Essential Planetary Core Fluid Dynamics

CIDER Meeting, KITP 7/10/14

Jonathan Aurnou
UCLA Earth & Space Sciences
aurnou@ucla.edu

IN POST:

- The movies shown at CIDER will be online later this summer at:
  - https://www.youtube.com/user/spinlabucla
- And downloadable from:
  - http://spinlab.ess.ucla.edu/?page_id=311

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core-Style Flows
    - And Beyond

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
    - And Beyond

Dynamos

- Generates a magnetic fields and then continually regenerate that field
- Natural dynamos: galactic, stellar, planetary, asteroidal
    - Magneto-hydrodynamic processes:
        - Convert kinetic energy of flowing metals/plasmas into magnetic energy

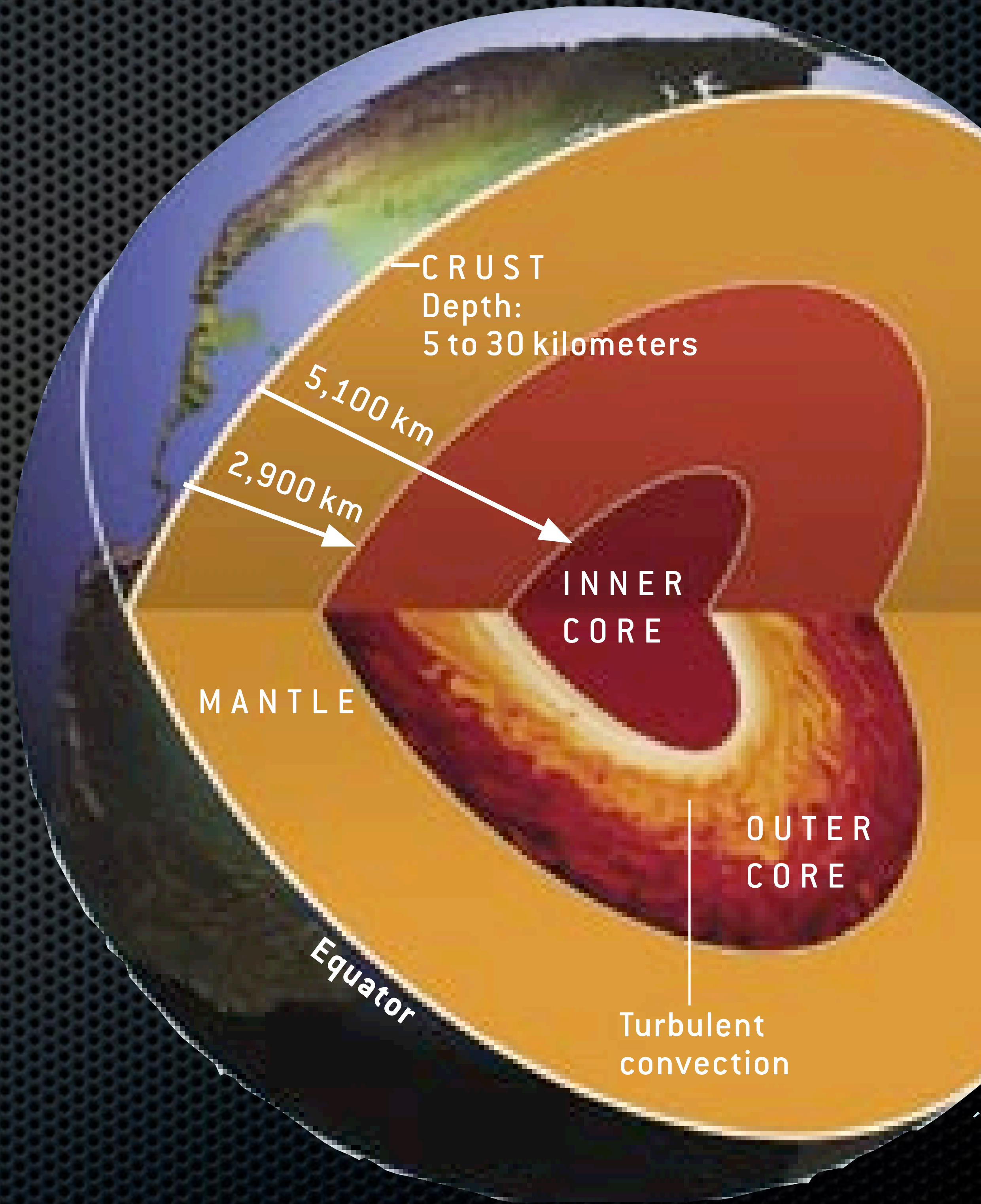
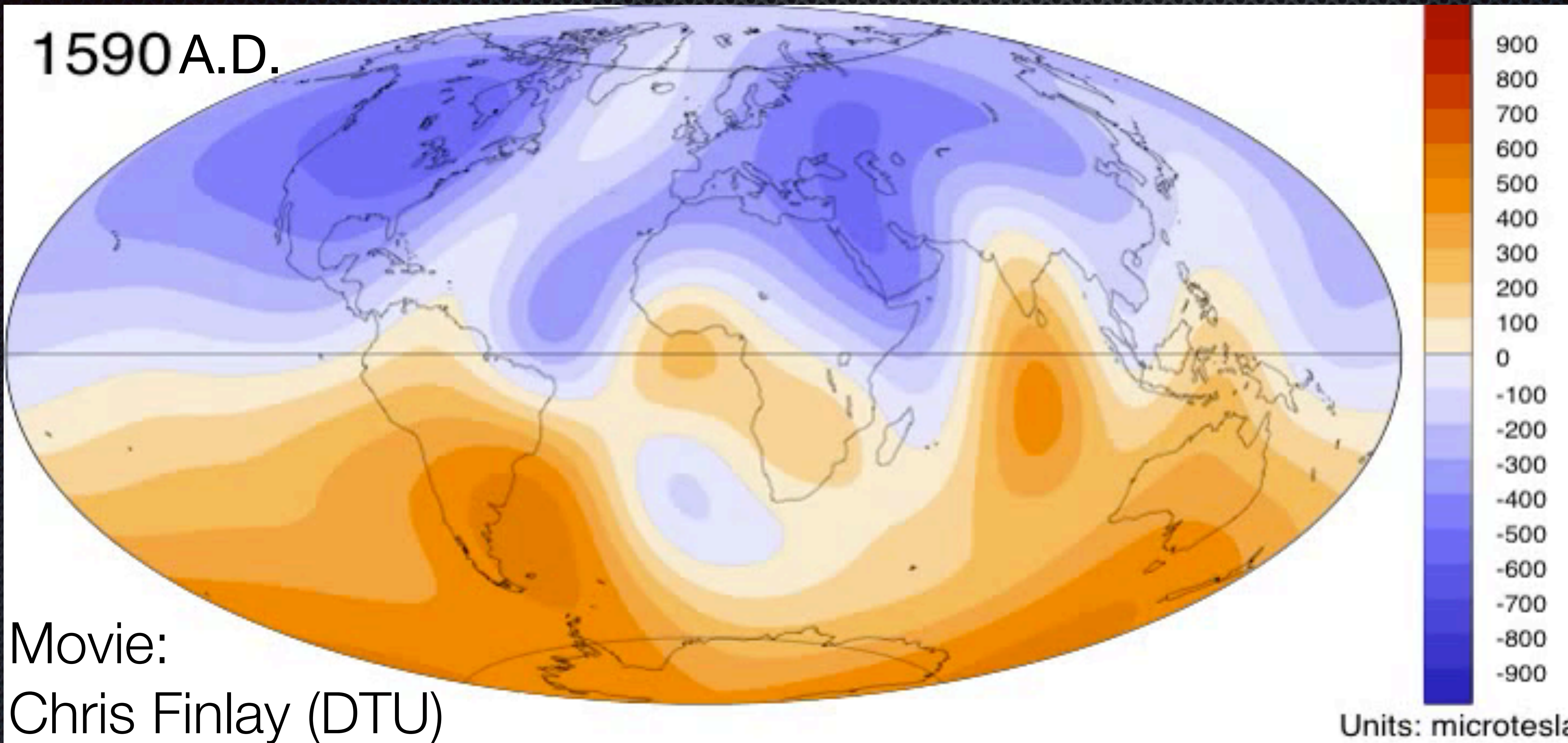


Image: Glatzmaier & Olson, SciAm 05

Core-Mantle Boundary *B_r*



- Field changes over relatively short times
    - Only present explanation: field tied to core fluid motions

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer (Movies)
- Planetary Core Columns
    - And Beyond

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer (Movies)
- Planetary Core Columns
  - And Beyond

Successful Modeling

- Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\frac{1}{\rho_o} \nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o} \mathbf{J} \times \mathbf{B}$$

- Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- Energy Equation

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + S$$

- Current Density

Ampere's Law: $\mathbf{J} = \frac{1}{\mu_o} \nabla \times \mathbf{B}$

- Continuity

$$\nabla \cdot \mathbf{u} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

Successful Modeling

- Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho_o}\nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o}\mathbf{J} \times \mathbf{B}$$

- Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- Energy Equation

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + S$$

- Current Density

Ampere's Law: $\mathbf{J} = \frac{1}{\mu_o}\nabla \times \mathbf{B}$

- Continuity

$$\nabla \cdot \mathbf{u} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

Successful Modeling

- Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho_o}\nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o}\mathbf{J} \times \mathbf{B}$$

- Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- Energy Equation

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + S$$

- Current Density

Ampere's Law: $\mathbf{J} = \frac{1}{\mu_o}\nabla \times \mathbf{B}$

- Continuity

$$\nabla \cdot \mathbf{u} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

Successful Modeling

- Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\frac{1}{\rho_o}\nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o}\mathbf{J} \times \mathbf{B}$$

- Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- Energy Equation

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + S$$

- Current Density

Ampere's Law: $\mathbf{J} = \frac{1}{\mu_o}\nabla \times \mathbf{B}$

- Continuity

$$\nabla \cdot \mathbf{u} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

Successful Modeling

- Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho_o} \nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o} \mathbf{J} \times \mathbf{B}$$

- Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- Energy Equation

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + S$$

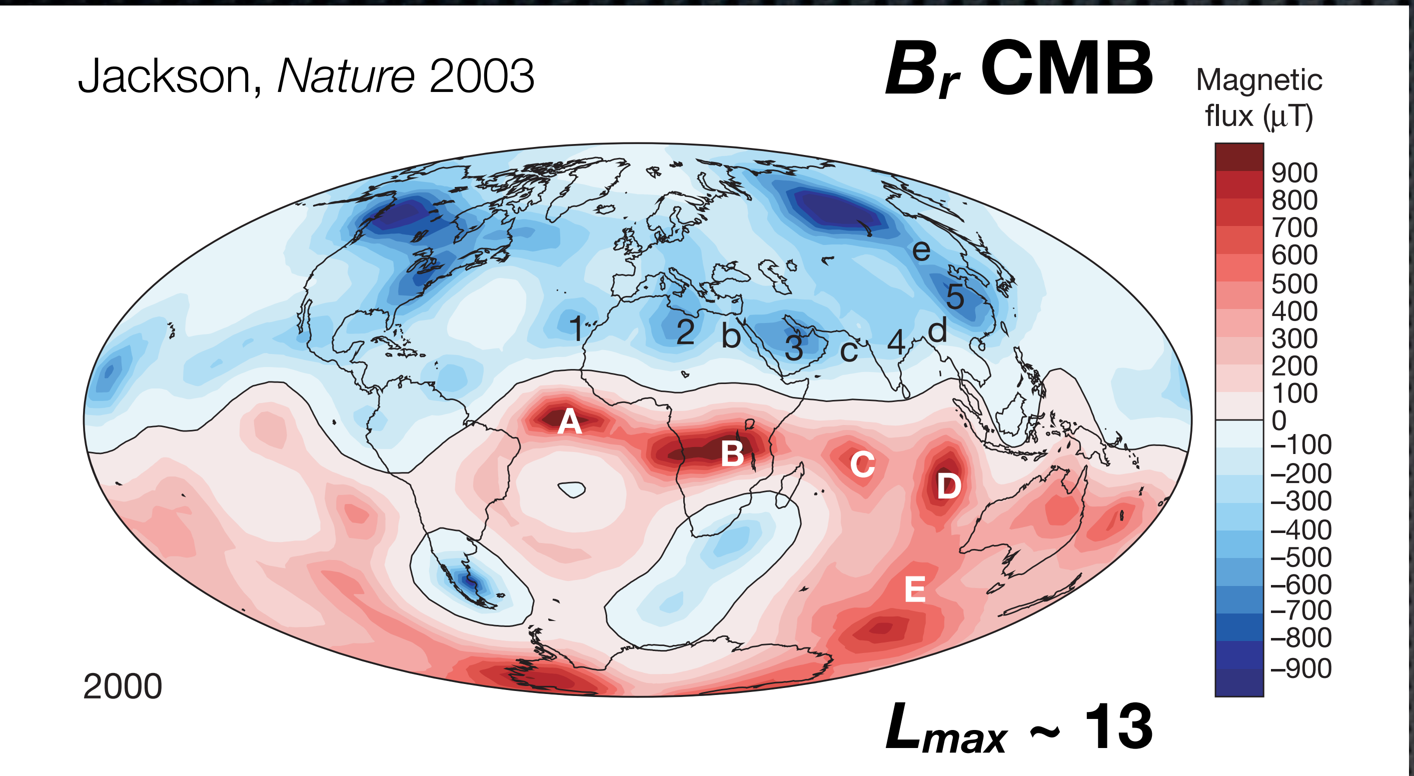
- Current Density

Ampere's Law: $\mathbf{J} = \frac{1}{\mu_o} \nabla \times \mathbf{B}$

- Continuity

$$\nabla \cdot \mathbf{u} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

Observations



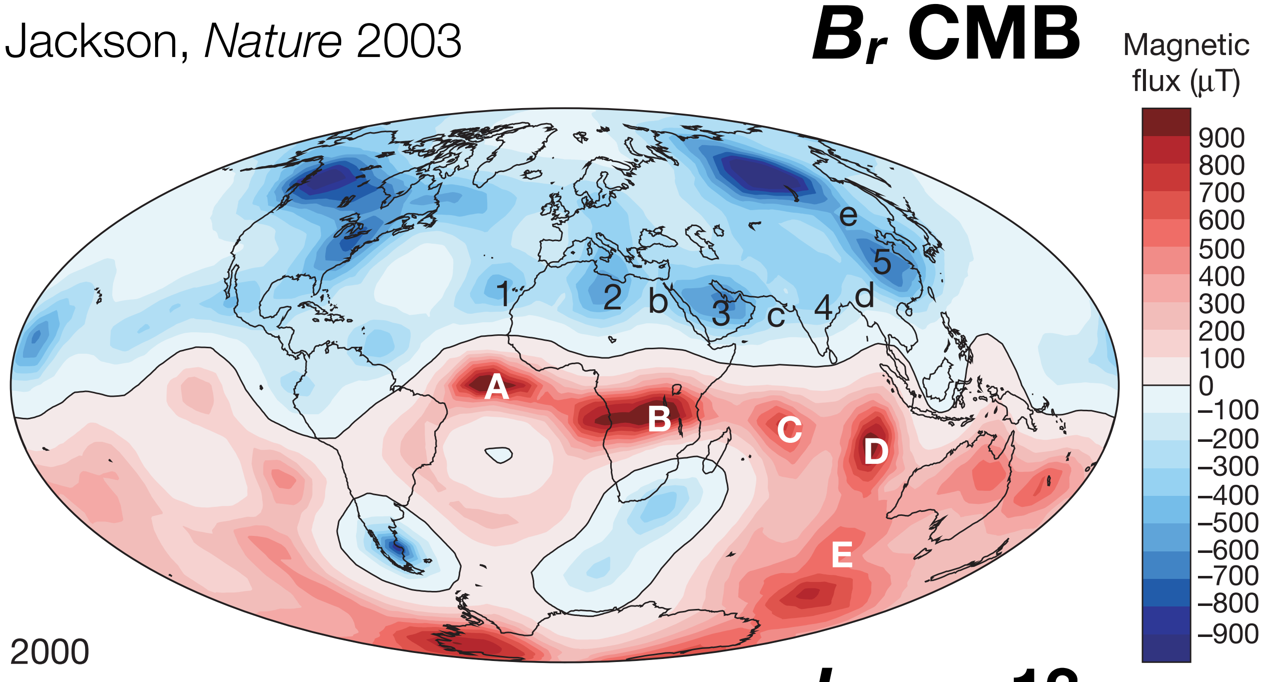
- Axial dipolar field
- High and low latitude large-scale flux patches

