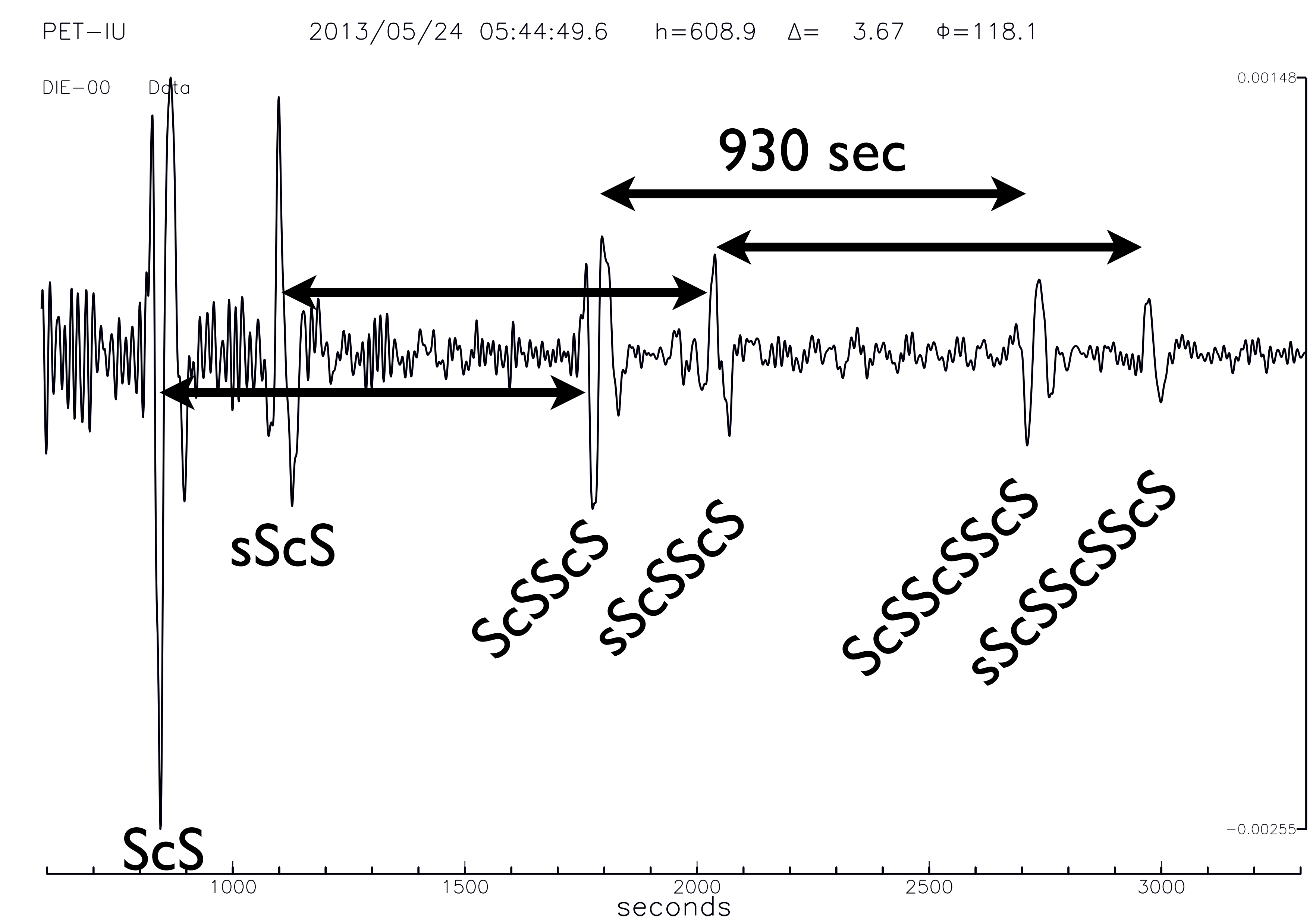

PET–IU
2013/05/24 05:44:49.6 h=608.9 Δ= 3.67 Φ=118.1
DIE–00 Data
Ground displacement at Petropavlovsk following the M8.4 Sea of Okhotsk earthquake
0.00399
–0.00461
–10
0
10
20
30
40
50
60
70
80
minutes


PET
2891 km



PET−IU
2013/05/24 05:44:49.6 h=608.9 Δ= 3.67 Φ=118.1
DIE−00 Data
0.00148
−0.00255
ScS
sScS
ScSScS
sScSScS
ScSScSScS
sScSScSScS
1000
1500
2000
2500
3000
seconds


2891 km


PET−IU
2013/05/24 05:44:49.6 h=608.9 Δ= 3.67 Φ=118.1
DIE−00 Data
0.00148
−0.00255
930 sec
ScS
sScS
ScSScS
sScSScS
ScSScSScS
sScSScSScS
1000
1500
2000
2500
3000
seconds

Travel time and distance gives us speed: $v=X/T$

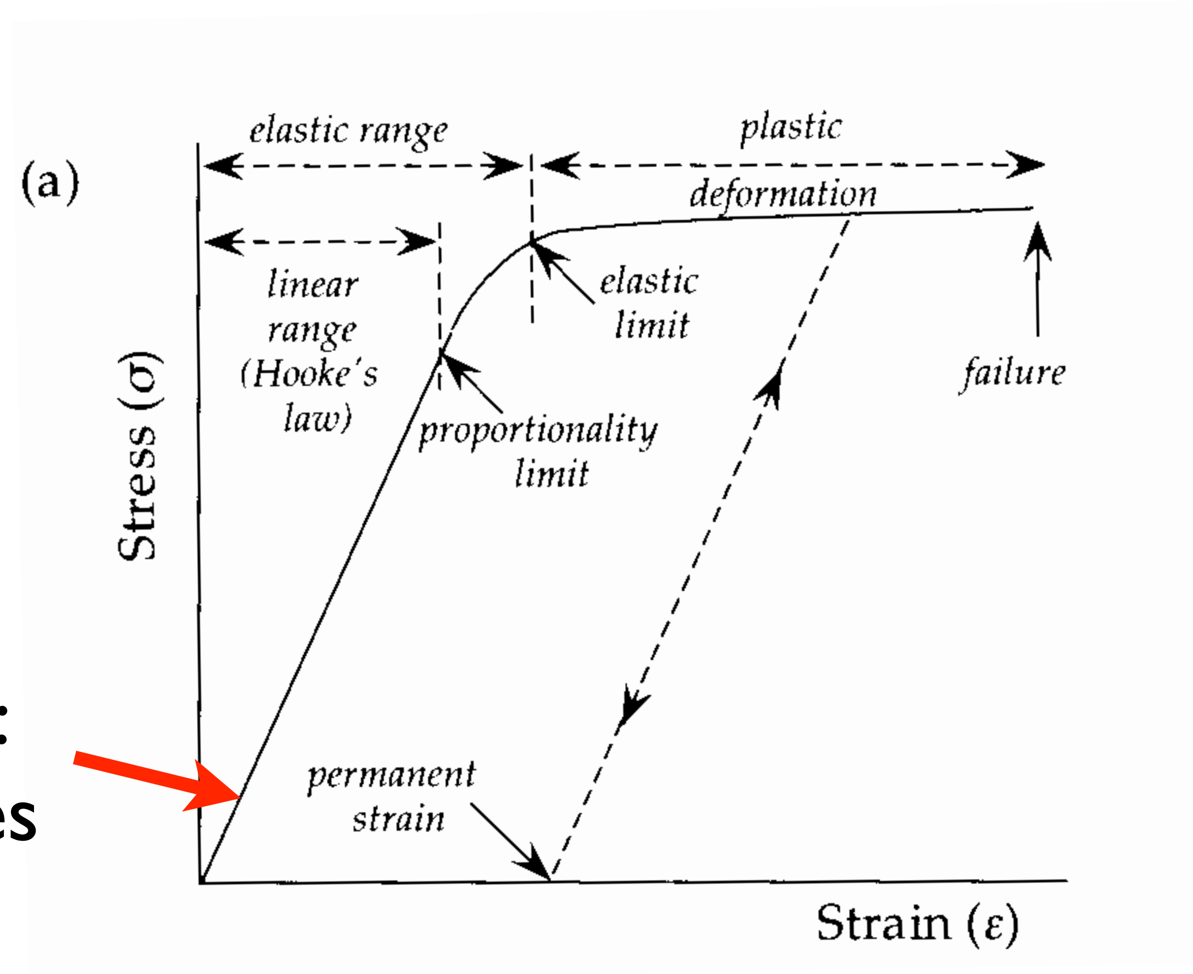
Average speed of the seismic wave through the mantle and crust:

$2 \times 2891 / 930 = 6.21$ km/sec

Basics Concepts of Seismology

Elasticity and P and S waves

Deformation of a solid



Seismology deals with small strains: Hooke's law applies

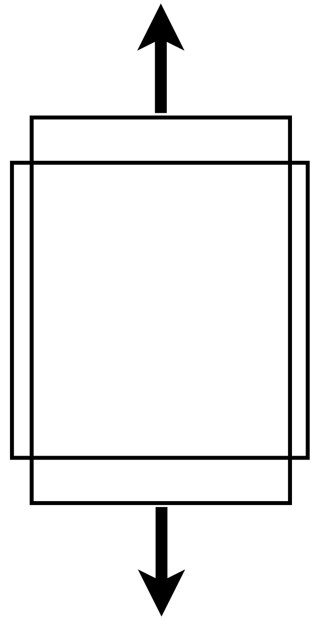
modified from Lowrie, 2007

Elastic moduli

Young's modulus:

$$E = \frac{\sigma_1}{\varepsilon_1}$$

with $\sigma_2 = \sigma_3 = 0$.



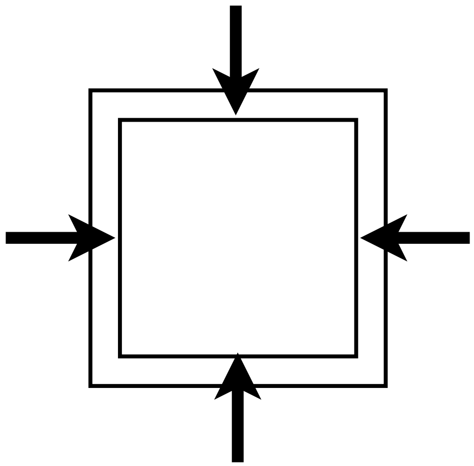
Poisson's Ratio:

$$\nu = -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{\varepsilon_3}{\varepsilon_1}$$

For much of the mantle, $\nu \approx 0.25$ (a 'Poisson solid').

Incompressibility (Bulk modulus):

$$K = -\frac{V \Delta P}{\Delta V}$$



If $\sigma_1 = \sigma_2 = \sigma_3 = P$,

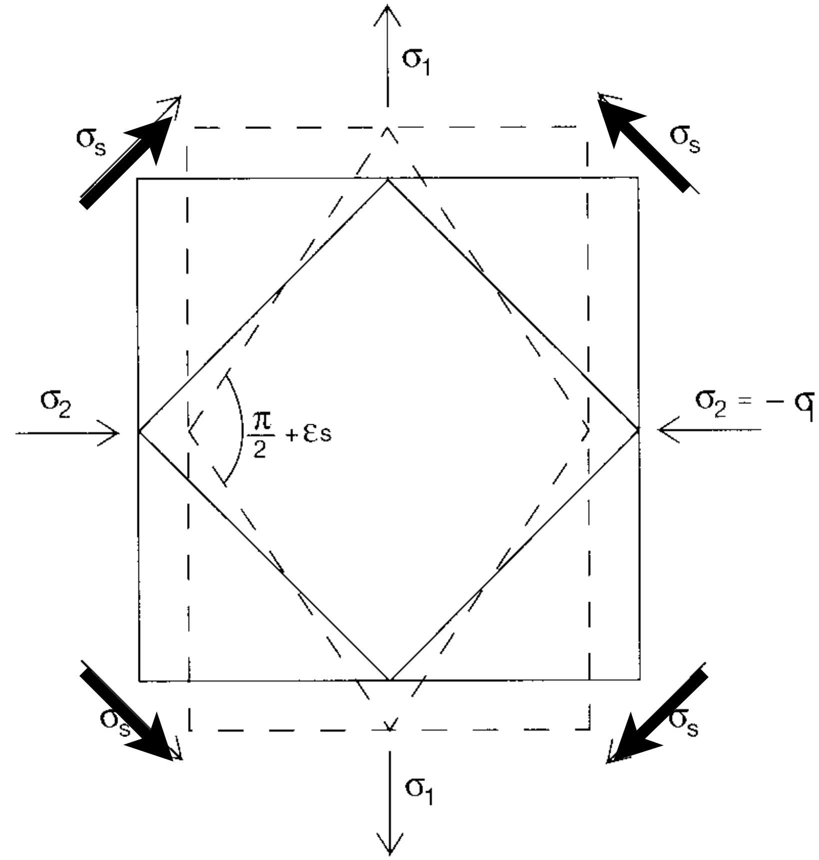
$$K = \frac{\sigma_1}{3\varepsilon_1}$$

with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \frac{1}{3}\Delta V / V$.

Rigidity (Shear modulus):

$$\mu = \frac{\sigma_S}{\varepsilon_S} = \frac{\sigma_1}{2\varepsilon_1}$$

$\sigma_S = \sigma_1 = -\sigma_2, \sigma_3 = 0; \varepsilon_S = 2\varepsilon_1 = -2\varepsilon_2, \varepsilon_3 = 0$

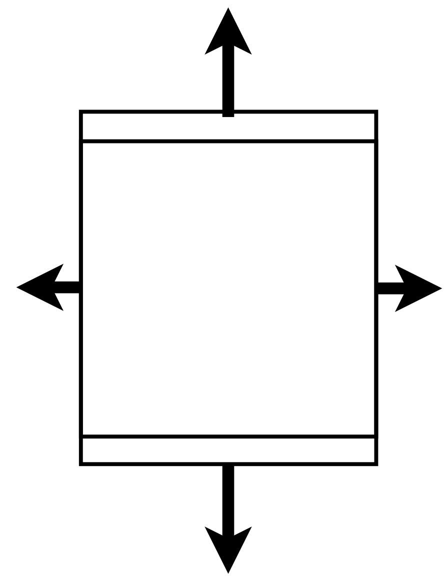


Modulus of simple longitudinal strain (Axial modulus):

$$\chi = \frac{\sigma_1}{\varepsilon_1}$$

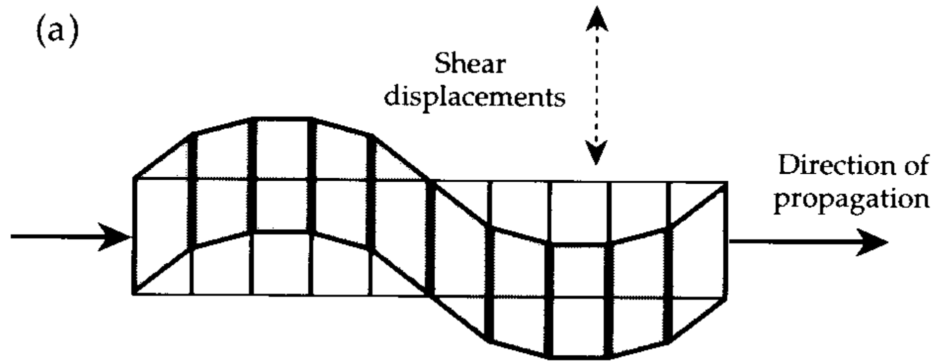
where $\varepsilon_2 = \varepsilon_3 = 0$.

$$\chi = K + \frac{4}{3}\mu$$



Equation of motion:

$$F = ma$$



Application to elastic deformation in shear:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2}$$ - the wave equation

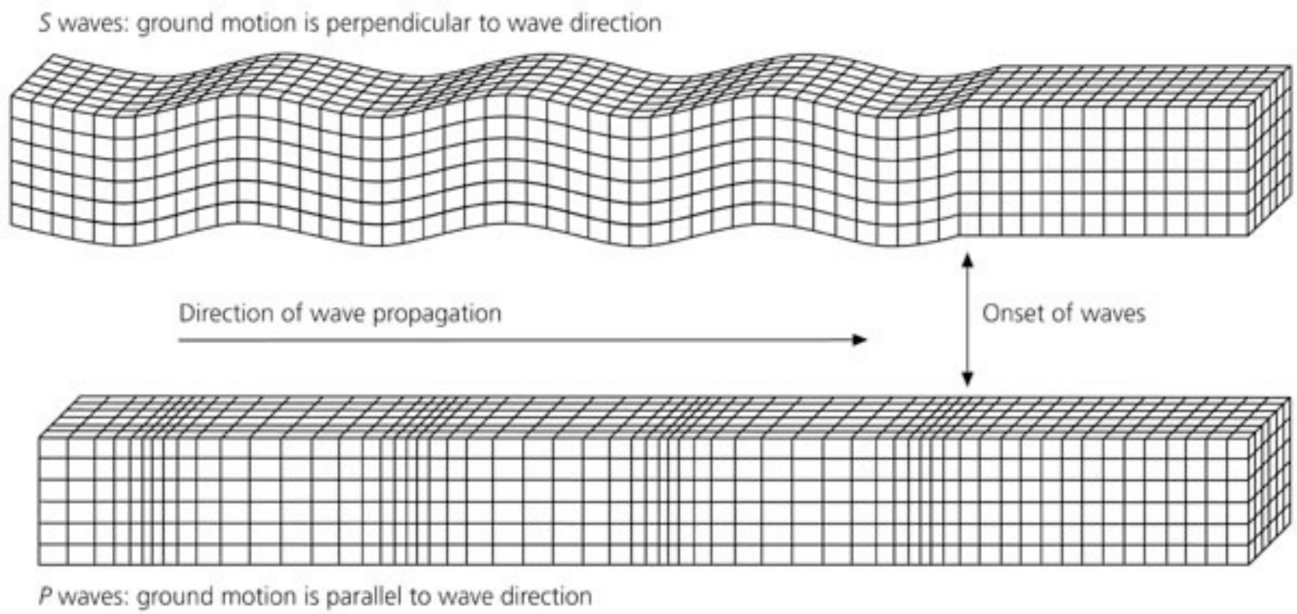
$$v = \sqrt{\frac{\mu}{\rho}}$$ - propagating wave speed

In an isotropic solid only two elastic moduli are independent, and there are two types of waves, P and S

Figure 2.4-3: Displacements for *P* and *S* waves.

$$\beta = v_S = \sqrt{\frac{\mu}{\rho}}$$

$$\alpha = v_P = \sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}}$$



modified from Stein and Wysession, 2009

Basics Concepts of Seismology

Seismic ‘rays’;
refraction, reflection, conversion

Huygen’s principle, wavefronts, rays

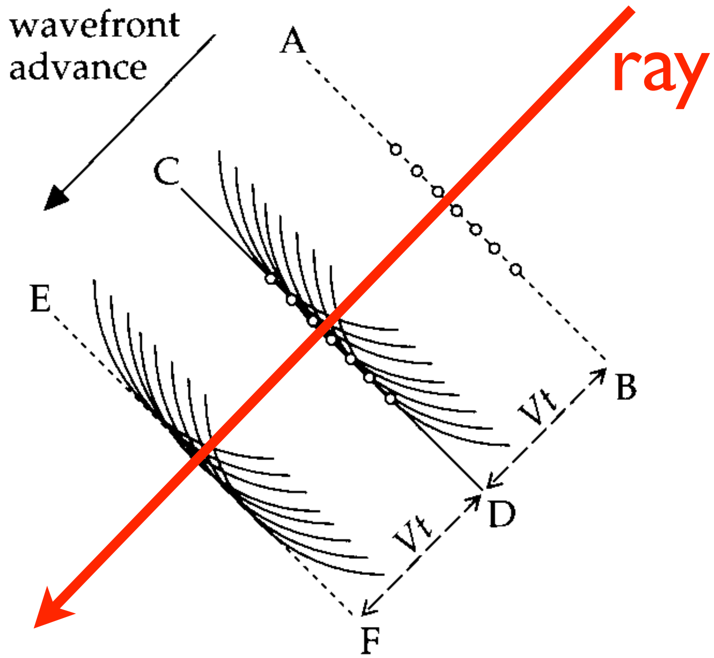
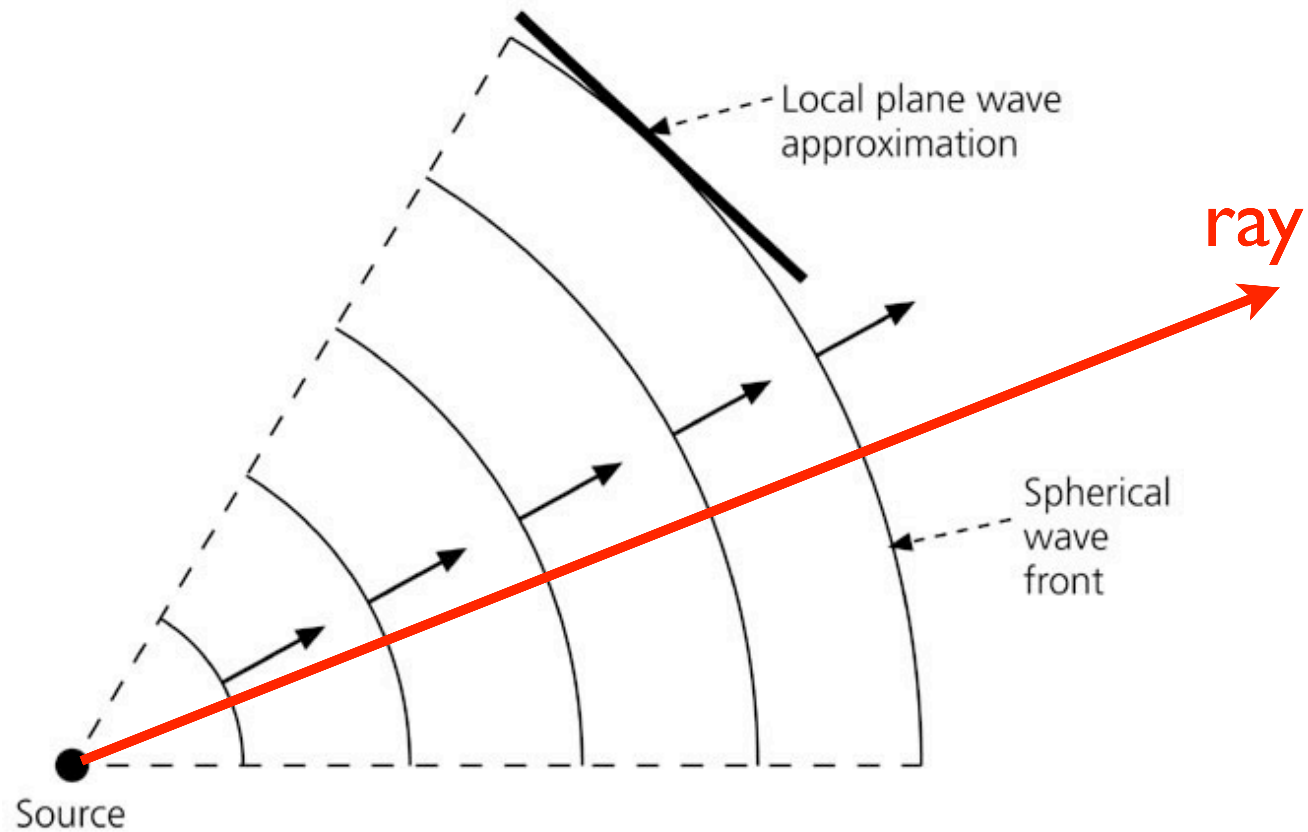


Fig. 3.48 Application of Huygens’ principle to explain the advance of a plane wavefront. The wavefront at CD is the envelope of wavelets set up by particle vibrations when the wavefront was at the previous position AB. Similarly, the envelope of wavelets set up by vibrating particles in the wavefront CD forms the wavefront EF.