Observations

Models

Jackson, *Nature* 2003

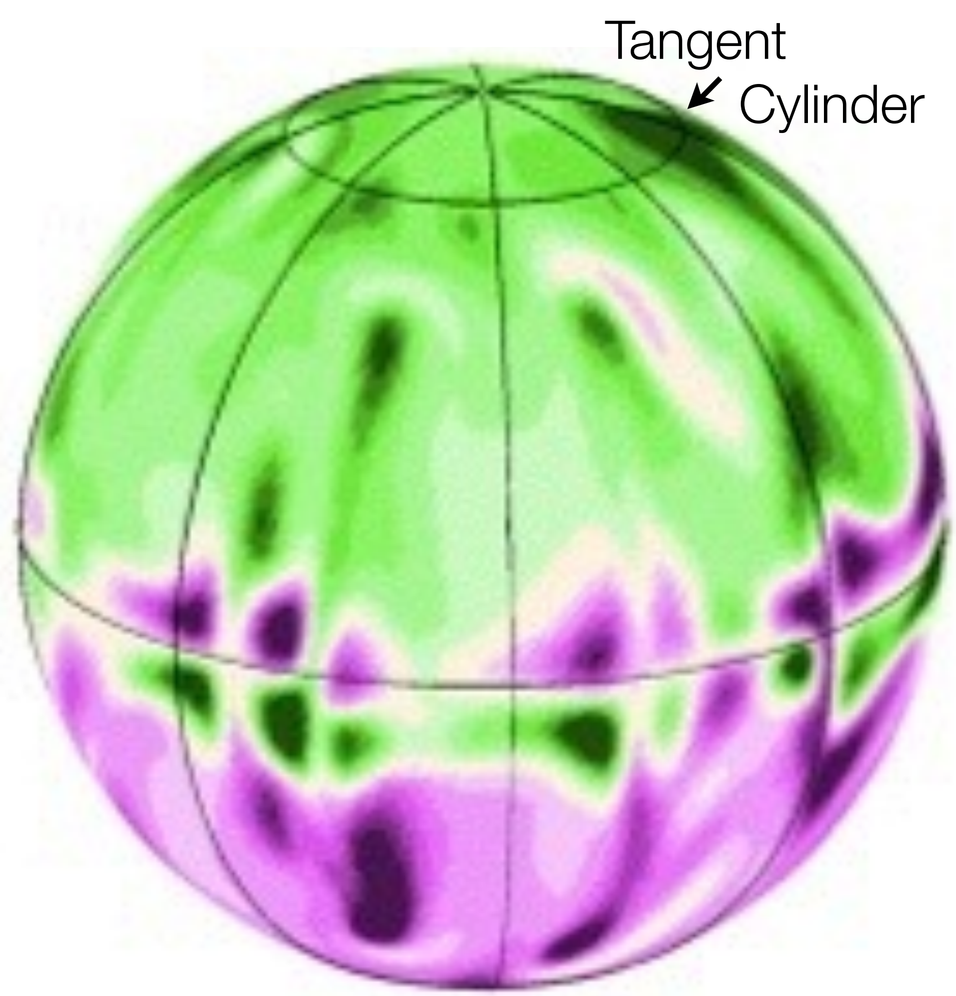
B_r CMB



L_{max} ~ 13

Soderlund et al. *EPSL* 2012

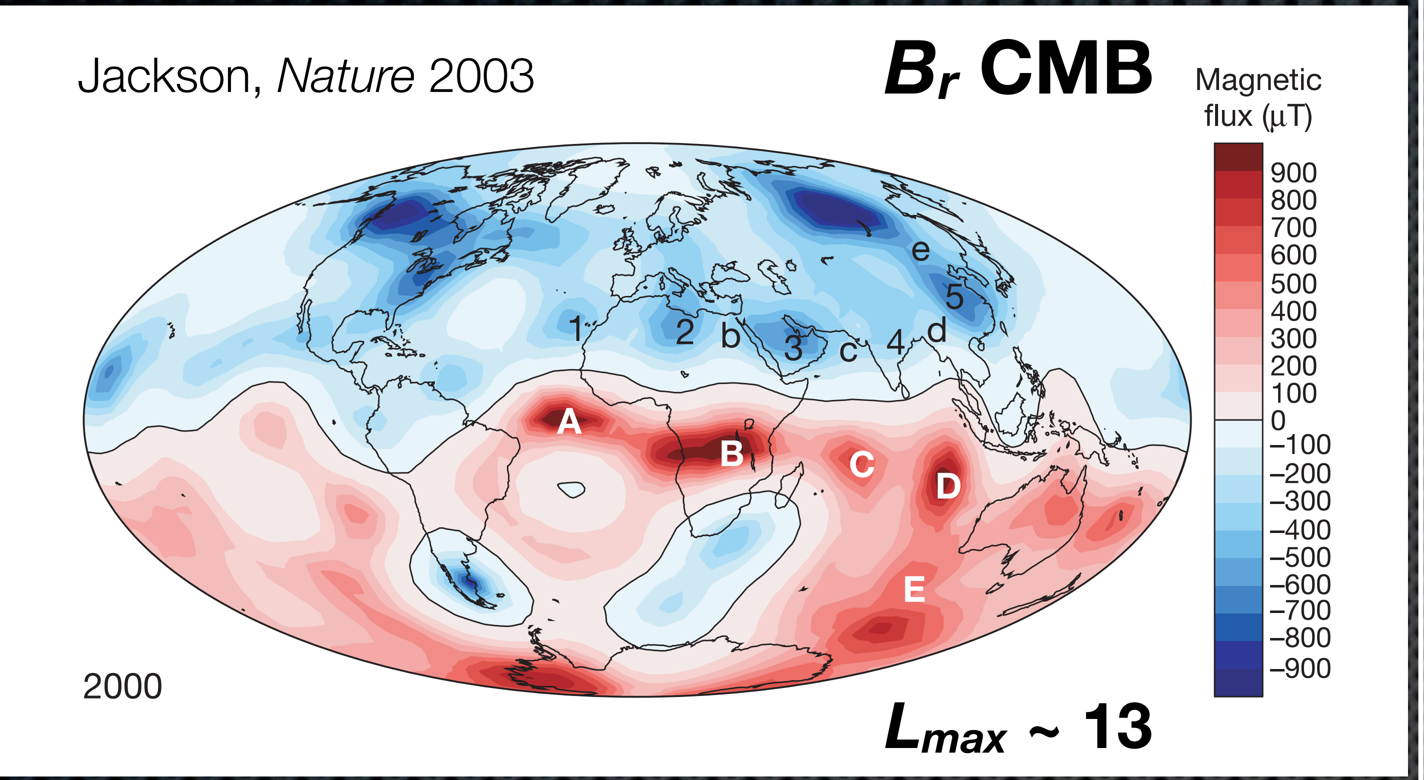
B_r CMB



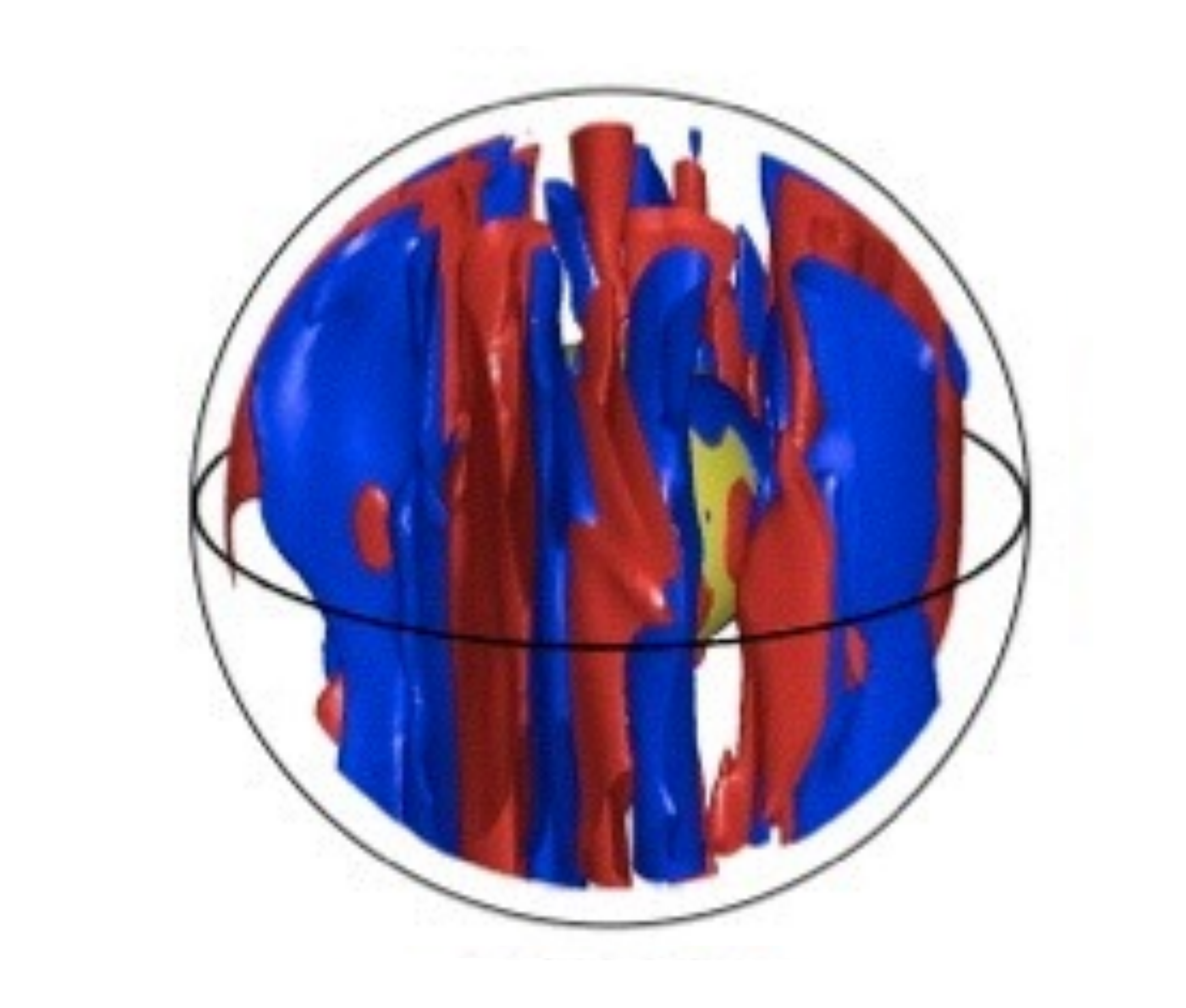
Observations

Models

Jackson, *Nature* 2003

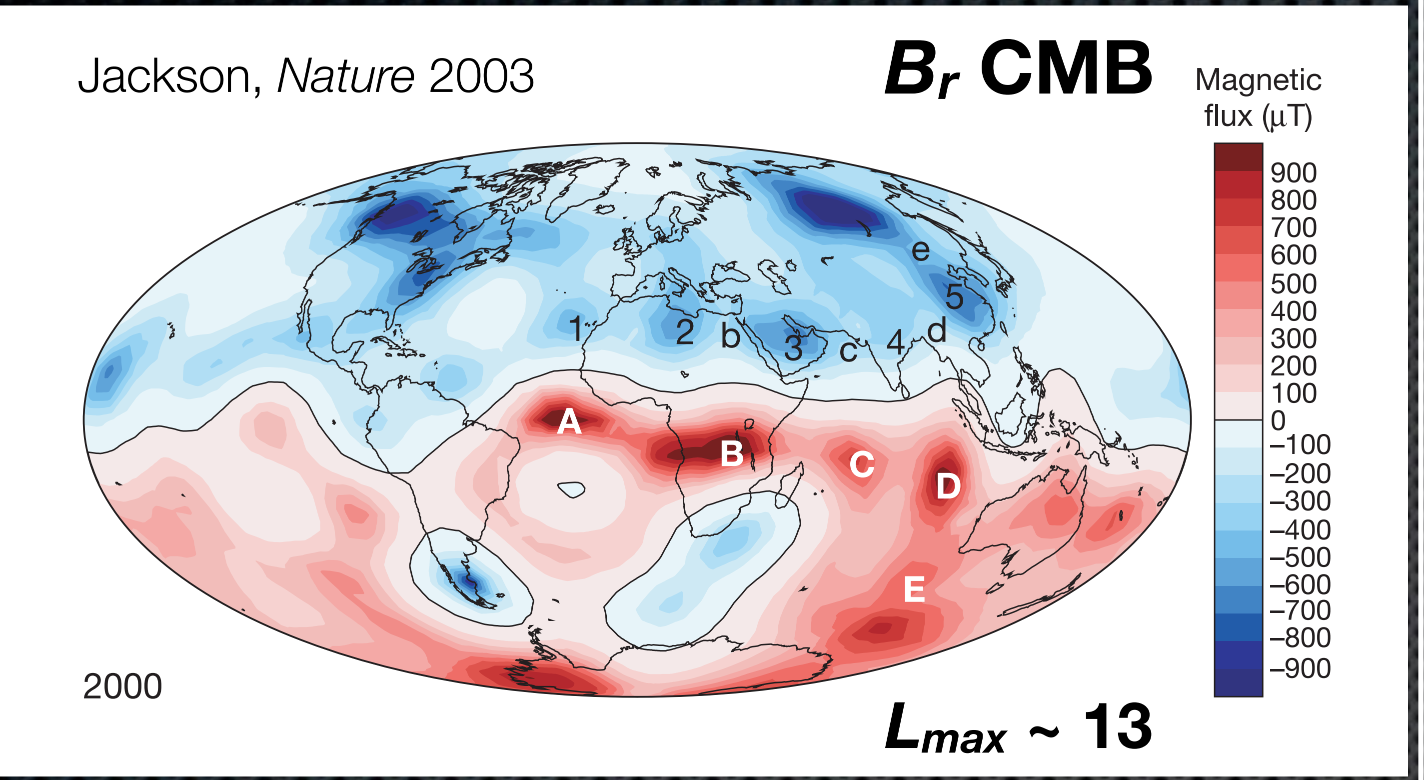


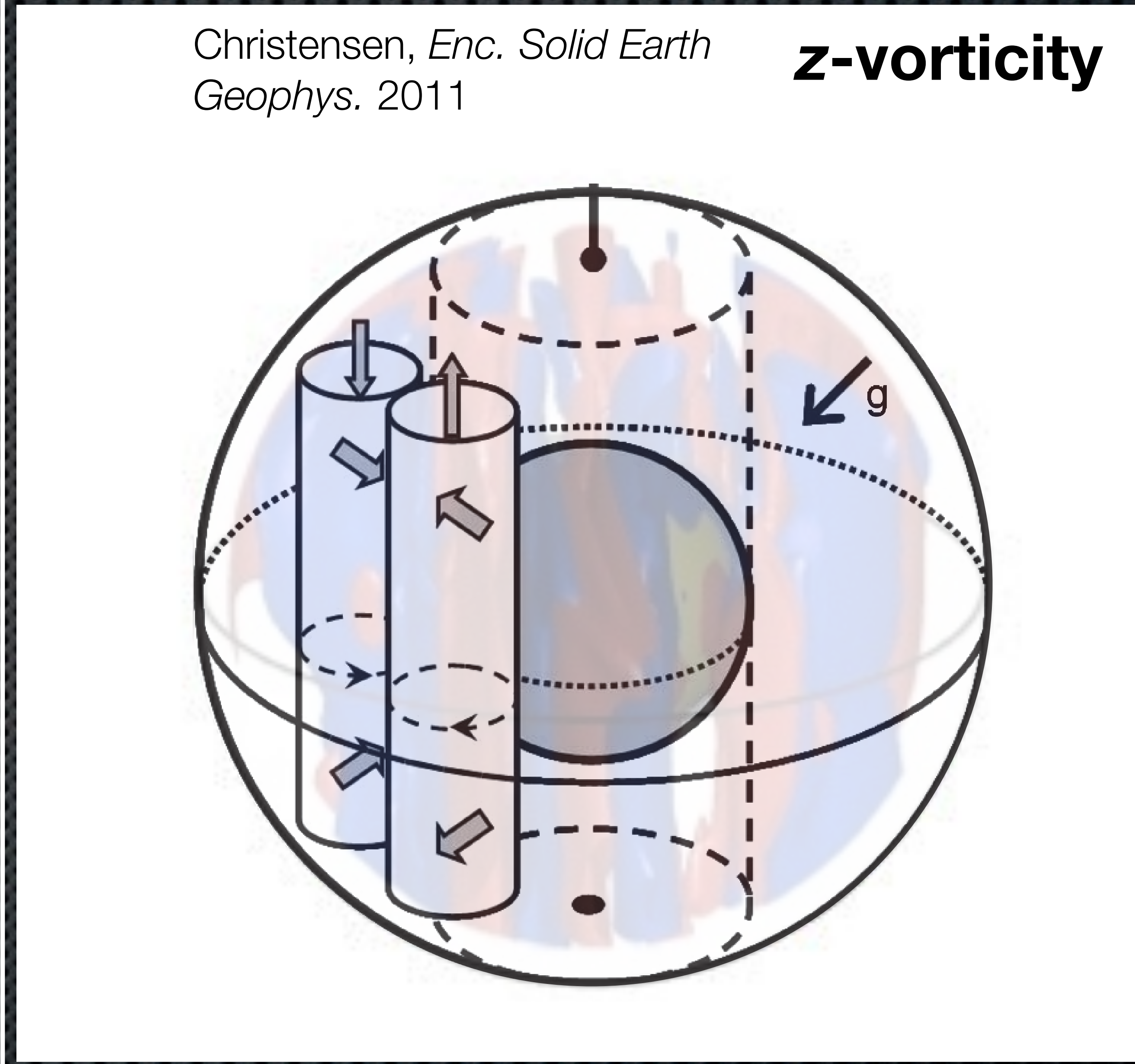
Soderlund et al. *EPSL* 2012



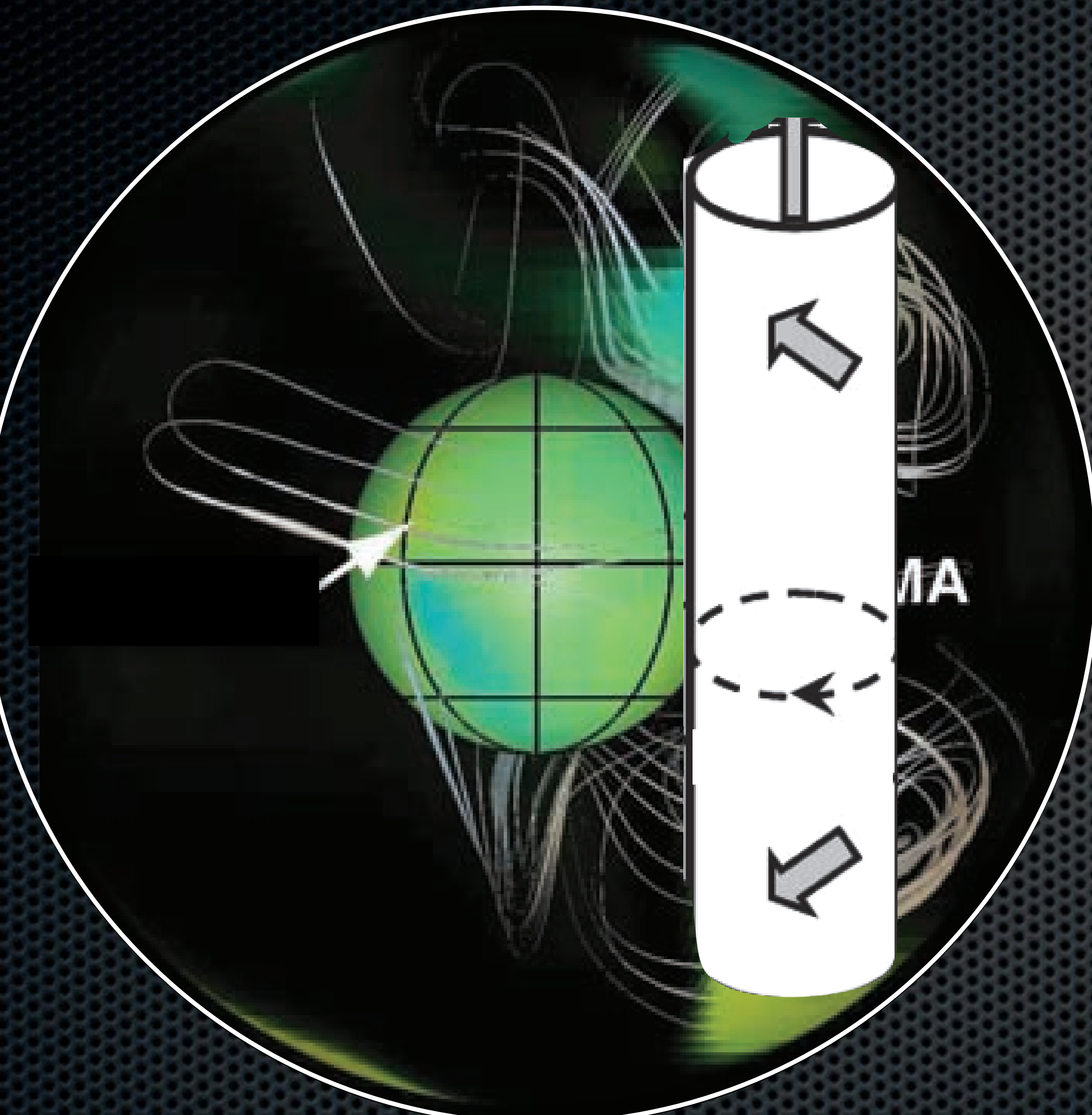
Observations

Models





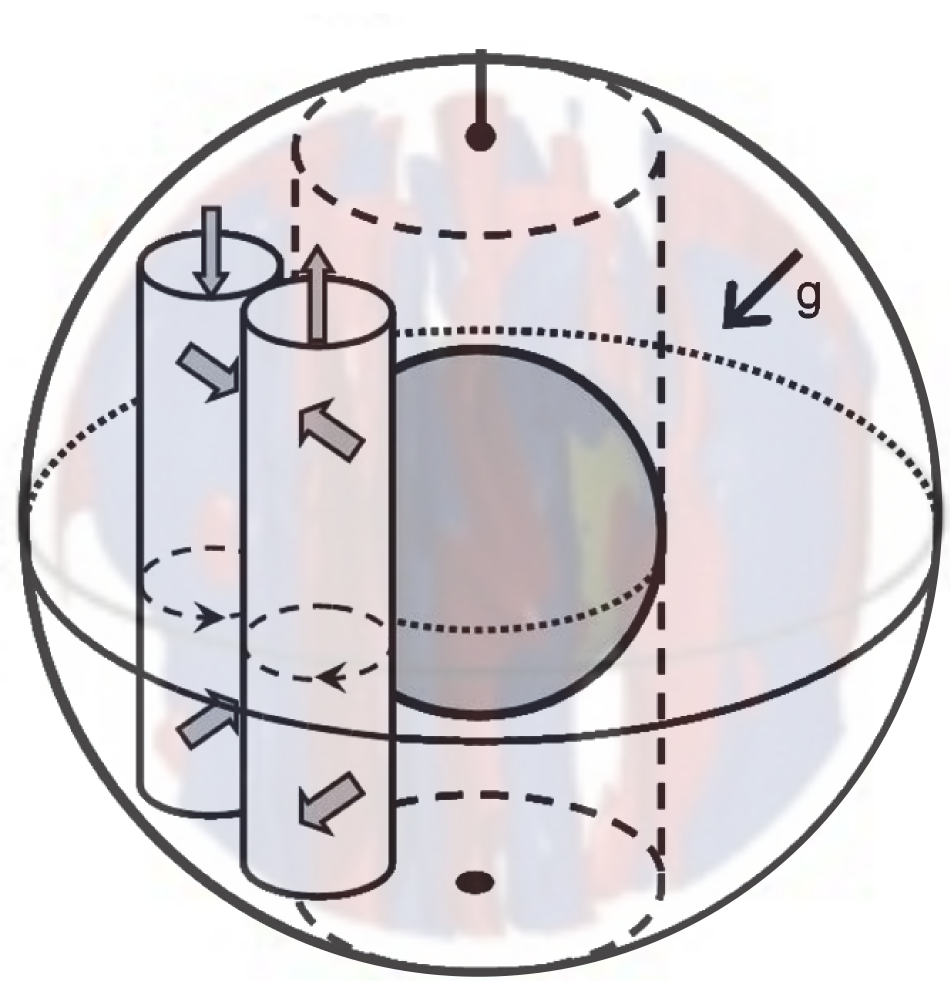
Models



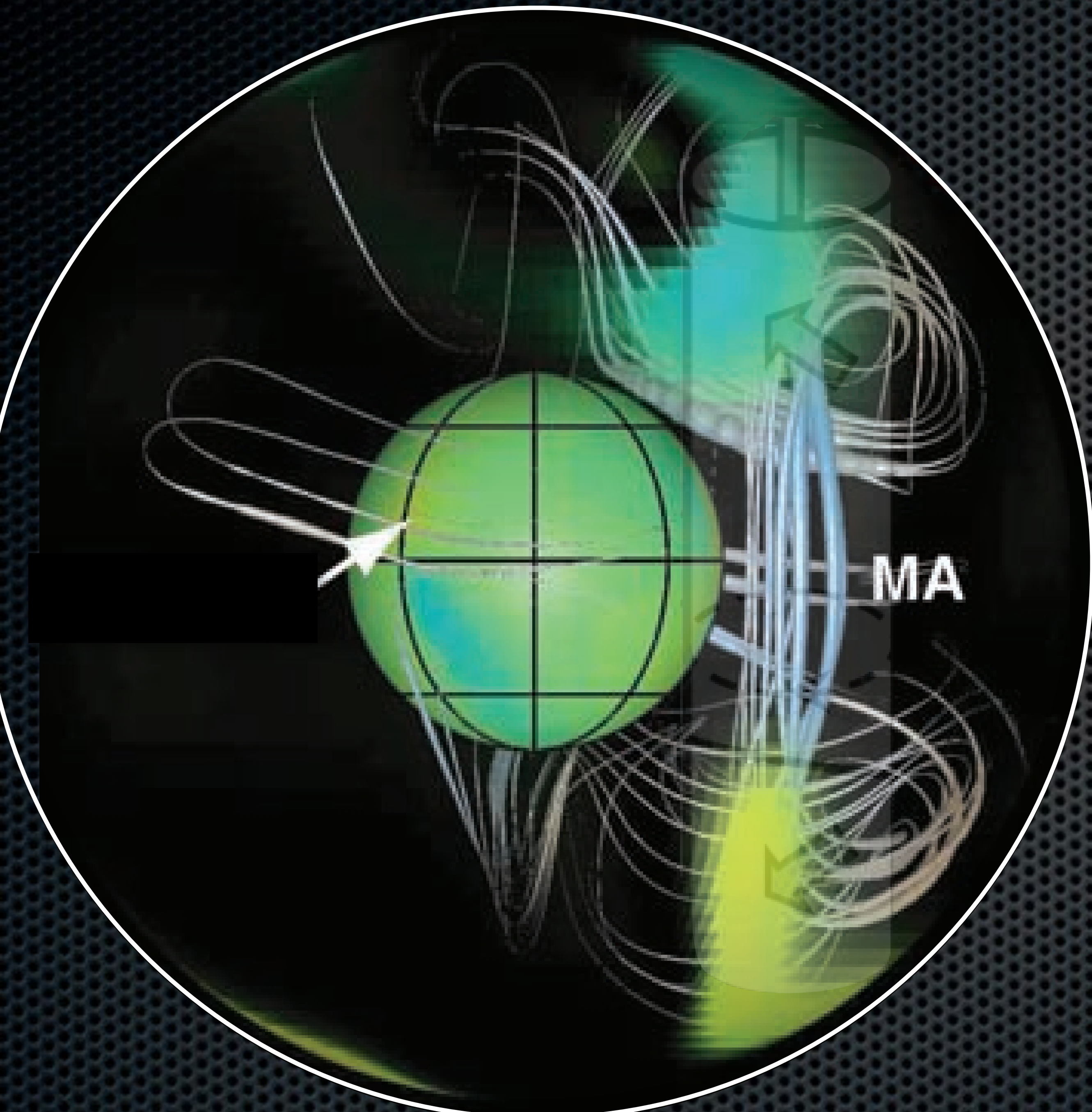
Aubert et al. *GJI* 2008

Christensen, *Enc. Solid Earth Geophys.* 2011

z-vorticity



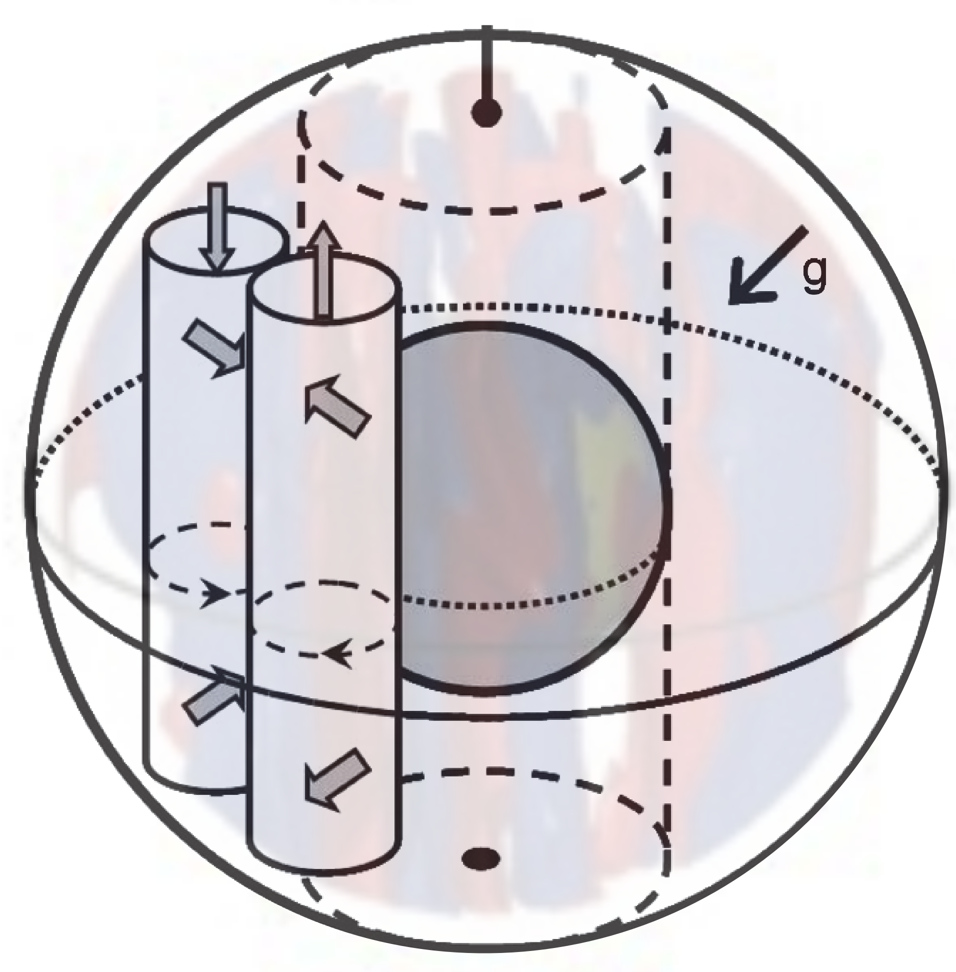
Models



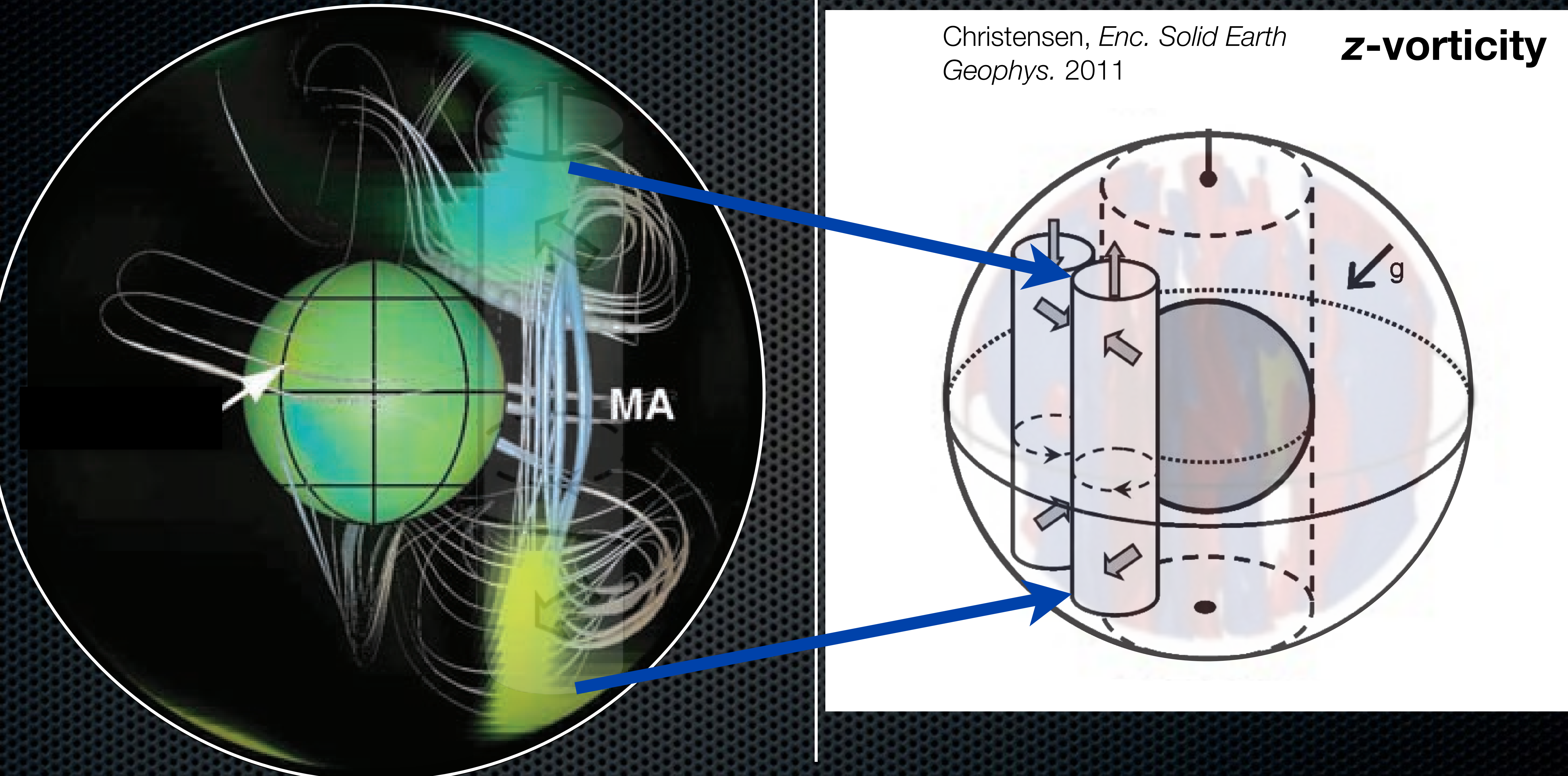
Aubert et al. *GJI* 2008

Christensen, *Enc. Solid Earth Geophys.* 2011

z-vorticity



Models

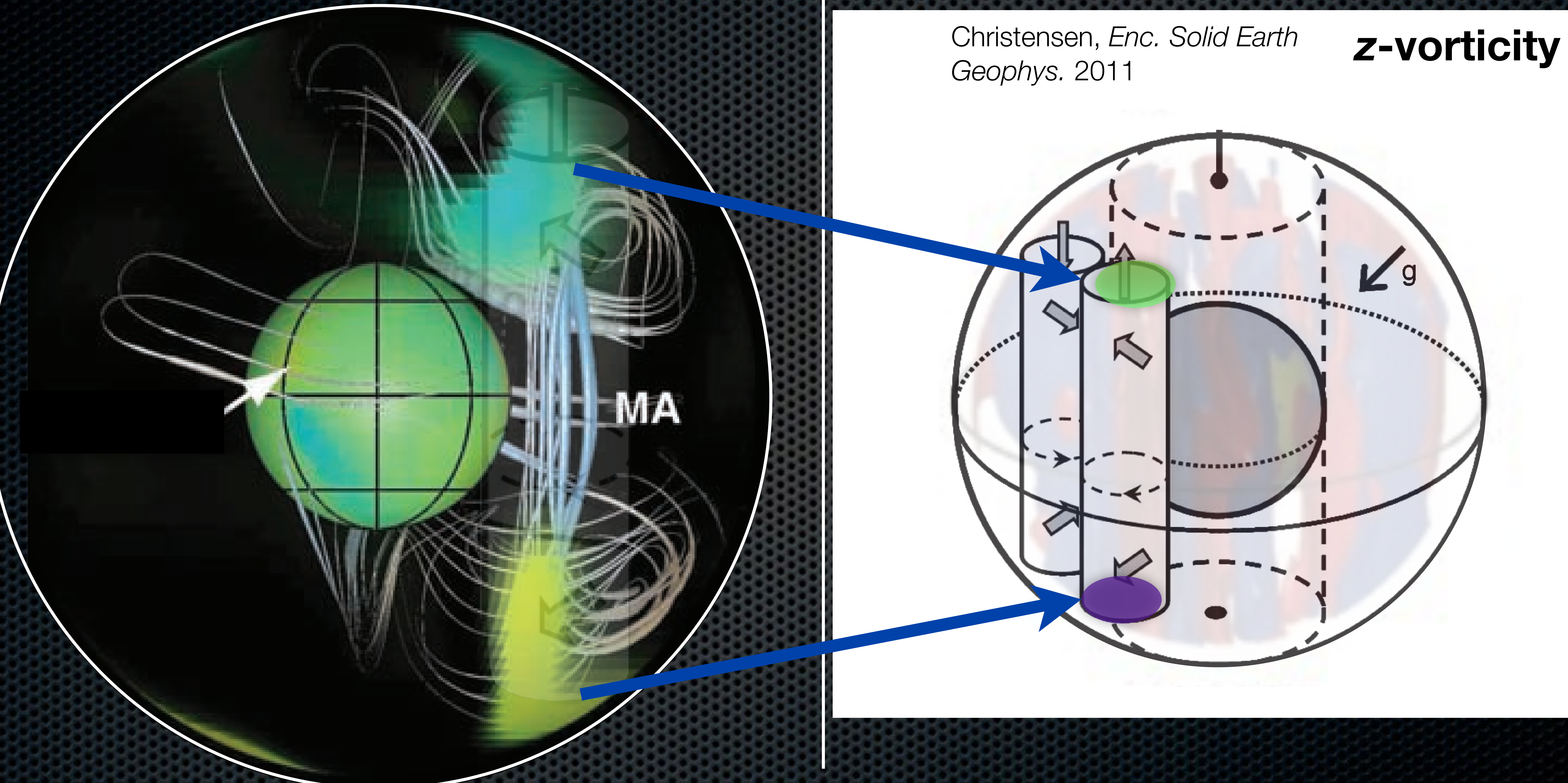


Christensen, *Enc. Solid Earth Geophys.* 2011

***z*-vorticity**

Aubert et al. *GJI* 2008

Models

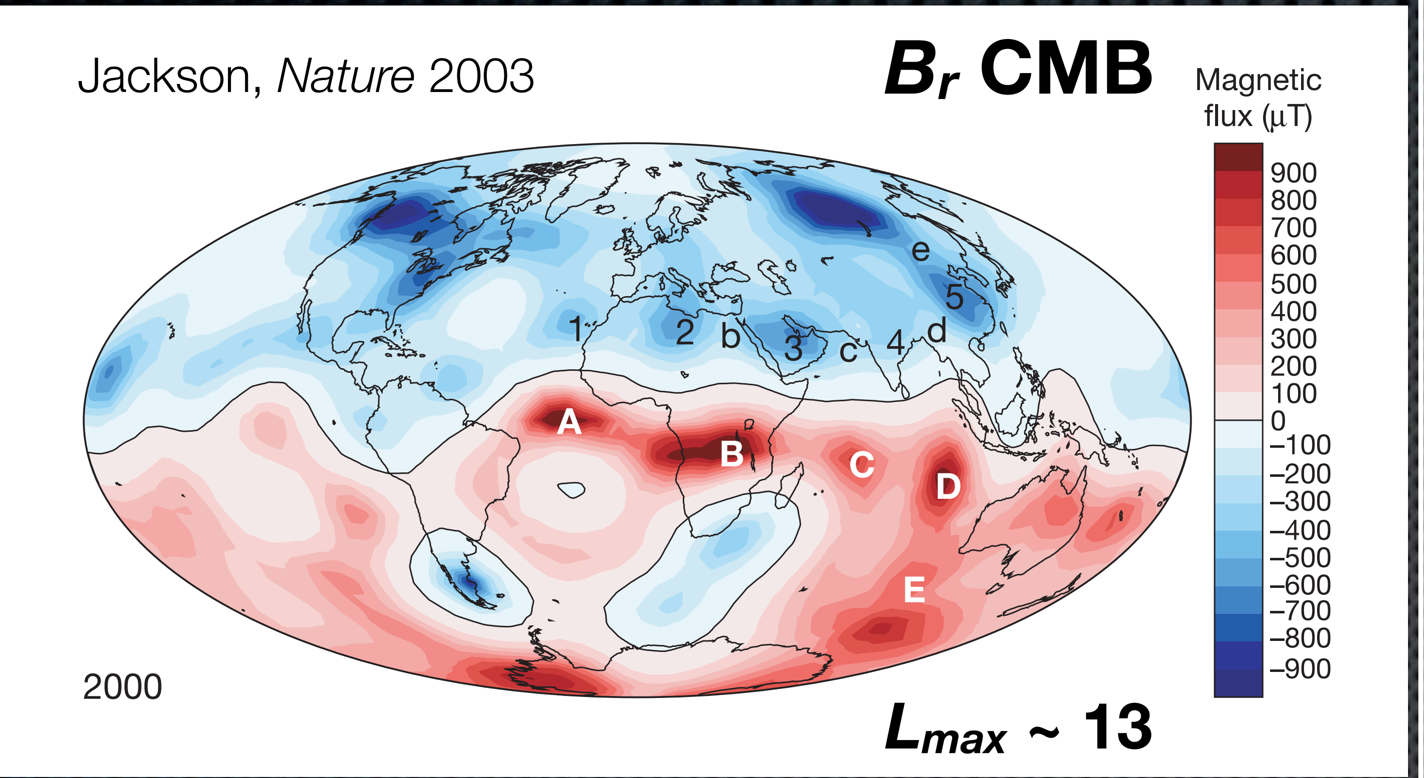


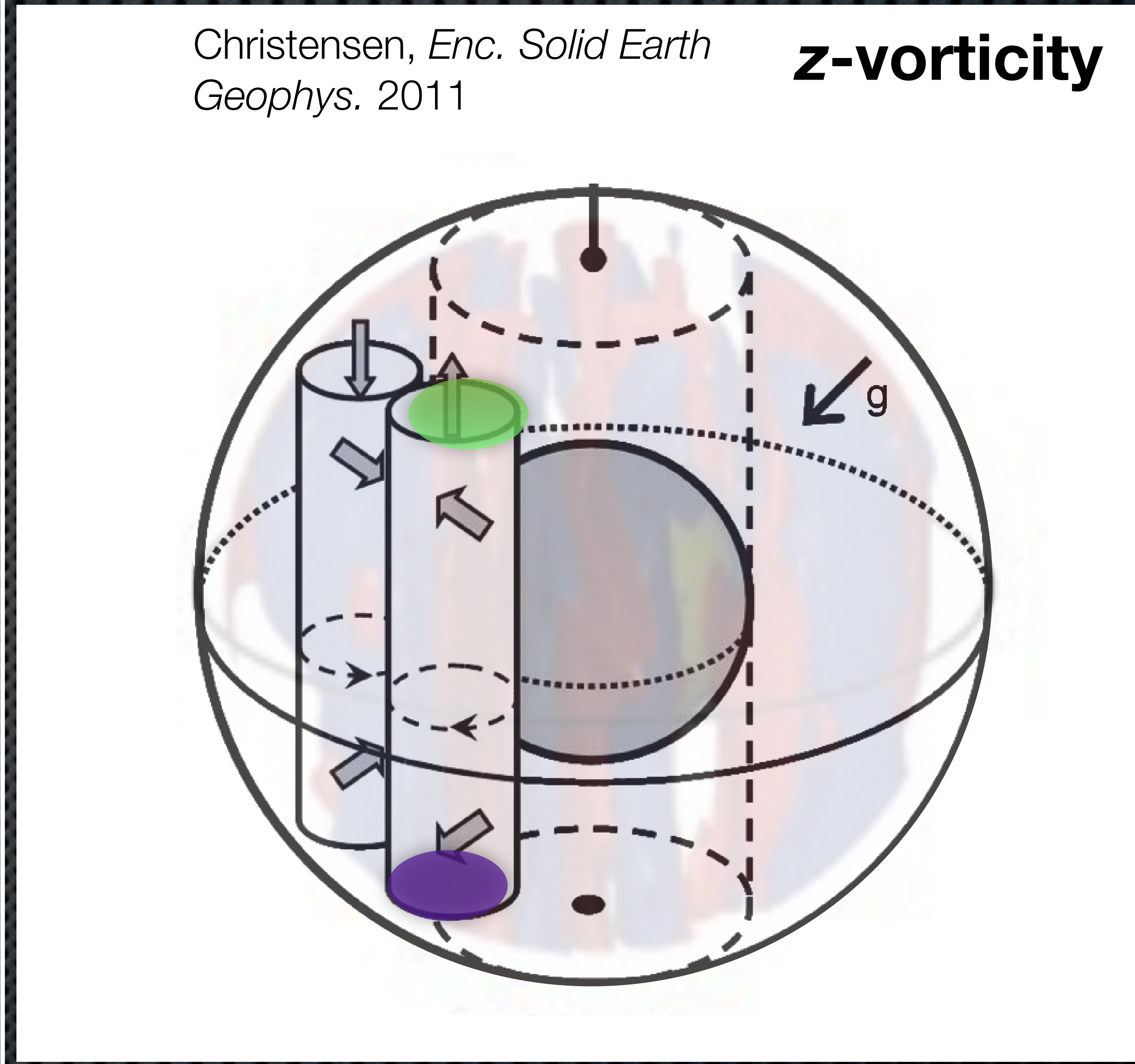
Christensen, *Enc. Solid Earth Geophys.* 2011

Aubert et al. *GJI* 2008

Observations

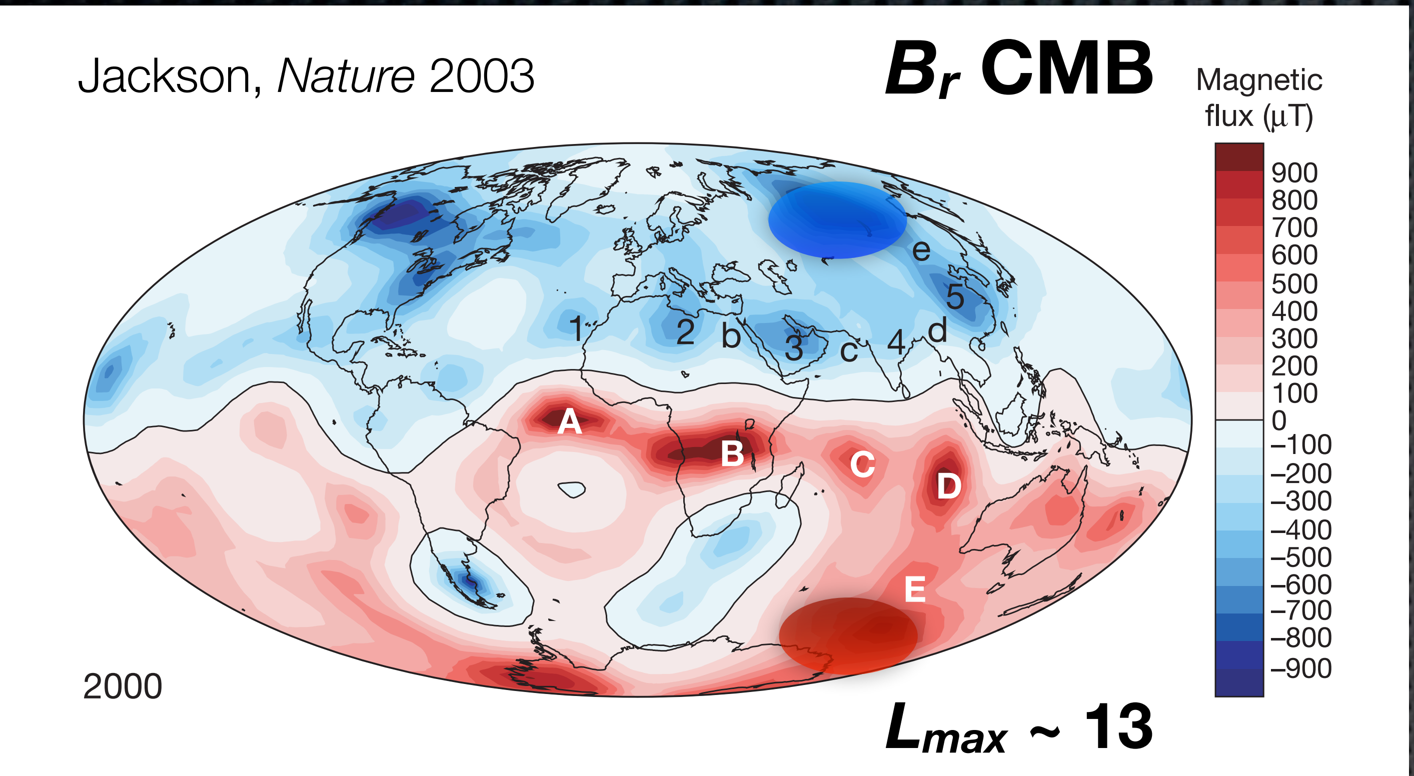
Models

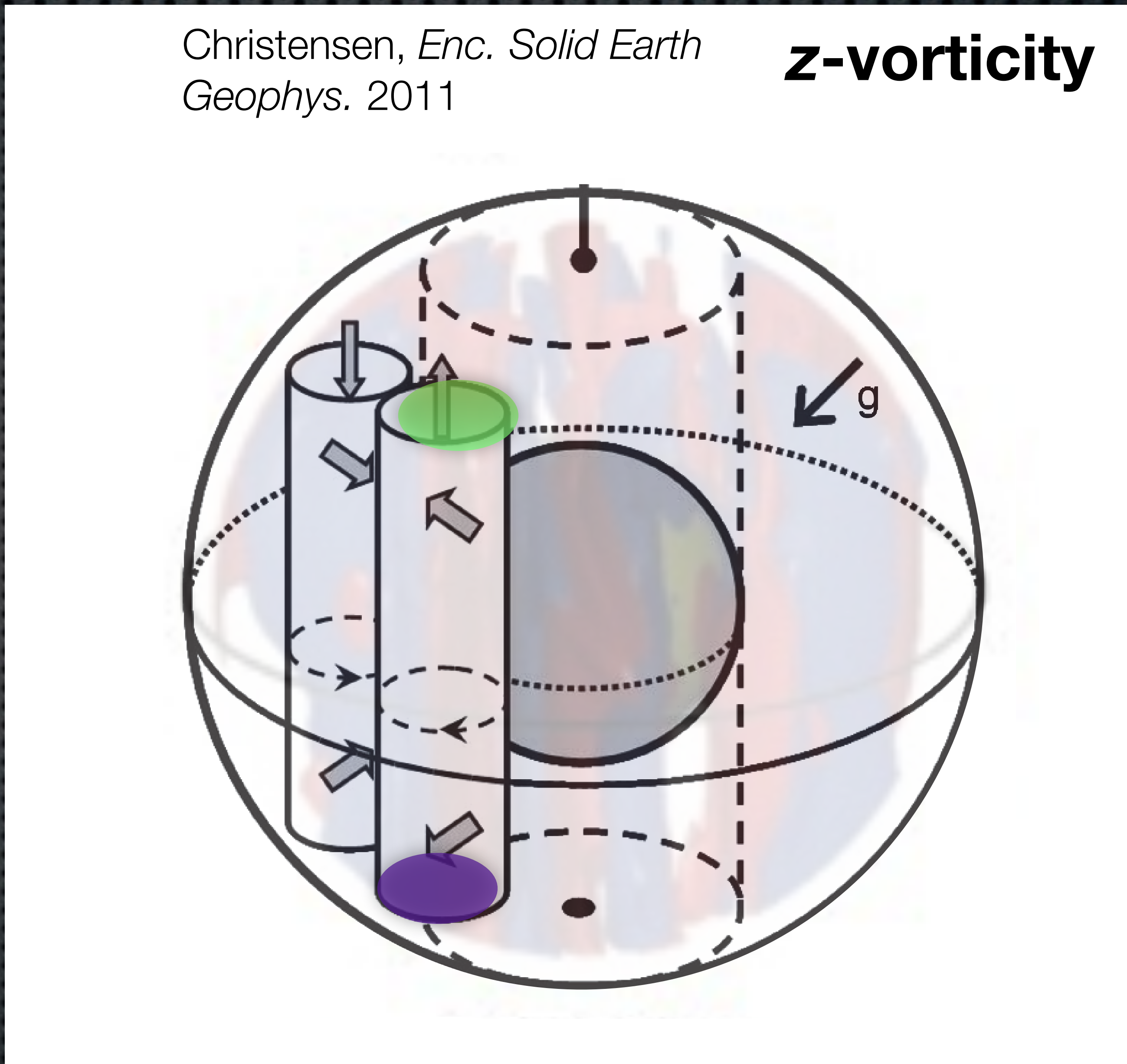




Observations

Models

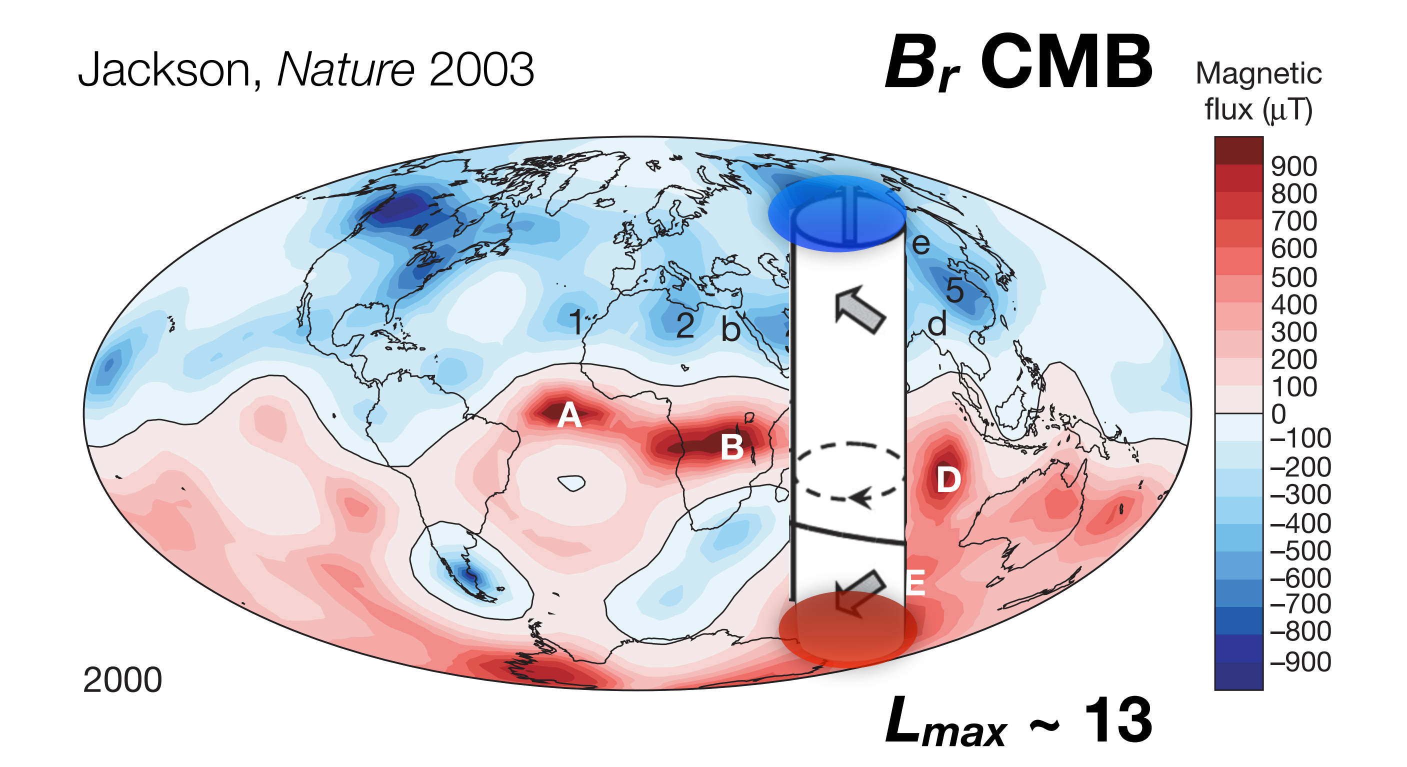


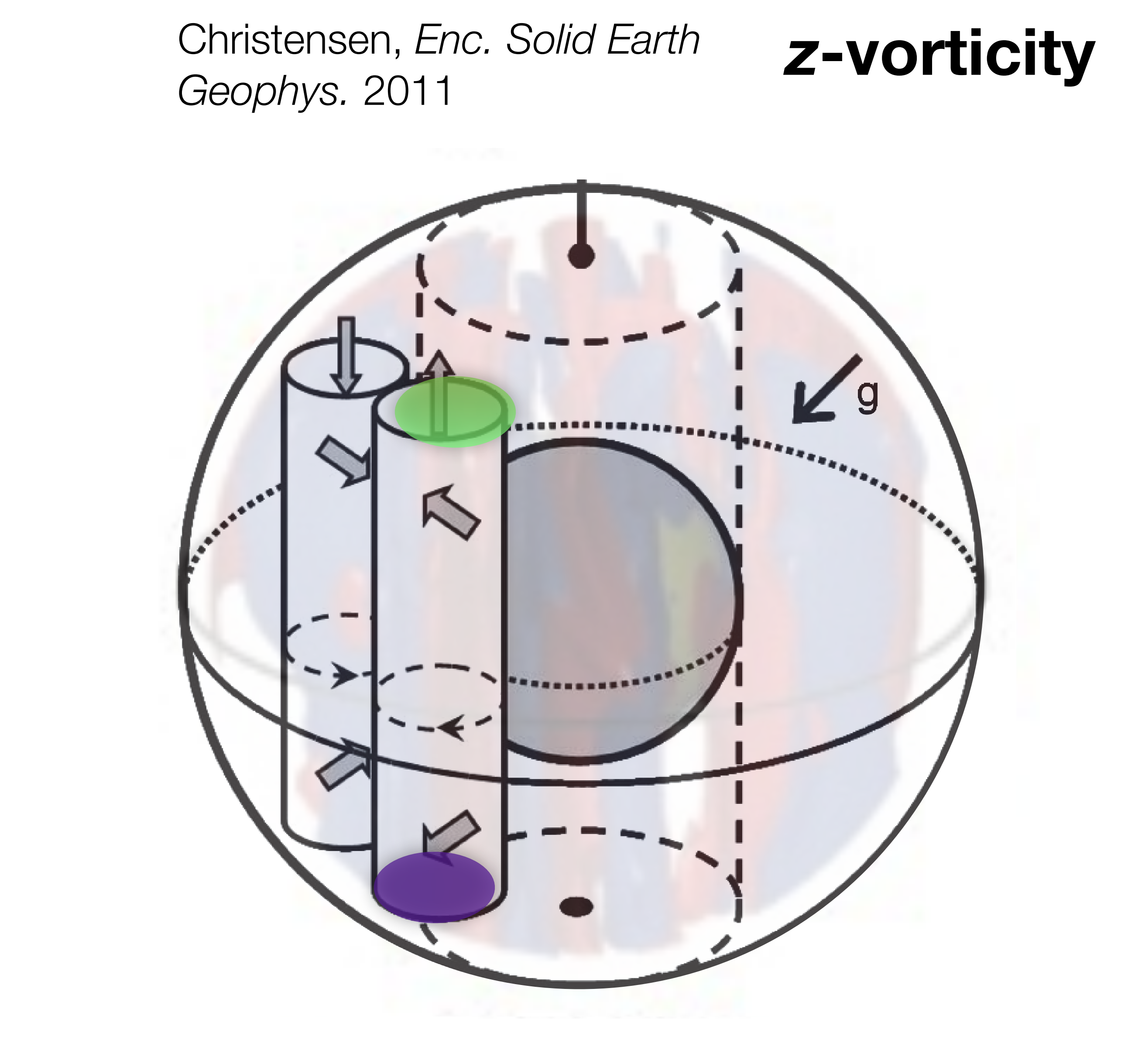


- Aligned flux patches in models & geomagnetic field

Observations

Models





- Aligned flux patches in models & geomagnetic field
- **Present paradigm: columns in models extrapolate to planetary cores; explain observations**

Talk Outline

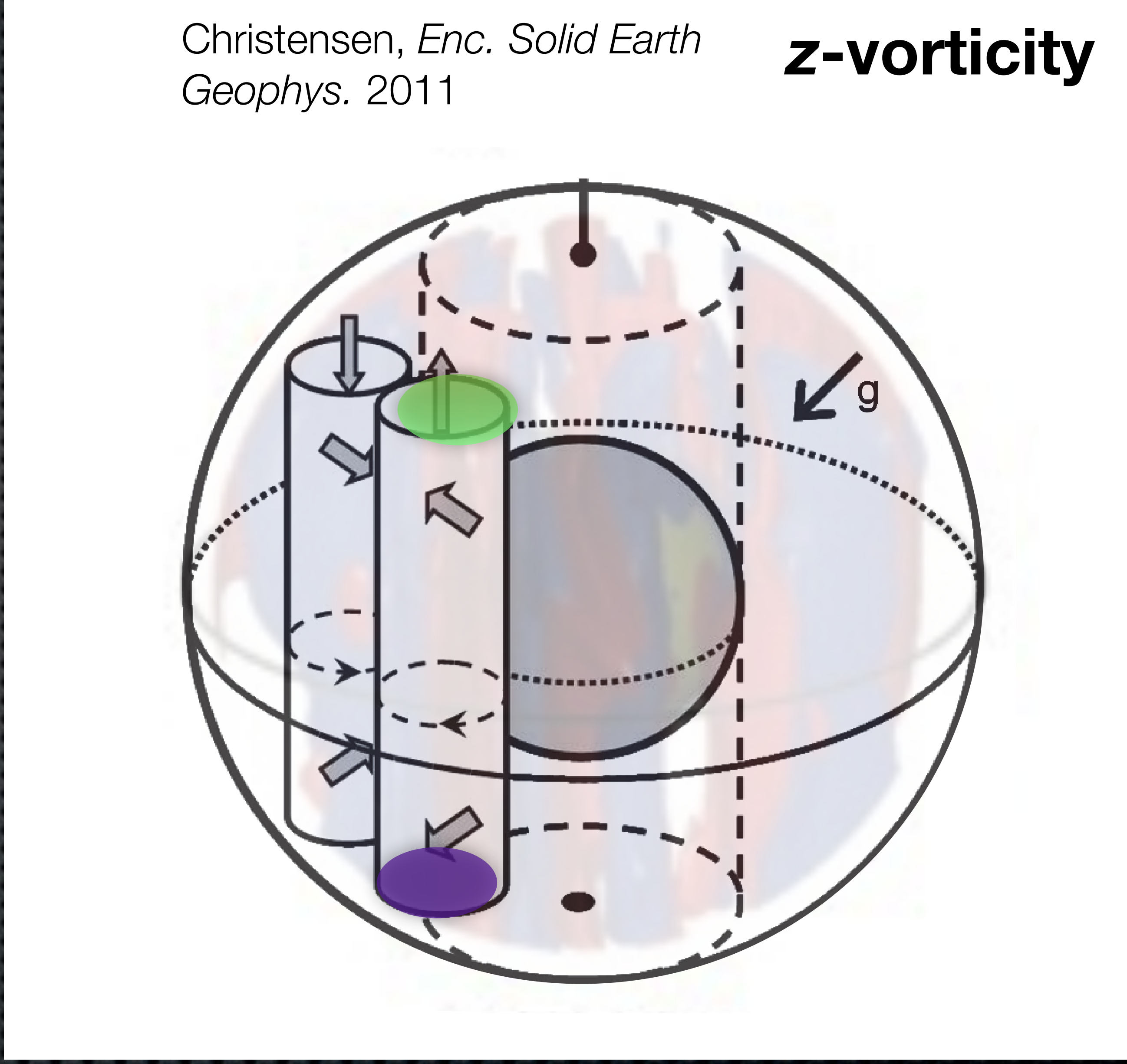
- Geodynamo Observations & <u>Models</u>
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
  - And Beyond

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
    - And Beyond

Fluids Primer

- Goal: Understand present columnar paradigm for core flows
  - Rotating fluid dynamics

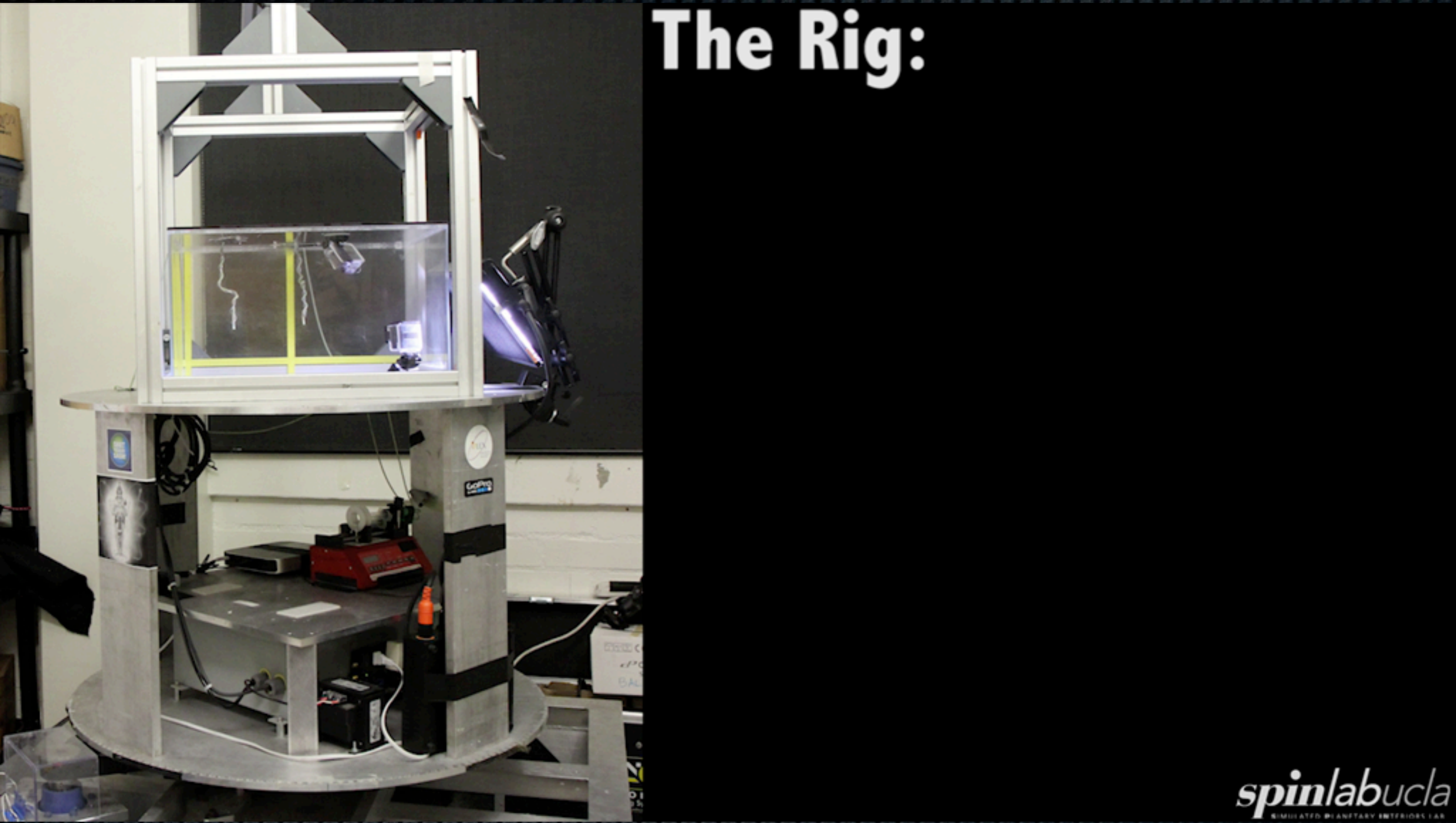


“Differencing” Experiments

- Step through a set of four related laboratory experiments
- Discuss relevant dynamics at each step
  - & build/take apart rotating Navier-Stokes…

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \rho \mathbf{g} - \nabla p + \rho \nu \nabla^2 \mathbf{u} + \rho \mathbf{u} \times 2\mathbf{\Omega} + \rho \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}$$

  - Heuristic approach: shortest time-scale process~ largest acceleration ~ dominates



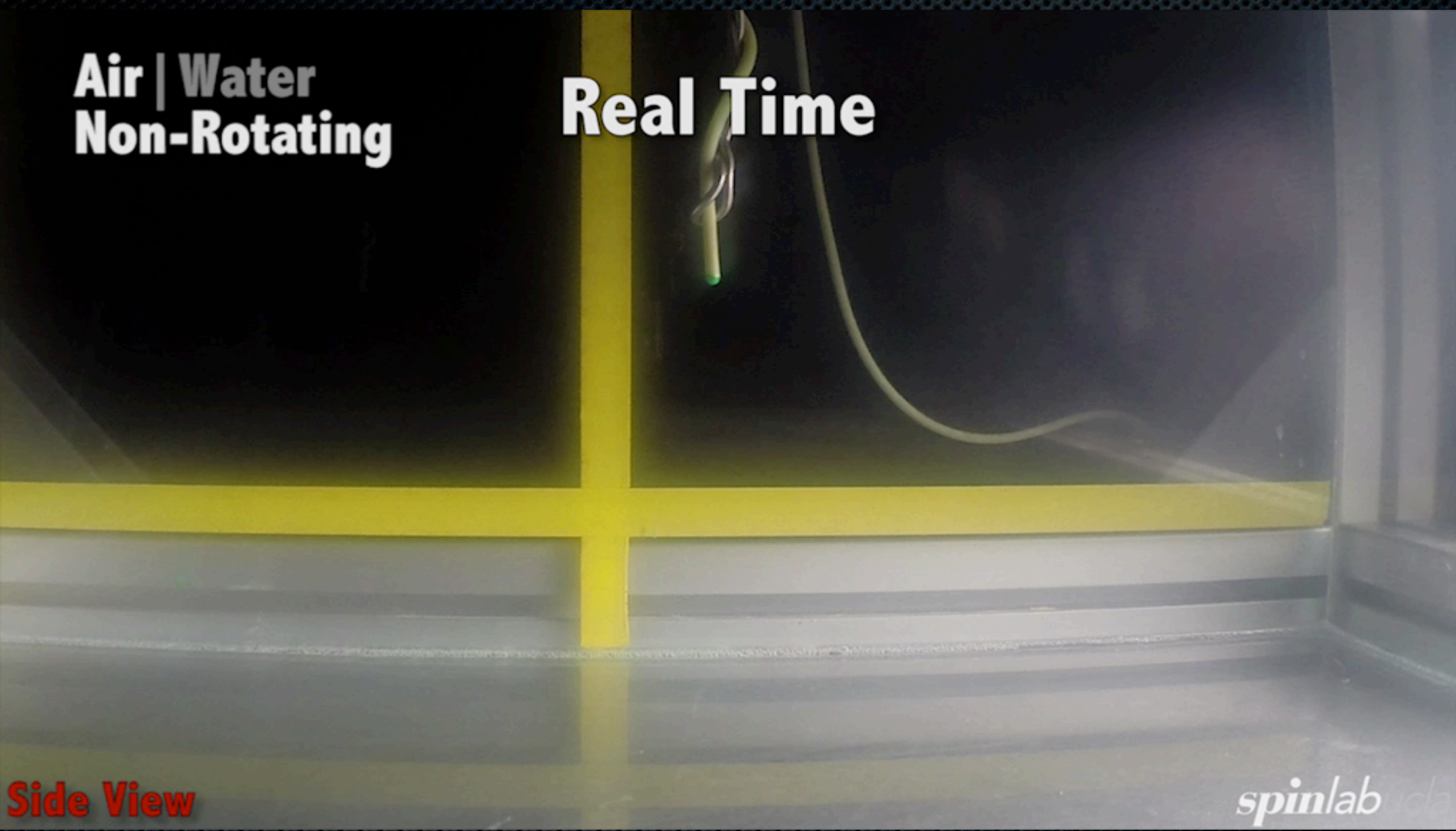
Jon Bridgeman, Mike Lavell, Steve Tomlinson & Eric King

“Differencing” Experiments

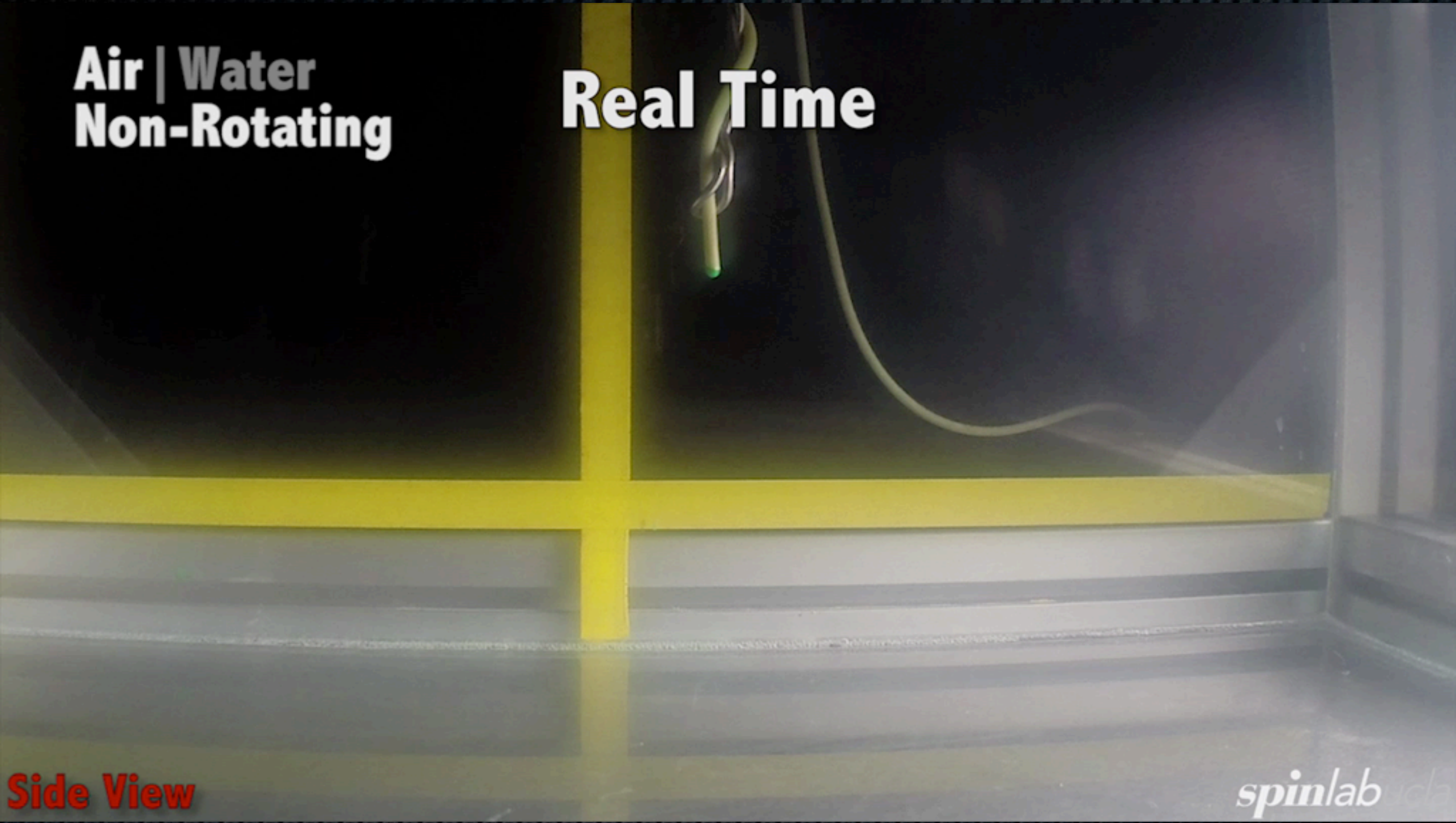
- Isolated droplets/particles (empty tank)
  - 1. Stationary vs. 2. Rotating
- Fluid Layer (water-filled tank)
  - 3. Stationary vs. 4. Rotating