Figure 2.4-2: Approximation of a spherical wave front as plane waves.



Refraction and Snell's law

Thus

$$\sin i_1 = BC / AB = v_1 \Delta t / AB$$

and

$$\sin i_2 = AD / AB = v_2 \Delta t / AB$$

so that

$$\frac{\sin i_1}{\sin i_2} = \frac{v_1}{v_2}$$

This is Snell's law of refraction.

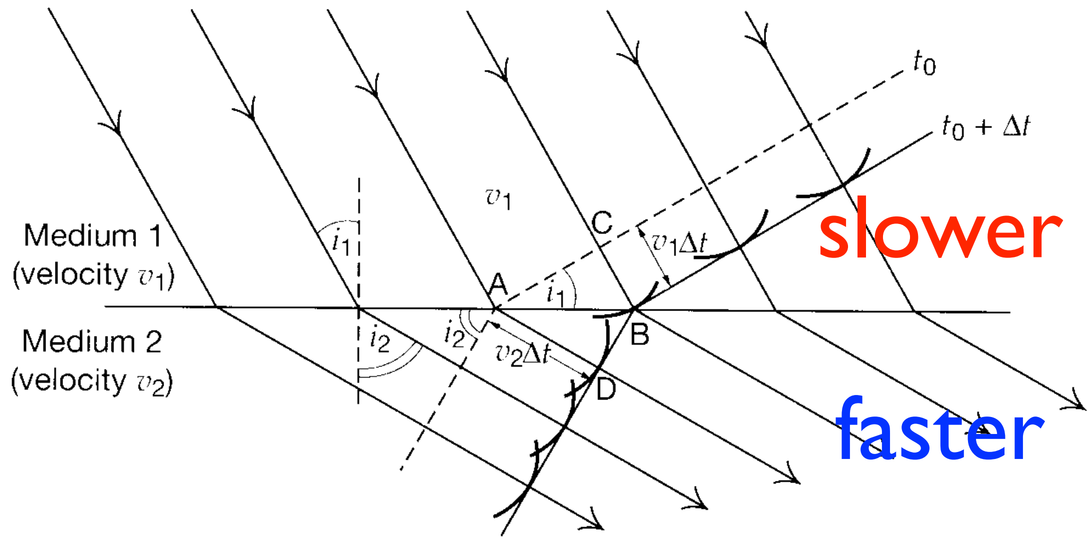
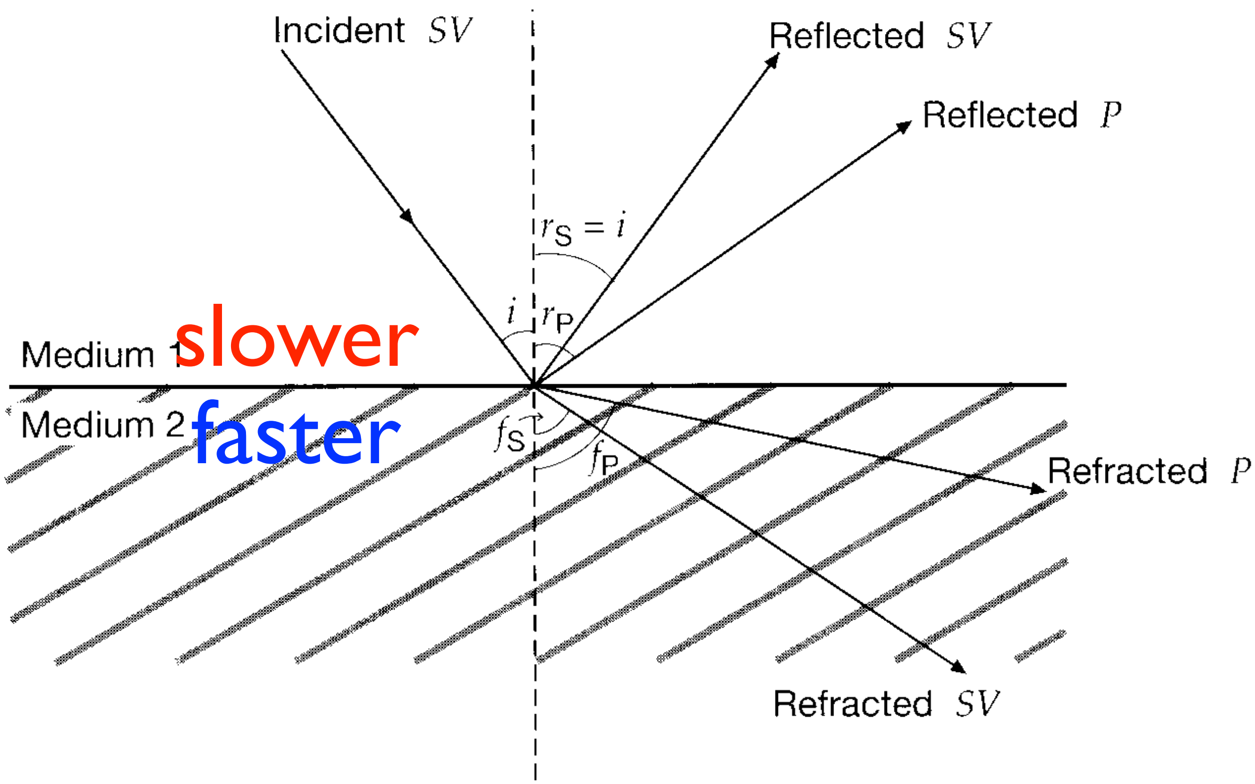


Figure 5.6. Huygens's construction for refraction of a wave at a plane boundary. Positions of a wavefront at times t_0 and $(t_0 + \Delta t)$ are shown by broken and solid lines and arrows on the rays show the direction of propagation. Each point on the wavefront at t_0 acts as a source of wavelets (shown as short arcs), the envelope of which is the wavefront at $(t_0 + \Delta t)$.

Geometry of refraction, reflection, and conversion at an interface (abrupt change)



Rays paths in a layered flat earth

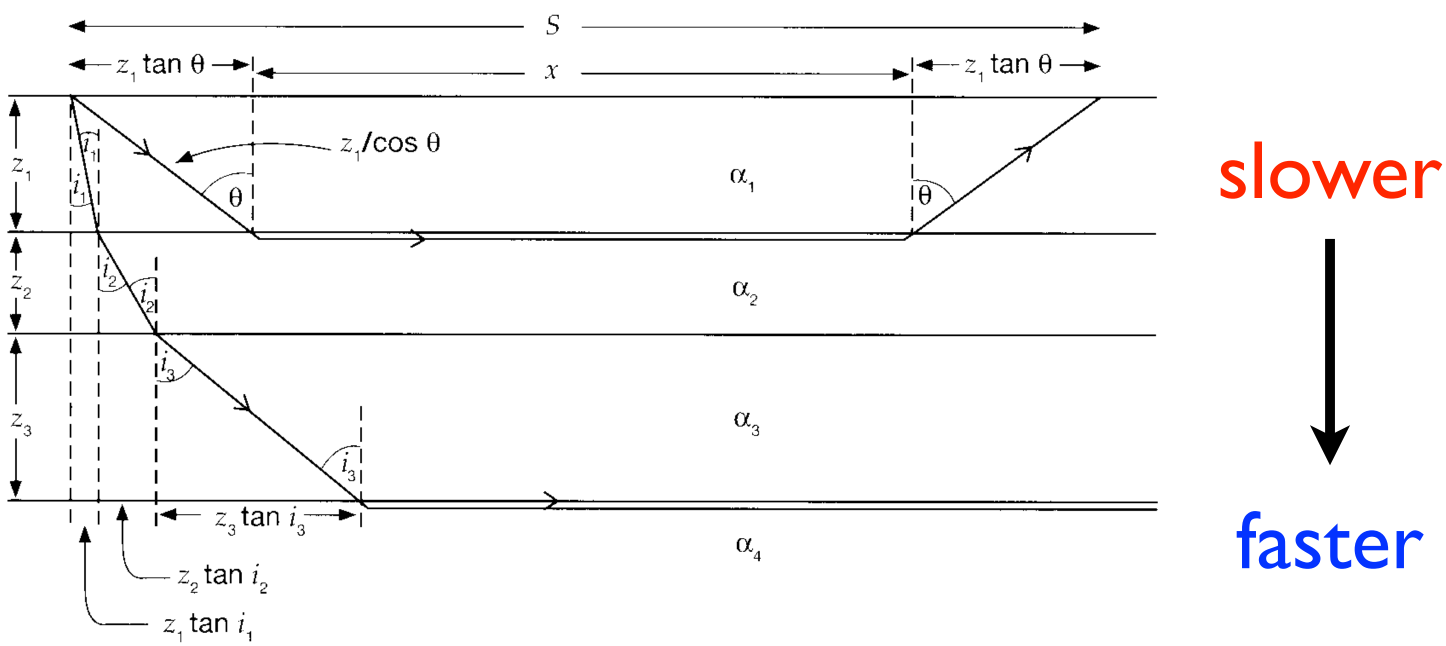
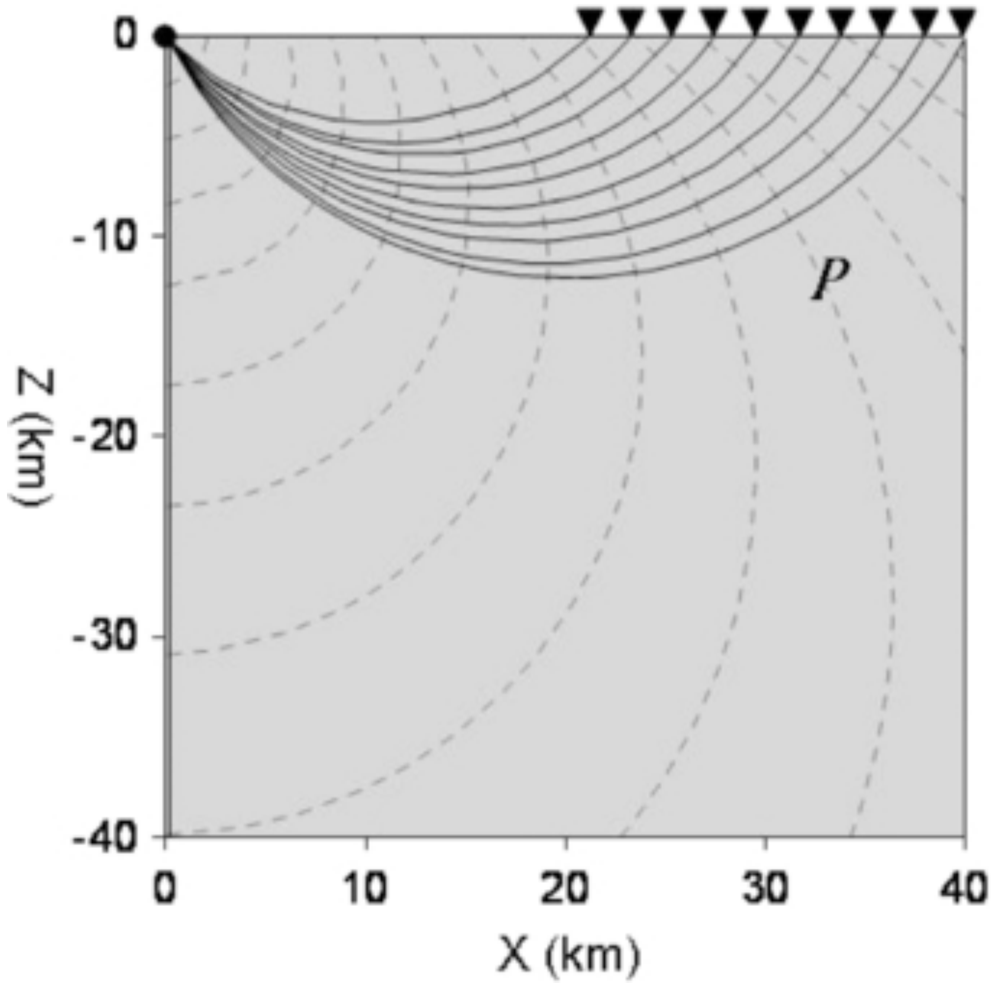


Figure 5.9. Geometry of seismic rays in a plane layered Earth model, having P wave speed, α, progressively increasing with depth. Waves that travel for much of their paths in deeper and faster layers, as illustrated, are known as head waves.

Ray refraction, smooth variations- no interfaces, no reflections, no conversions



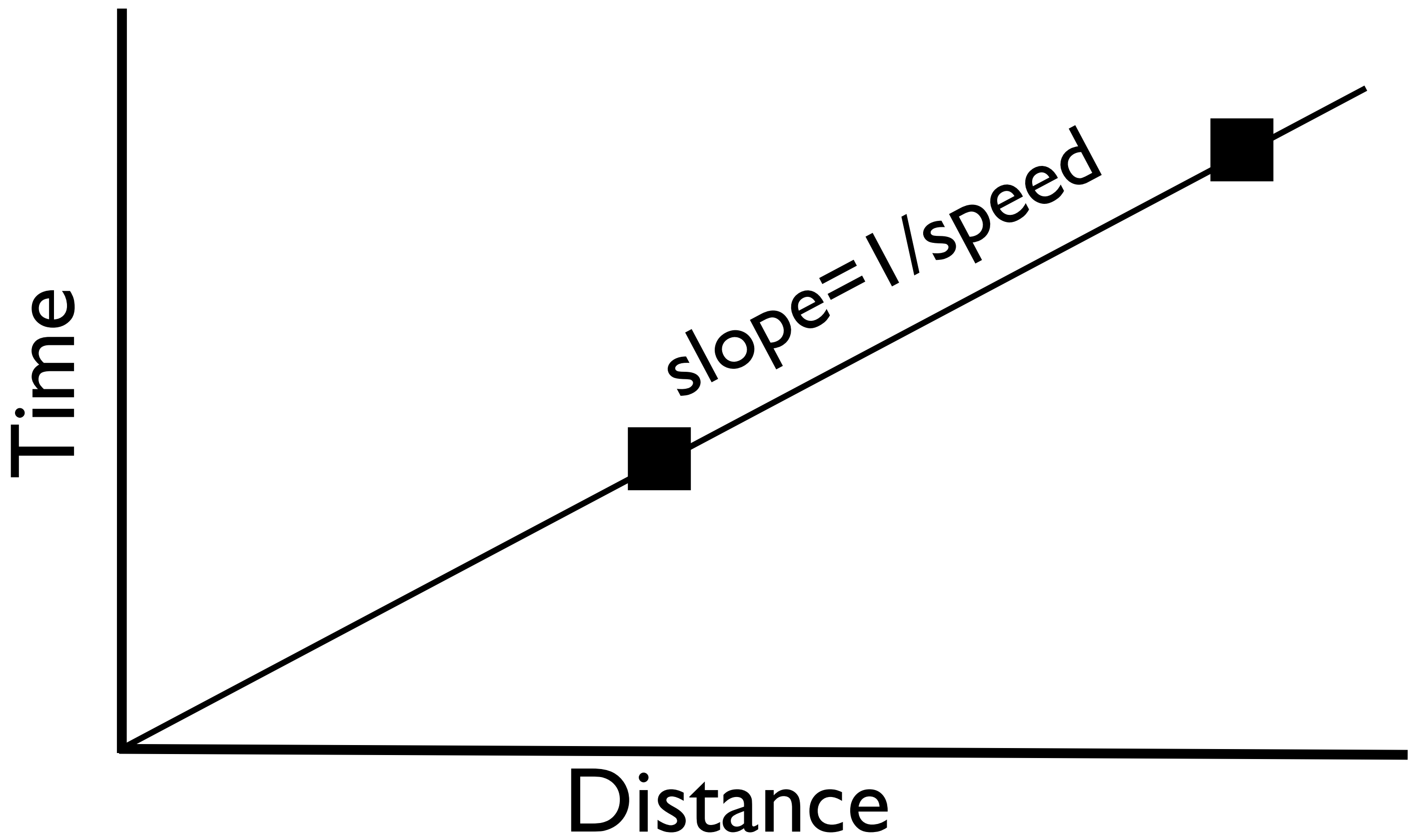
slower

faster

Basics Concepts of Seismology

Travel-time curve

Travel time curve for horizontal rays



Travel times in a layered, flat Earth

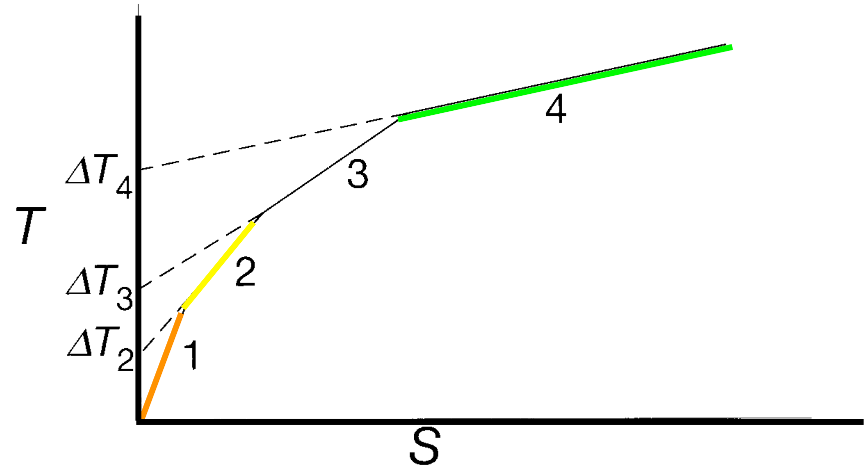


Figure 5.10. Travel-time curve for first arriving pulses from a layered structure, as in Fig. 5.8. Numbers on segments indicate the deepest layer penetrated by each "family" of rays.

Ray parameter (flat Earth)

(constant along the ray):

$$p = \frac{\sin i}{v}$$

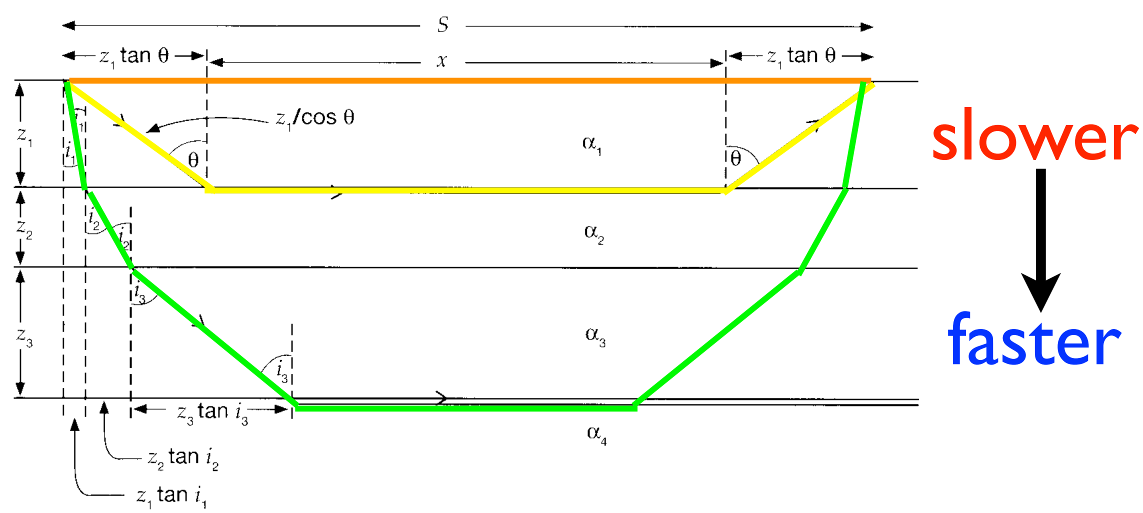
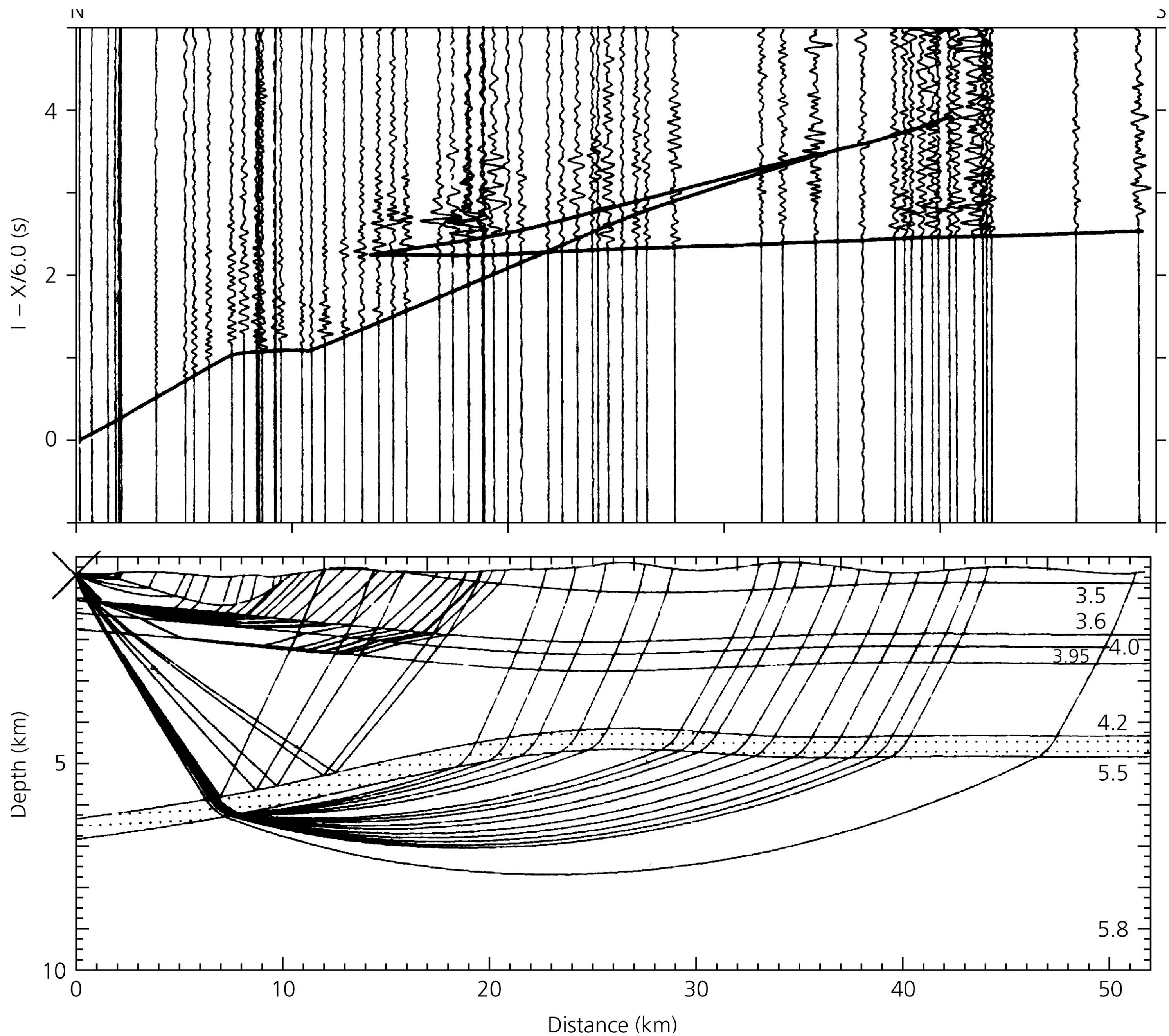
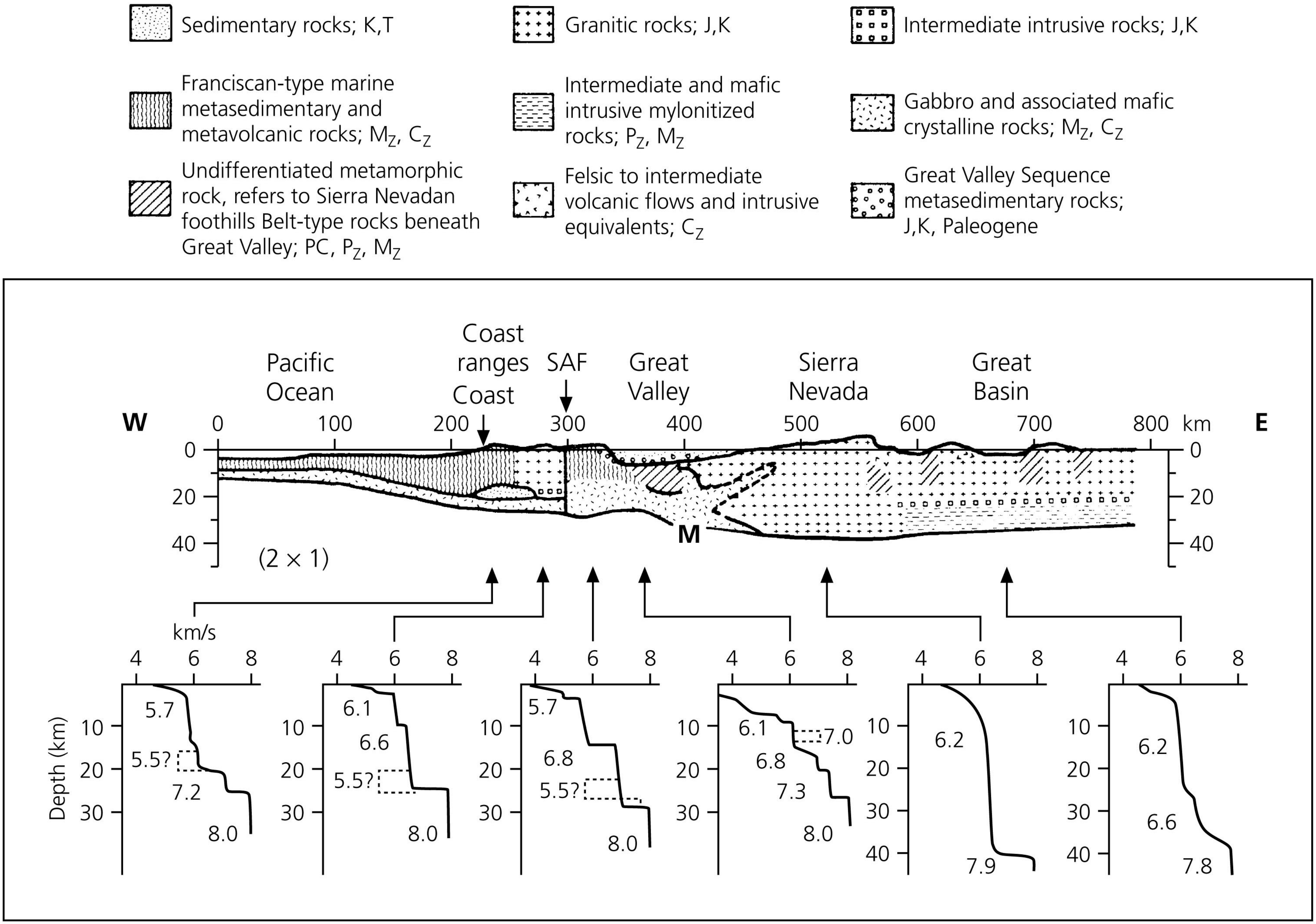


Figure 5.9. Geometry of seismic rays in a plane layered Earth model, having P wave speed, α, progressively increasing with depth. Waves that travel for much of their paths in deeper and faster layers, as illustrated, are known as head waves.

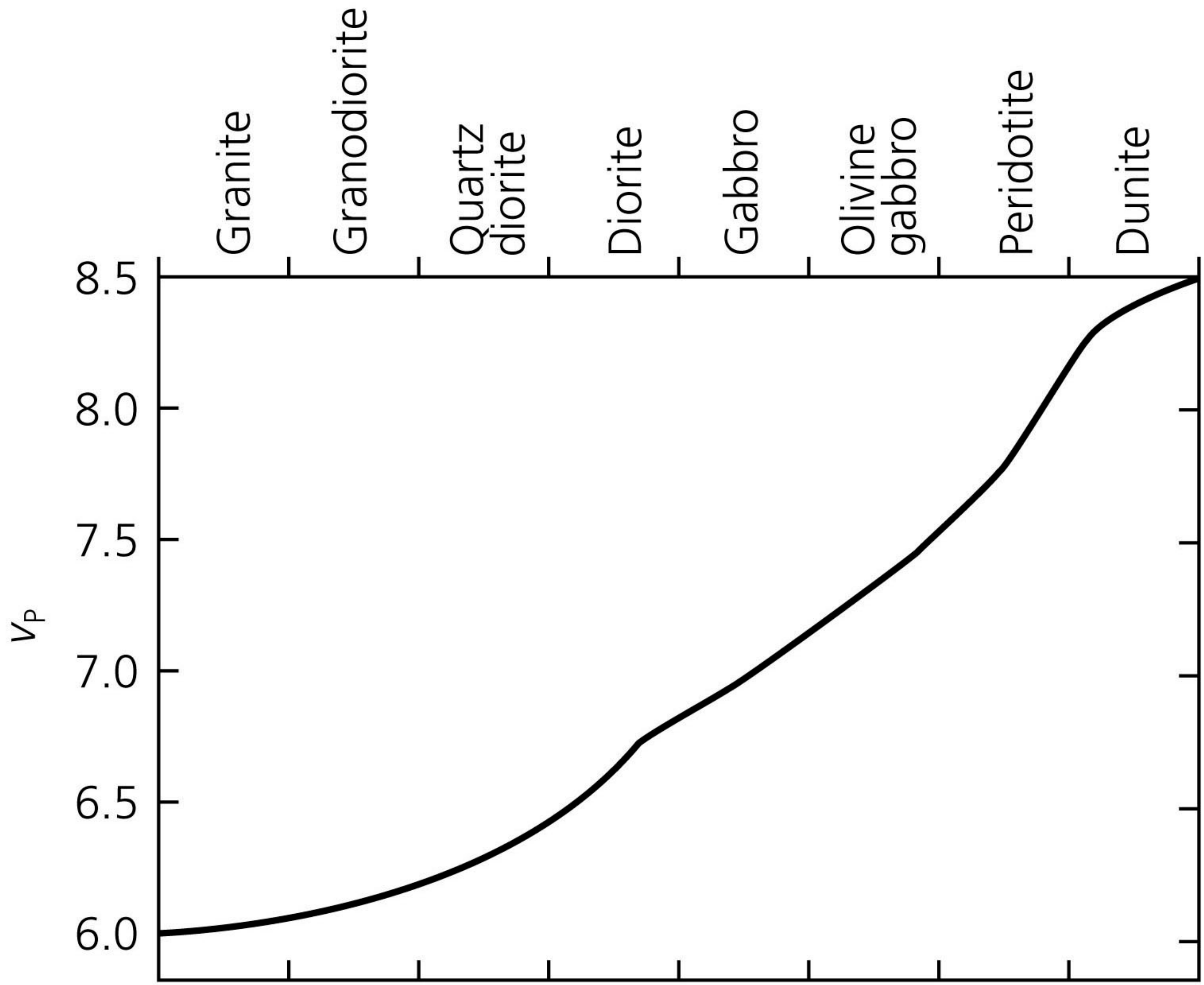
Refracted rays and a velocity model





modified from Stein and Wysession (2009)

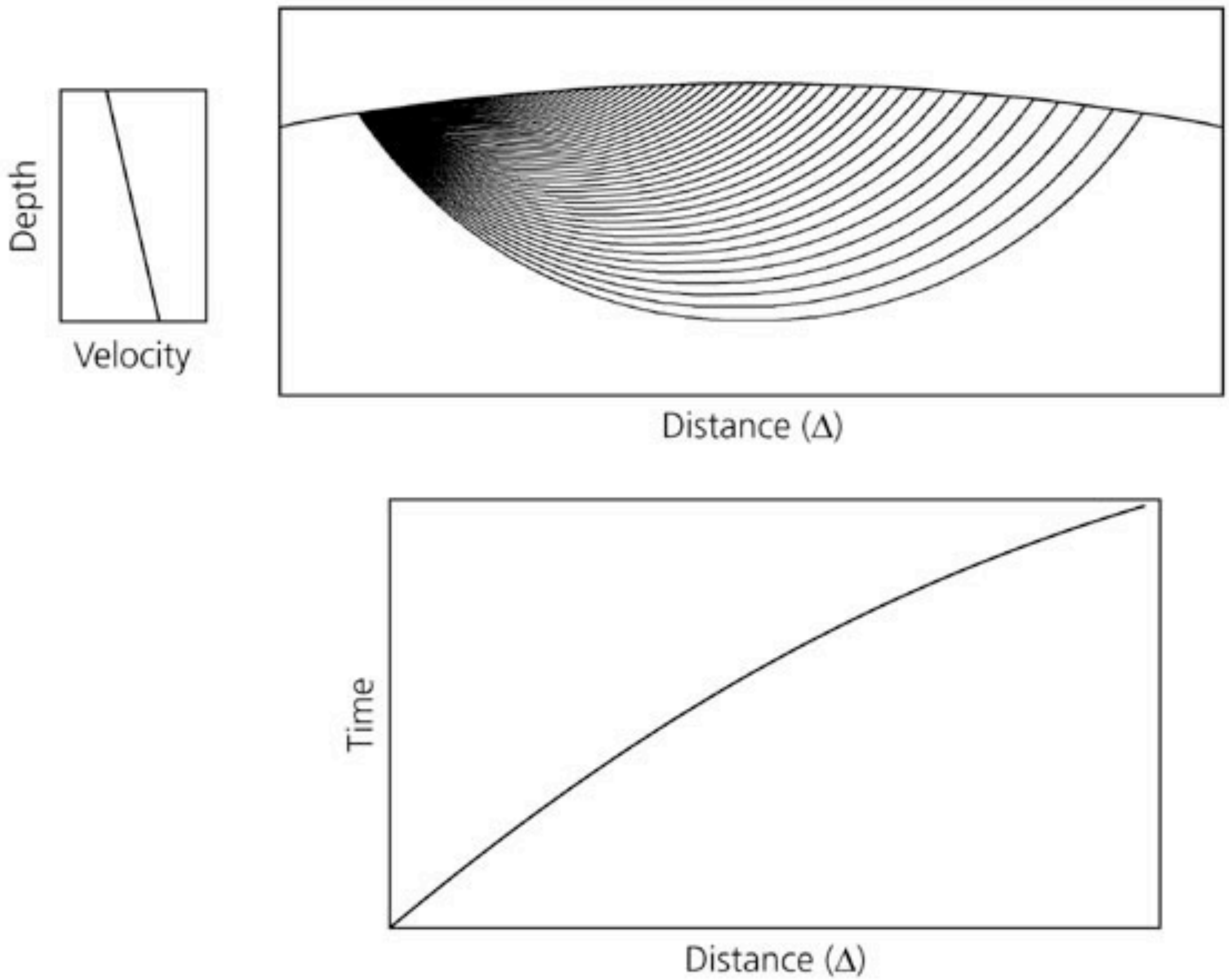
Figure 3.2-22: Variation of *P*-wave velocity with lithology.



Basics Concepts of Seismology

Triplications, shadow zones

Figure 3.4-5: Ray path effects for increasing velocity.



Why does velocity typically increase with depth?

$$\beta = v_S = \sqrt{\frac{\mu}{\rho}}$$

Why does velocity typically increase with depth?

$$\beta = v_S = \sqrt{\frac{\mu}{\rho}}$$

1. Weaker rock types at shallow depth (crust)
2. Pressure effects on elastic moduli dominate over pressure effects on density
2. Pressure effects dominate over temperature effects
3. Phase changes make rocks stiffer (generally)

(but temperature, melt, and water can lead to a velocity decrease)

Figure 3.4-5: Ray path effects for increasing velocity.

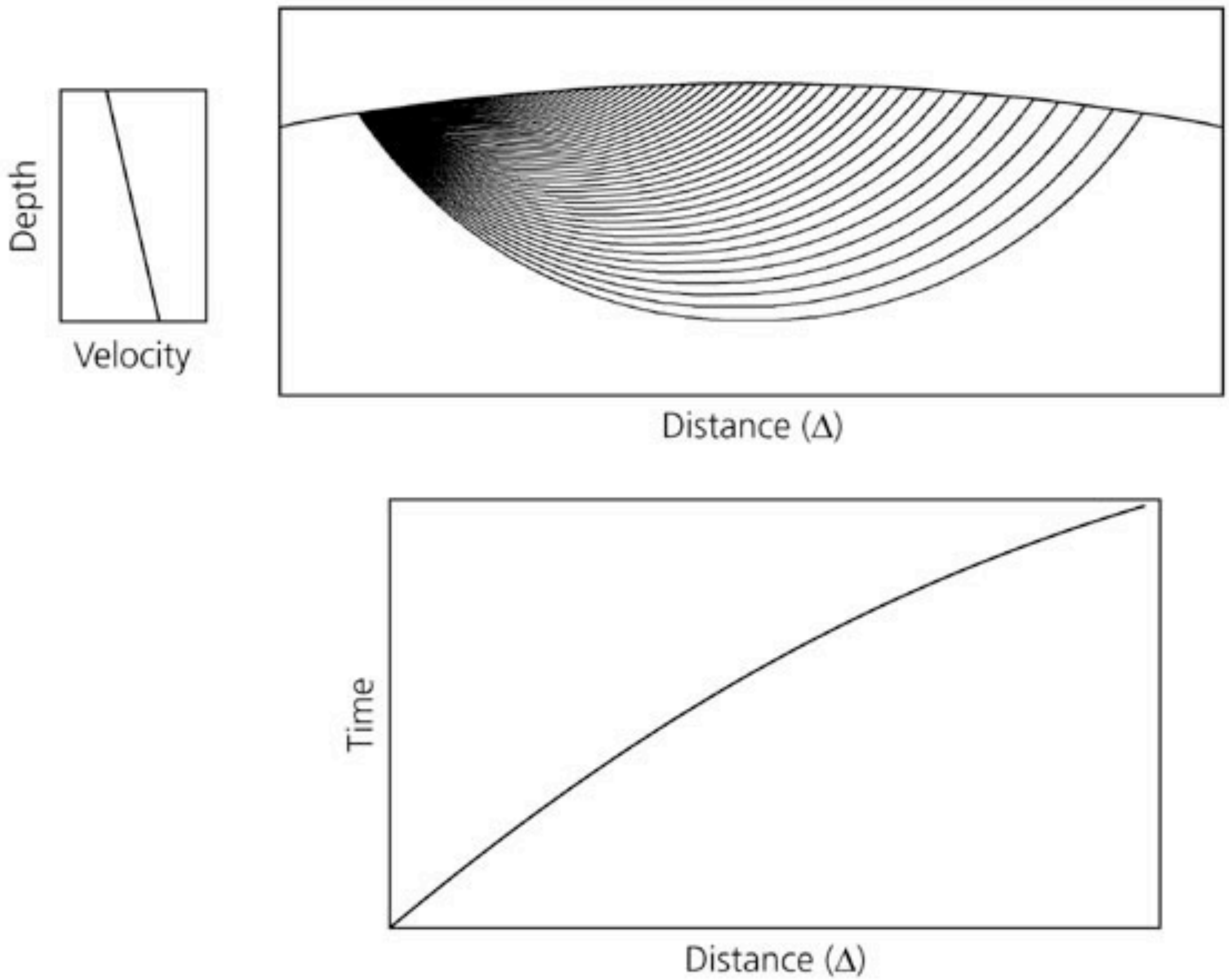
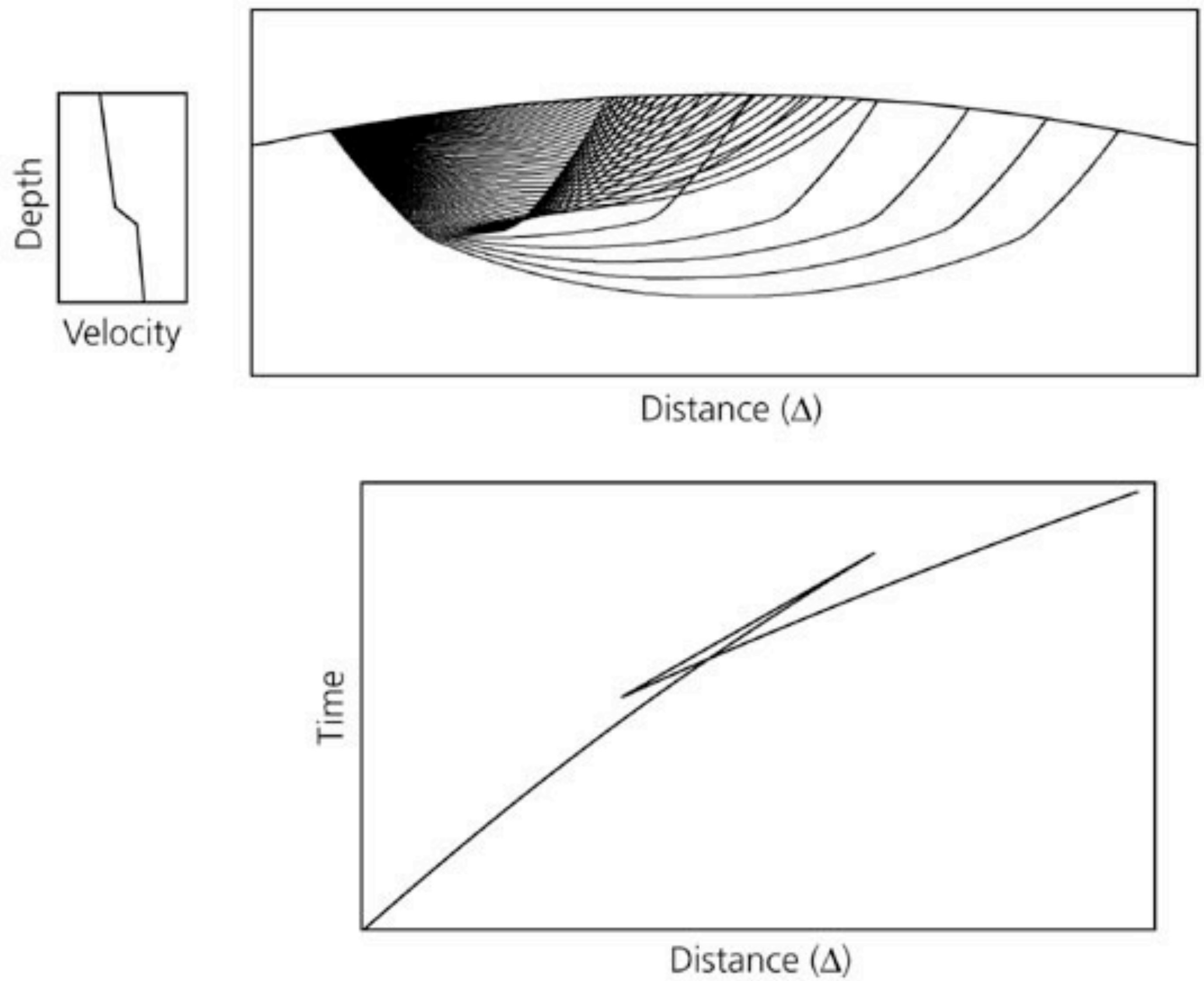
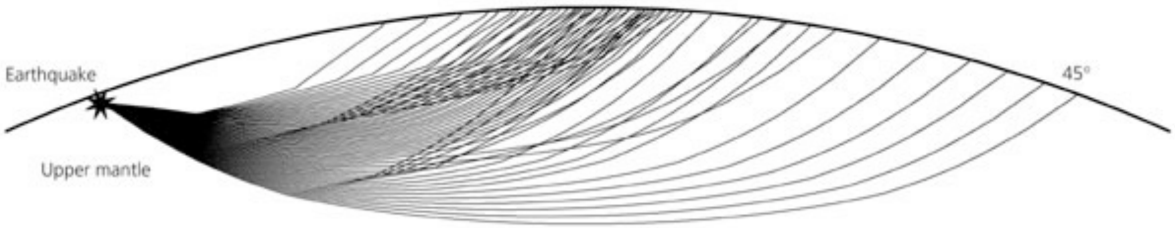


Figure 3.4-6: Ray path triplication effects for a velocity increase.