Experiment 1

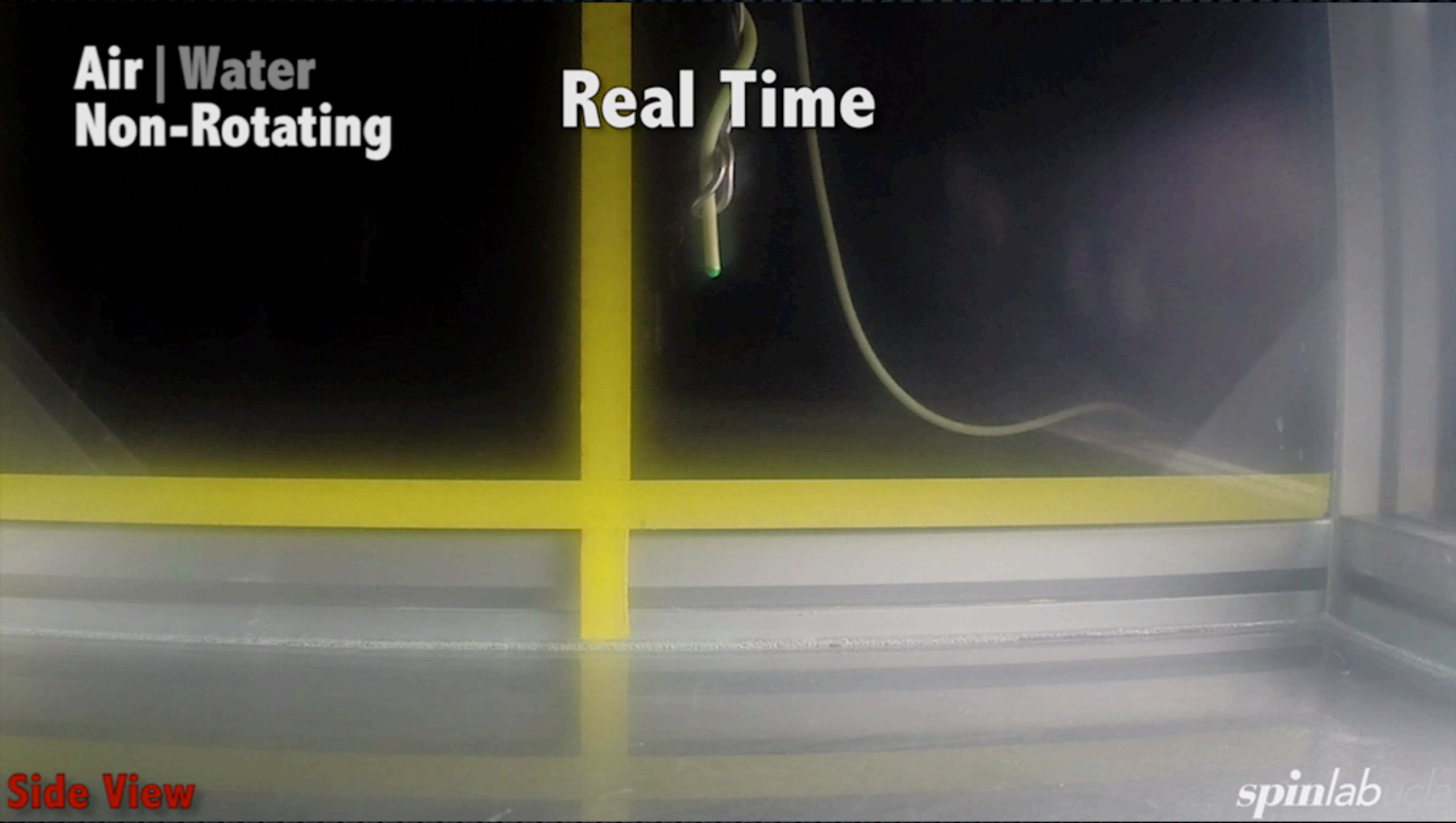
Stationary “Empty” Tank



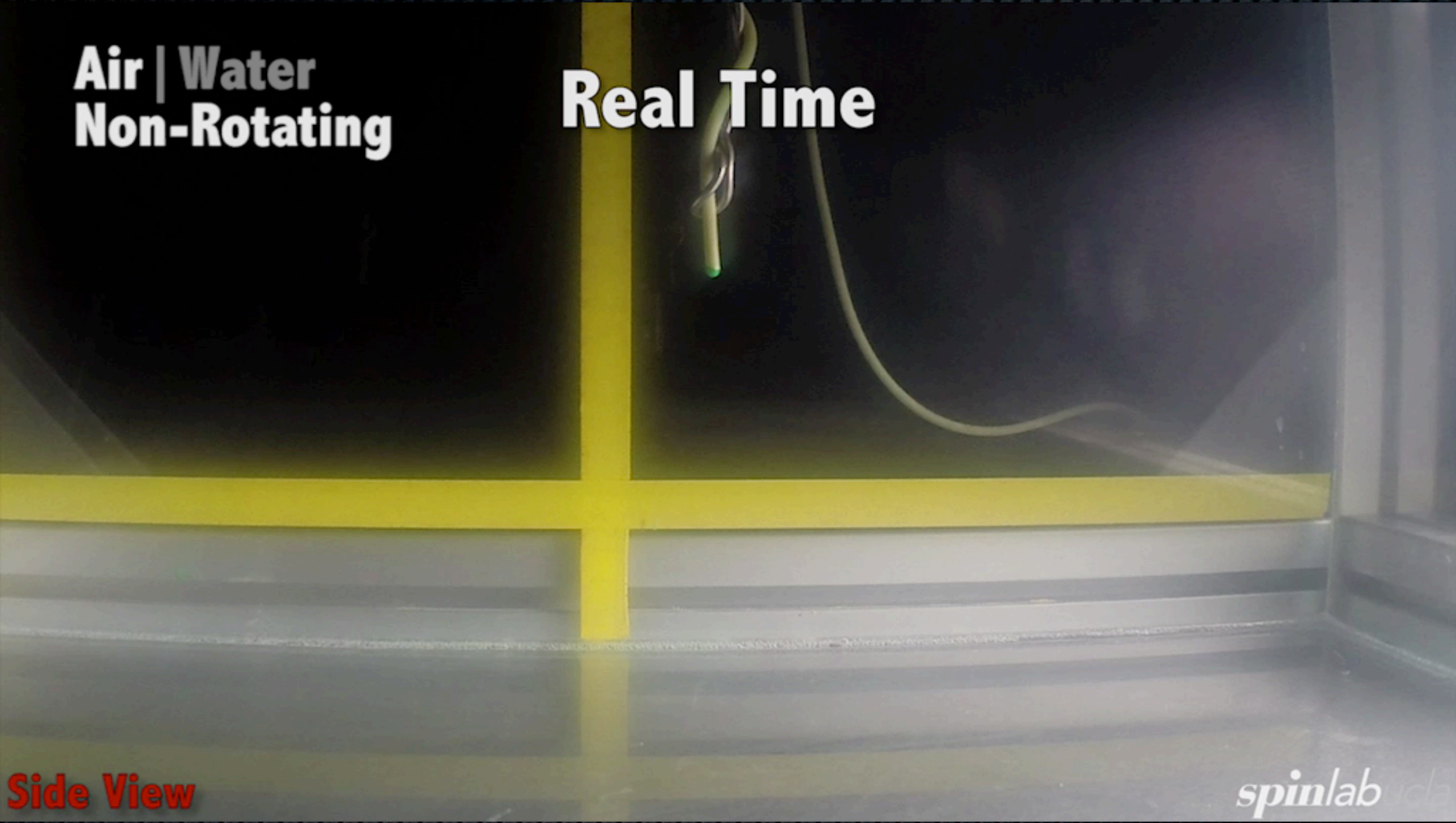
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} =$$



$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \rho \mathbf{g}$$

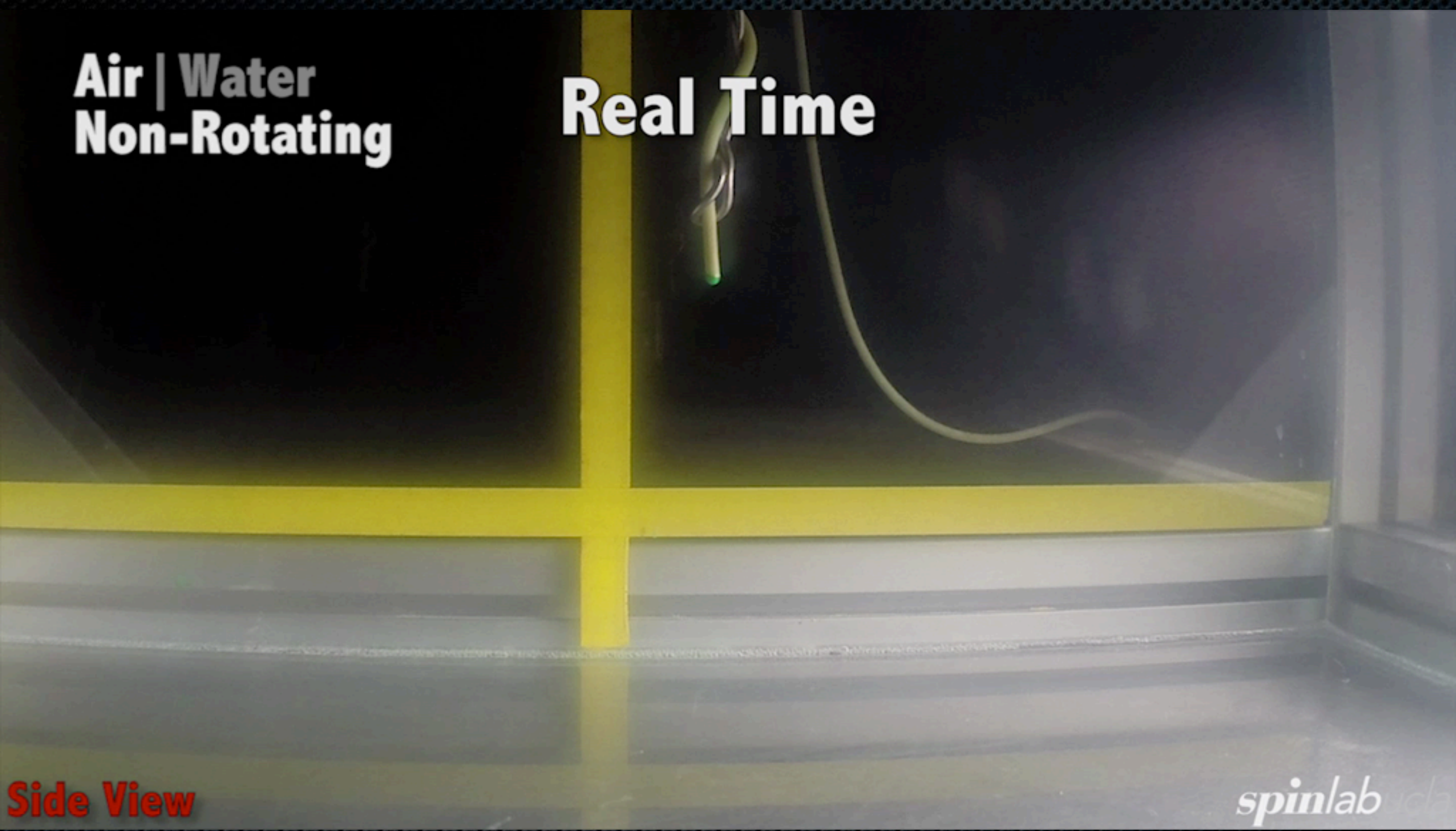


$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = \rho \mathbf{g}$$



$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = \rho \mathbf{g}$$

$$u \sim \sqrt{gH} \to \tau \sim \sqrt{H/g}$$



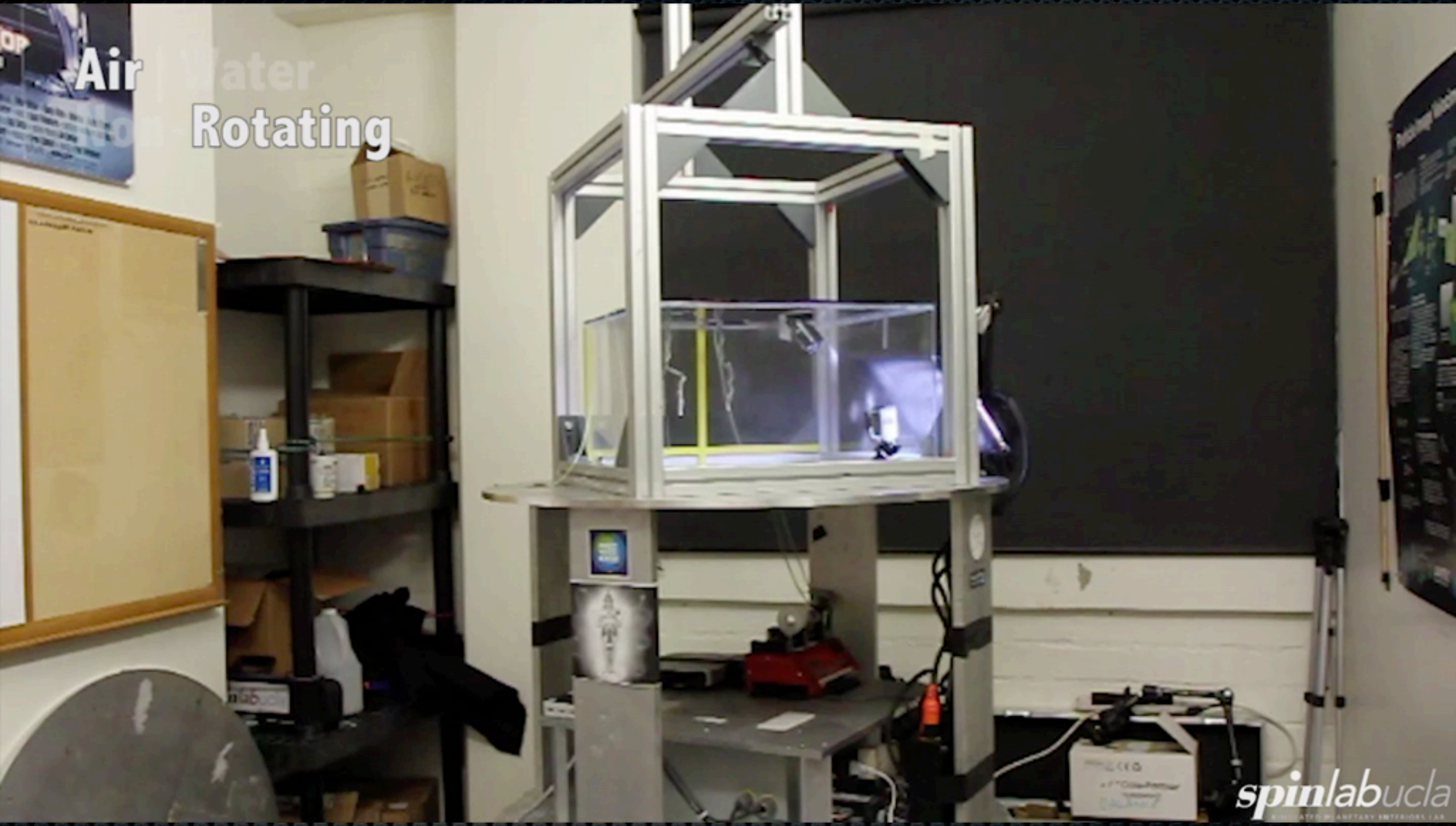
$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = \rho \mathbf{g}$$

$$u \sim \sqrt{gH} \rightarrow \tau \sim \sqrt{H/g} = 0.12 \text{ s}$$

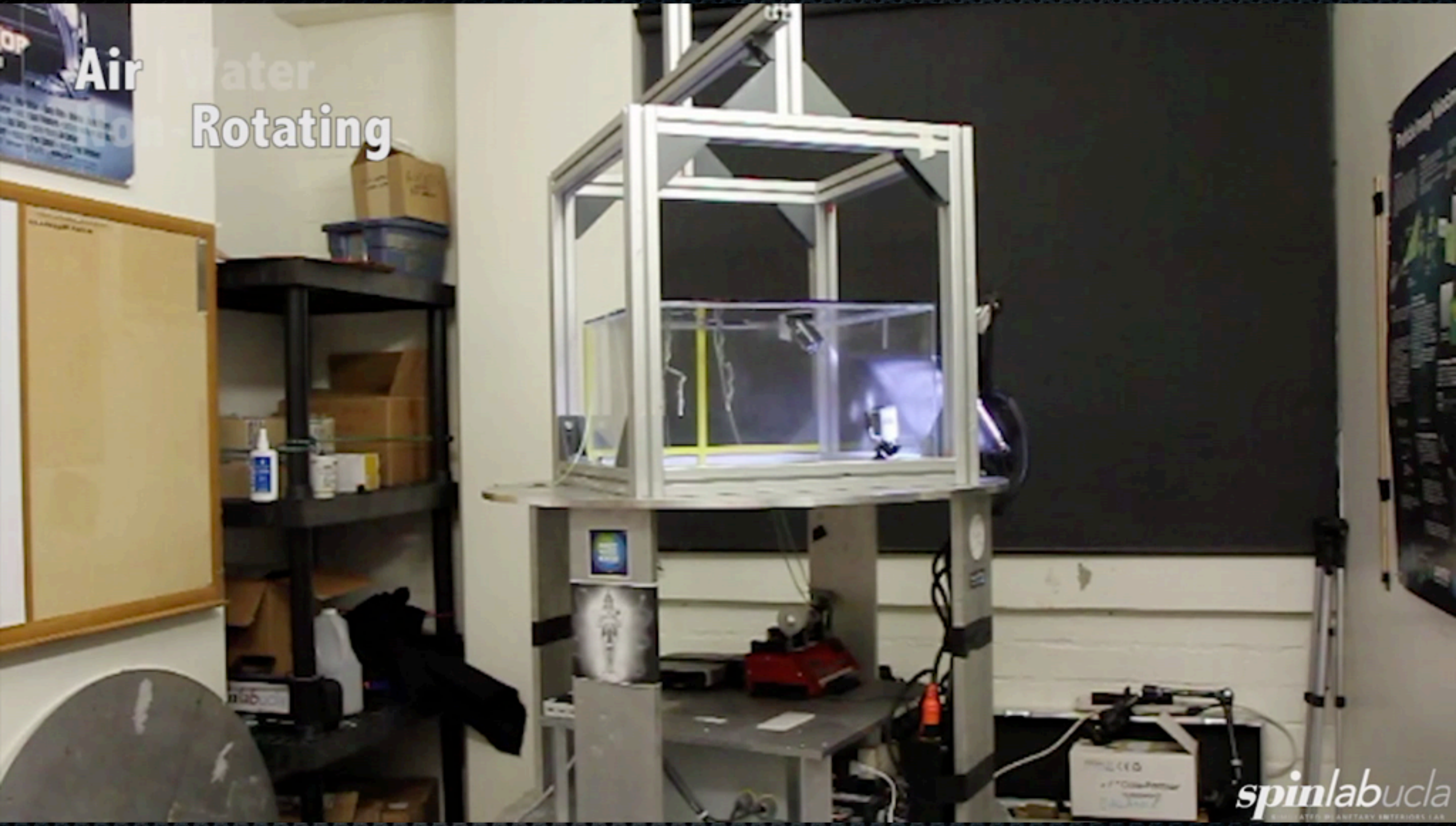
Free-Fall Estimates

Experiment 2

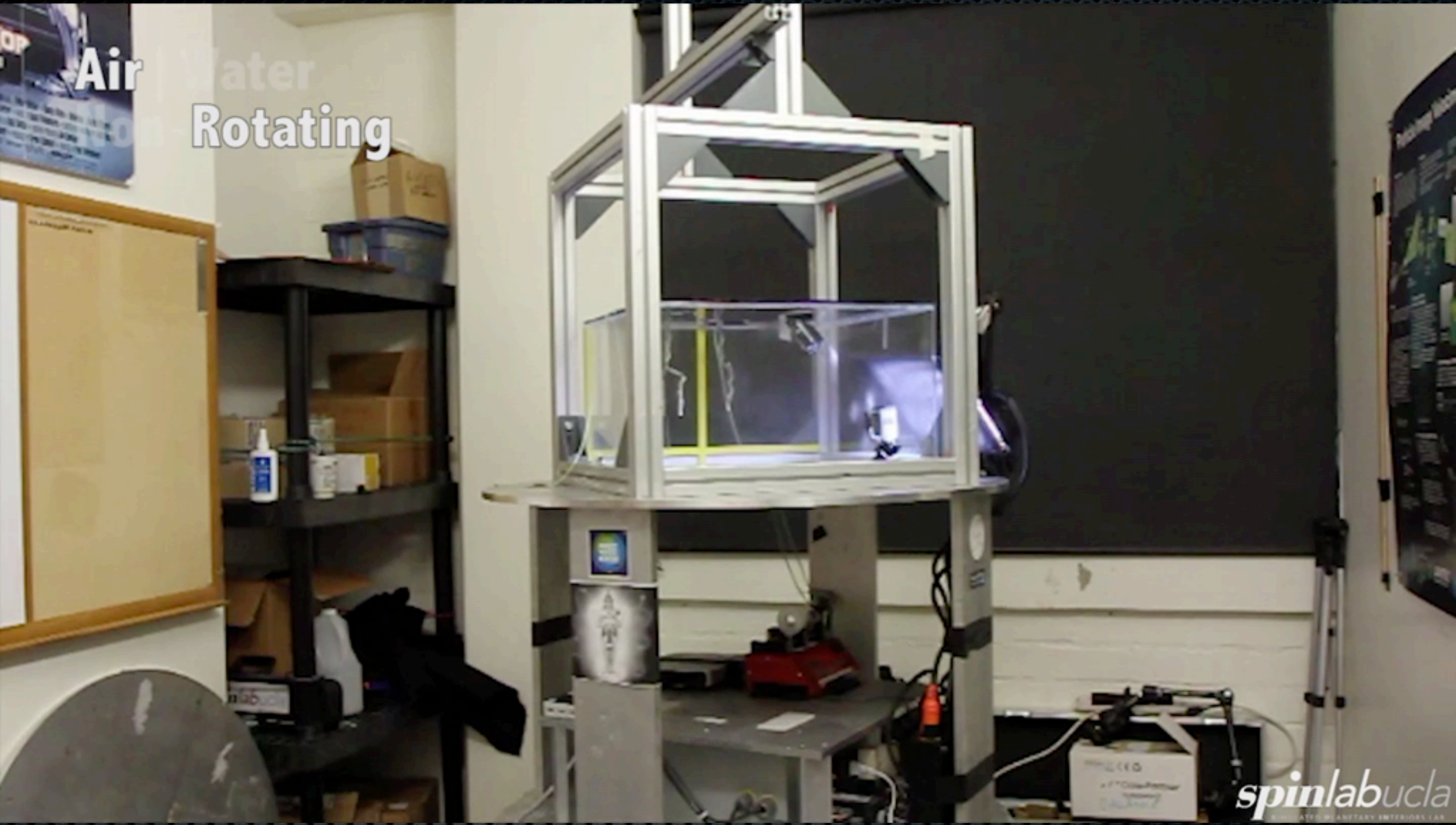
Rotating “Empty” Tank



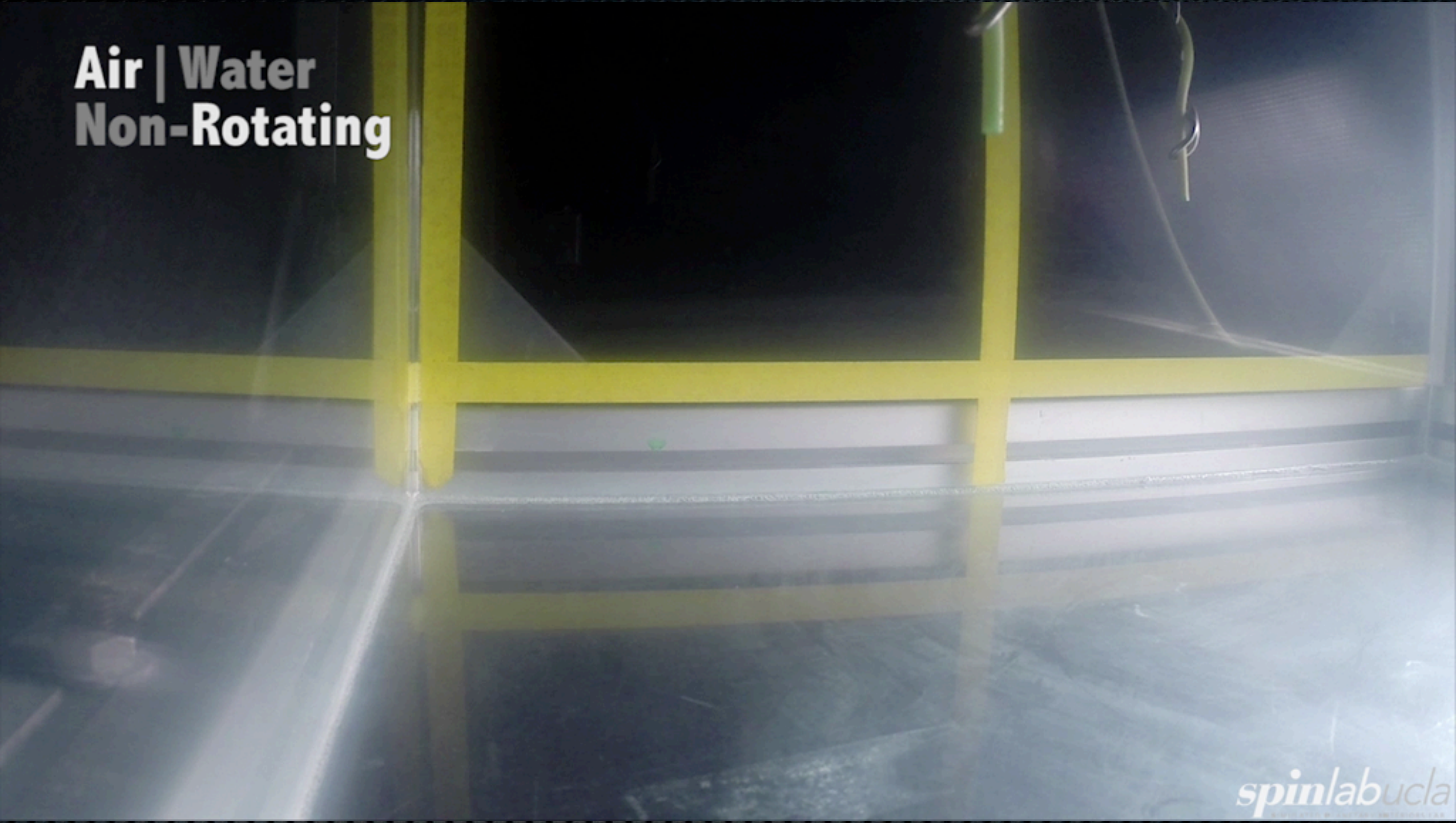
$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} = -\rho g\hat{z} + \rho\Omega^2 s\hat{s} + \rho\mathbf{u}\times 2\mathbf{\Omega}$$



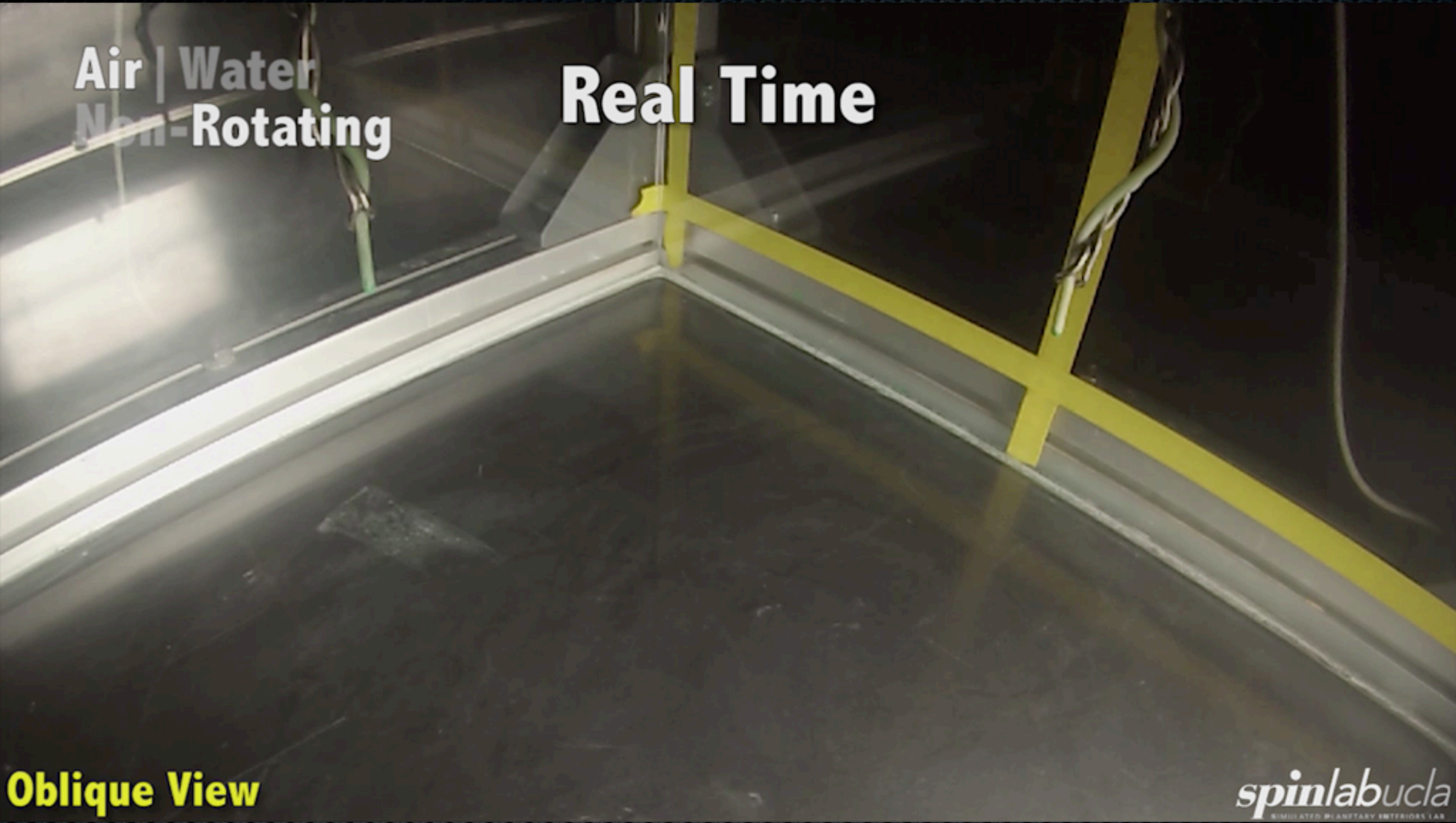
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\rho g \hat{z} + \underbrace{\rho \Omega^2 s \hat{s} + \rho \mathbf{u} \times 2\boldsymbol{\Omega}}_{\text{Non-inertial Forces}}$$



$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \underbrace{-\rho g \hat{z} + \rho \Omega^2 s \hat{s}}_{\text{Effective Gravity}} + \rho \mathbf{u} \times 2\boldsymbol{\Omega}$$

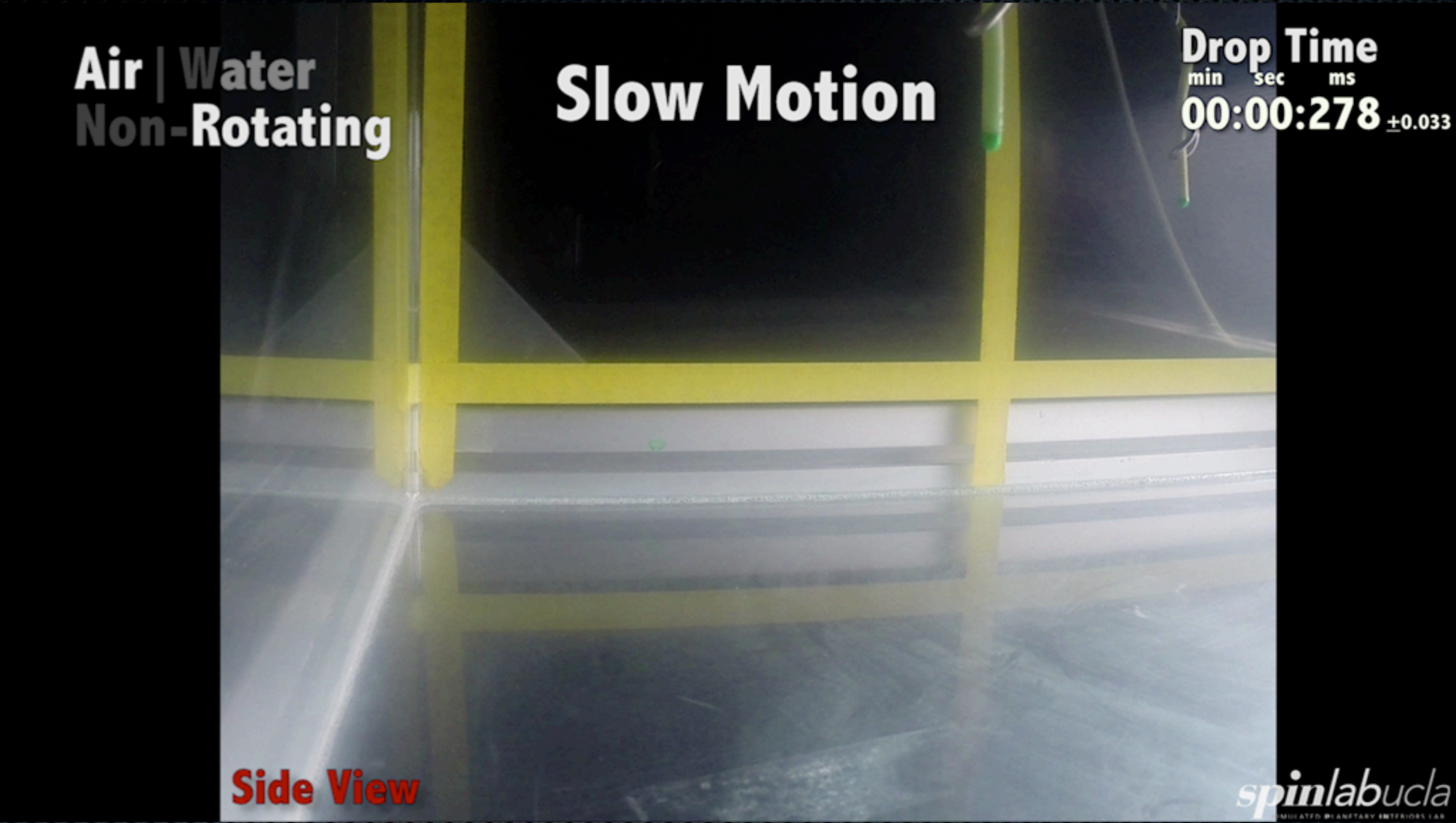


$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} = \underbrace{-\rho g\hat{z} + \rho\Omega^2 s\hat{s}}_{\text{Effective Gravity}} + \rho\mathbf{u}\times 2\boldsymbol{\Omega}$$