Triplications in the upper mantle

Figure 3.5-12: Ray paths for *P* waves through the upper mantle.



Rapid velocity increases at 400 km and 650 km due to mineralogical phase changes

ity models.

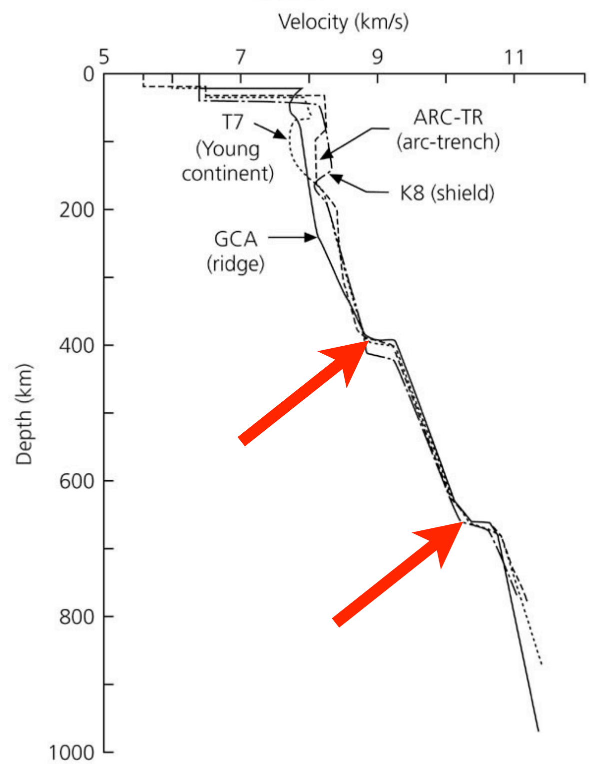
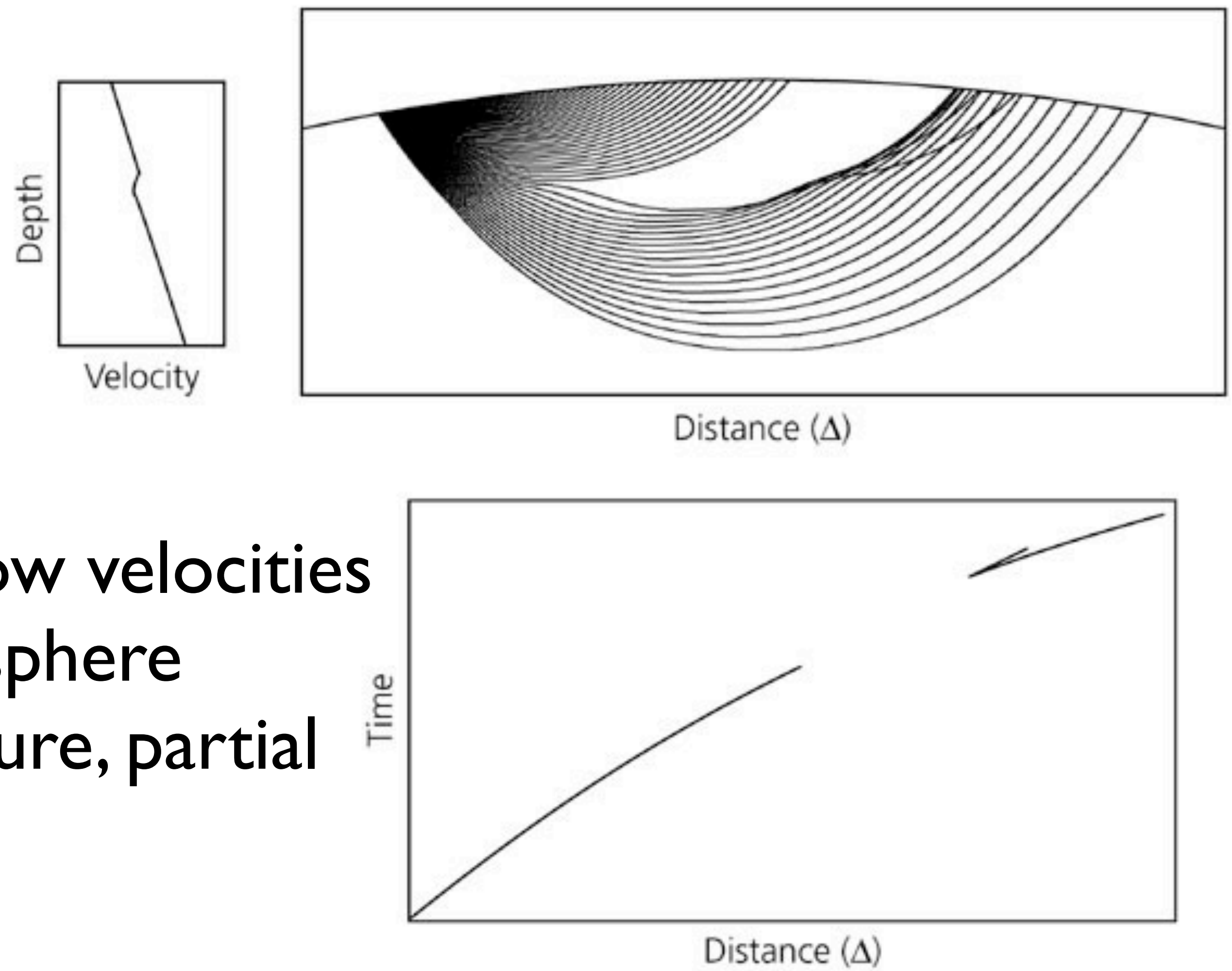


Figure 3.4-7: Ray path shadow-zone effects for a velocity decrease.



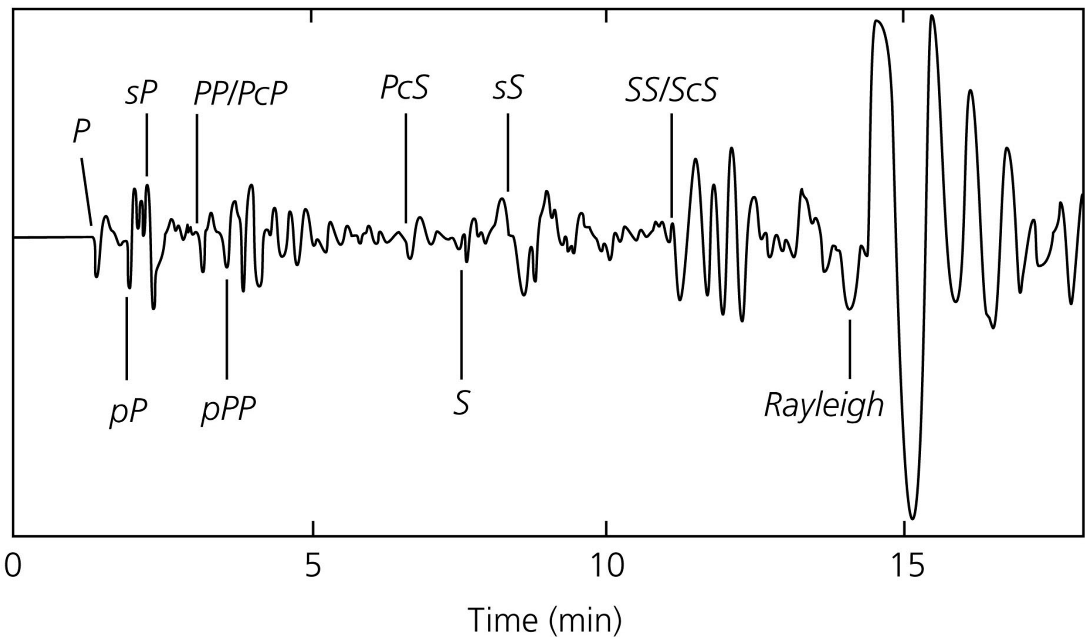
For example, low velocities in the asthenosphere (high temperature, partial melting?)

Basics Concepts of Seismology

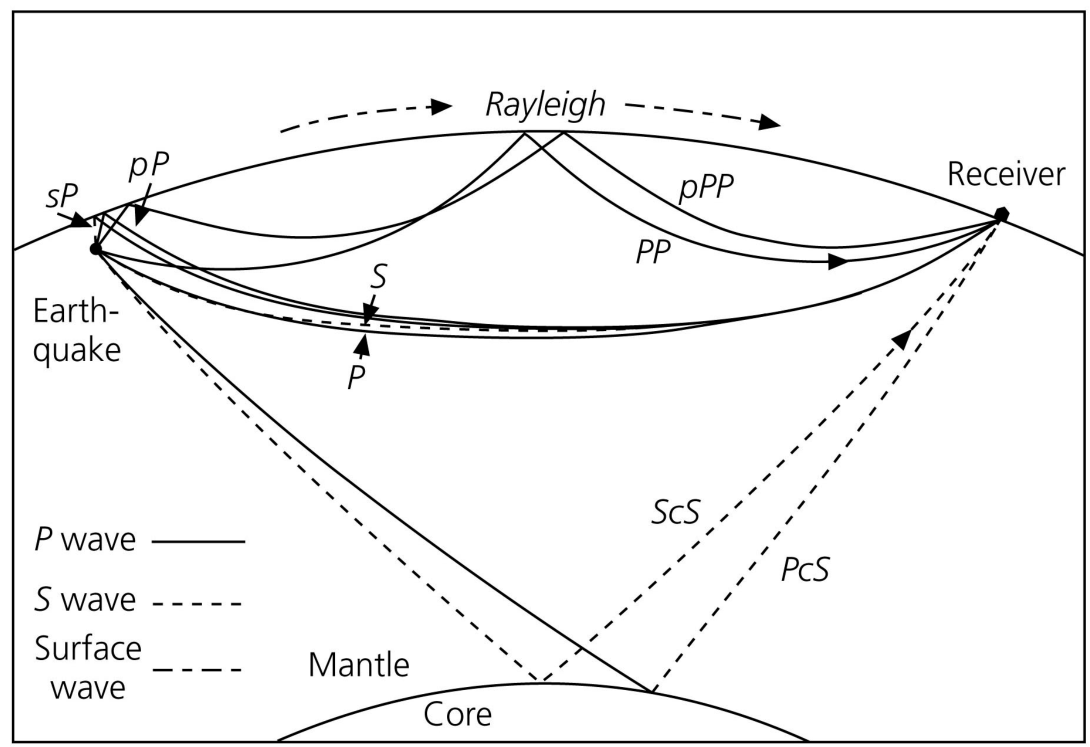
Gross Earth structure

Phases identified in seismogram

Figure 3.5-2: Selection of body phases and their ray paths.



Associated with paths through the Earth



Travel-time curve for the Earth

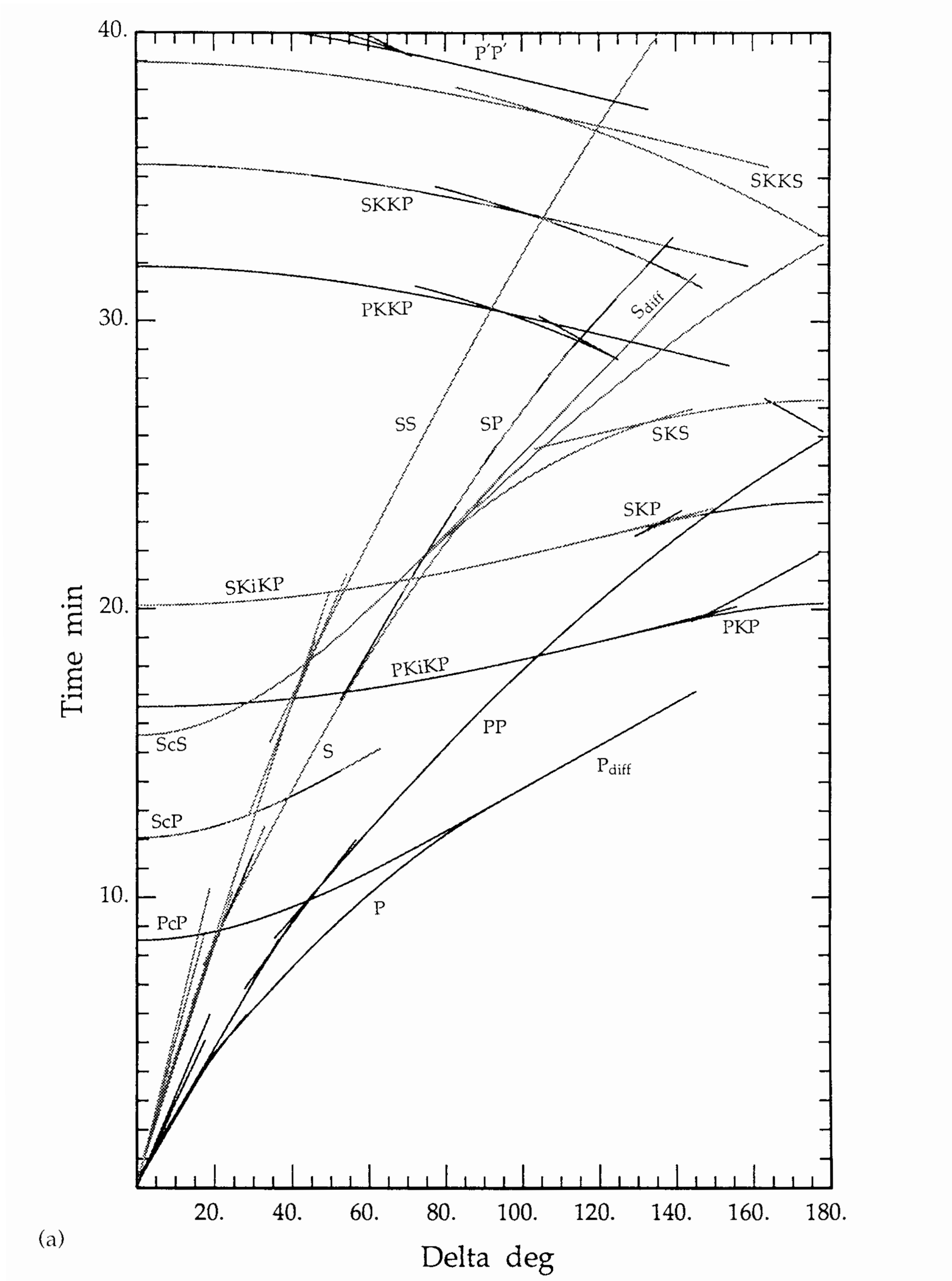
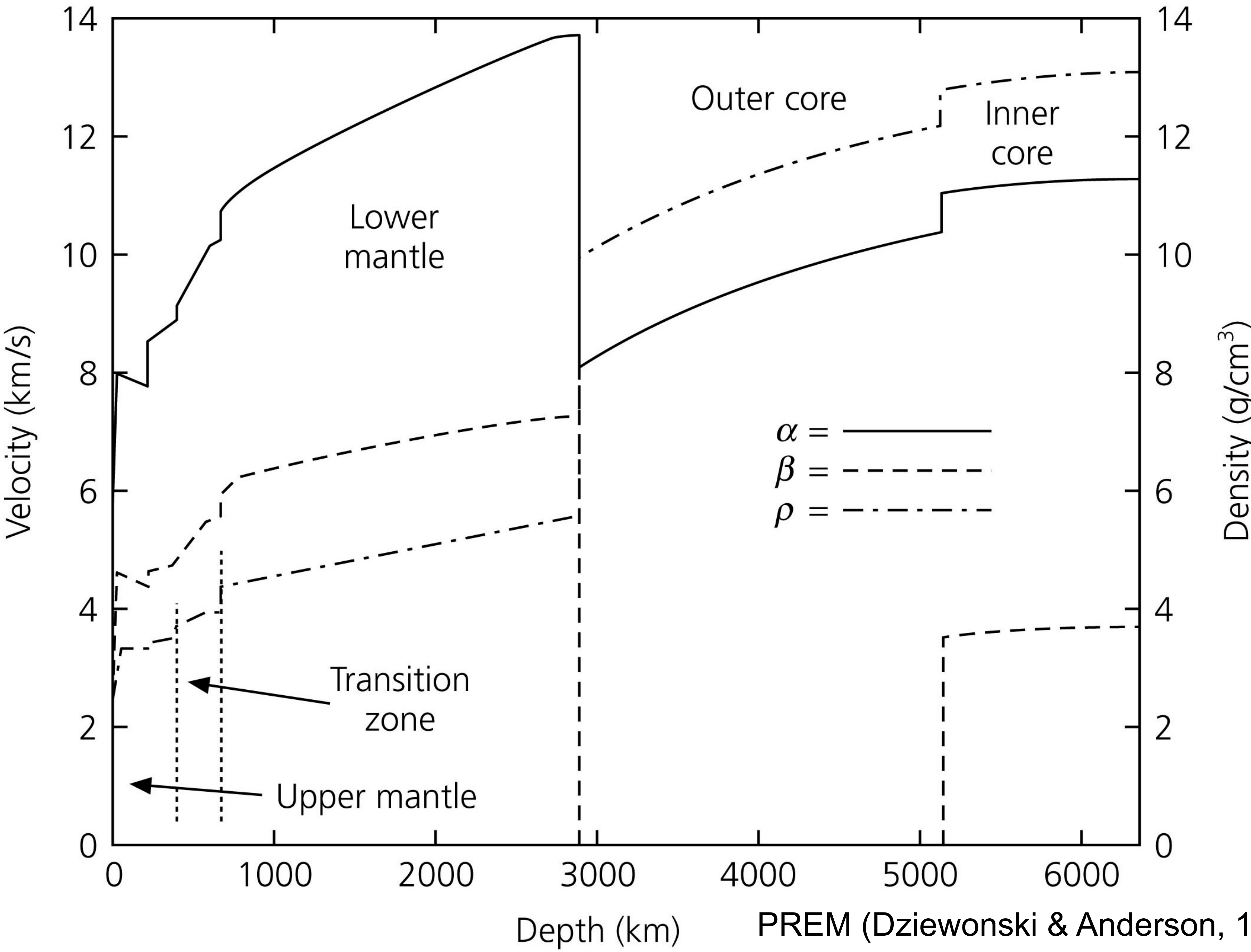
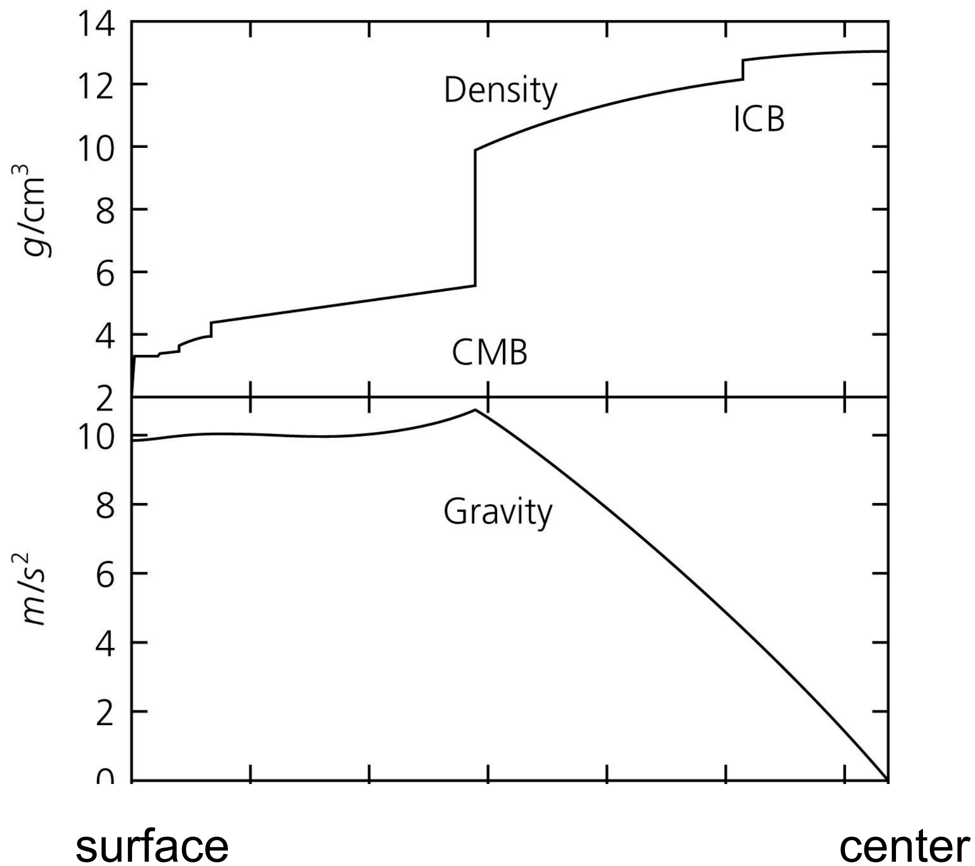


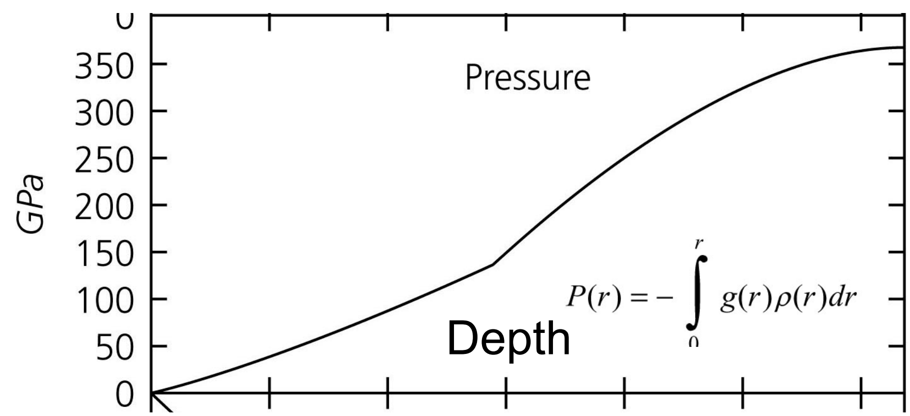
Figure 3.8-4: Preliminary Reference Earth Model.



PREM (Dziewonski & Anderson, 1981)

Density is part of PREM
This gives pressure at all depths





Elastic moduli in the Earth

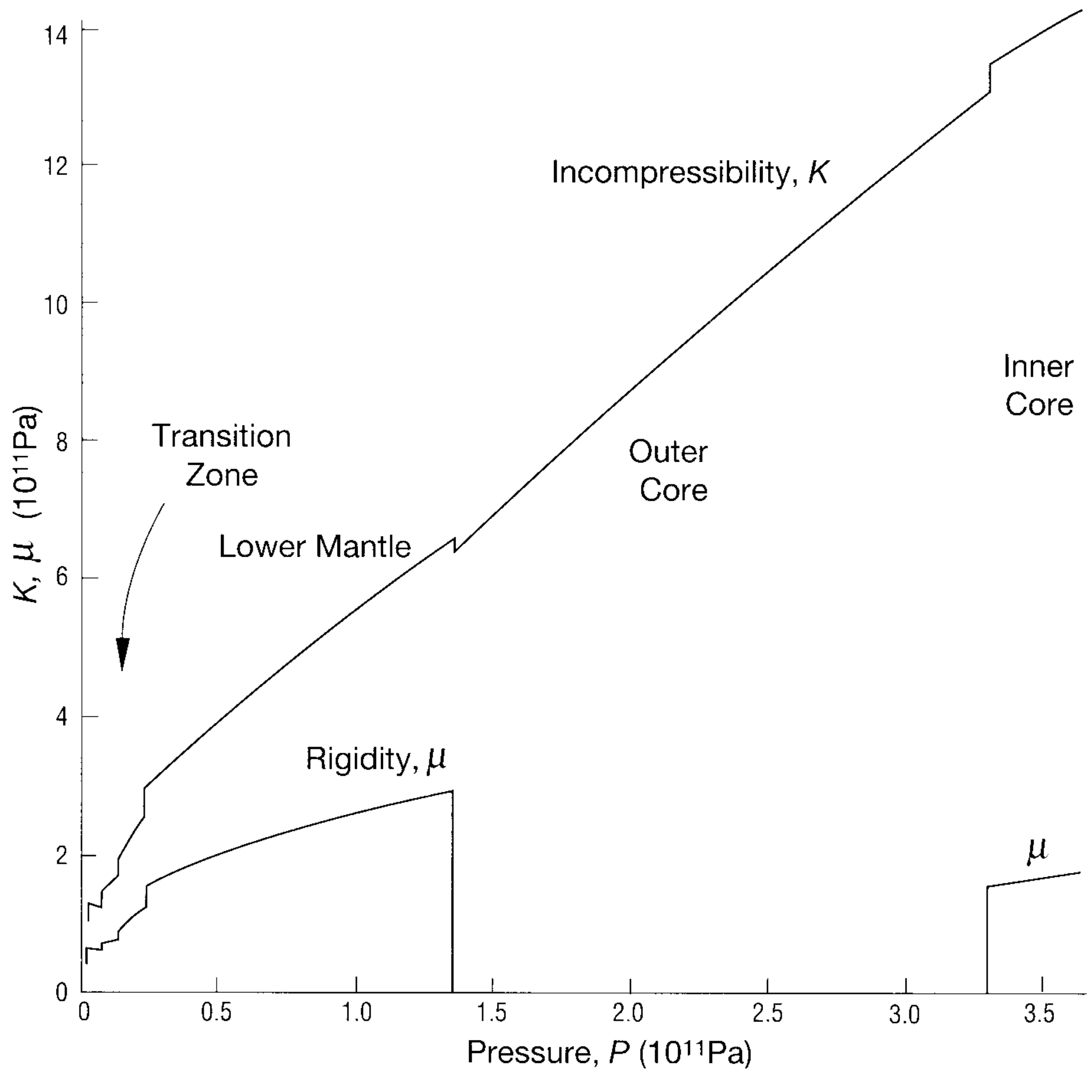
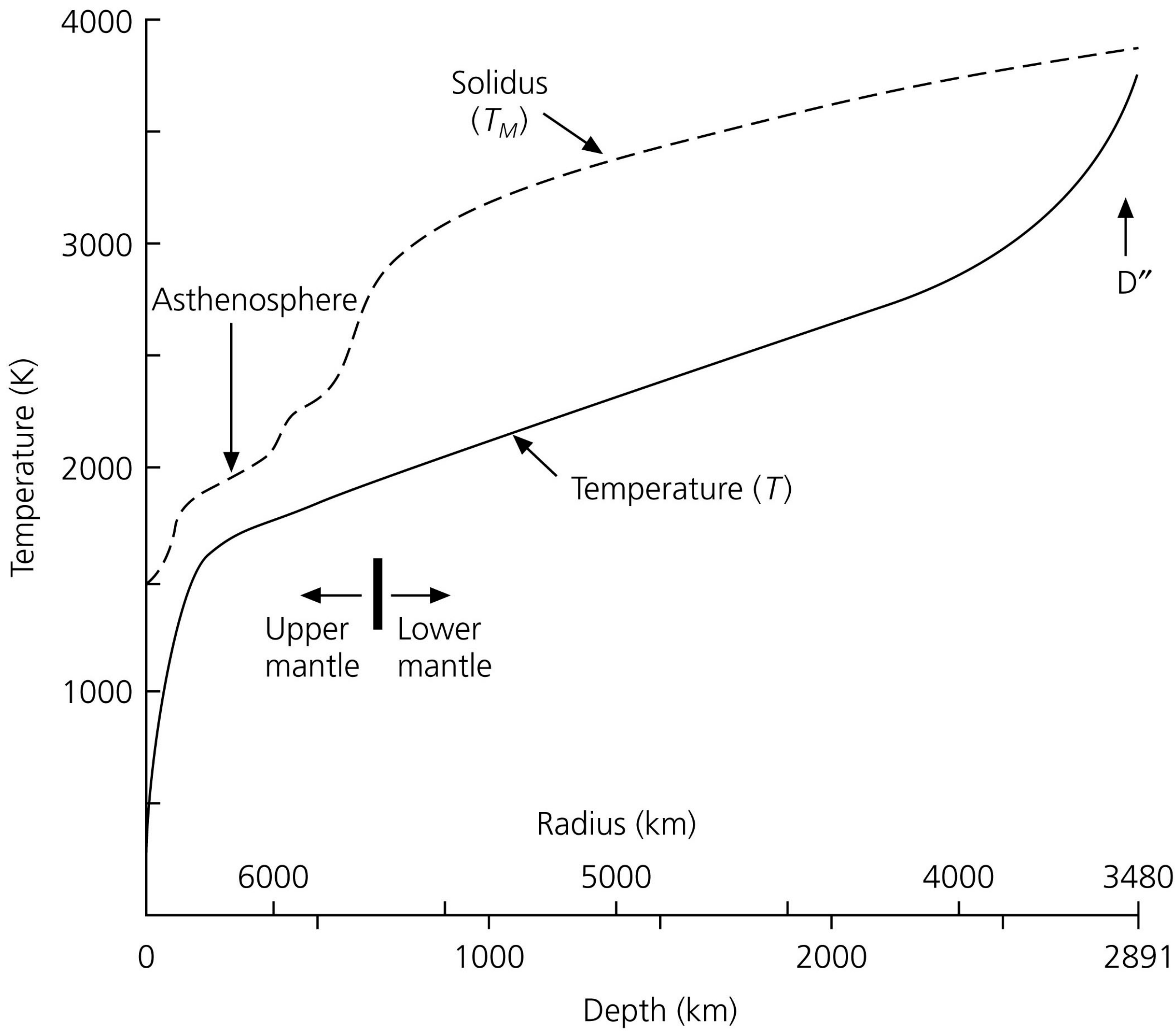


Figure 5.20. Variations of the elastic moduli, K and μ, with pressure, P, for the PREM earth model.

Figure 3.8-6: Geotherm and solidus for the mantle.



Relaxed (low pressure, low temperature) density

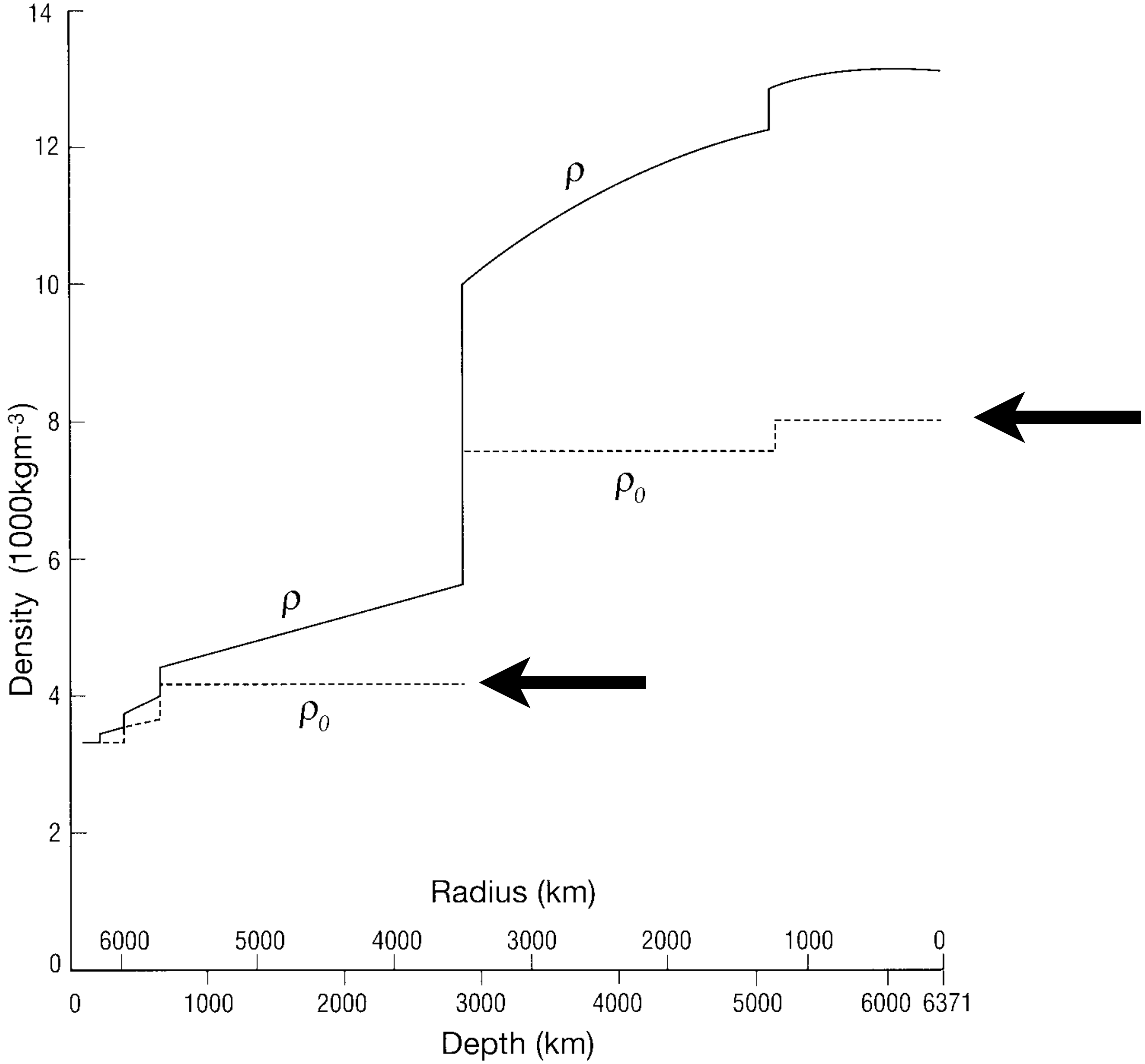
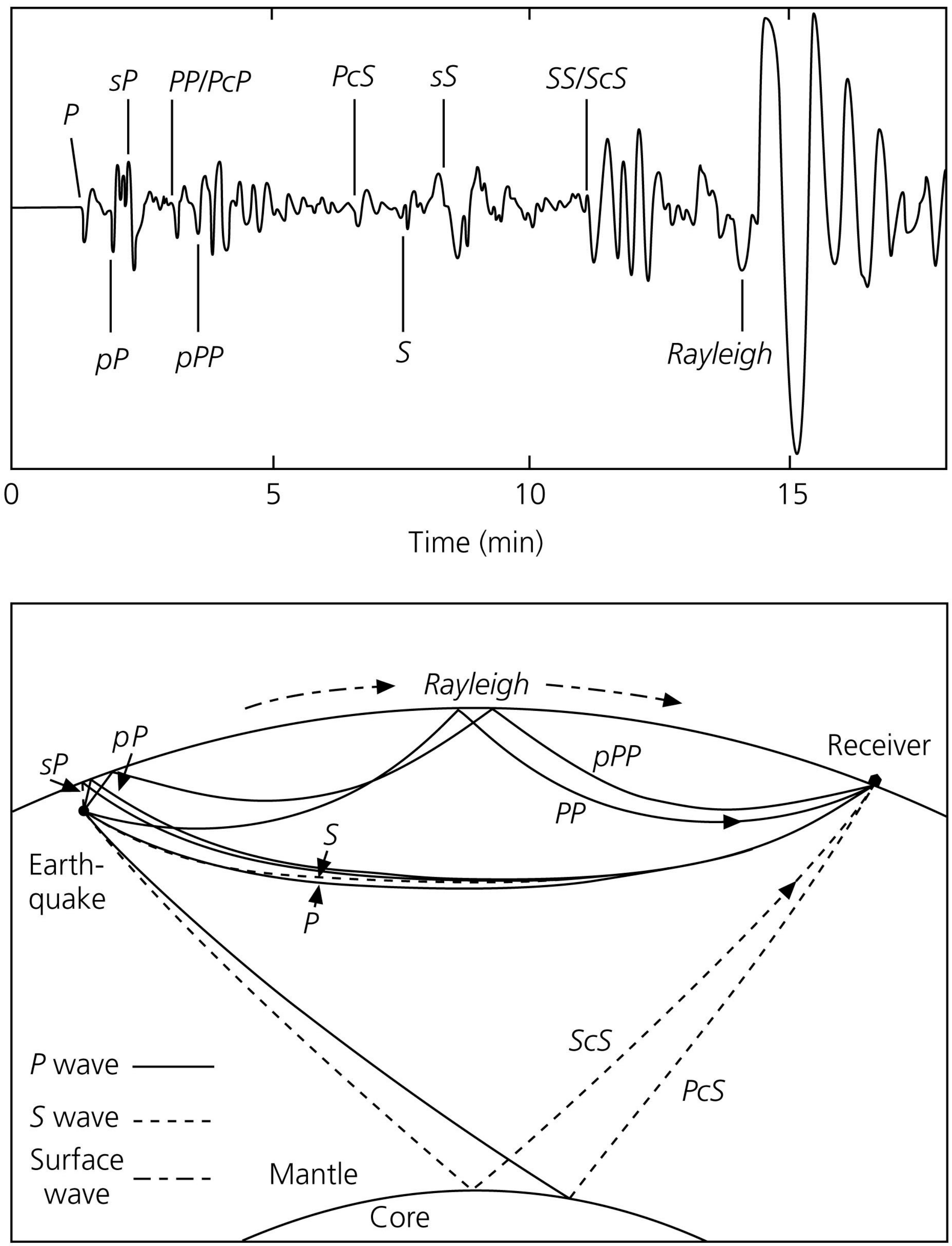


Figure 5.19(a). Profile of density, ρ, through the earth model PREM with corresponding zero pressure, low temperature density, estimated by finite strain theory.

Basics Concepts of Seismology

Surface waves, dispersion

Figure 3.5-2: Selection of body phases and their ray paths.



Surface waves

1. travel along the Earth's surface
2. exist as Love waves and Rayleigh waves, each with a distinct particle motion
3. have speeds that depend mainly on the rigidity (shear modulus) of the rock
4. are dispersive

Seismic surface waves - analogy with water waves is apt

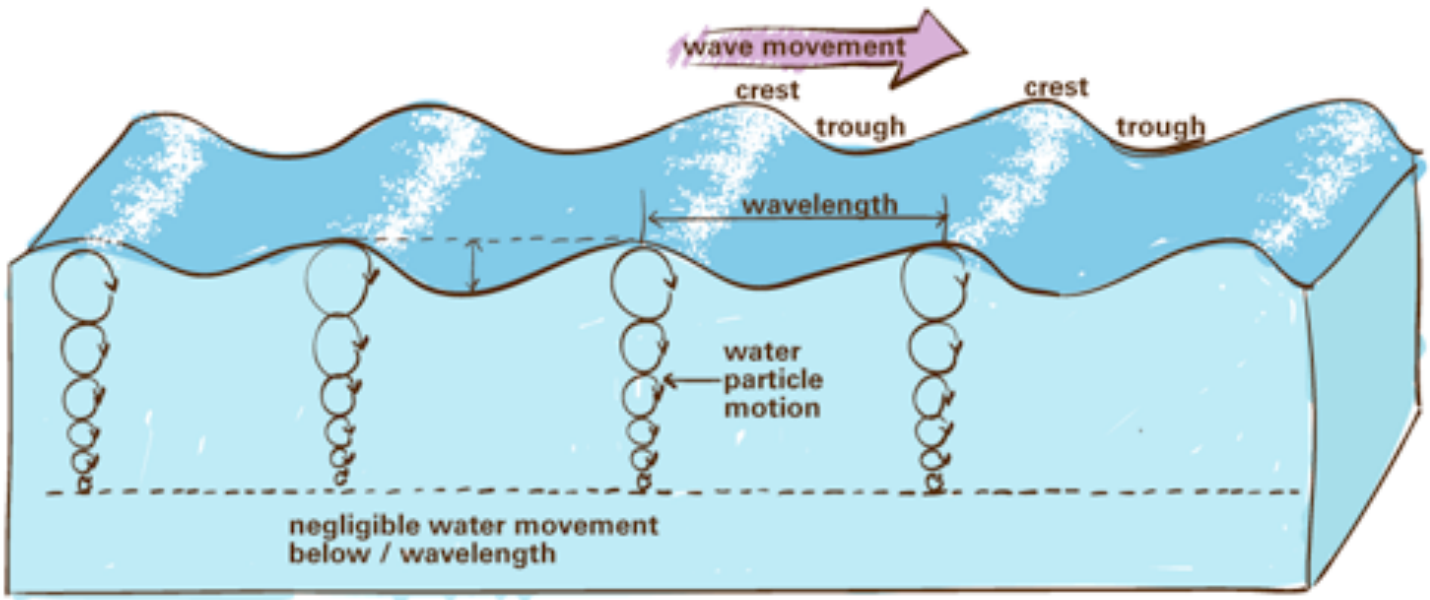


Figure 2.7-1: Seismograms recorded at a distance of 110°, showing surface waves.