$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \underline{-\rho g \hat{z} + \underline{\rho \Omega^2 s \hat{s}} + \rho \mathbf{u} \times 2\mathbf{\Omega}}$$

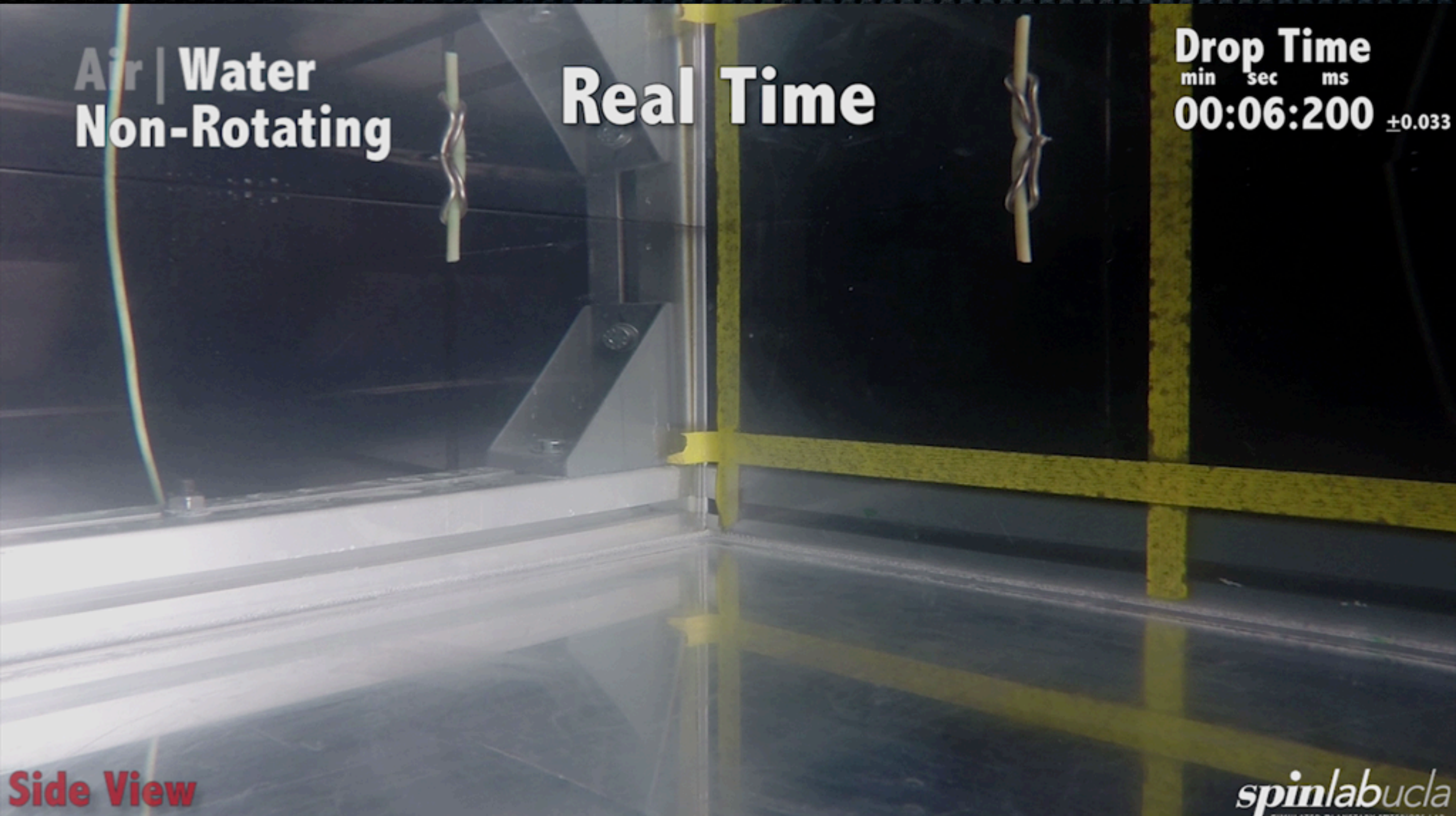
Effective Gravity



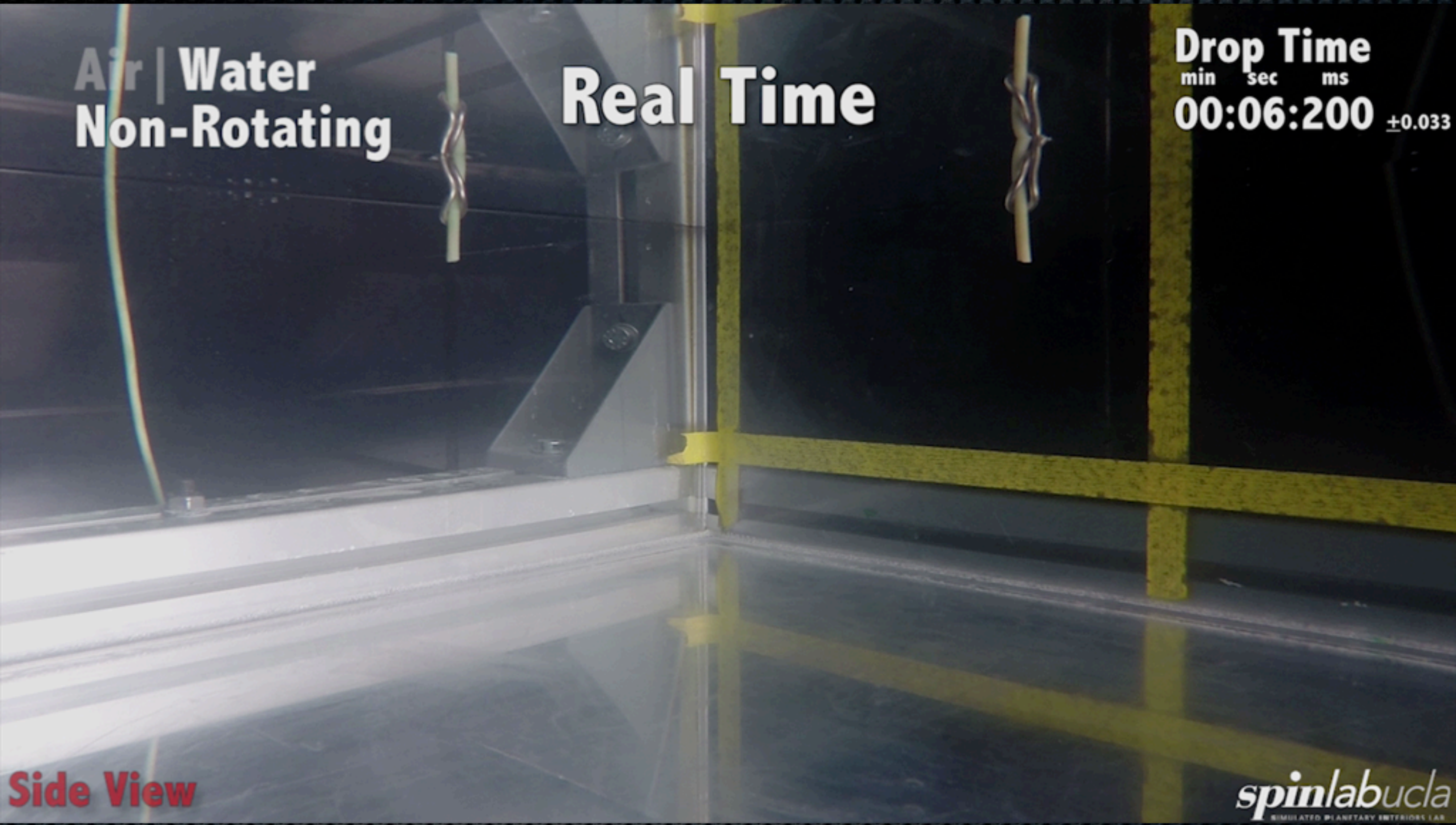
Rossby, $Ro = \frac{\tau_{rot}}{\tau_{fall}} \sim \frac{u}{\Omega H}$ ~ **5.4** ***Gravity dominates***

Experiment 3

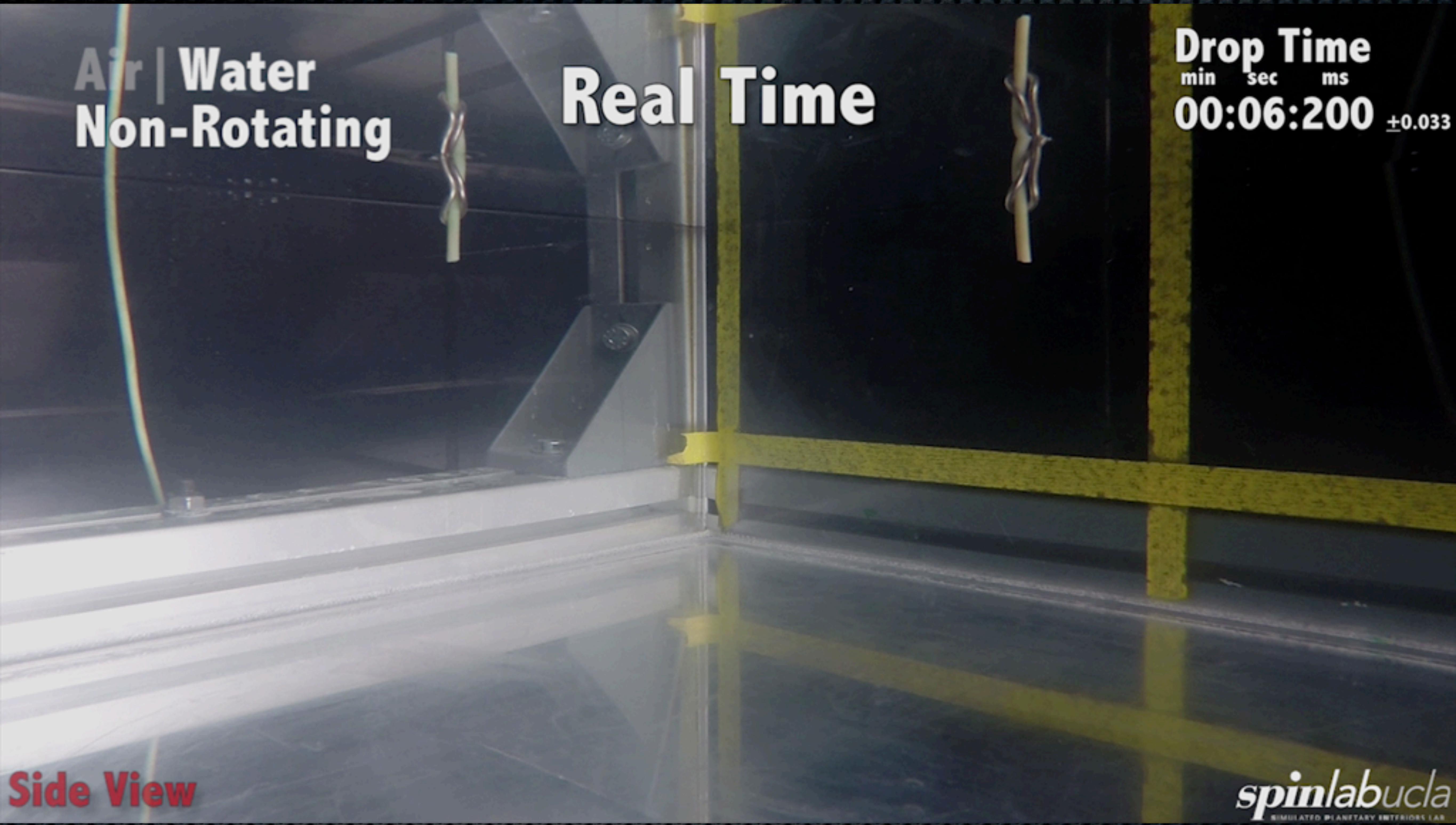
Stationary Water-Filled Tank



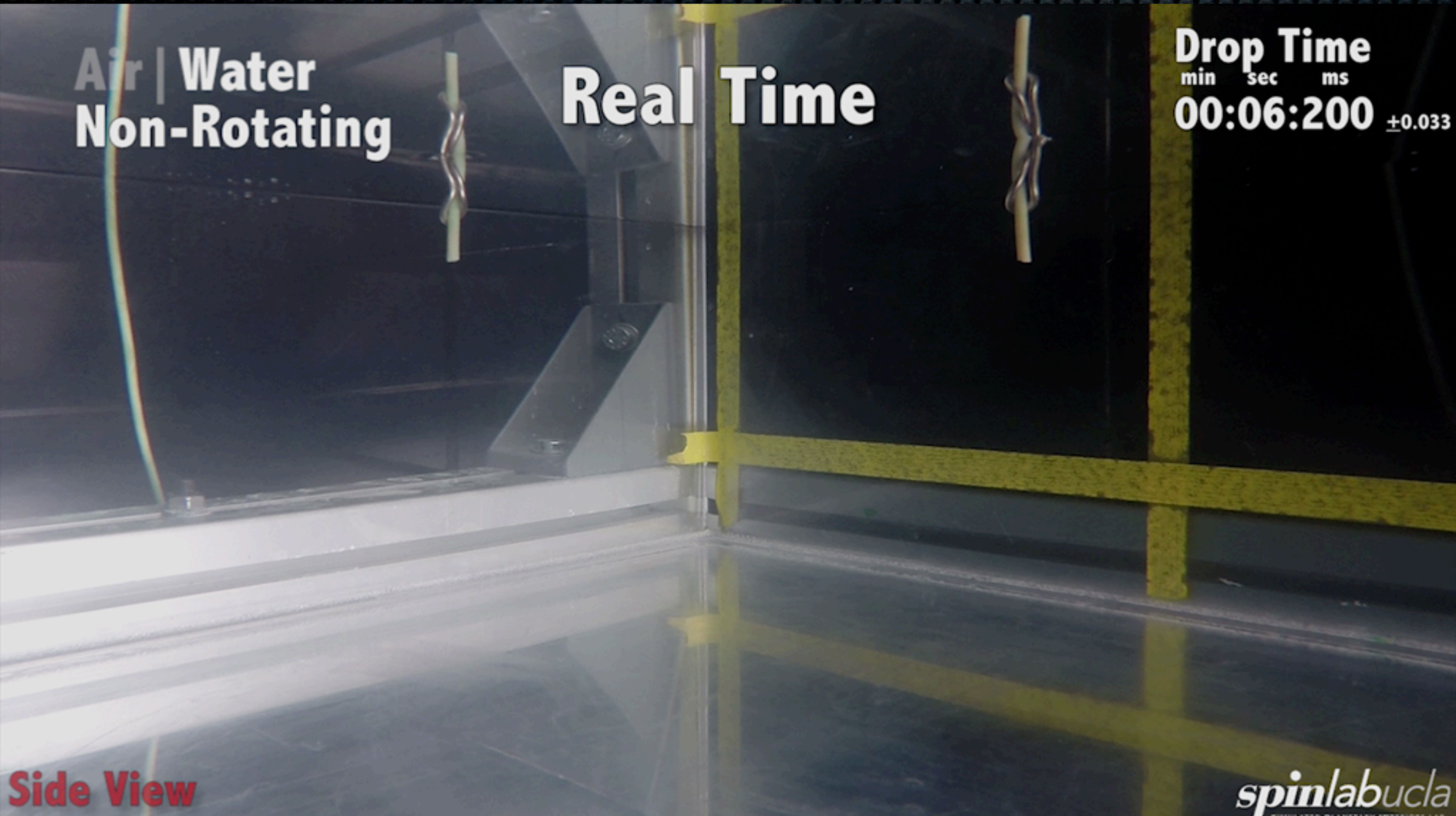
$$\rho_w \frac{\partial \mathbf{u}}{\partial t} + \rho_w \mathbf{u} \cdot \nabla \mathbf{u} = -\rho_w g \hat{z}$$


Air | Water
Non-Rotating
Real Time
Drop Time
min sec ms
00:06:200 ±0.033
Side View
spinlabucla

$$0 = -\rho_w g \hat{z}$$

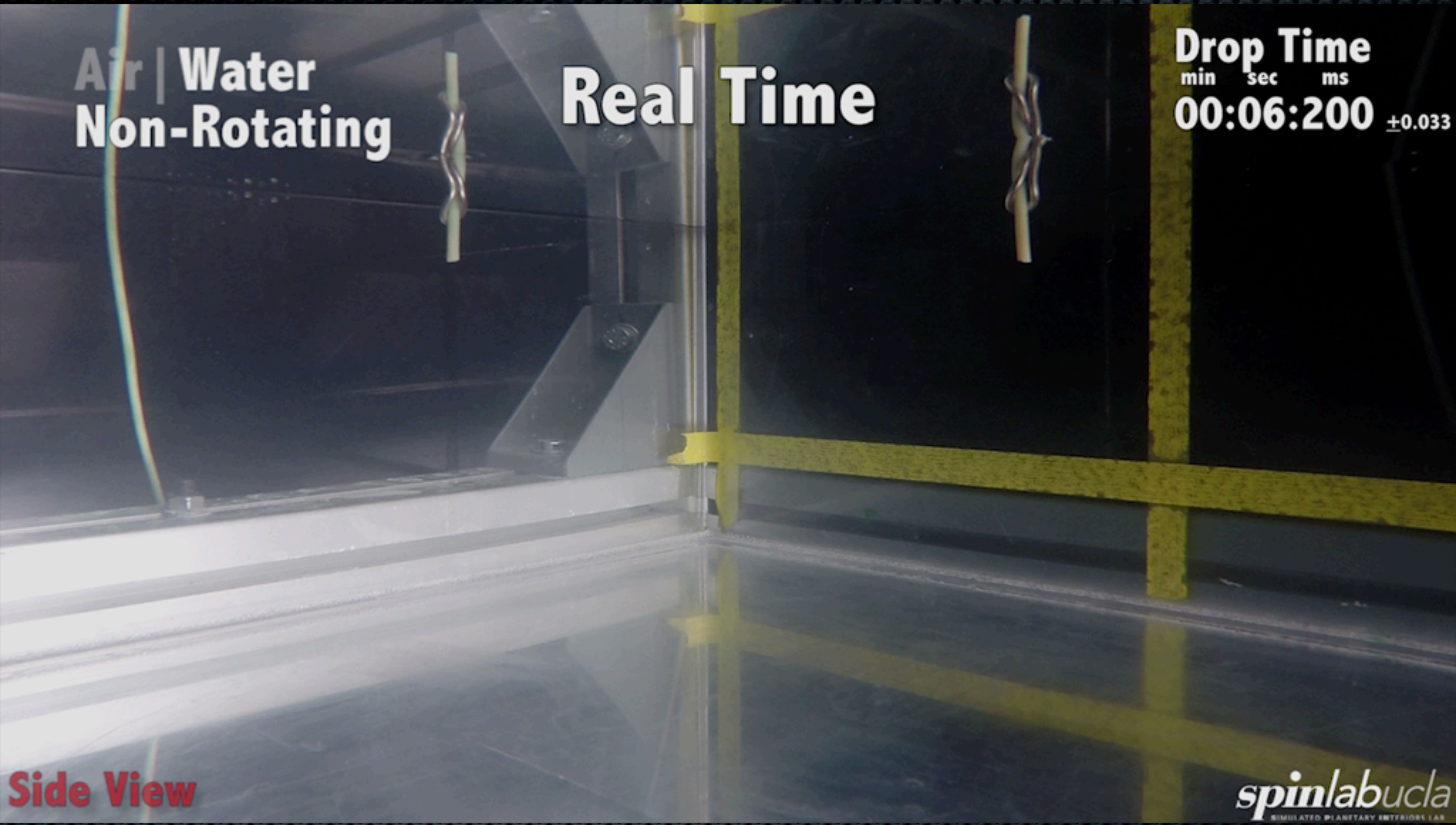


$$0 = -\rho_w g \hat{z} - \nabla p_{hs}$$ **hydrostatic balance**



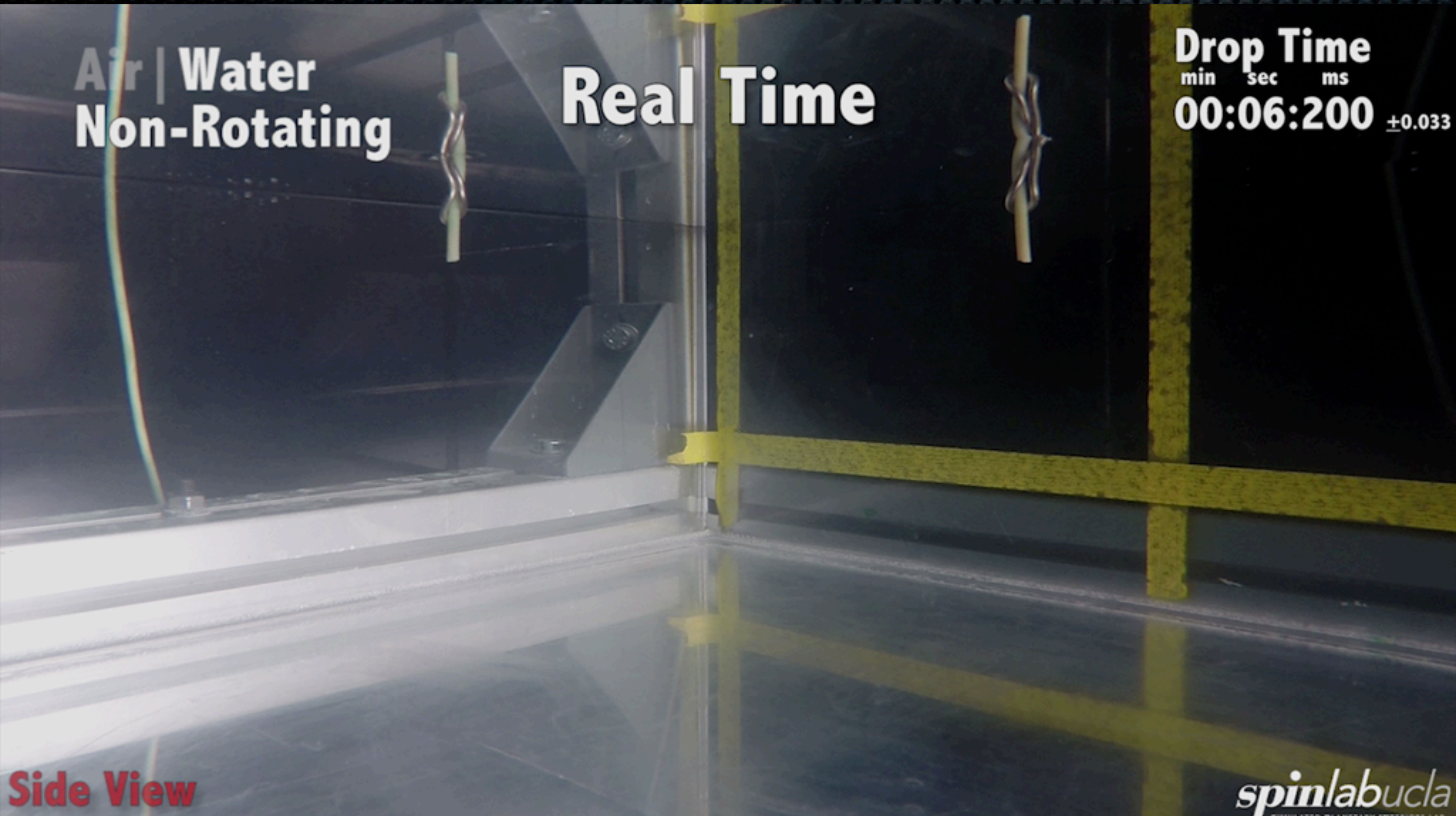
$$0 = -\rho_w g \hat{z} - \nabla p_{hs}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\rho_c - \rho_w}{\rho} g \hat{z} - \frac{1}{\rho} \nabla p_d$$



$$0 = -\rho_w g\hat{z} - \nabla p_{hs}$$

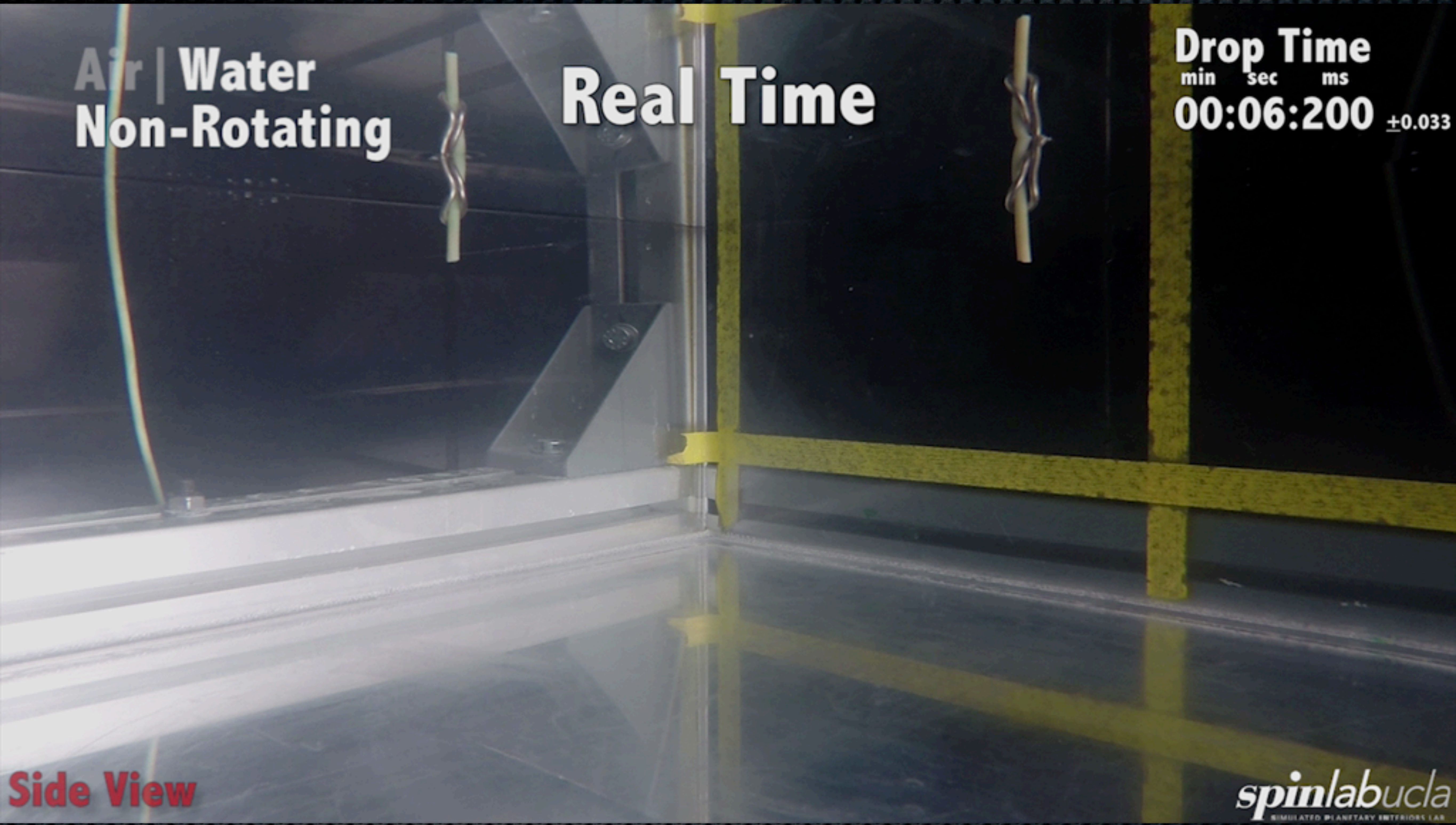
$$\frac{\partial \mathbf{u}}{\partial t} + \boxed{\mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\rho_c-\rho_w}{\rho} g\hat{z}} - \frac{1}{\rho}\nabla p_d$$



$$0 = -\rho_w g\hat{z} - \nabla p_{hs}$$

$$u \sim \sqrt{(\delta\rho/\rho)gH} \to \tau \sim \sqrt{H/(\delta\rho/\rho)g}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \boxed{\mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\rho_c-\rho_w}{\rho}g\hat{z}} - \frac{1}{\rho}\nabla p_d$$



$$0 = -\rho_w g\hat{z} - \nabla p_{hs}$$

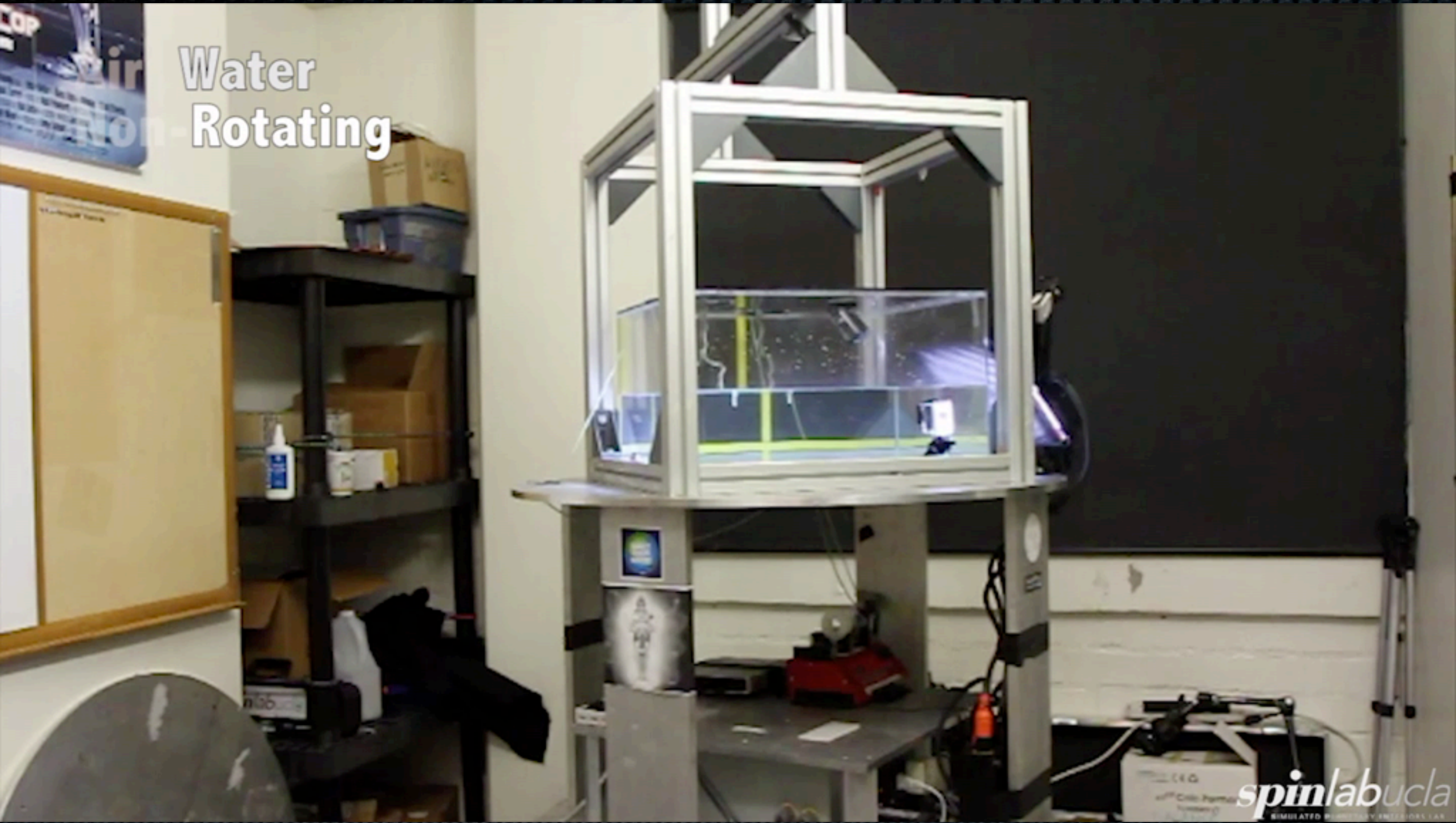
$$u \sim \sqrt{(\delta\rho/\rho)gH} \rightarrow \tau \sim \sqrt{H/(\delta\rho/\rho)g}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \boxed{\mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\rho_c - \rho_w}{\rho} g\hat{z}} - \frac{1}{\rho}\nabla p_d$$