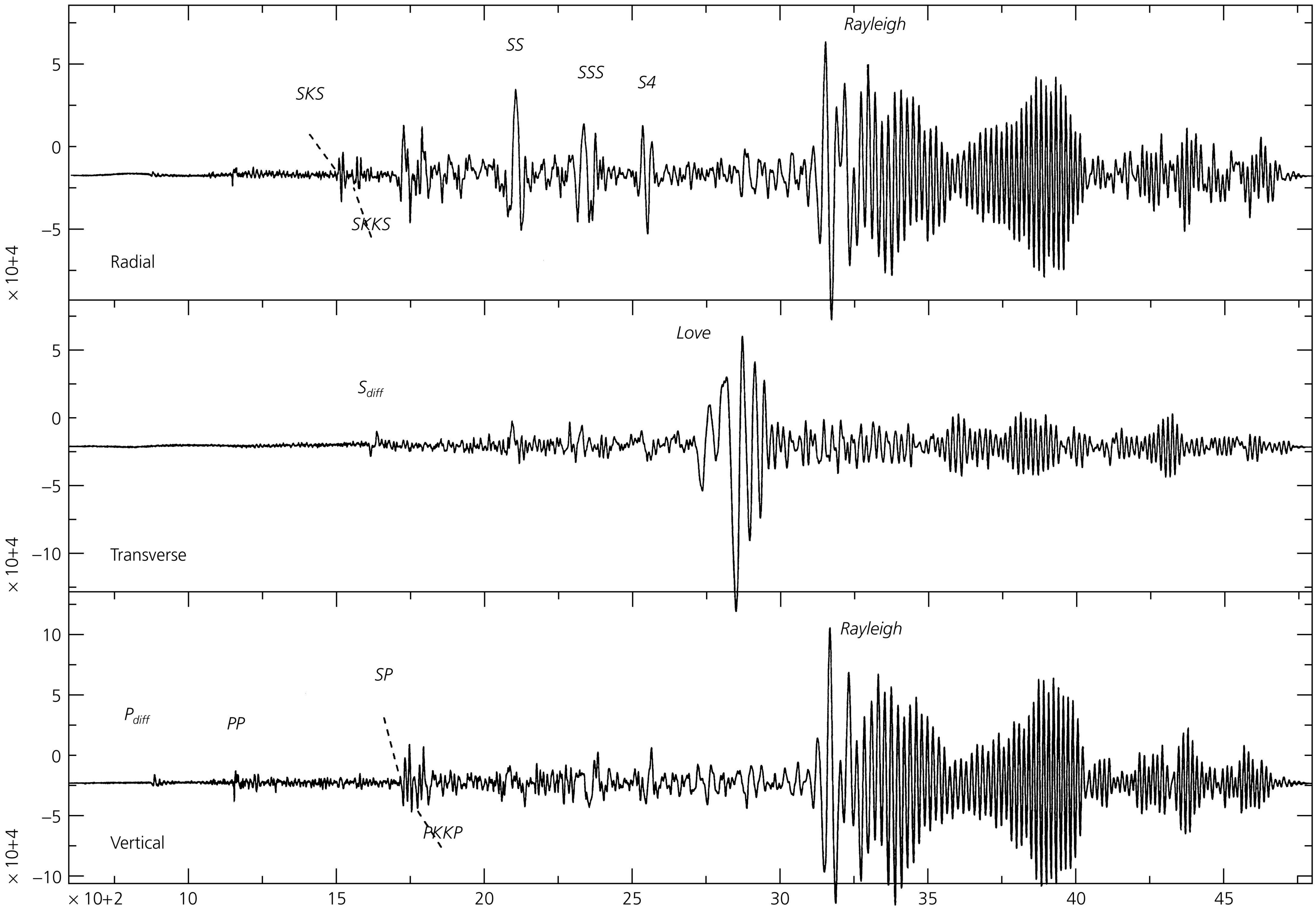
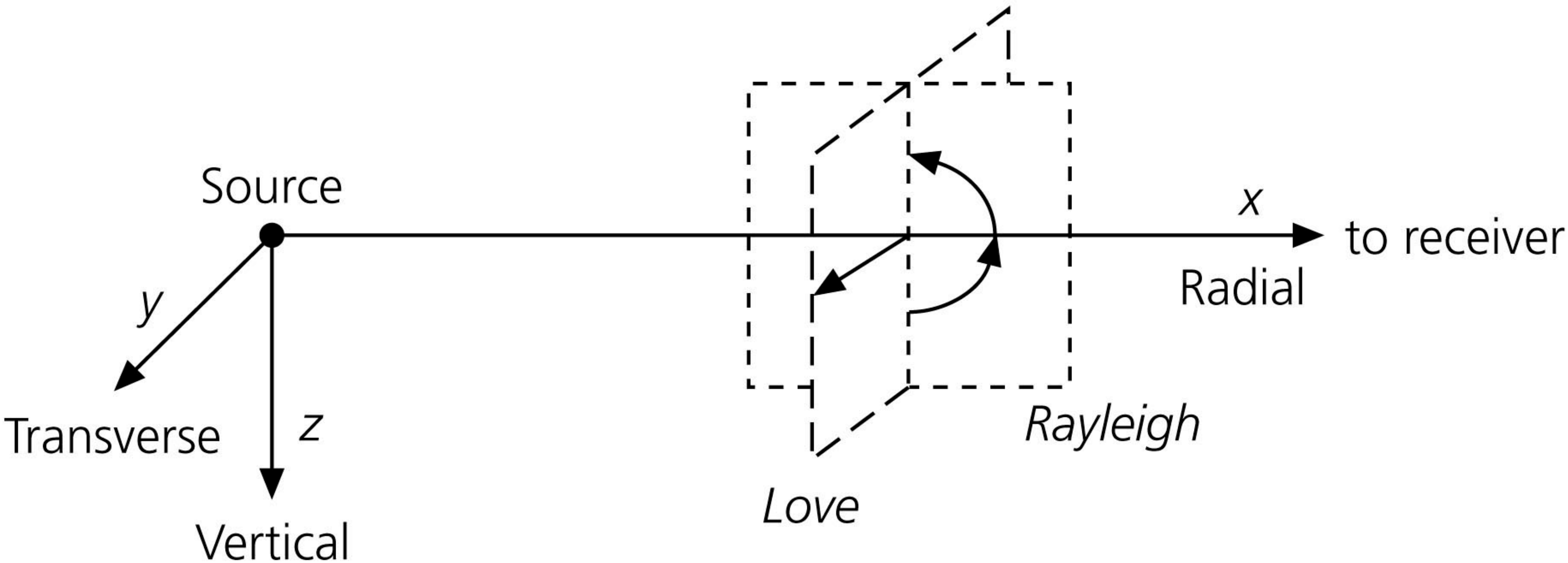
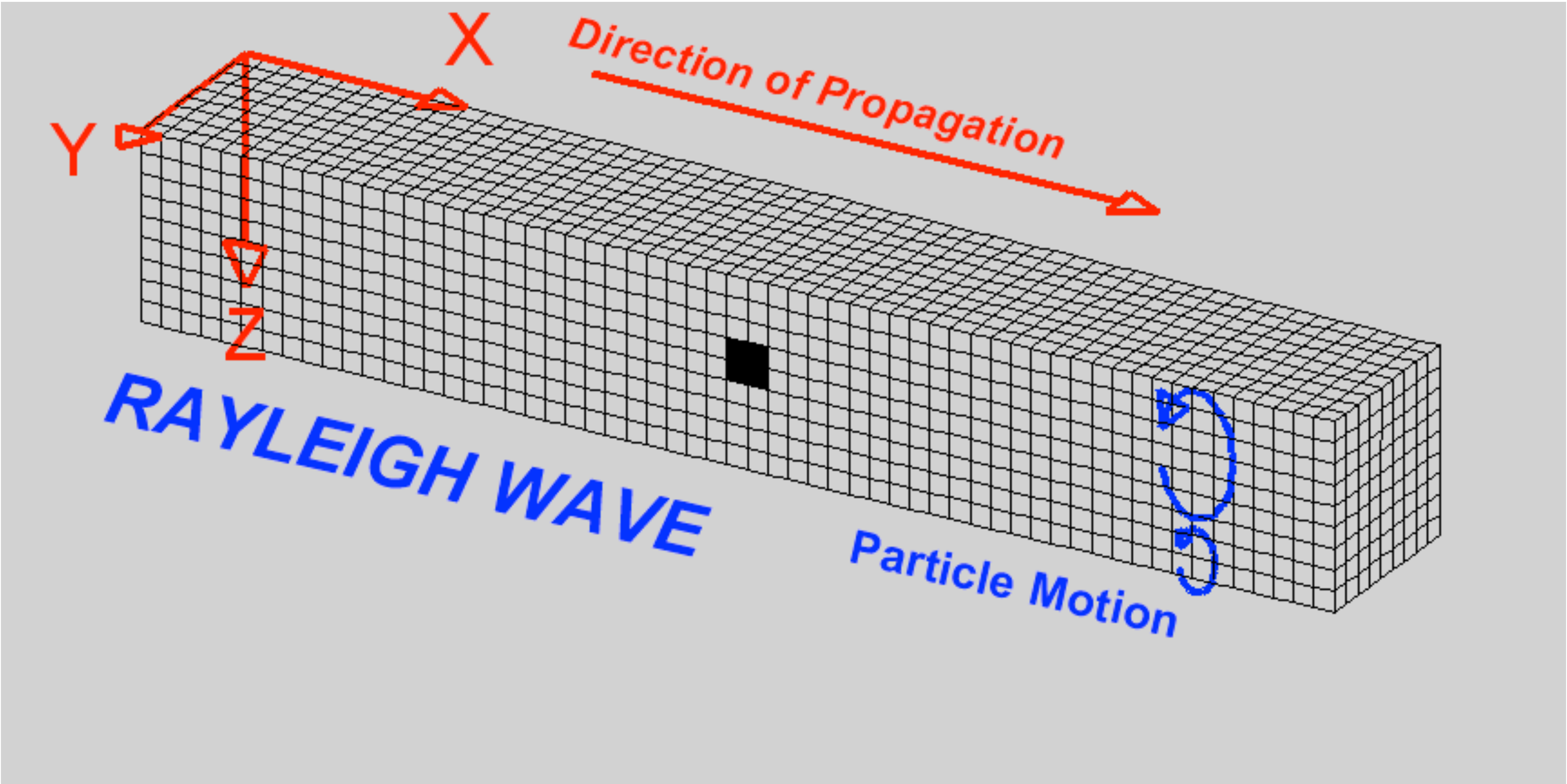


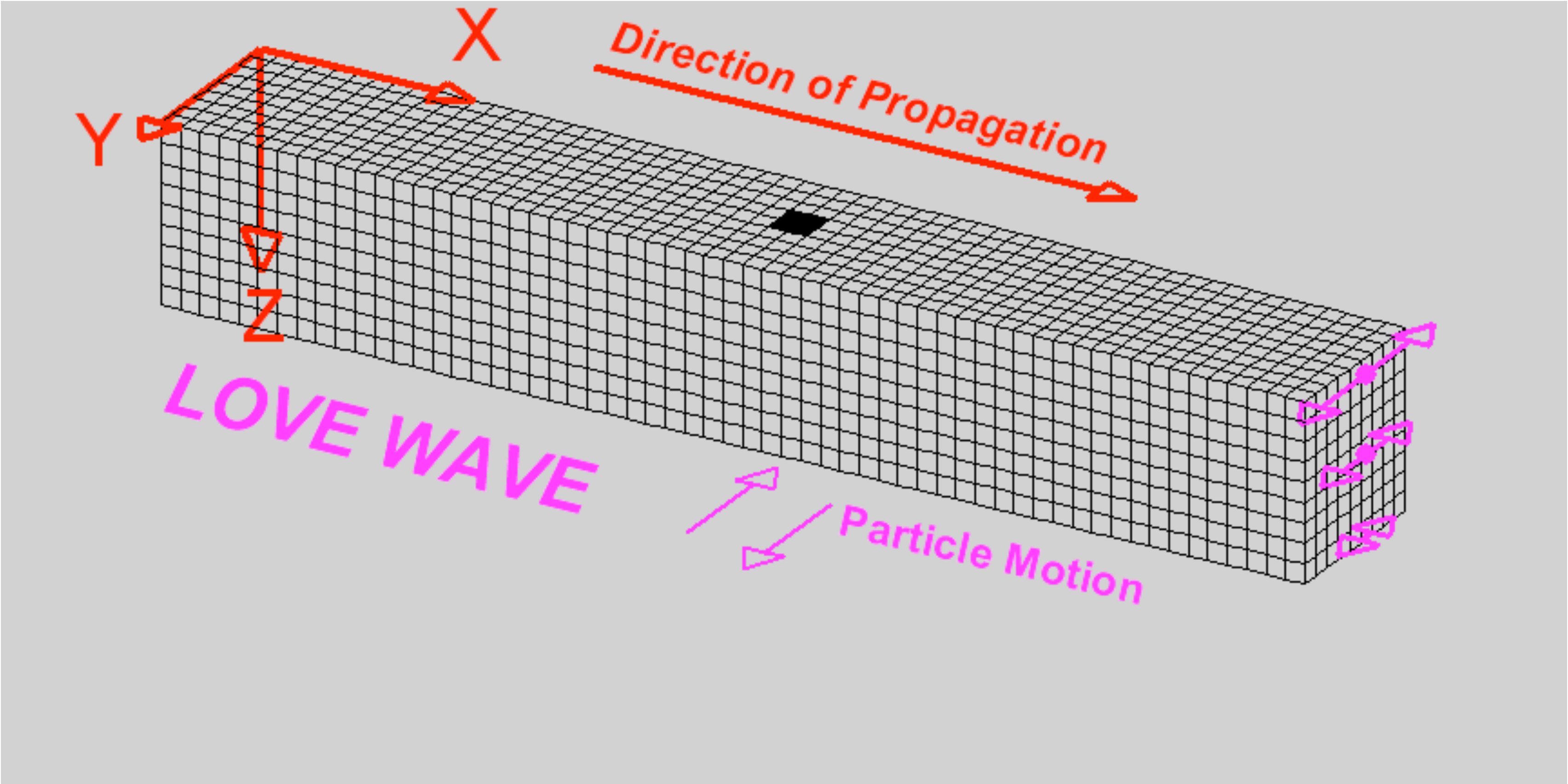
Figure 2.7-2: Geometry for Love and Rayleigh wave motions.



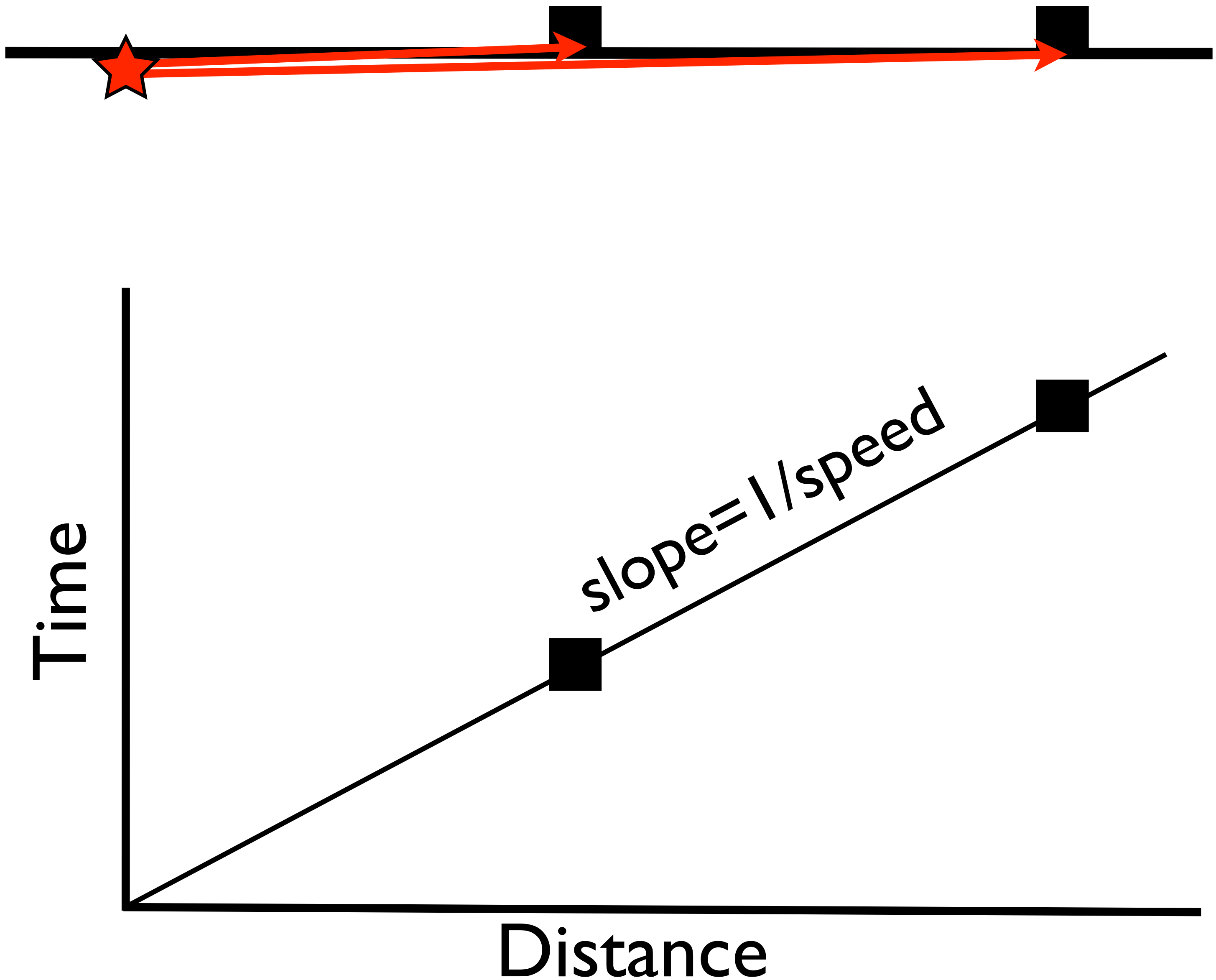
Rayleigh wave



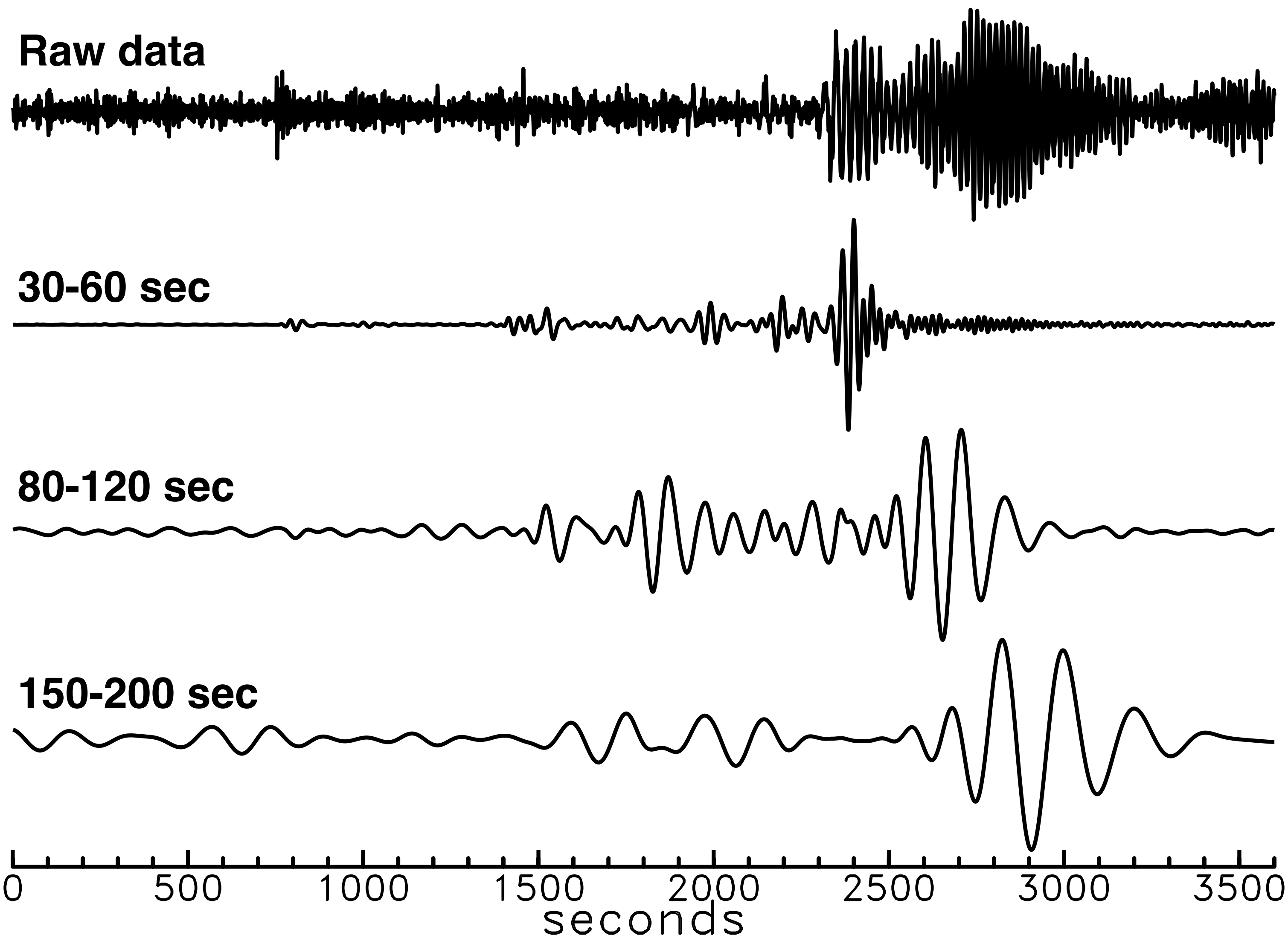
Love wave



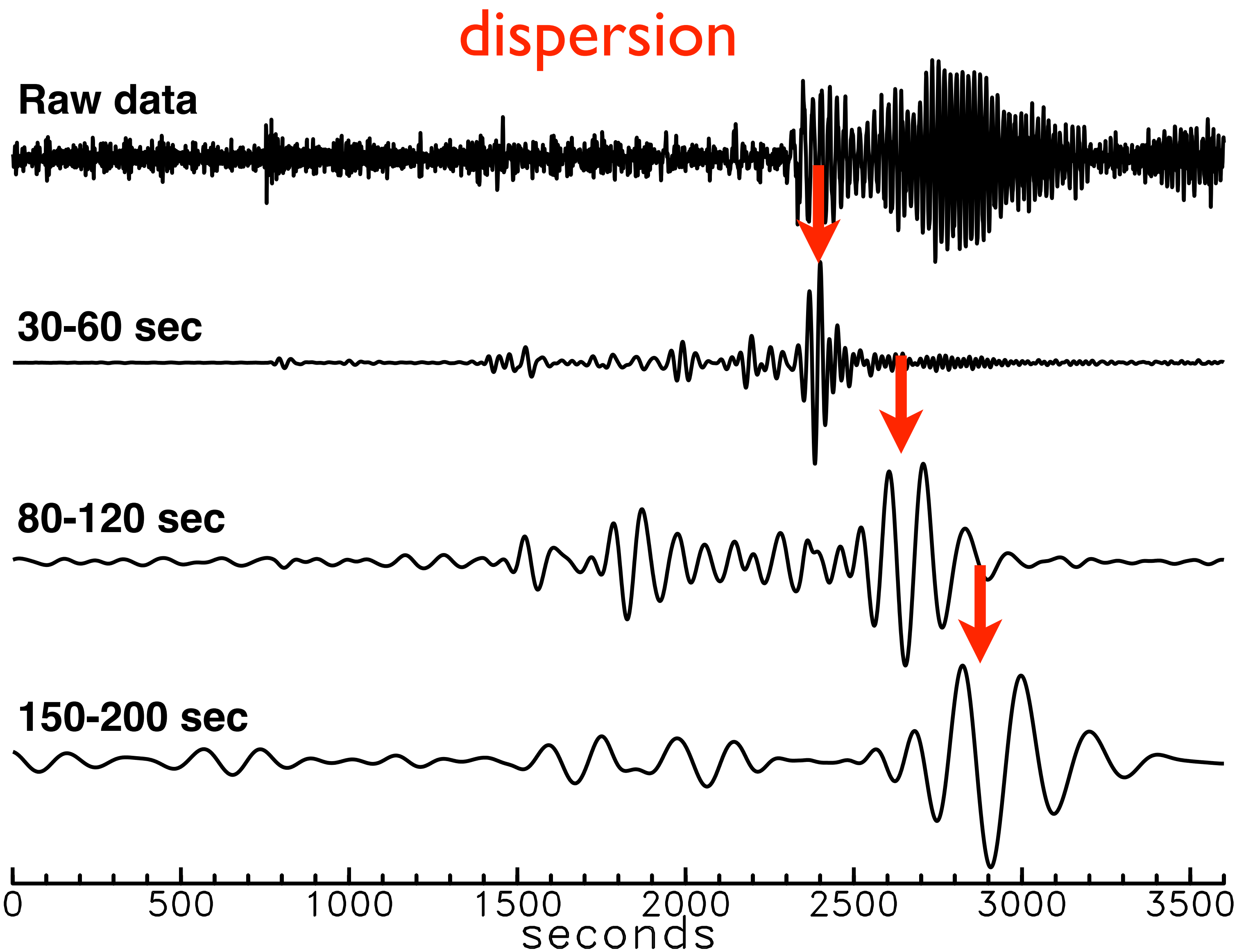
Travel time curve for horizontal rays



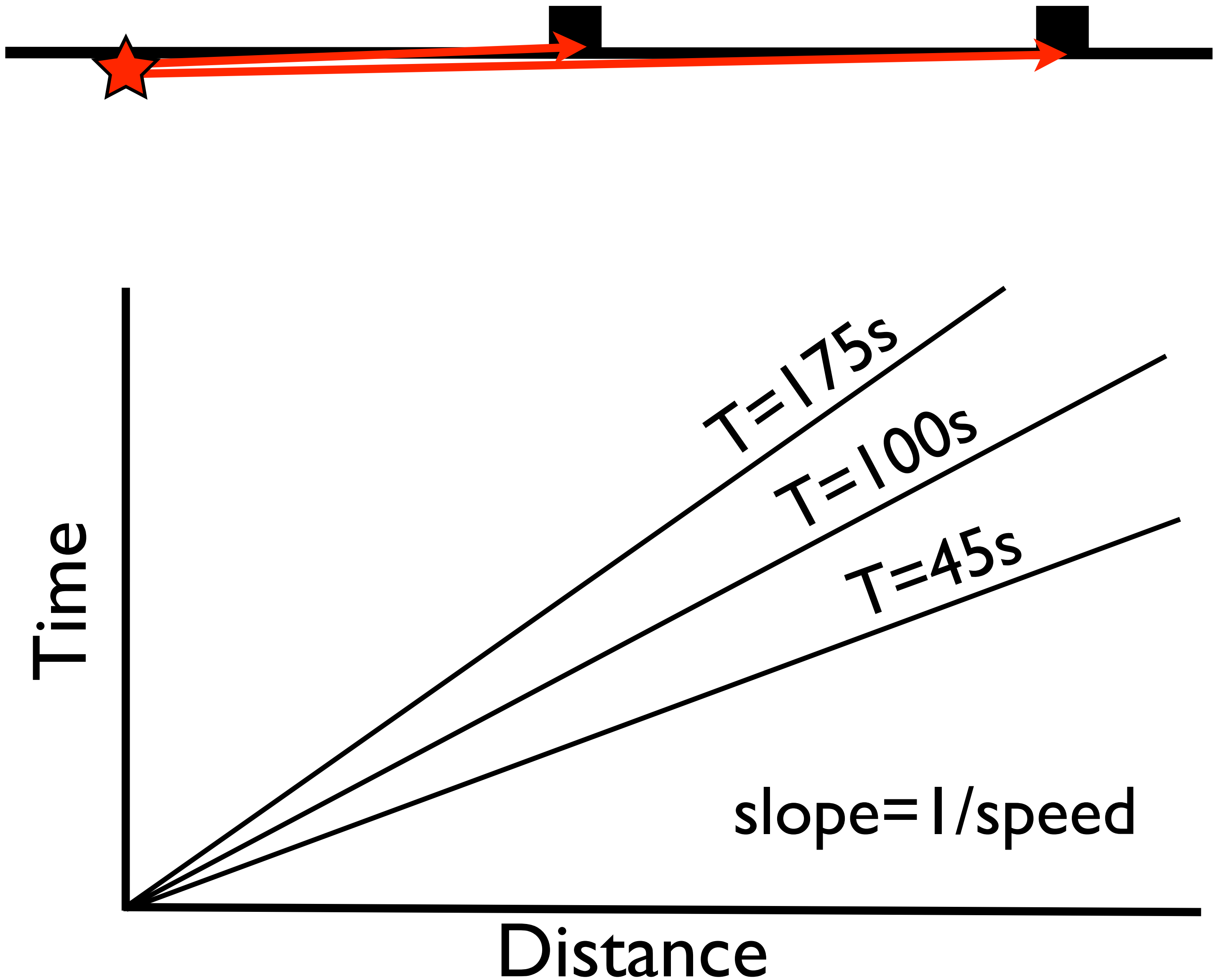
Kermadec to Pasadena, $\Delta = 85^{\circ}$



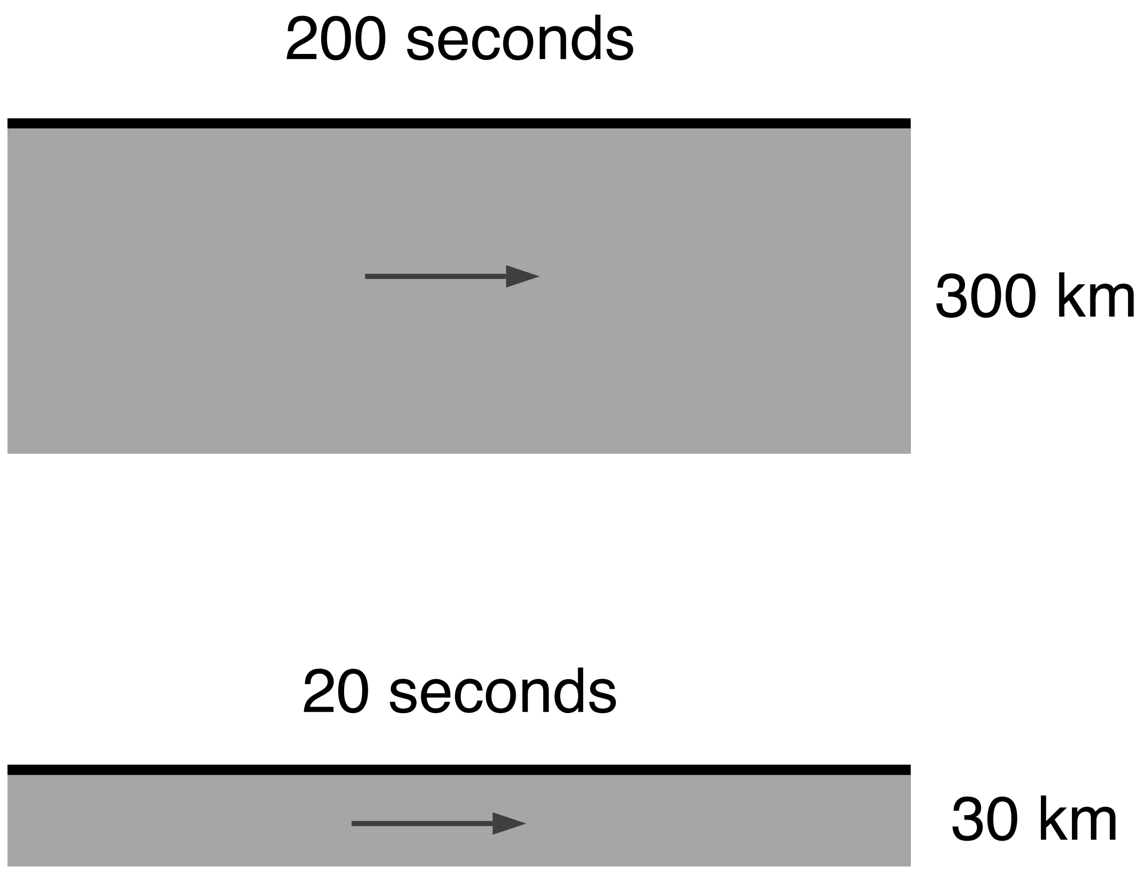
Kermadec to Pasadena, $\Delta = 85^\circ$



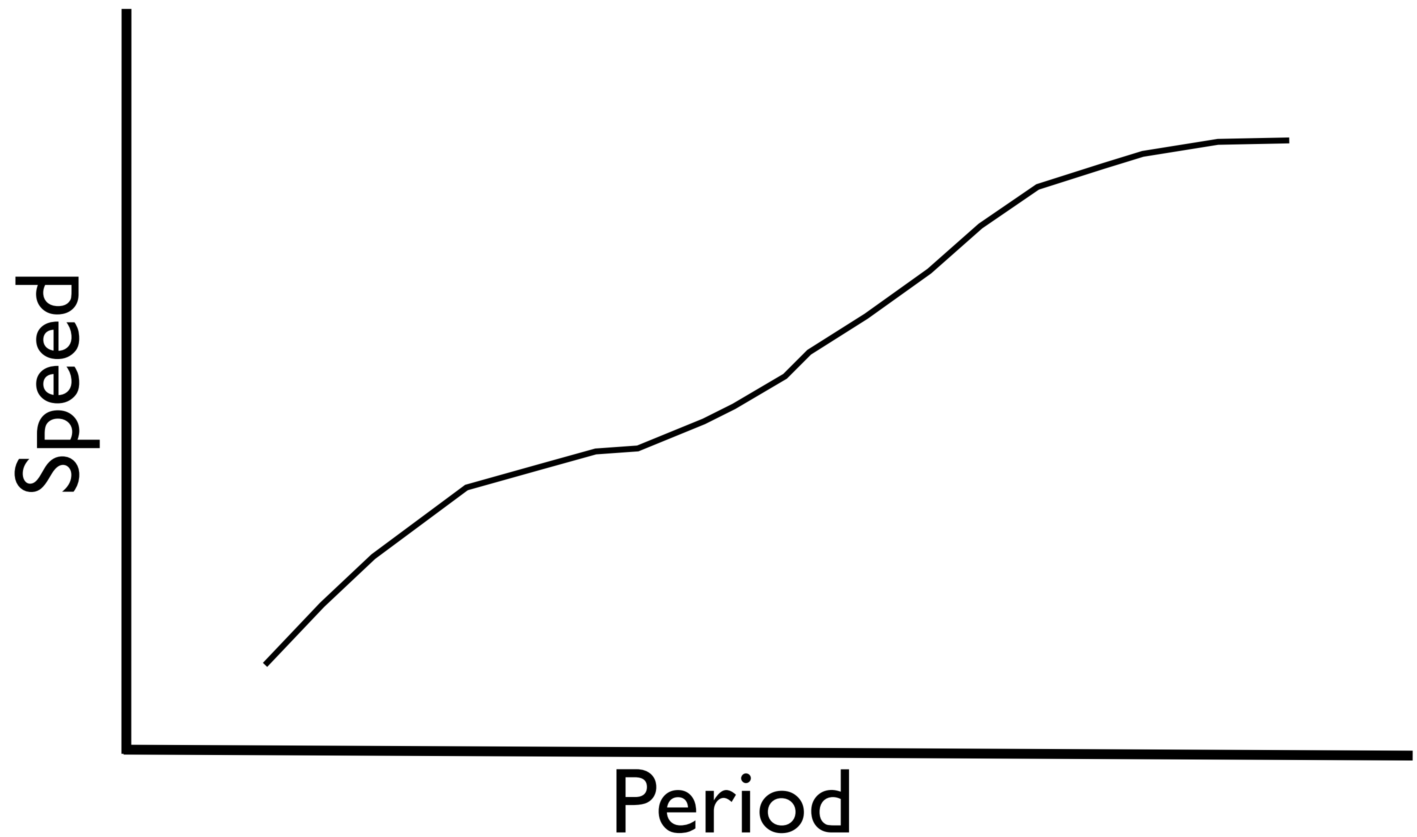
Waves of different periods travel at different speeds



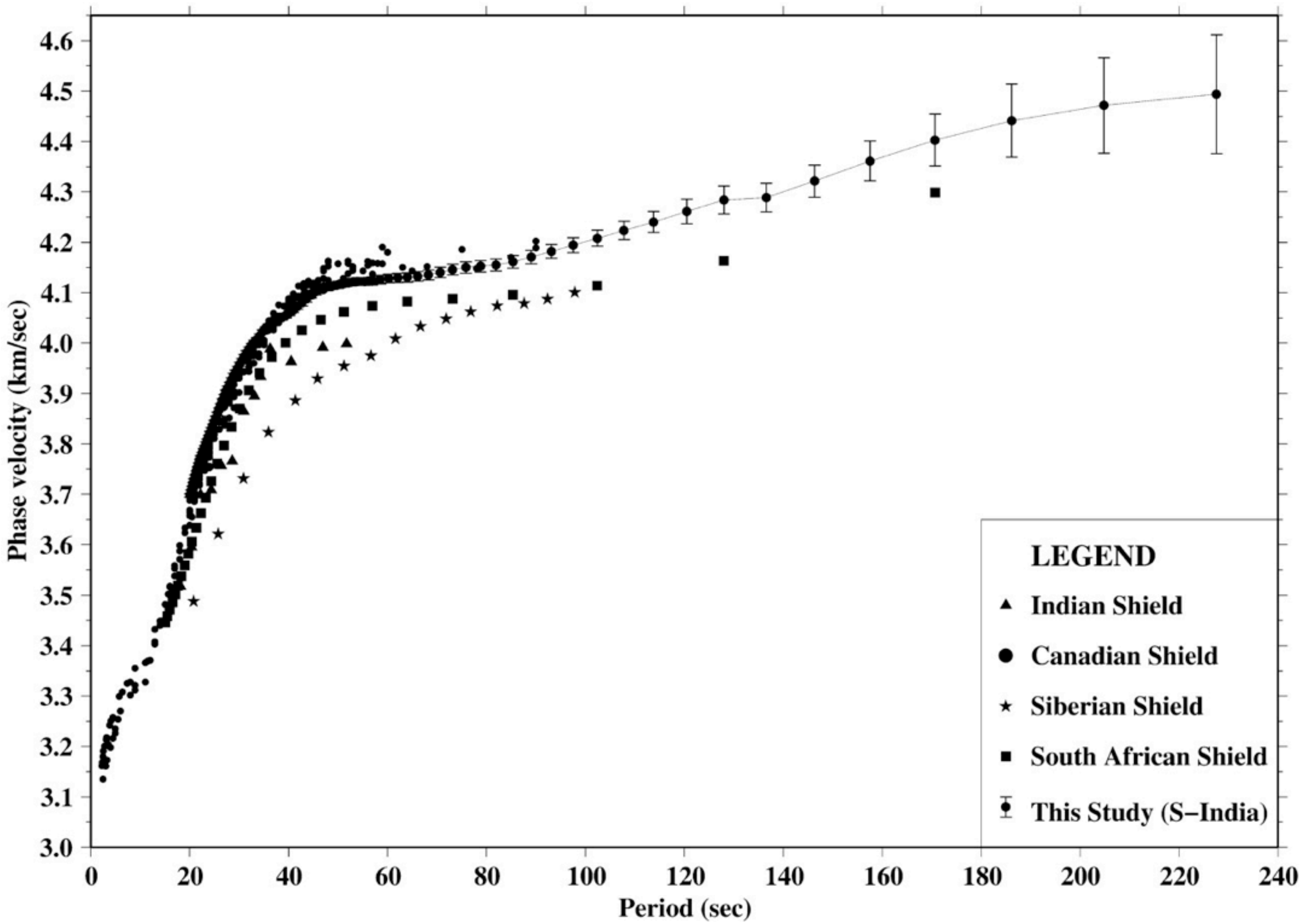
Sensitivity of surface wave velocities to elastic stucture at depth



Dispersion curve for an elastic Earth profile



Measured disperion curve (example)



Mitra et al., 2006

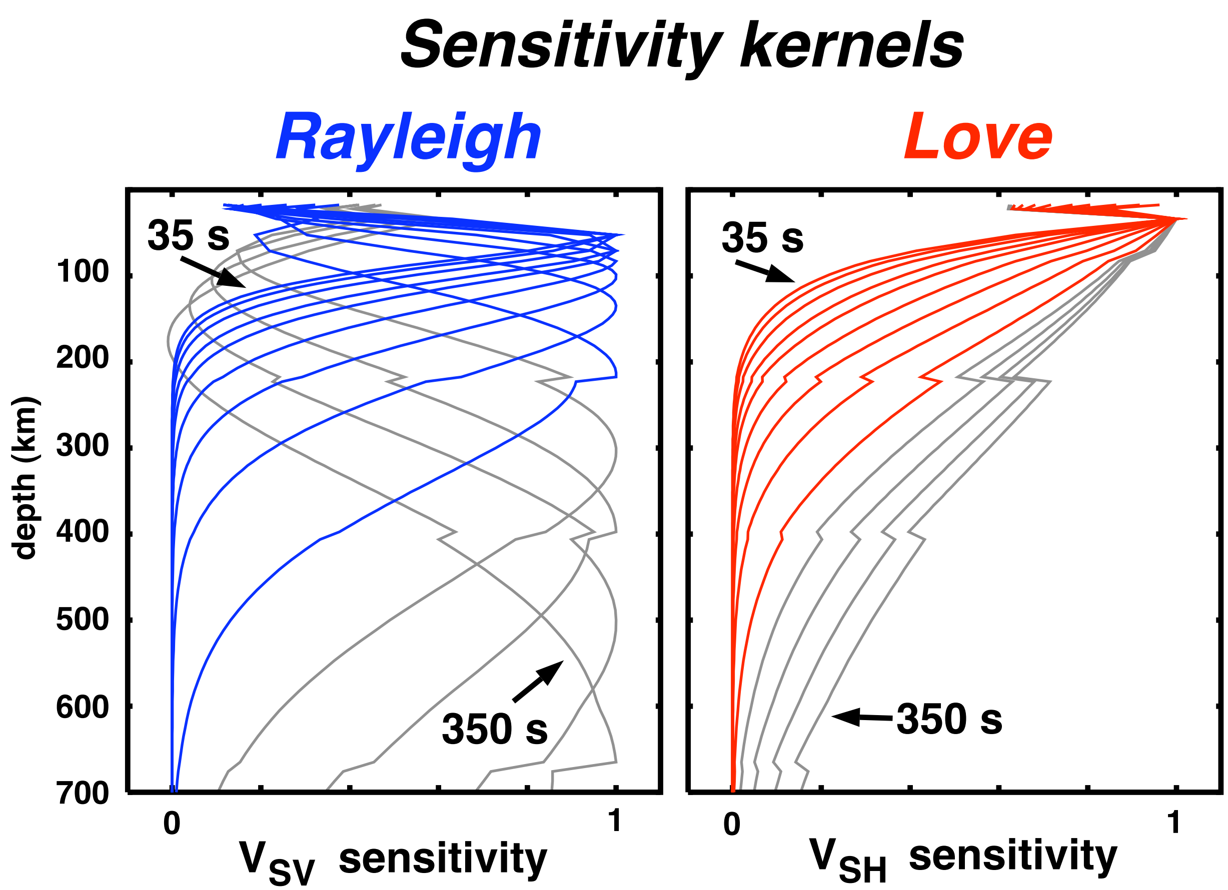


Figure 2.1: Sensitivity functions for fundamental-mode Rayleigh and Love waves at the frequencies measured for this work, calculated for the spherically symmetric Earth model PREM (Dziewonski and Anderson, 1981). The bulk of the dataset consists of measurements at 35–150 s, made using the method of Ekström et al. (1997); kernels for these periods are shown in blue and red. A smaller set of phase-velocity measurements at longer periods (200–350 s) made by Nettles et al. (2000) is also included; kernels for these periods are shown in grey.