= 0.9 s

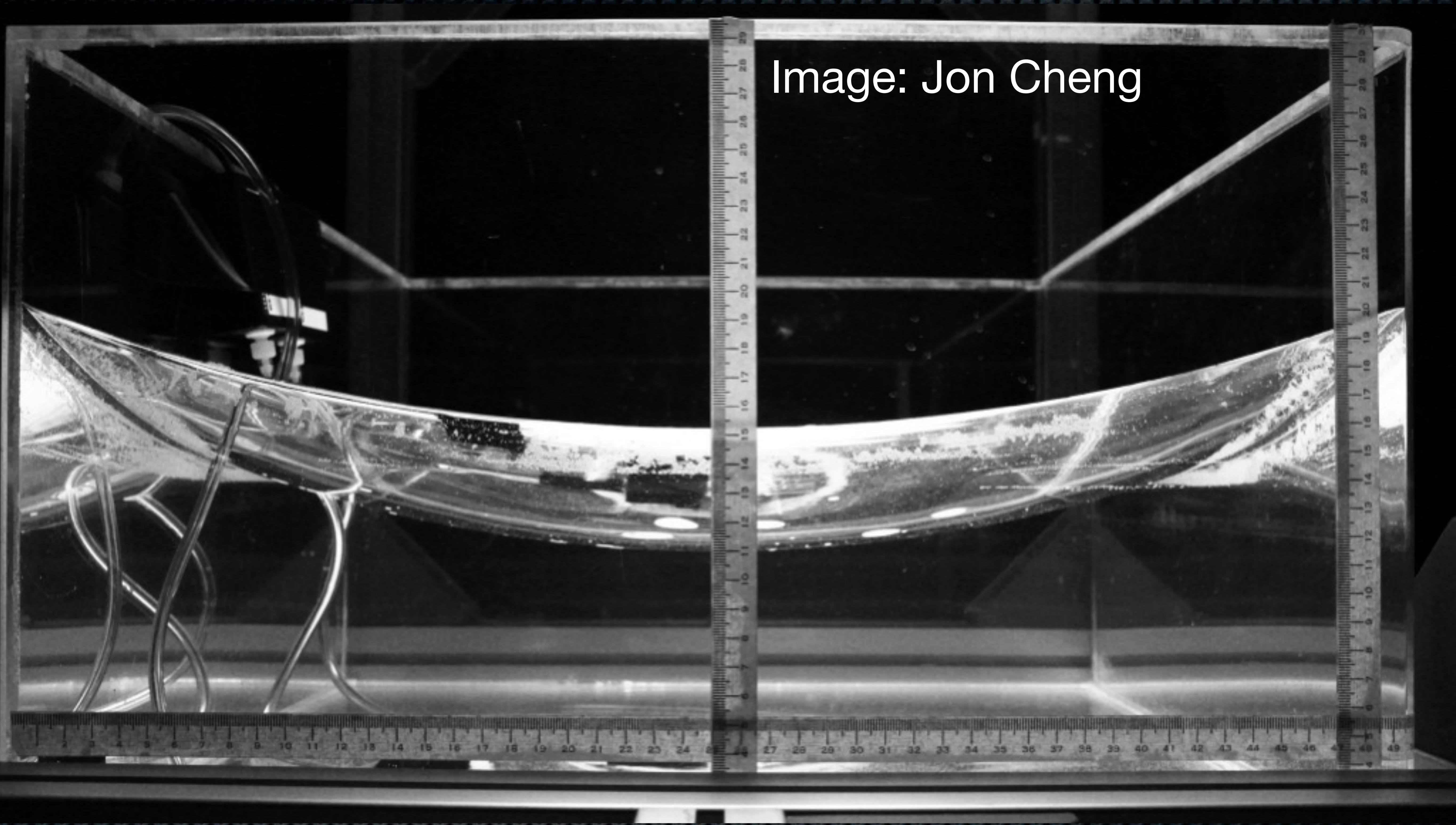
Experiment 4

Rotating Water-Filled Tank

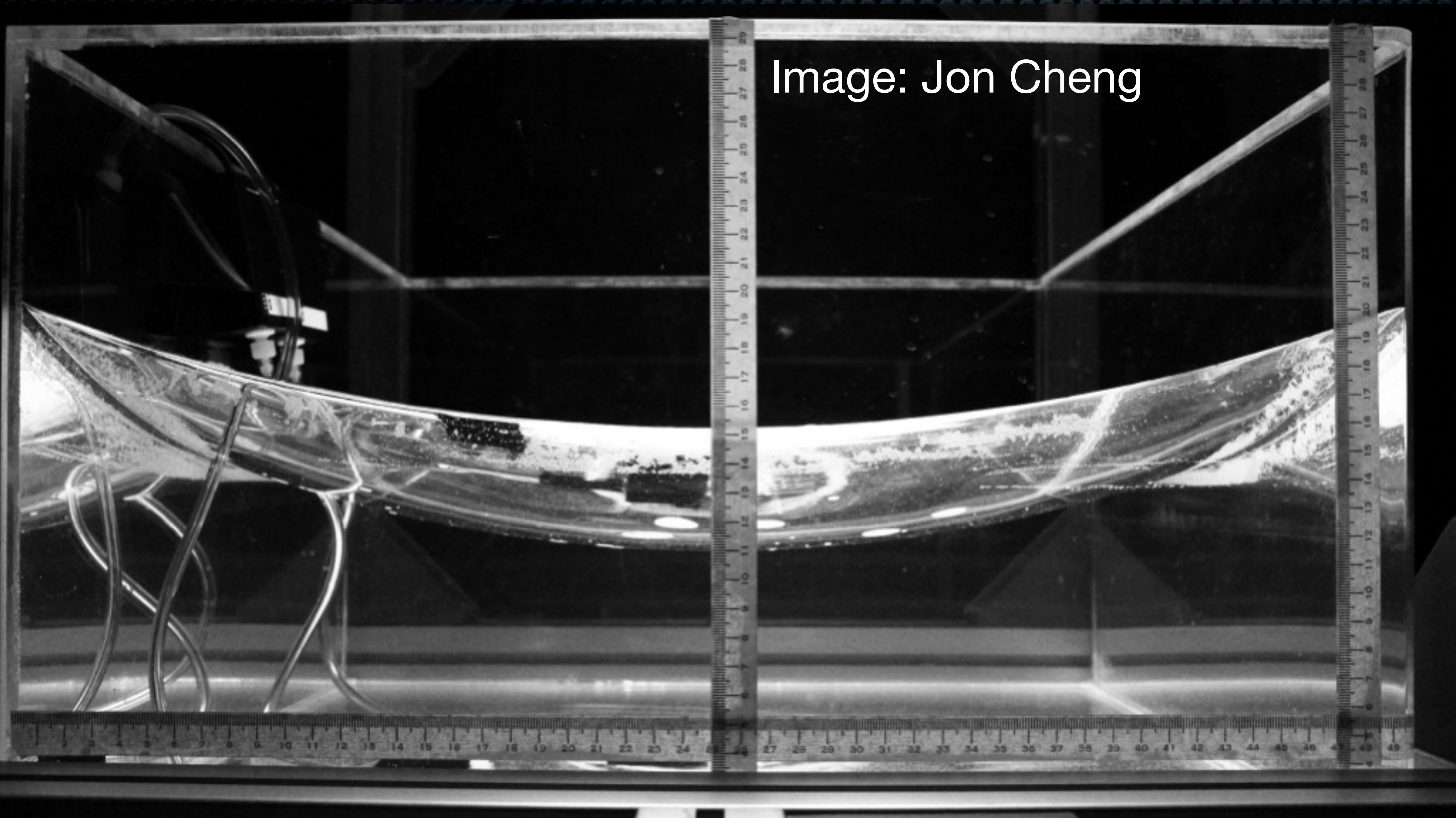


$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \underline{\rho \nu \nabla^2 \mathbf{u} - \nabla p} - \rho g \hat{z} + \rho \Omega^2 s \hat{s} + \rho \mathbf{u} \times 2\boldsymbol{\Omega}$$

Spin-up by viscous & pressure terms (~ 10 minutes)

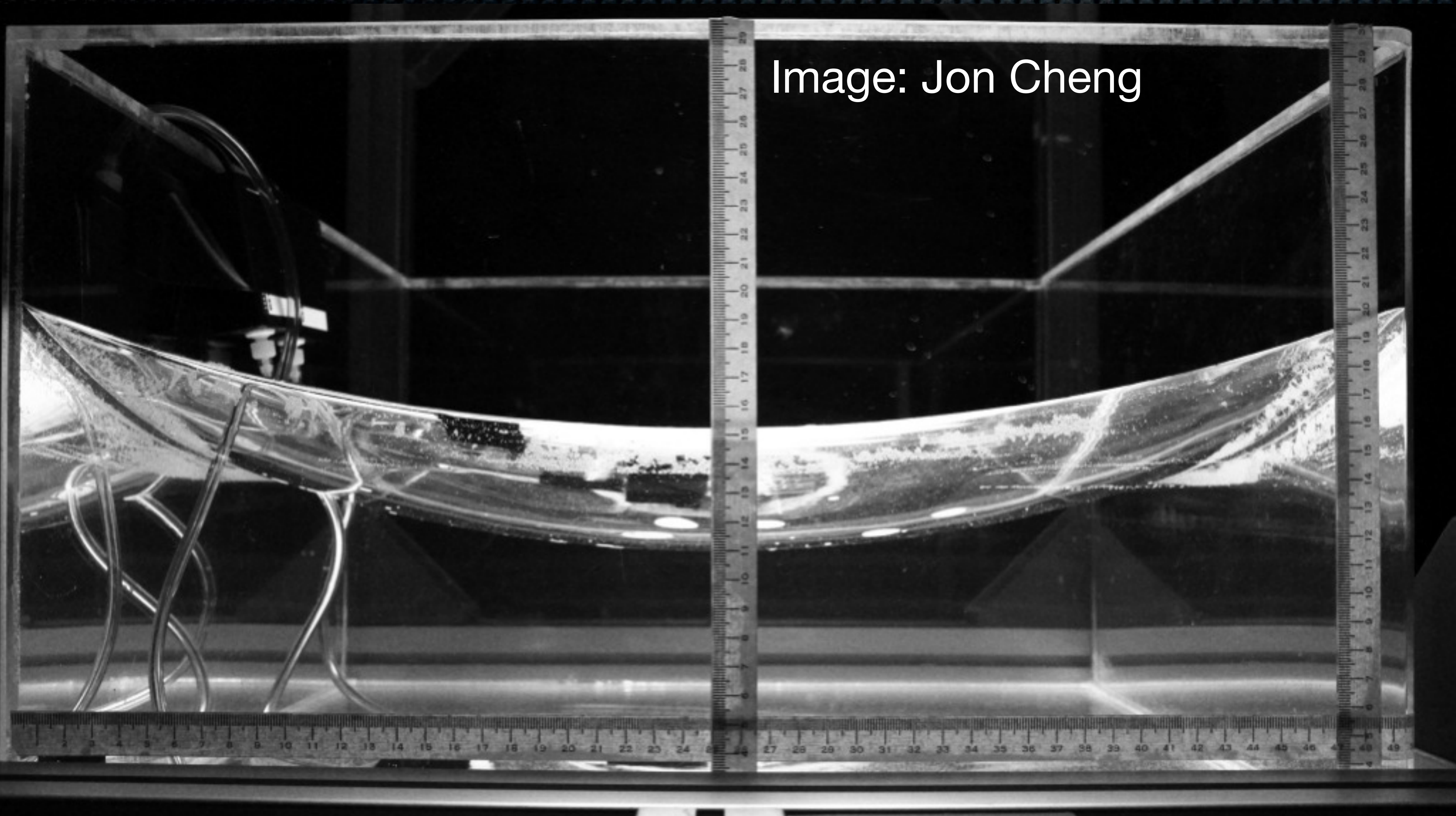


hydrostatic balance: $0 = -\nabla p - \rho g \hat{z} + \rho \Omega^2 s \hat{s}$



hydrostatic balance: $0 = -\nabla p - \rho g \hat{z} + \rho \Omega^2 s \hat{s}$

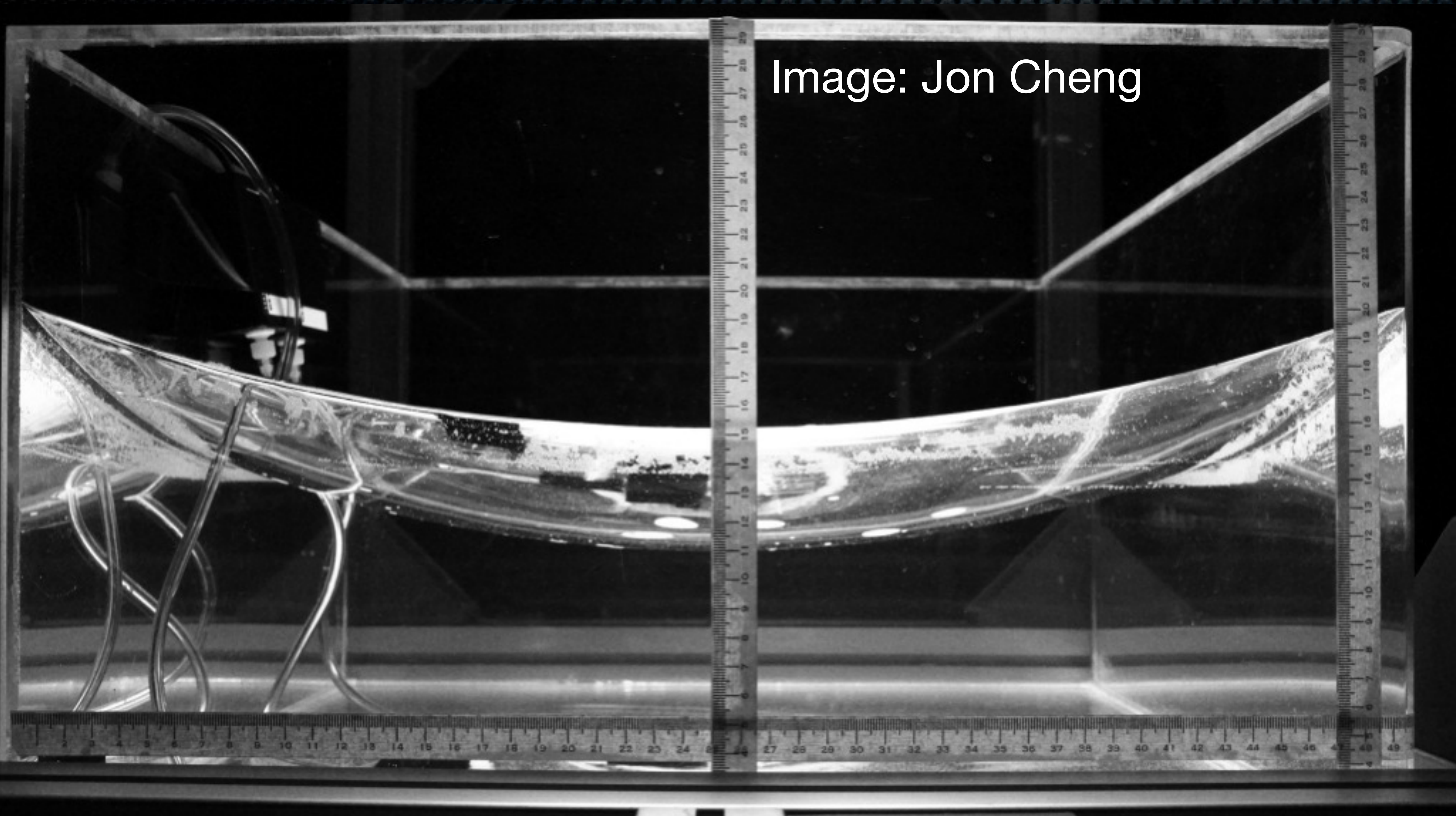
$$\partial p / \partial z = -\rho g$$



hydrostatic balance: $0 = -\nabla p - \rho g \hat{z} + \rho \Omega^2 s \hat{s}$

$$\partial p / \partial z = -\rho g$$

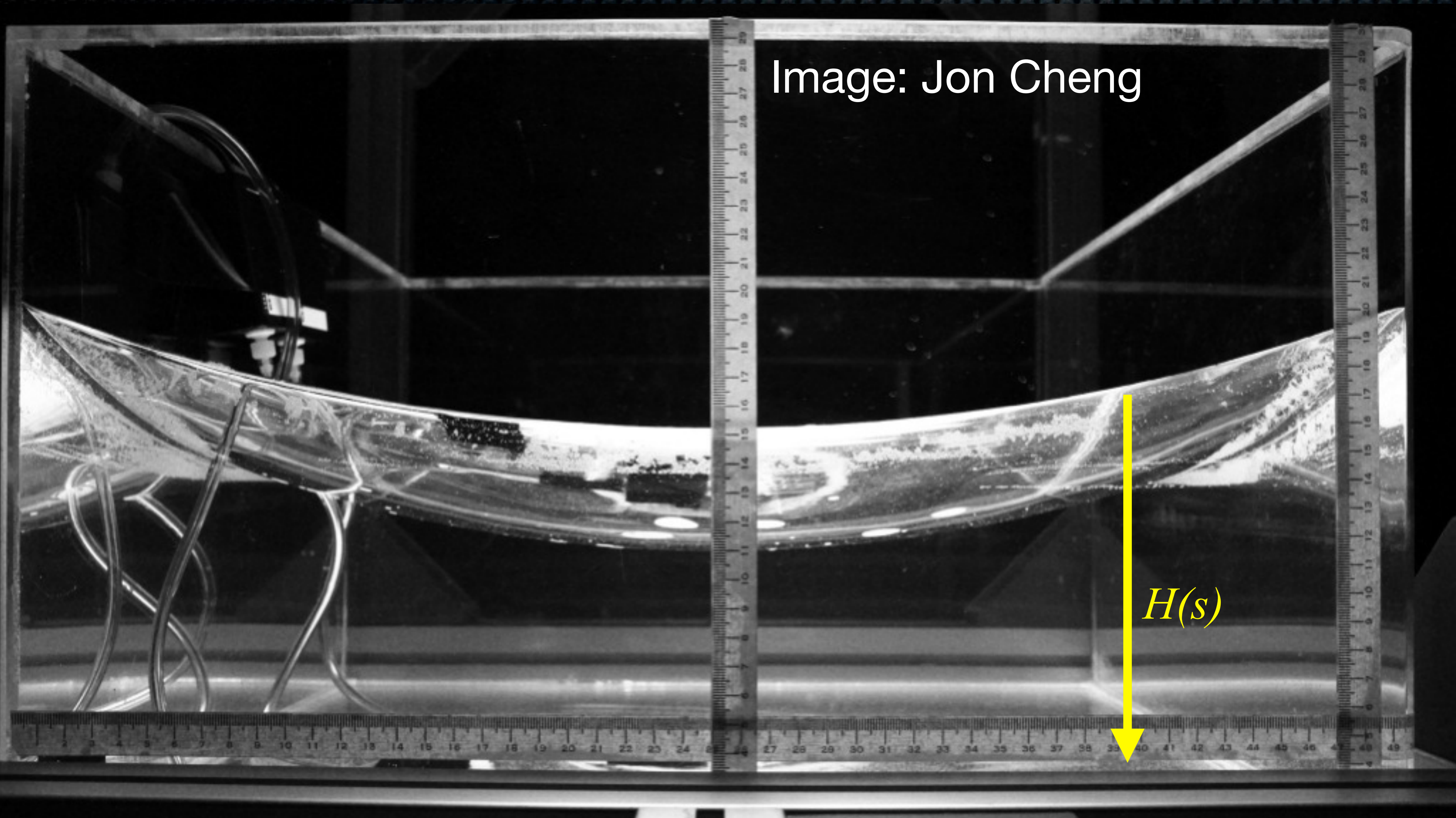
$$p = p_{atm} + \rho g H(s)$$



hydrostatic balance: $0 = -\nabla p - \rho g \hat{z} + \rho \Omega^2 s \hat{s}$

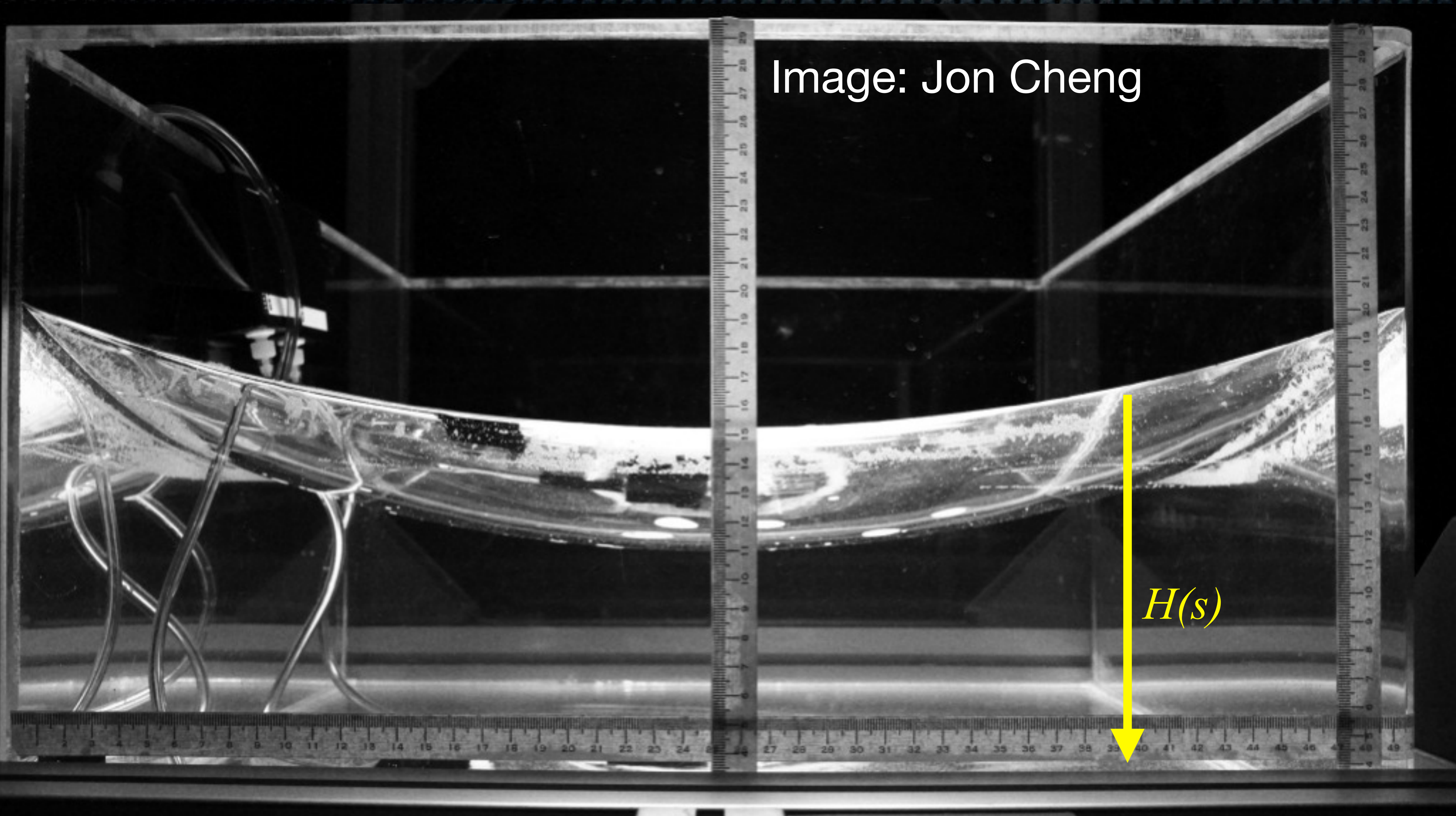
$$\partial p / \partial z = -\rho g \qquad \partial p / \partial s = \rho \Omega^2 s$$

$$p = p_{atm} + \rho g H(s)$$



hydrostatic balance: $0 = -\nabla p - \rho g \hat{z} + \rho \Omega^2 s \hat{s}$

$\partial p/\partial z = -\rho g$	$\partial p/\partial s = \rho \Omega^2 s$
$p = p_{atm} + \rho g H(s)$	$H(s) = H_{s0} + (\Omega^2/2g)s^2$

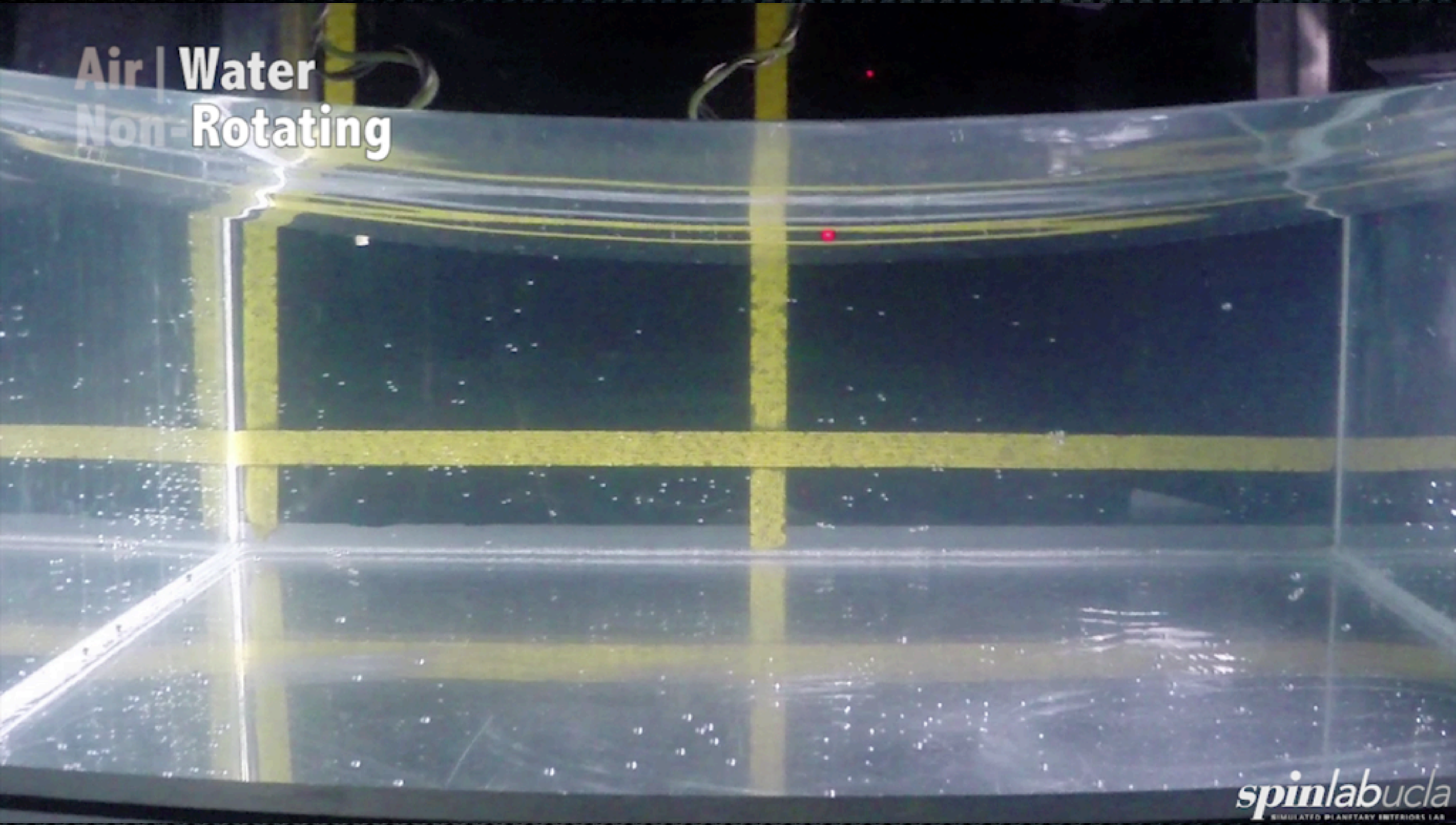


hydrostatic balance: $0 = -\nabla p - \rho g \hat{z} + \rho \Omega^2 s \hat{s}$

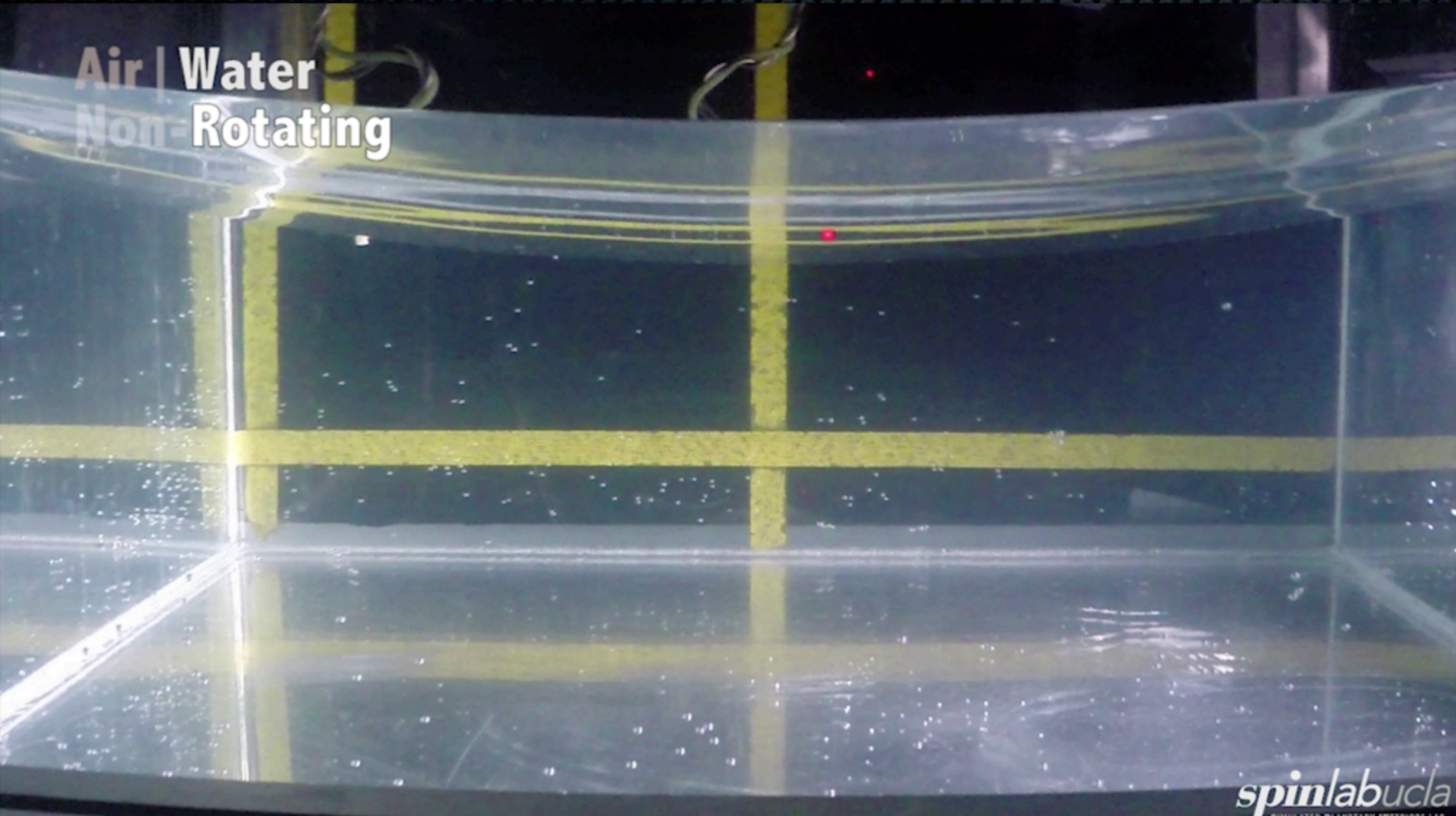
$$\partial p/\partial z = -\rho g \qquad \partial p/\partial s = \rho \Omega^2 s$$

$$p = p_{atm} + \rho g H(s) \qquad H(s) = H_{s0} + (\Omega^2/2g)s^2$$

Parabolic equipotential surfaces

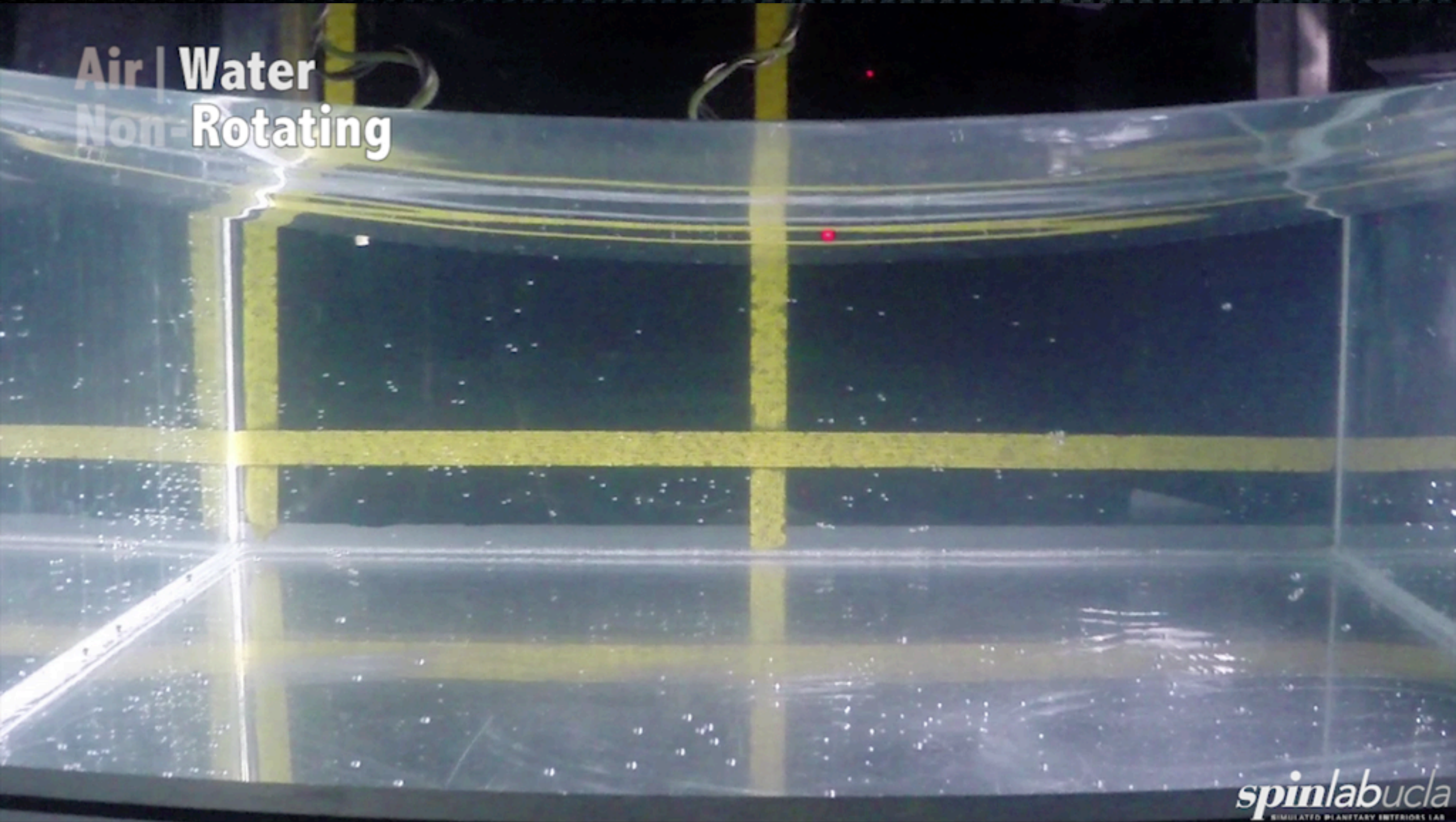


$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = (\delta\rho/\rho)\mathbf{g}_{eff} - \frac{1}{\rho}\nabla p + \mathbf{u} \times 2\Omega$$



$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = (\delta\rho/\rho)\mathbf{g}_{eff} - \frac{1}{\rho}\nabla p + \mathbf{u}\times 2\Omega \quad \text{Rossby, } Ro = \frac{\tau_{rot}}{\tau_{fall}} \simeq 0.3$$

Coriolis dominates



$$0 = -\frac{1}{\rho}\nabla p + \mathbf{u} \times 2\Omega \quad \text{Rossby, } Ro = \frac{\tau_{rot}}{\tau_{fall}} \simeq 0.3$$

<u>**Geostrophic Balance**</u>

<u>***Coriolis dominates***</u>

Geostrophic Flows

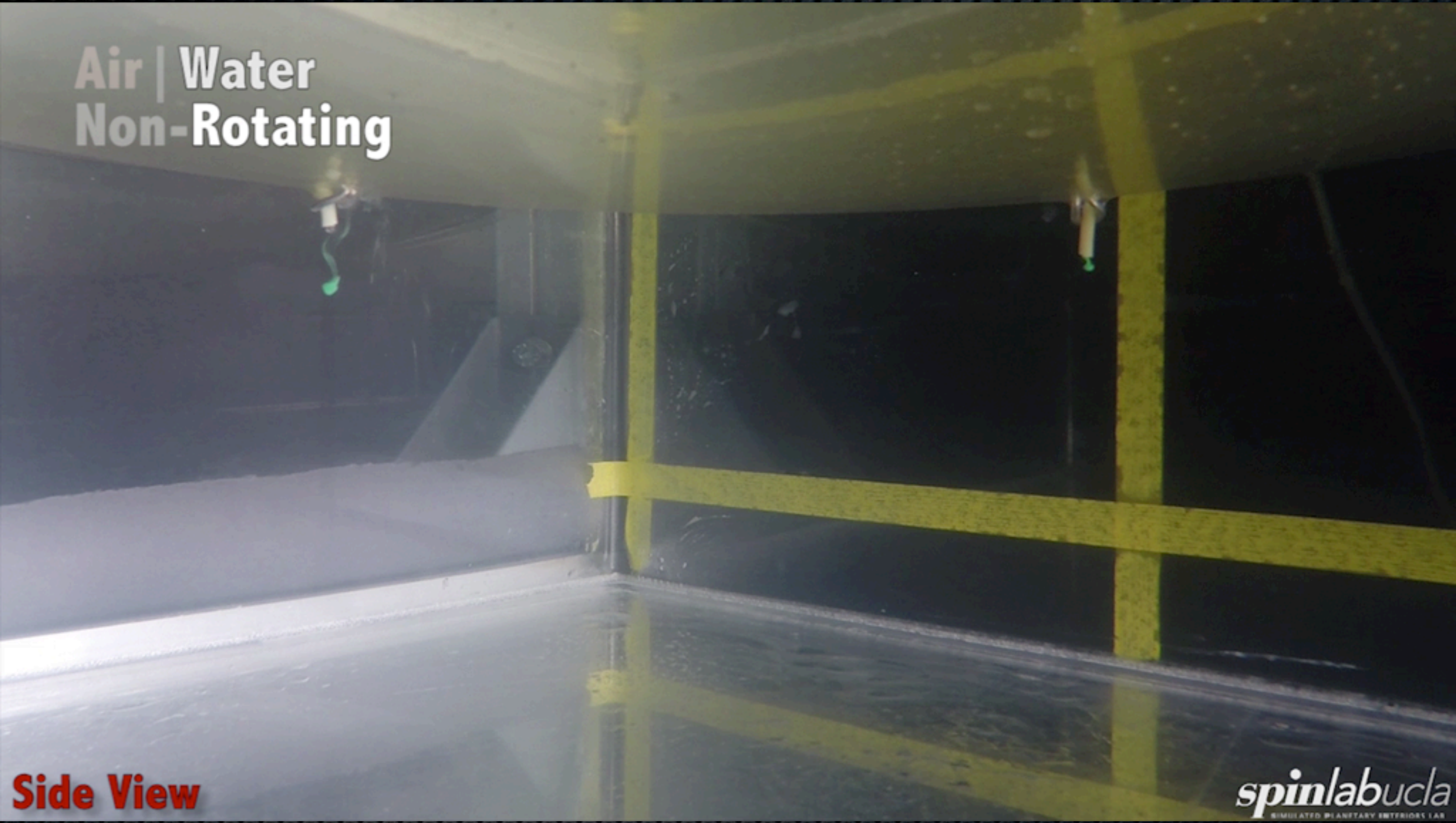
- Invariant along rotation axis

$$\nabla \times \left(-\frac{1}{\rho} \nabla p + \mathbf{u} \times 2\Omega \hat{z} \right) = 0$$

$$\boldsymbol{\Omega} \cdot \nabla \mathbf{u} = 0$$

Taylor-Proudman Theorem

$$\boxed{\frac{\partial \mathbf{u}}{\partial z} = 0}$$



$$Ro = \frac{\tau_{rot}}{\tau_{fall}} \simeq 0.3\,; \quad Ro_{core} \sim 10^{-6}$$

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer (Movies)
- Planetary Core Columns
    - And Beyond

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
    - And Beyond

Rotating Convection Columns

- **Ekman number, E**
  - Scaled ***viscous*** force

$$E = \frac{\tau_{rot}}{\tau_{visc}} = \frac{\nu}{\Omega L^2}$$

Image:
J. Cheng

Rotating Convection Columns

- **Ekman number, *E***
  - Scaled ***viscous*** force
- **Rayleigh number, *Ra***
  - Scaled ***buoyancy*** force

$$E = \frac{\tau_{rot}}{\tau_{visc}} = \frac{\nu}{\Omega L^2}$$

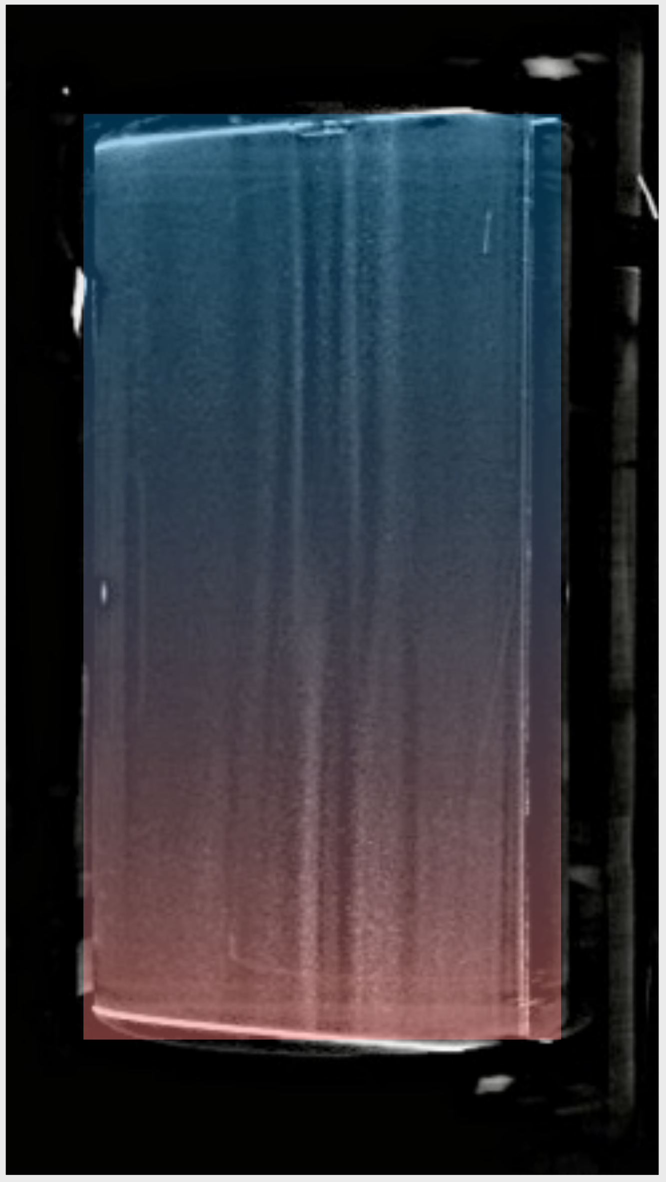


Image:
J. Cheng

Rotating Convection Columns

- **Ekman number, *E***
  - Scaled ***viscous*** force

$$E = \frac{\tau_{rot}}{\tau_{visc}} = \frac{\nu}{\Omega L^2}$$

- **Rayleigh number, *Ra***
  - Scaled ***buoyancy*** force
  - Onset Forcing:

$$Ra_C \sim E^{-4/3}$$

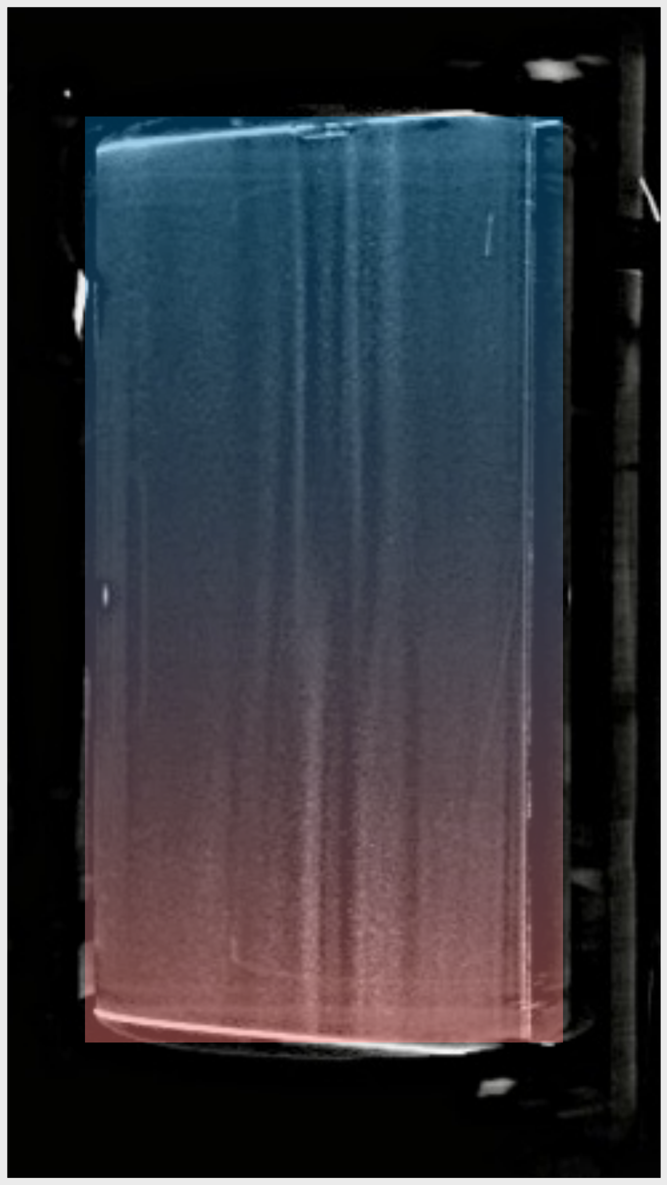


Image: J. Cheng

Rotating Convection Columns

- **Ekman number, E**
  - Scaled ***viscous*** force

$$E = \frac{\tau_{rot}}{\tau_{visc}} = \frac{\nu}{\Omega L^2}$$

- **Rayleigh number, Ra**
  - Scaled ***buoyancy*** force
  - Onset Forcing:

$$Ra_C \sim E^{-4/3}$$

- **Onset column *width*:**

$$\ell_C \sim E^{1/3}$$

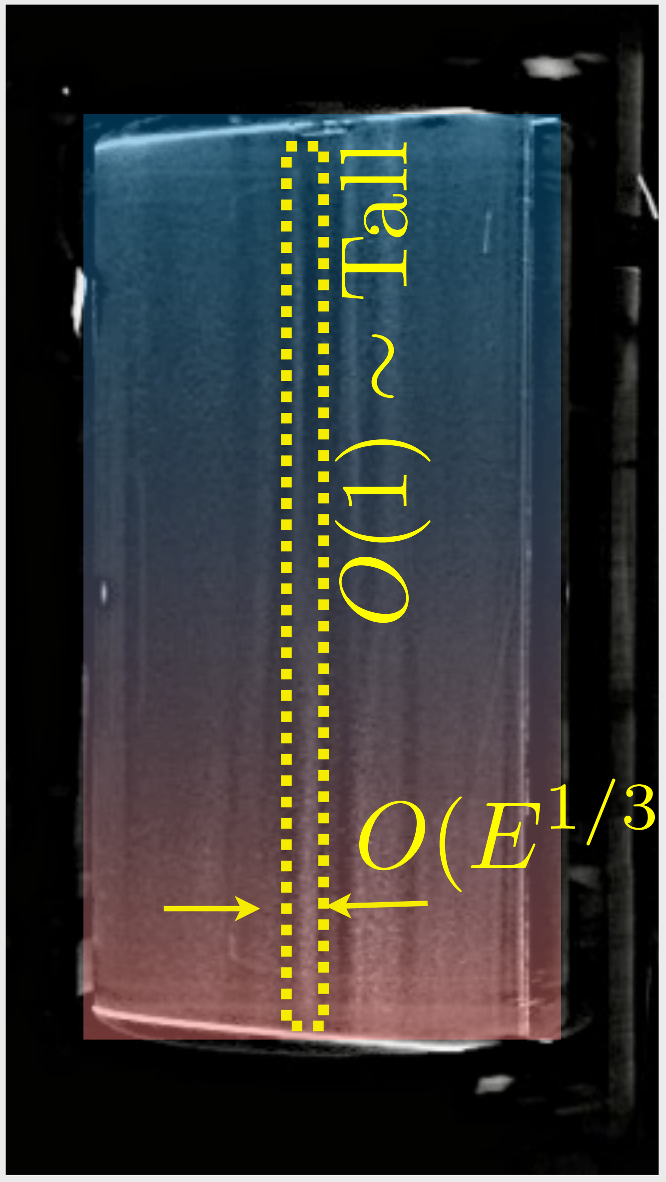
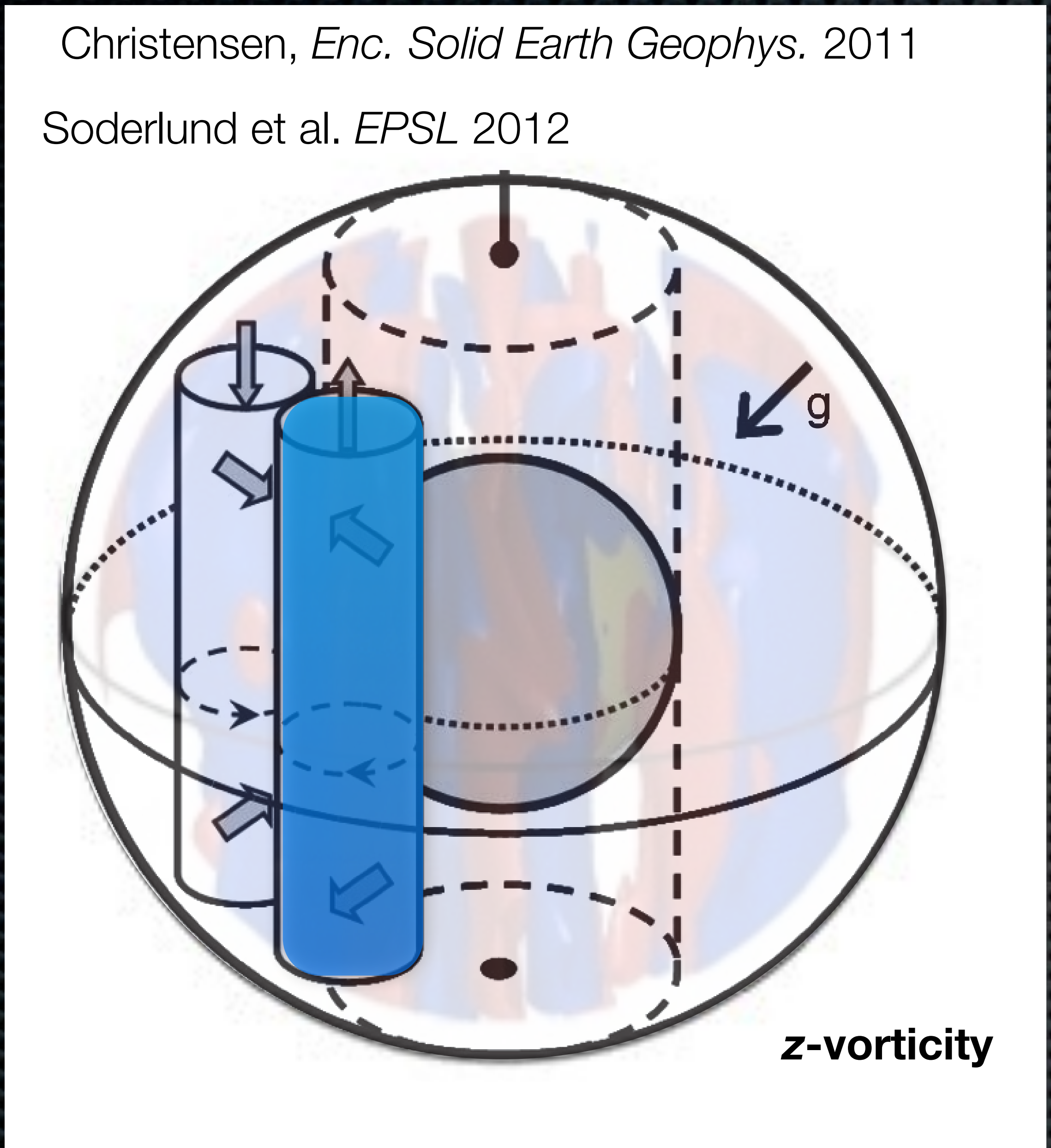


Image:
J. Cheng

Rotating Convection Columns



$$E = \frac{\tau_{rot}}{\tau_{visc}} = \frac{\nu}{\Omega L^2}$$

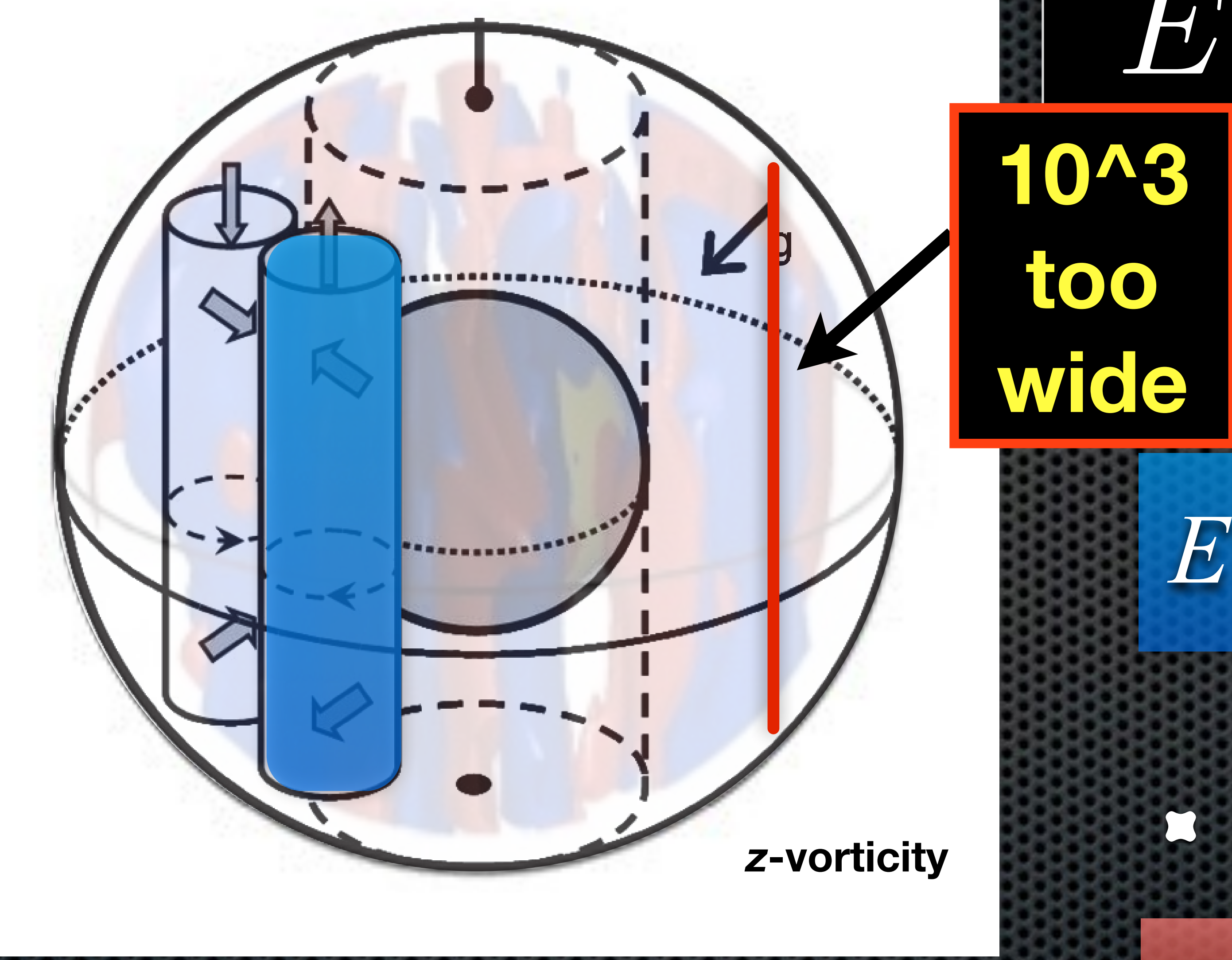
- **Models:**

E ~ 1e-4; l_c ~ 0.1

Rotating Convection Columns

Christensen, *Enc. Solid Earth Geophys.* 2011

Soderlund et al. *EPSL* 2012



$$E = \frac{\tau_{rot}}{\tau_{visc}} = \frac{\nu}{\Omega L^2}$$

- **Models:**

E ~ 1e-4; l_c ~ 0.1

- **Earth's Core:**

E ~ 1e-15; l_c ~ 1e-5

(i.e., 10^4 x smaller than scale of flux patches)

Columns: Models vs. Cores

- *Model* $E^{1/3}$ **columns: generate large-scale flux patches**

- ***Planetary Cores*** $E^{1/3}$ **columns: ~10 m wide**
  - Tall and wafer thin; local $Rm << 1$(aka, non-inductive)
  - **Unobservably small: Cannot explain large-scale core flux patches**
  - **Model-style columns magnetically ineffective in planetary cores**

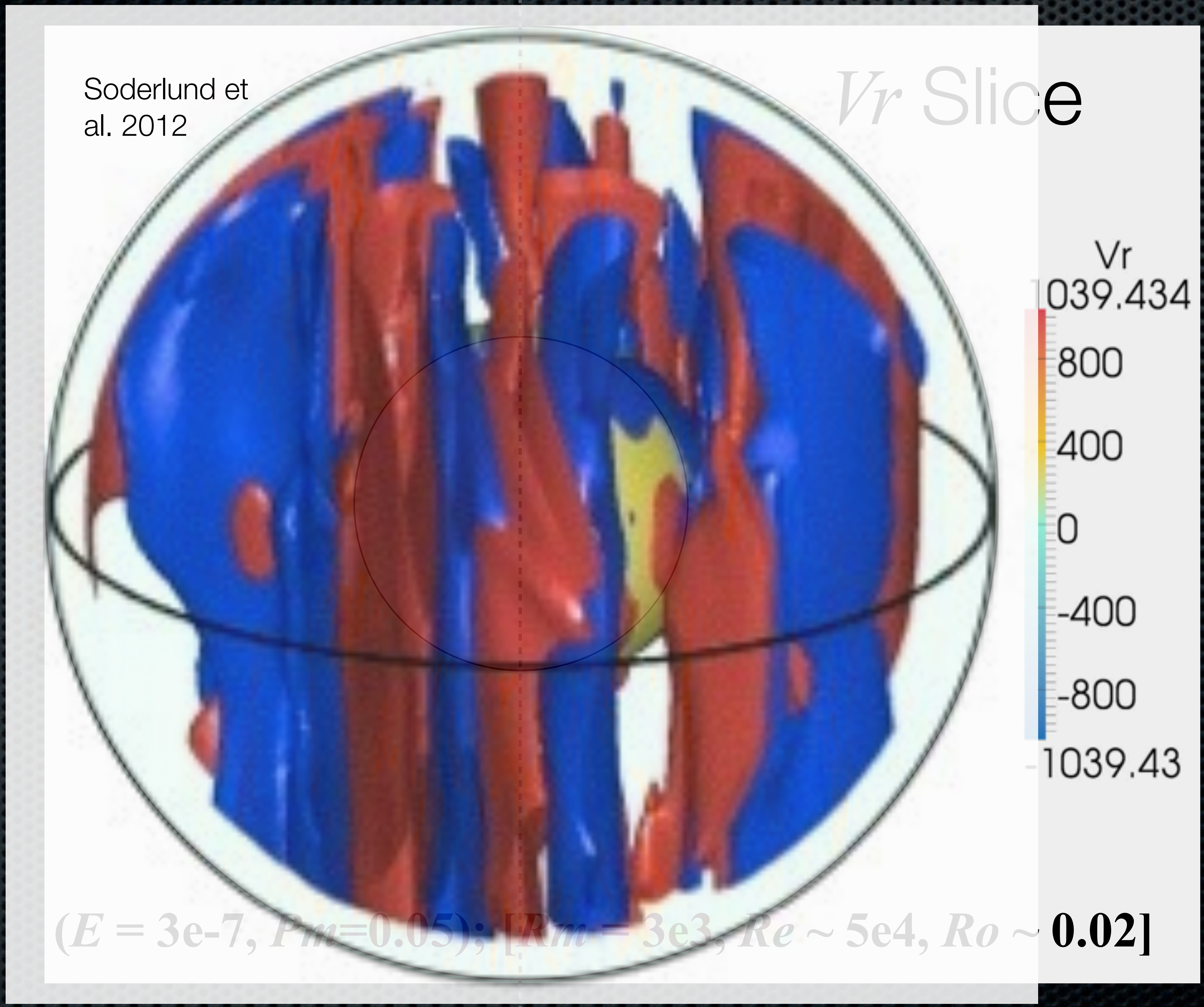
Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
    - And Beyond