Nettles, 2005

Basics Concepts of Seismology

Standing waves, normal modes

Great Circle Wave Propagation

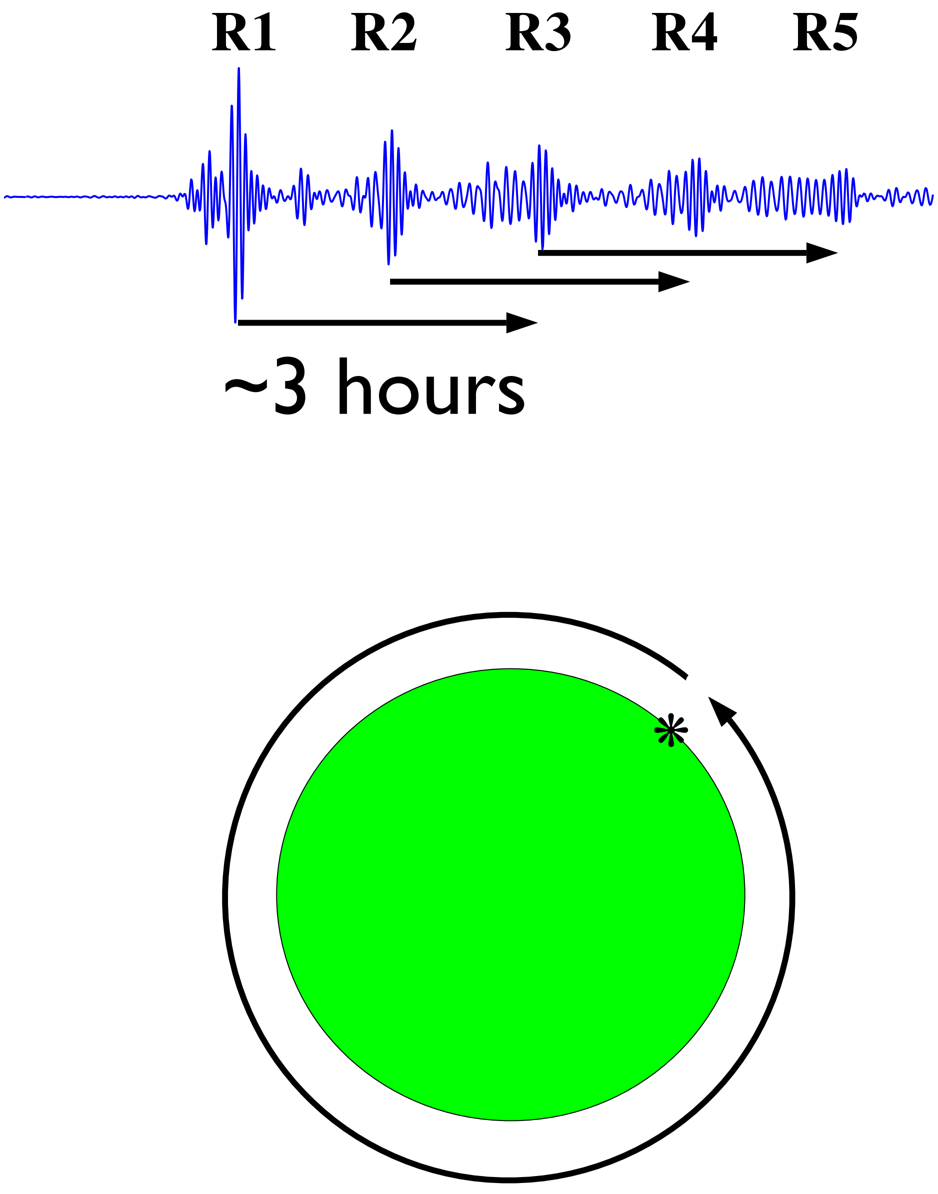
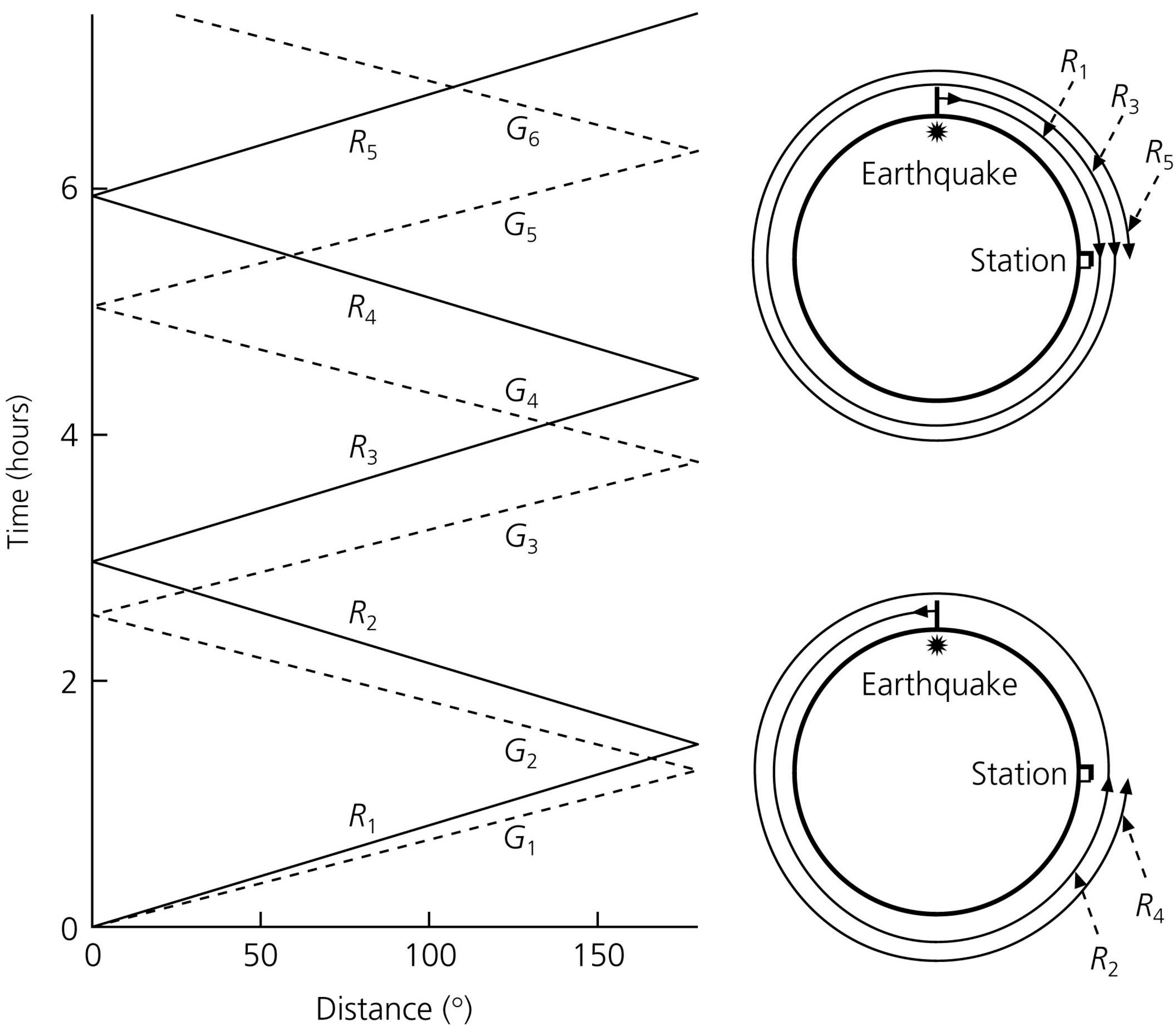
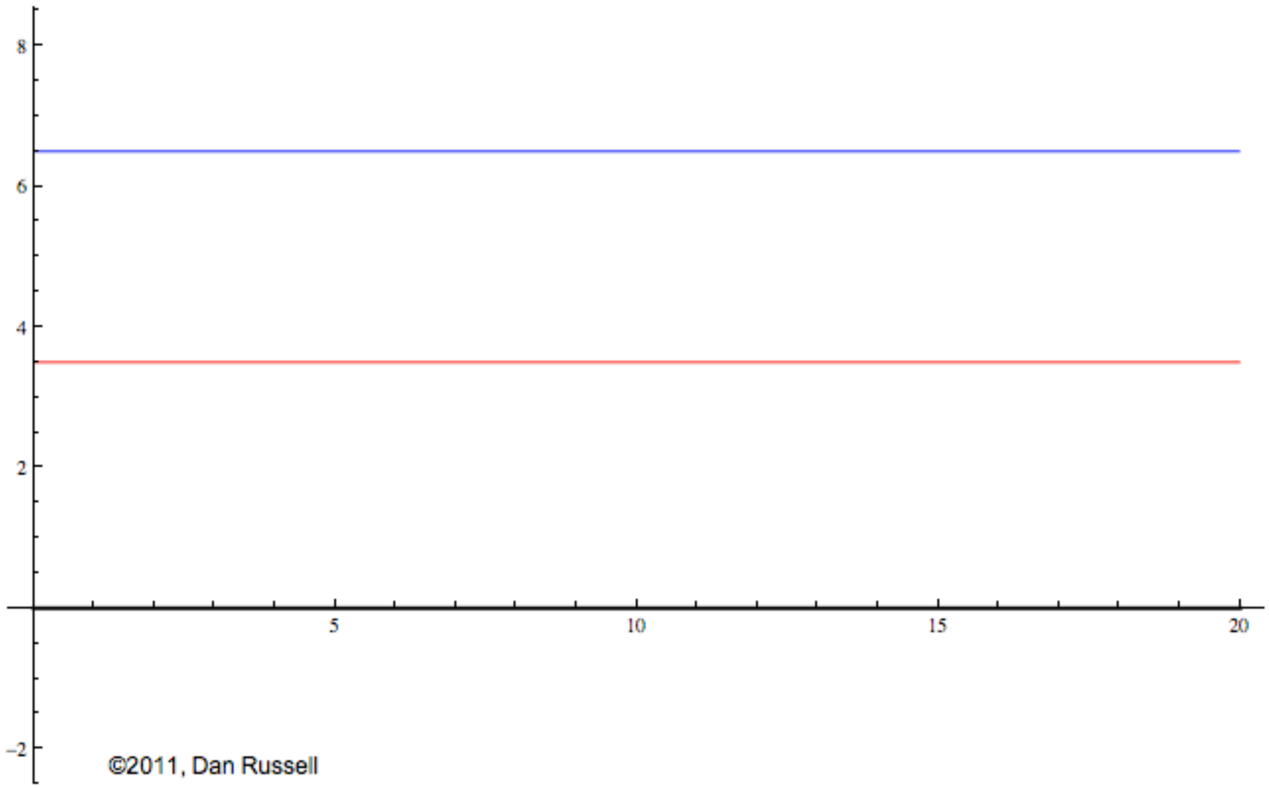


Figure 2.7-3: Multiple surface waves circle the earth.



Standing wave as result of traveling-wave interference



Animation courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State

Back to the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2}$$

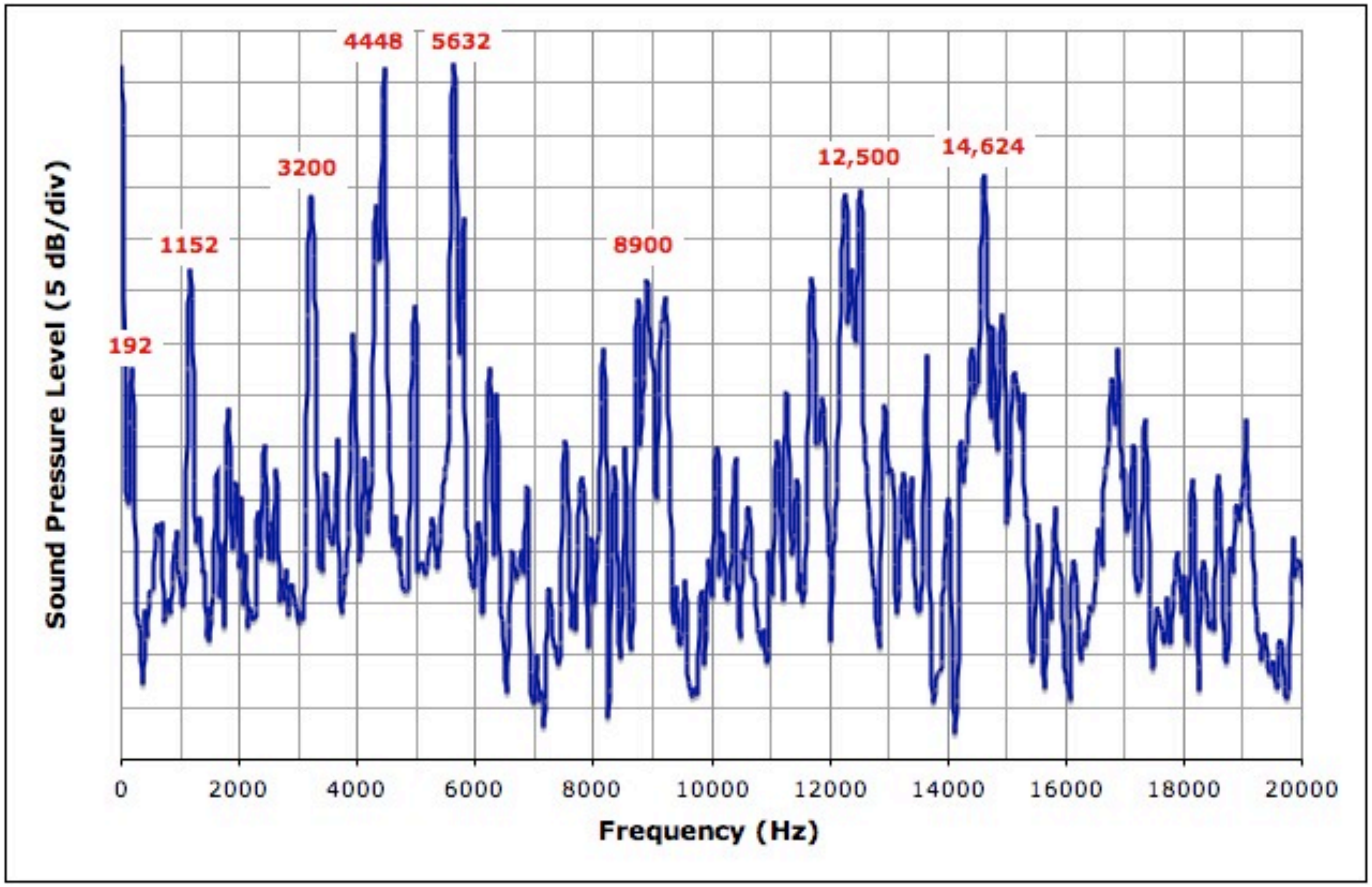
This equation has standing-wave mode solutions:

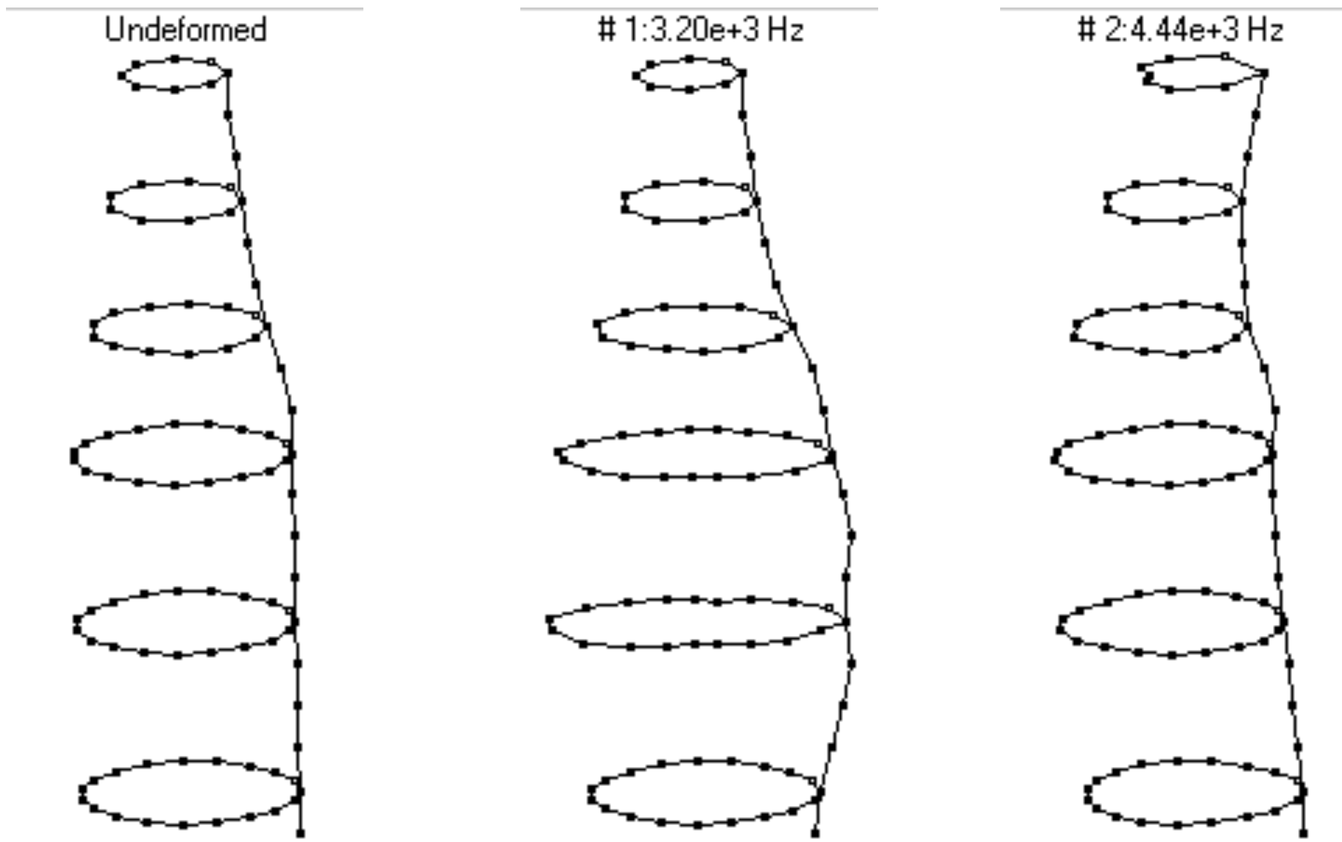
$$U_n(x)\cos(\omega_n t)$$

Any motion can be represented as a sum modes:

$$u(x,t) = \sum_{n=1}^{N} a_n U_n(x)\cos(\omega_n t)$$

Frequency spectrum of a beer bottle



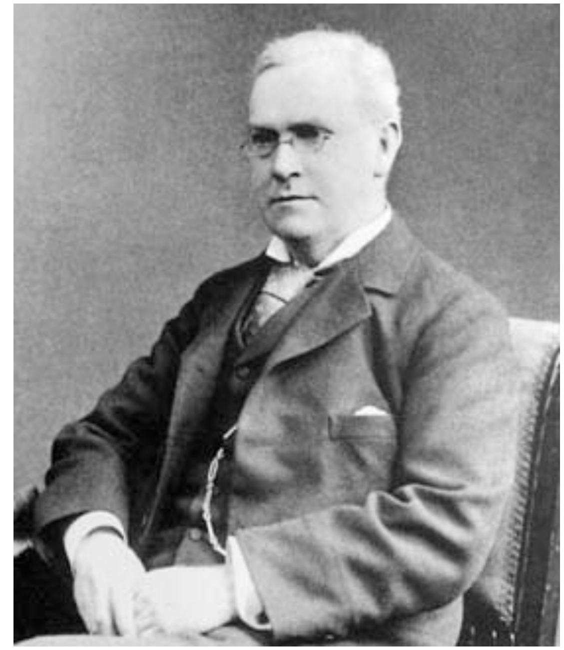


Animation courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State

On the Vibrations of an Elastic Sphere. *By* HORACE LAMB, M.A.

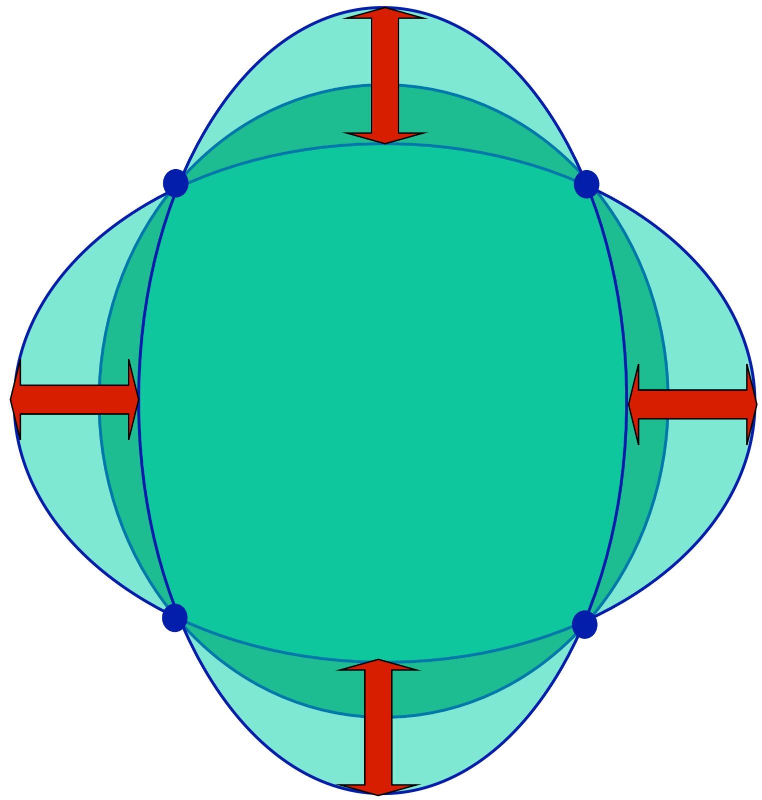
[*Read May* 11*th*, 1882.]

The following paper contains an examination into the nature of the fundamental modes of vibration of an elastic sphere by the method employed in a previous communication, "On the Oscillations of a Viscous Spheroid."* The problem here considered is one of considerable theoretic interest, being as yet the only case in which the vibrations of an elastic solid whose dimensions are all finite have been discussed with any attempt at completeness. I have therefore thought it worth while to go into considerable detail in the interpretation of the results, and have endeavoured, by numerical calculations and the construction of diagrams, to present these in as definite and intelligible a form as possible. I find that some of my results (the most important being the general classification of the fundamental modes given in § 5 below) have been already obtained by Paul Jaerisch, of Breslau, in *Crelle*, t. lxxxviii. (1879); but the methods employed, as well as the form in which the results in question are expressed, are entirely different in the two investigations.



Sir Horace Lamb

$_0S_2$ - the football mode

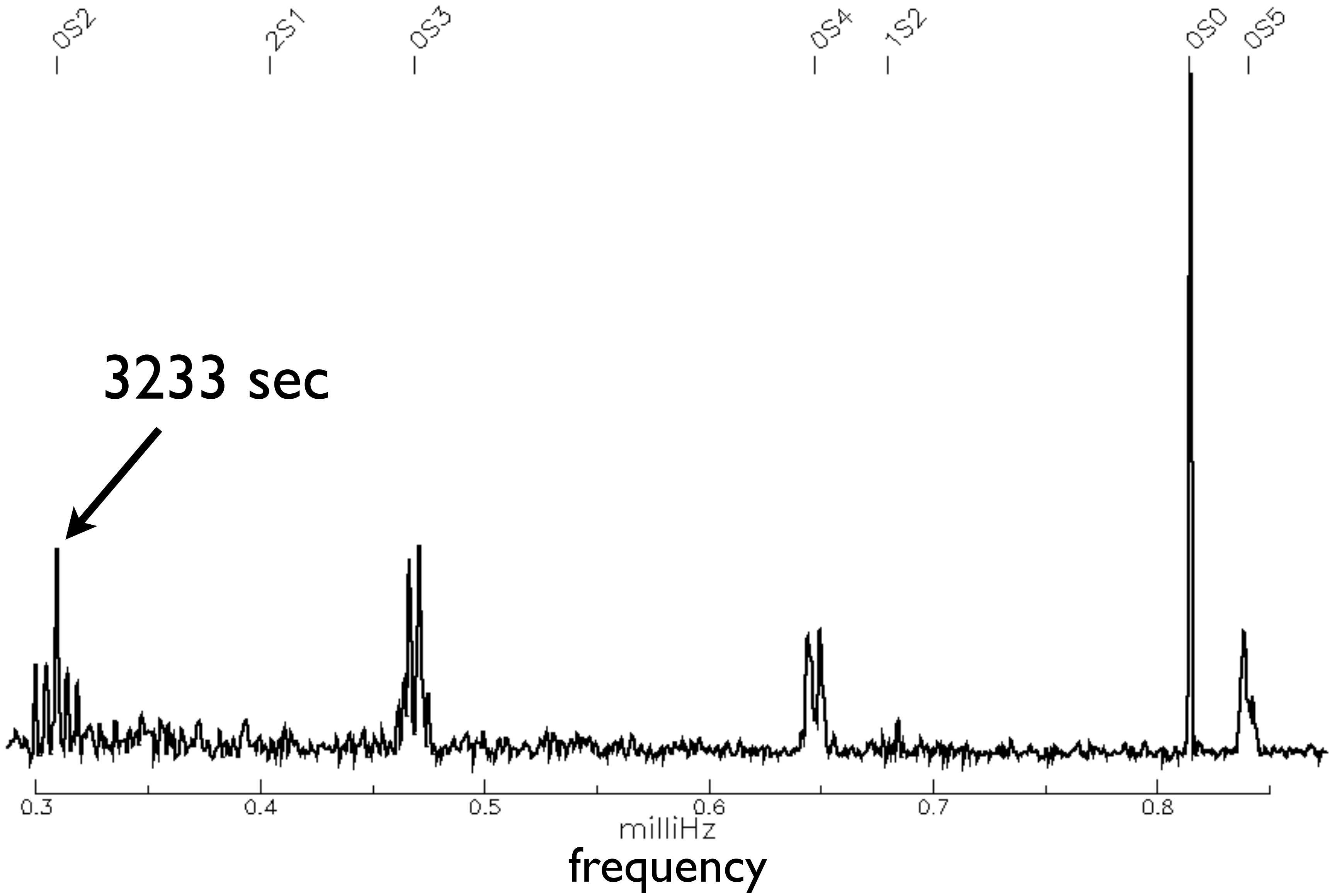


Lamb's estimate of $_0S_2$

13. As an application of the preceding results we may calculate the frequency of vibration of a steel ball one centimetre in radius, for the slowest of those fundamental modes in which the surface oscillates in the form of a harmonic spheroid of the second order. In § 12 we obtained for this case $ka/\pi = \cdot 842$. Now, $ka/\pi = T_0/\tau$, where τ is the period, and $T_0 = 2a/\sqrt{(n\rho^{-1})}$. Making then $a=1$, and adopting from Everett* the values $n = 8 \cdot 19 \times 10^{11}$, $\rho = 7 \cdot 85$, in C. G. S. measure, I find that the frequency $\tau^{-1} = 136000$, about. For a steel globe of any other dimensions, this result must be divided by the radius in centimetres. For a globe of the size of the earth $[a = 6 \cdot 37 \times 10^8]$, I find that the period $\tau = 1$ hr. 18 m.†

$$0.842 \times \text{period} = 2 \times R_0/v_s$$

for steel Earth, 78 minutes



0S2
2S1
0S3
0S4
1S2
0S0
0S5
3233 sec
0.3
0.4
0.5
0.6
0.7
0.8
milliHz
frequency