Talk Outline

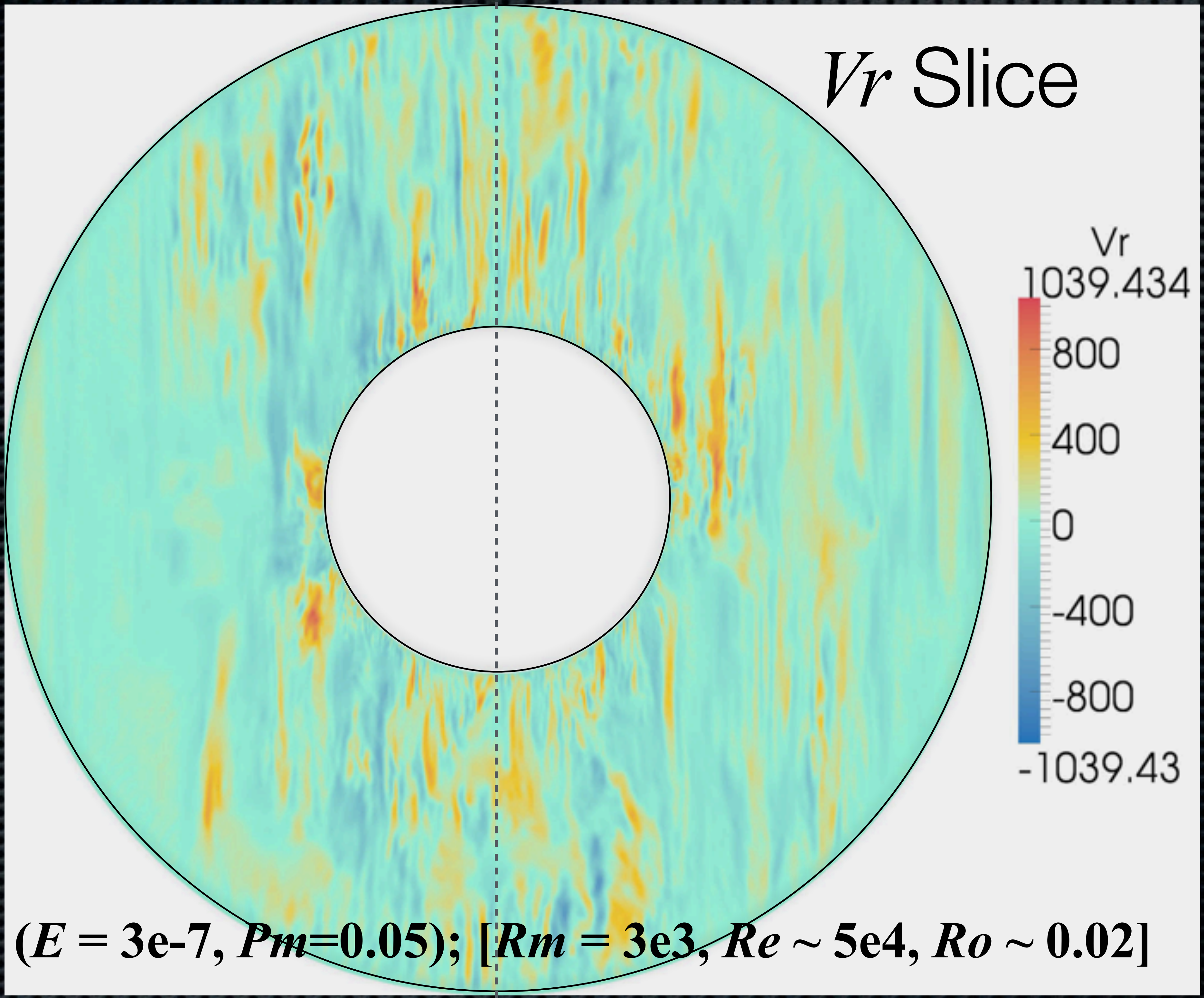
- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
    - And Beyond

Sheyko's Turbulent Dynamo



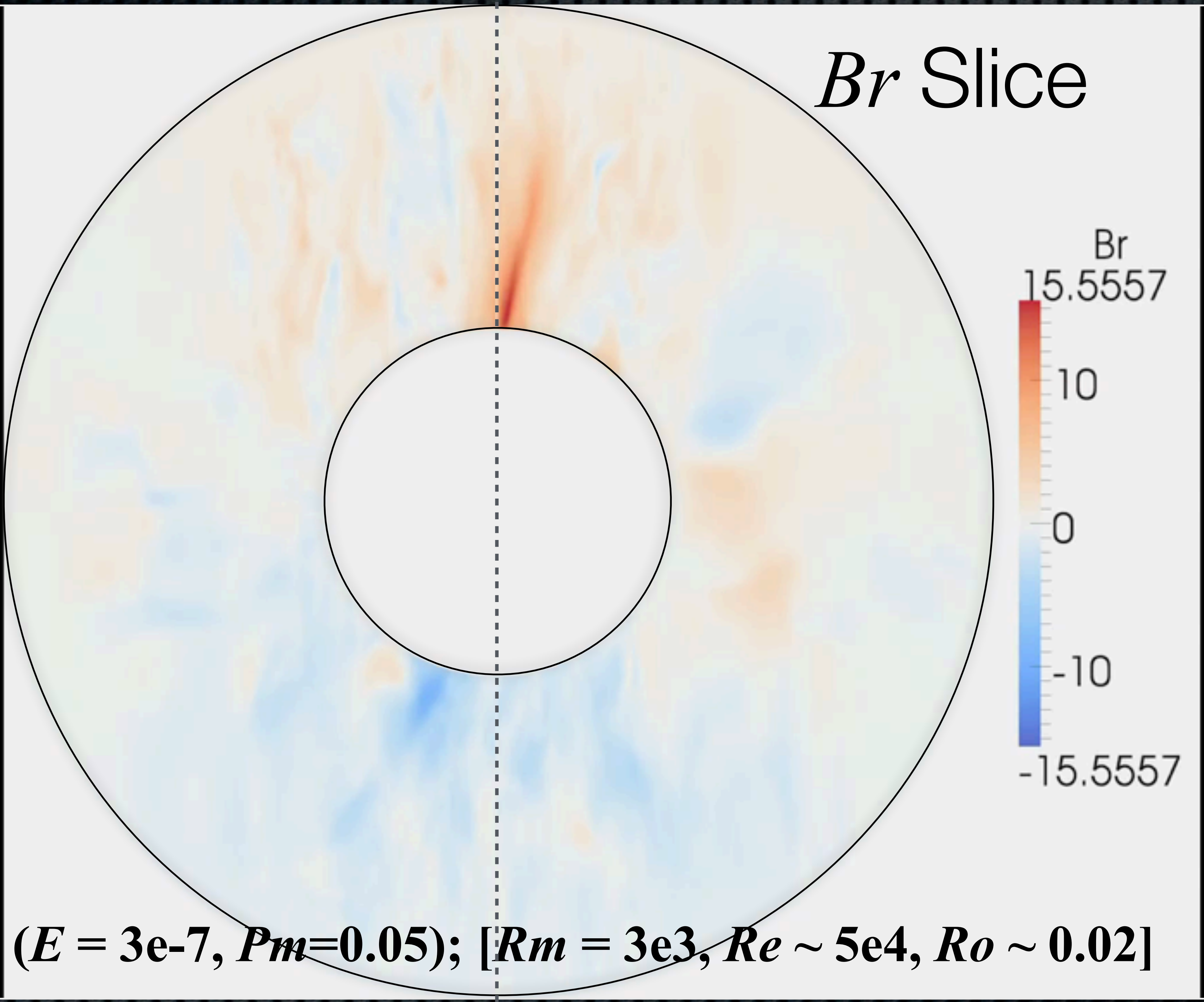
Andrey Sheyko et al., in prep. (2014)

Sheyko's Turbulent Dynamo



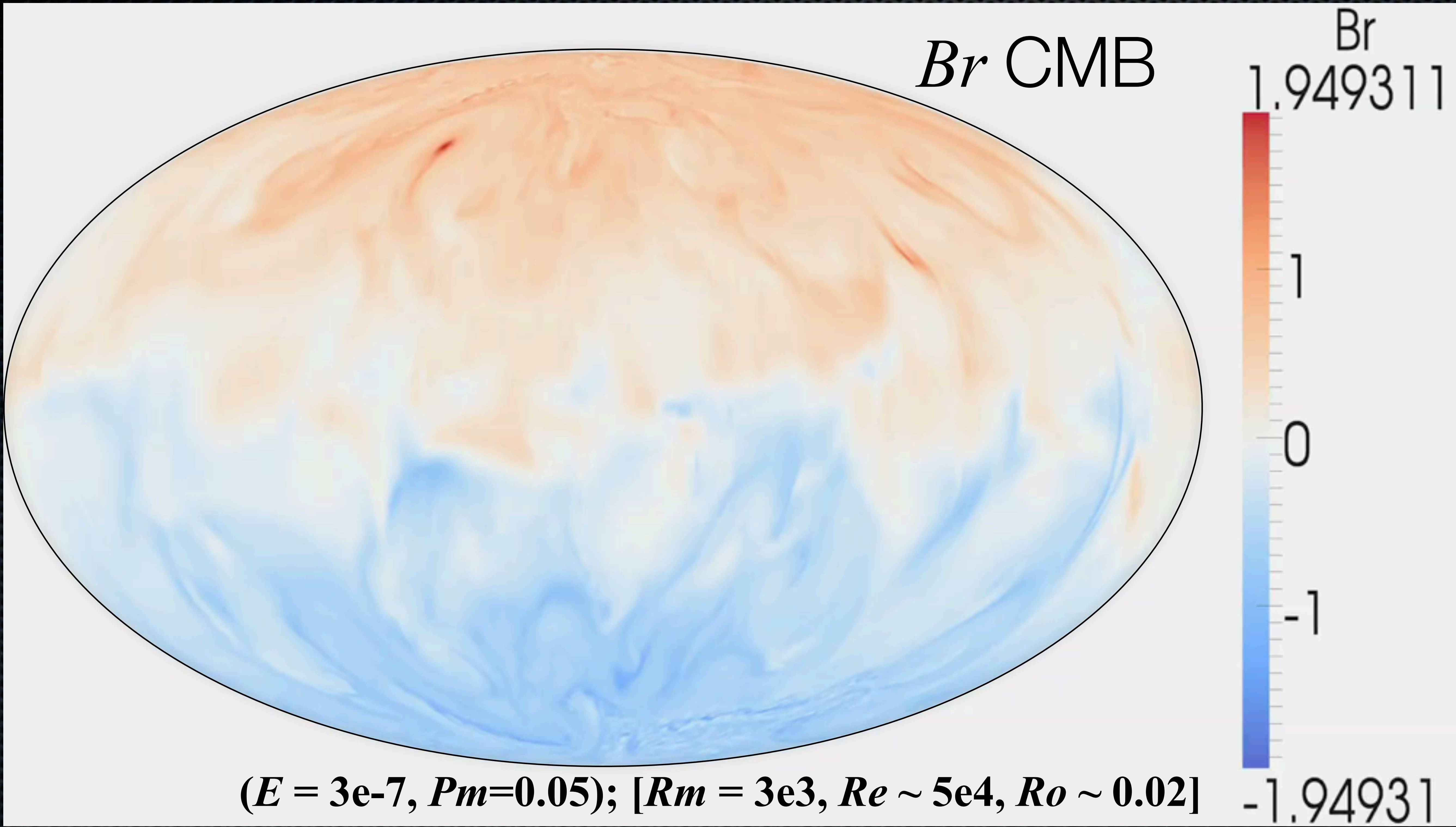
Andrey Sheyko et al., in prep. (2014)

Sheyko's Turbulent Dynamo



Andrey Sheyko et al., in prep. (2014)

Sheyko's Turbulent Dynamo



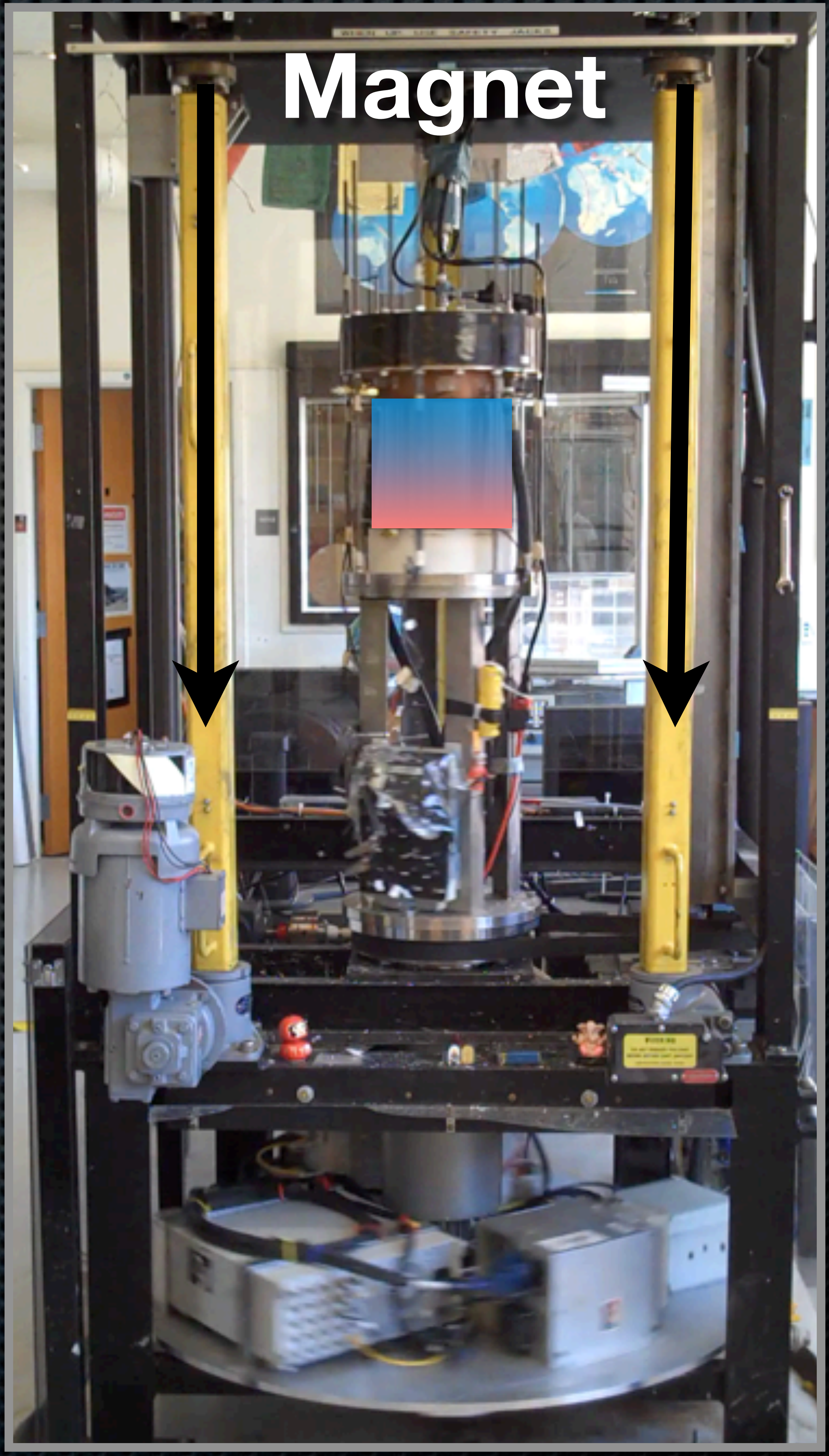
Andrey Sheyko et al., in prep. (2014)

Sheyko’s Turbulent Dynamo

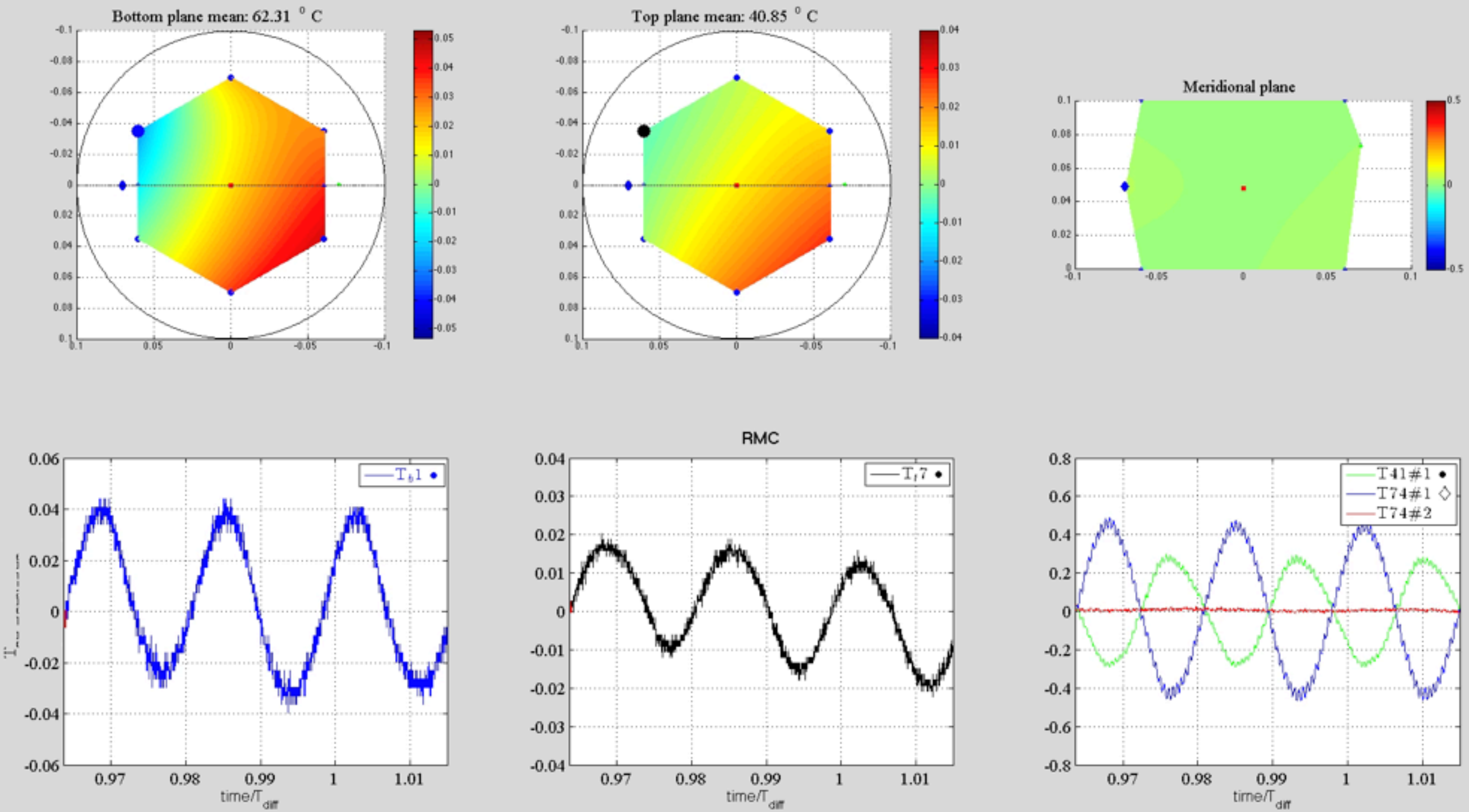
- Fully turbulent model: magnetic flux bundles no longer directly tied to columns (scale separation)
- Axialization still essential

MHD Experiments

- Rotating magneto-convection in liquid gallium
- Strong magnetic field can balance Coriolis
- Fundamentally different behaviors predicted

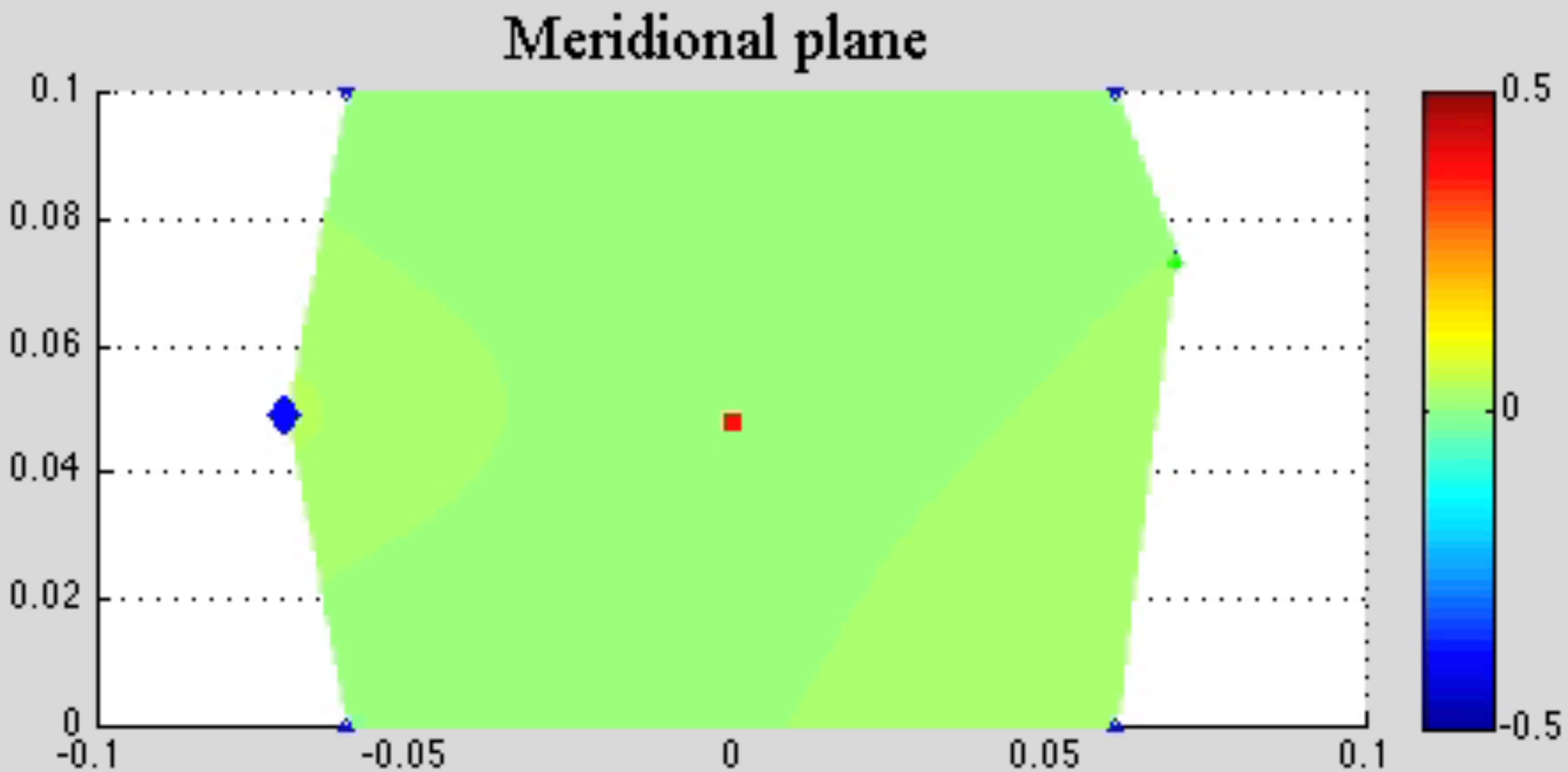


Adolfo Ribeiro, Guillaume Fabre



- Dominant mode: m=1 precessing structure

Movie: Guillaume Fabre



- Dominant mode: m=1 precessing structure

Movie: Guillaume Fabre

Lab MHD Experiments

- In liquid metal, columns not found when imposed fields provide Lorentz ~ Coriolis
- Dominant container-scale precessing wave mode
  - Fundamentally different physics than exists in present dynamo models…

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
    - And Beyond

Summary

- Successful present-day models
  - Large-scale columns generate axial field with large flux patches
- Axialized flows in Coriolis-dominated systems
- Need for lower viscosity, liquid metal dynamo models, requiring HPC efforts

Talk Outline

- Geodynamo Observations & Models
- Rotating Fluid Dynamics Primer
- Planetary Core Columns
  - And Beyond

THANKS...

Appendix

Geostrophic Flows

- Perpendicular to pressure gradients
- Invariant along rotation axis

$$0 = \hat{z} \times \left(-\frac{1}{\rho} \nabla p + \mathbf{u} \times 2\Omega \hat{z} \right)$$

$$\mathbf{u}_H = \frac{1}{2\rho\Omega} (\hat{z} \times \nabla p)$$

$$(u_x, u_y) = \frac{1}{2\rho\Omega} \left(-\frac{\partial p}{\partial y}, \frac{\partial p}{\partial x} \right)$$

Horizontally non-divergent flow

$$\partial_z (u_x, u_y) = 0$$

for hydrostatic balance in z

Taylor-Proudman Theorem

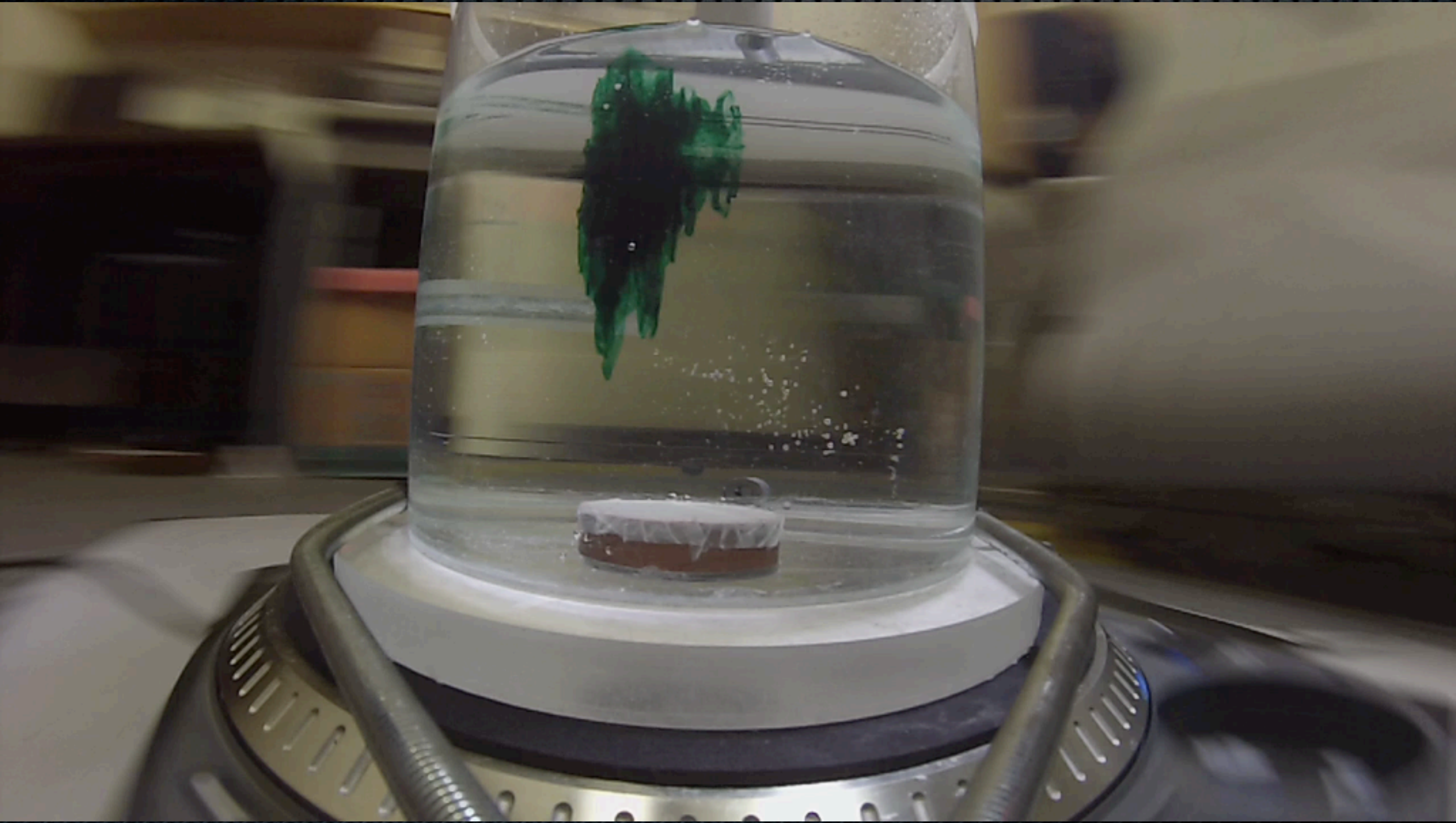
- Requires axially invariant flow around a localized obstacle

Taylor-Proudman Theorem

- Requires axially invariant flow around a localized obstacle

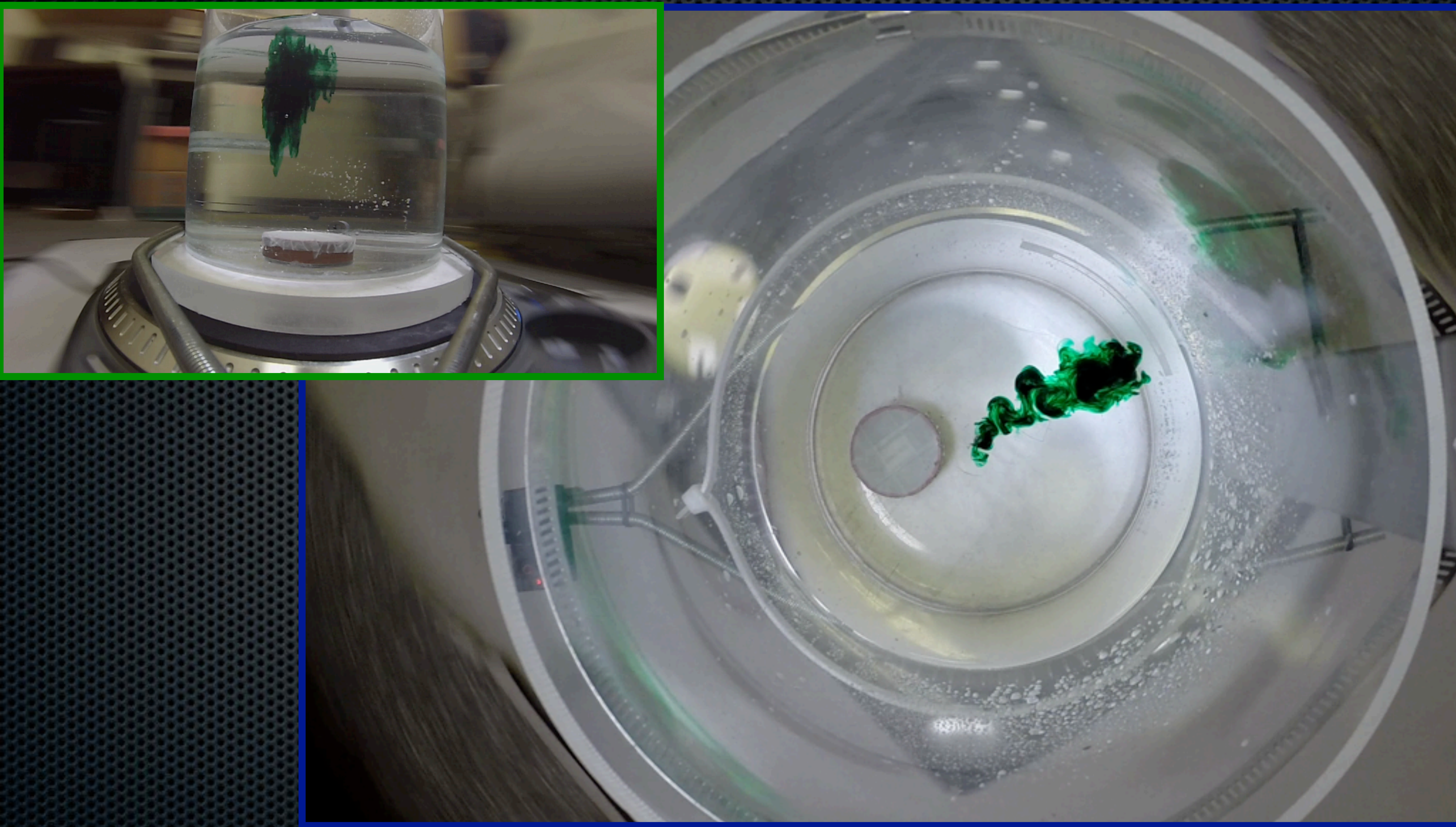
Taylor-Proudman Theorem

- Requires axially invariant flow around a localized obstacle



Taylor-Proudman Theorem

- Requires axially invariant flow around a localized obstacle



Taylor-Proudman Theorem

- Requires axially invariant flow around a localized obstacle